A pedagogical review of electroweak symmetry breaking scenarios

Gautam Bhattacharyya

Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Kolkata 700064, India

Abstract

We review different avenues of electroweak symmetry breaking explored over the years. This constitutes a timely exercise as the world's largest and the highest energy particle accelerator, namely, the Large Hadron Collider (LHC) at CERN near Geneva, has started running whose primary mission is to find the Higgs or some phenomena that mimic the effects of the Higgs, i.e. to unravel the mysteries of electroweak phase transition. In the beginning, we discuss the Standard Model Higgs mechanism. After that we review the Higgs sector of the minimal supersymmetric Standard Model. Then we take up three relatively recent ideas: little Higgs, gauge–Higgs unification and Higgsless scenarios. For the latter three cases, we first present the basic ideas and restrict our illustration to some instructive toy models to provide an intuitive feel of the underlying dynamics, and then discuss, for each of the three cases, how more realistic scenarios are constructed and how to decipher their experimental signatures. Wherever possible, we provide pedagogical details, which beginners might find useful.

(Some figures in this article are in colour only in the electronic version)

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Contents

1. Introduction

The theory of beta decay, which manifests weak interaction, was first formulated by Fermi. Below we write down the effective Lagrangian of beta decay. In doing so, we use modern notation and rely on the $(V - A)$ structure of currents [1]:

$$\mathscr{L}_{\text{eff}} = \frac{G_{\text{F}}}{\sqrt{2}}(\bar{p}\gamma_\mu(1 + \alpha\gamma_5)n)(\bar{e}\gamma_\mu(1 - \gamma_5)\nu). \qquad (1.1)$$

Since every fermion field has mass dimension 3/2 (which follows from power counting in Dirac Lagrangian), the prefactor G_{F} has clearly a mass dimension -2. From the neutron decay width and angular distribution, one obtains $\alpha \simeq -1.2$ and the 'Fermi scale' $G_{\text{F}}^{-1/2} \simeq 300$ GeV. Particle physics has gone through a dramatic evolution since the time of Fermi [2]. The success of the Yang–Mills theory revolutionized the whole scenario [3]. The charged $W^{\pm}$ boson was eventually predicted by the Standard Model (SM) [4] having a mass of around 80 GeV, which was later experimentally confirmed by direct detection by the UA1 collaboration at CERN. This $W^{\pm}$ boson induces radioactivity by mediating the beta decay process. However, a full understanding of the dynamics that controls the Fermi scale and hence the W boson mass still remains an enigma. This is the scale of electroweak phase transition, and understanding the origin of electroweak symmetry breaking (EWSB) constitutes the primary goal of the Large Hadron Collider (LHC) at CERN. The readers are strongly recommended to follow references [5–10] to have a broad overview of different possible EWSB mechanisms.

The SM reigns supreme at the electroweak scale. But it cannot account for a few experimentally established facts: neutrino mass, dark matter and the right amount of baryon asymmetry of the universe. Any viable scenario beyond the SM that is expected to trigger EWSB and to answer one or more of the above questions must pass the strict constraints imposed by the electroweak precision tests (EWPT) carried out mainly at the Large Electron Positron (LEP) collider at CERN. Non-abelian gauge theory as the theory for weak interaction has been established to a very good accuracy: (*i*) the ZWW and γWW vertices have been measured to a per cent accuracy at LEP-2 implying that the SU(2) × U(1) gauge theory is unbroken at the vertices, (*ii*) accurate measurements of the Z and W masses have indicated that gauge symmetry is broken in masses. Precision measurements at LEP have shown that the ρ-parameter (introduced in section 3.6.4) is unity to a very good accuracy. This attests the 'SU(2)-doublet' nature of the scalar employed in the SM for spontaneous electroweak breaking. Any acceptable new physics scenario should be in accordance with the above observations. CMS and ATLAS are the two

general purpose detectors of the LHC which are expected to answer a lot of such questions by hunting not only the Higgs but also any possible ruler of the teraelectron volt (TeV) regime.

The primary concern is the following. Is the Higgs mechanism as portrayed in the SM a complete story? Bluntly speaking, nobody believes so! Then, what is the nature of the more fundamental underlying dynamics? A more pointed question is if the Higgs exists, is it *elementary* or *composite*? The advantage of working with an elementary Higgs, as in the SM, is that the two issues of generating gauge boson masses and fermion masses are solved in one stroke. Also, as it turned out, a theory relying on elementary Higgs is perfectly comfortable with EWPT. The disadvantage is that the Higgs mass receives quadratically divergent quantum correction which inevitably calls for new physics, e.g. supersymmetry, to solve the hierarchy problem by taming the unruly quantum behavior. On the other hand, when the Higgs is a composite object, e.g. in technicolor, the hierarchy problem is not there any way because the composite Higgs *dissolves* at the scale where new heavy fermions (e.g. technifermions) condense to break EWSB. But a major disadvantage of technicolor is that such models, in general, inflict unacceptably large flavor changing neutral currents (FCNC) and induce large contributions to the oblique electroweak parameters T (or $\Delta\rho$) and S. Although the FCNC problem can be evaded by going to some more complicated versions of technicolor models, general inconsistency with EWPT in the post-LEP era has put technicolor far behind supersymmetry in terms of acceptability. But the idea of technicolor was too elegant to die. It simply went into slumber only to reappear some years later in a different guise through the AdS/CFT correspondence [11] as dual to some extra-dimensional theories. Many modern non-supersymmetric ideas, which we shall discuss in this review, are reminiscent of technicolor, but sufficiently advanced and equipped over the traditional versions to meet the FCNC and EWPT challenges. At this point it is fair to say that supersymmetry and the new *avatars* of technicolor/compositeness are the two most attractive general *classes of theories* that may dictate the EWSB mechanism and are expected to be observed at the LHC. Therefore, before we get going into a systematic but incremental elaboration of how the idea of EWSB evolved and how the different concerns at different stages were sorted out, we briefly touch upon the main features of two most important conceptual pillars on which many specific models were built, namely, supersymmetry and technicolor.

1.1. Supersymmetry

Supersymmetry is arguably the most favored extension of physics beyond the SM. It all started more than 30 years ago

Table 1. The particle content of the minimal supersymmetric SM: superparticles are marked by a superscript 'tilde'.

Particles/superparticles	Spin 0	Spin 1/2	$SU(3)_C \times SU(2)_L \times U(1)_Y$
leptons, sleptons (L)	$(\widetilde{\nu}, \widetilde{e}_L)$	(ν, e_L)	$(1, 2, -1/2)$
(in 3 families) (E^c)	$\widetilde{e}_R^*$	e_L^c	$(1, 1, 1)$
quarks, squarks (Q)	$(\widetilde{u}_L, \widetilde{d}_L)$	(u_L, d_L)	$(3, 2, 1/6)$
(in 3 families) (U^c)	$\widetilde{u}_R^*$	u_L^c	$(\bar{3}, 1, -2/3)$
(D^c)	$\widetilde{d}_R^*$	d_L^c	$(\bar{3}, 1, 1/3)$
Higgs, higgsinos (H_u)	(H_u^+, H_u^0)	$(\widetilde{H}_u^+, \widetilde{H}_u^0)$	$(1, 2, 1/2)$
(up, down types) (H_d)	(H_d^0, H_d^-)	$(\widetilde{H}_d^0, \widetilde{H}_d^-)$	$(1, 2, -1/2)$
particles/superparticles	Spin 1	Spin 1/2	$SU(3)_C \times SU(2)_L \times U(1)_Y$
gluon, gluino	g	$\widetilde{g}$	$(8, 1, 0)$
W bosons, winos	$W^\pm, W^0$	$\widetilde{W}^\pm, \widetilde{W}^0$	$(1, 3, 0)$
B boson, bino	B^0	$\widetilde{B}^0$	$(1, 1, 0)$

from theoretical works pursued independently by Golfand and Likhtman [12], Volkov and Akulov [13], and Wess and Zumino [14]. For historical developments of the idea of supersymmetry and subsequent model building and phenomenology, we recommend the text books [15] and reviews [16, 17]. We briefly outline the concept below.

Supersymmetry is a new space–time symmetry interchanging bosons and fermions, relating states of different spins. We first recall that Poincaré group is a semi-direct product of translations and the Lorentz transformations (which involve rotations and boosts), while a super-Poincaré group additionally includes supersymmetry transformations linking bosons and fermions. More specifically, the Poincaré group is generalized to the super-Poincaré group by adding two anticommuting generators Q and $\bar{Q}$, to the existing p (linear momentum), J (angular momentum) and K (boost), such that $\{Q, \bar{Q}\} \sim \gamma^\mu p_\mu$. Haag, Lopuszanski and Sohnius generalized the work of O'Raifeartaigh and by Coleman and Mandula to show that the most general symmetries of the S-matrix are a direct product of super-Poincaré group with the internal symmetry group. Since the new symmetry generators linking bosons and fermions are spinors, not scalars, supersymmetry is not an internal symmetry. Years ago, Dirac postulated a doubling of states by introducing an antiparticle to every particle in an attempt to reconcile special relativity with quantum mechanics. In a Stern–Gerlach experiment, an atomic beam in an inhomogeneous magnetic field splits due to doubling of the number of electron states into spin-up and -down modes indicating a doubling with respect to angular momentum. So it is no surprise that Q causes a further splitting into particle and superparticle ($f \xrightarrow{Q} f, \widetilde{f}$) [18]. Since Q is spinorial, the superpartners differ from their SM partners in spin. The superpartners of fermions are scalars and are called 'sfermions', while the superpartners of gauge bosons are fermions and are called 'gauginos'. Put together, a particle and its superpartner form a supermultiplet. The two irreducible supermultiplets which are used to construct the supersymmetric SM are the 'chiral' and the 'vector' supermultiplets. The chiral supermultiplet contains a scalar (e.g. selectron) and a 2-component Weyl fermion (e.g. left-chiral electron). The vector supermultiplet contains a gauge field (e.g. photon) and a 2-component Majorana fermion (e.g. photino). We should remember that

(i) there is an equal number of bosonic and fermionic degrees of freedom in a supermultiplet, and (ii) since p^2 commutes with Q, the bosons and fermions in a supermultiplet have the same mass.

But, why don't we see the superpartners? According to supersymmetry every fermion should have a bosonic partner and vice versa. Then the superpartner of electron which is a scalar with the same mass as that of the electron should have been found. This simply means that supersymmetry is not only broken but very badly broken and the superpartners are heavy enough to have escaped detection so far. There are quite a few ideas as to how supersymmetry is broken. Supersymmetry breaking can be mediated by supergravity, or by gauge interactions, or superconformal anomaly, and so on. Although we do not know exactly how it is broken, we know very well how to parametrize this breaking. Recall that the SM has 18 parameters, but the minimal supersymmetric standard model (MSSM) contains 106 additional parameters (see table 1 for the particle content). But once we assume a particular mechanism of supersymmetry breaking many of these parameters will be related. The next question is how long the superparticles can hide themselves? How good is the chance of finding them at the LHC? In other words, is there a reason for expecting them to appear at the TeV scale? An interesting observation is that the gauge couplings measured at LEP do not unify at a high scale when extrapolated using renormalization group (RG) equations containing beta functions computed with the SM particle content. But if we use supersymmetric RG equations, i.e. with beta functions computed with the supersymmetric particle content, the couplings do unify at a high (grand unification) scale (M_{GUT}) provided that the superparticle masses lie in the 100 GeV–10 TeV range. Moreover, this GUT scale is somewhat higher than what is obtained in non-supersymmetric scenarios which makes the prediction of proton lifetime more consistent with its non-observation. A very attractive property of all supersymmetric models with conserved R-parity is that they all include a stable electrically and color neutral massive ($\sim$100 GeV) particle which could be an excellent candidate of the observed dark matter of the universe. If R-parity is violated, even the gravitino could make a reasonable dark matter candidate. Furthermore, supersymmetry provides a framework to turn on gravity, as

when global supersymmetry is promoted to a local one we get supergravity. Supersymmetric theories have adapted very well with the LEP data, because they are *decoupling* theories in the sense that superparticle induced loop corrections to electroweak observables, in general, rapidly decouple with increasing superparticle masses.

Supersymmetry provides an important prediction on the Higgs mass. In the (two-Higgs doublet) MSSM the lightest Higgs cannot be heavier than about 135 GeV or so provided the superparticles weigh around a TeV. If we do not find any Higgs within that limit, the minimal version will be seriously disfavored. We have discussed in detail the properties of both neutral and charged scalars of the supersymmetric Higgs sector in section 6.3. But in this review we refrain from discussing their collider search strategies—for detailed search studies see Djouadi's review in [17].

1.2. Technicolor

Here we present an outline of the main idea behind technicolor theories. For a detailed survey of the historical development and the evolution of different concepts of dynamical electroweak symmetry breaking (DWSB) the readers are recommended to go through the early papers of Susskind [19] and Weinberg [20] and consult the reviews on DWSB breaking and technicolor [21–23]. We also recommend the readers to subsequently follow two recent papers on Higgs as a pseudo-Goldstone boson which discuss from a modern perspective as to how the difficulties of traditional technicolor models are overcome [24, 25].

QCD provides a strong force that binds the colored quarks. Can it induce EWSB by creating a bound state of strongly interacting sector which receives a non-zero expectation value in the vacuum? This is the central theme of technicolor (TC). Let us for the moment consider only $SU(3)_C$ interaction and switch off the electroweak gauge force of the SM. Let us assume only one generation of massless quark doublet, both left-handed and right-handed: $Q_L = (u, d)_L^T$ and $Q_R = (u, d)_R^T$. The QCD Lagrangian is invariant under a global chiral symmetry

$$SU(2)_L \times SU(2)_R.$$

The symmetry is spontaneously broken down to the diagonal subgroup $SU(2)_{L+R}$, which corresponds to isospin symmetry, when

$$\langle \bar{u}u \rangle_{\mathrm{vac}} = \langle \bar{d}d \rangle_{\mathrm{vac}} \neq 0.$$

This chiral symmetry breaking is accompanied by three massless pseudoscalars which are identified with the pions. These are associated with three axial currents ($q \equiv (u, d)^T$)

$$j_{Aa}^\mu = f_\pi \partial^\mu \pi_a = \bar{q} \gamma^\mu \gamma^5 \frac{\tau^a}{2} q,$$

where τ^a are the three Pauli matrices ($a = 1, 2, 3$) and f_π is the pion decay constant. When the electroweak interaction is switched on, the massless pions are eaten up by the as yet massless gauge bosons to form the longitudinal components

of those gauge bosons which in turn become massive. The W and Z boson masses are given by

$$M_W = g f_{\pi^\pm}/2, \qquad M_Z = \sqrt{g^2 + g'^2} f_{\pi^0}/2.$$

Isospin symmetry guarantees that $f_\pi \equiv f_{\pi^\pm} = f_{\pi^0}$. This picture is not phenomenologically acceptable as by putting $f_\pi \sim 93$ MeV, we obtain $M_W \sim 30$ MeV, while in reality $M_W \sim 80$ GeV. So the QCD force of the SM is not strong enough to generate the correct EWSB scale. TC does precisely this job. It is a scaled-up version of QCD, where $f_\pi \to F_\pi \sim v \approx 246$ GeV. So the W and Z bosons do not eat up the ordinary pions but the technipions. The beauty of this theory is that the hierarchy problem is solved by dimensional transmutation. Recalling that the QCD beta function is negative ($\beta < 0$), the electroweak scale (v) is dynamically generated when the TC gauge coupling g_{TC} diverges in the infrared, in complete analogy with the dynamical generation of Λ_{QCD}:

$$\frac{\mathrm{d}g_{TC}(\mu)}{\mathrm{d}\ln \mu} = \frac{\beta}{16\pi^2} g_{TC}^3(\mu) \Rightarrow v = M_{Pl} \exp\left(\frac{8\pi^2/\beta}{g_{TC}^2(M_{Pl})}\right).$$

The next important question is how fermion masses are generated [26]. Let us consider an example by enlarging the TC group G_{TC} to an extended technicolor (ETC) group G_{ETC} in which both $SU(3)_C$ and G_{TC} are embedded:

$$G_{ETC} \supset SU(3)_C \times G_{TC}.$$

It is assumed that G_{ETC} is spontaneously broken at a scale Λ_{ETC}. The gauge bosons corresponding to broken ETC generators would connect ordinary quarks (q) which transform under $SU(3)_C$ to the TC quarks (Ψ_{TC}) which transform under G_{TC}, and would generate effective four-fermion operators (after appropriate Fierz transformations)

$$\frac{g_{ETC}^2}{\Lambda_{ETC}^2} (\bar{q}q)(\bar{\Psi}_{TC}\Psi_{TC}).$$

At a lower scale Λ_{TC}, a condensation takes place: $\langle \bar{\Psi}_{TC}\Psi_{TC} \rangle \sim \Lambda_{TC}^3 \sim F_\pi^3 \sim v^3$. This immediately generates the ordinary quark mass

$$m_q \sim \Lambda_{TC} \left(\frac{\Lambda_{TC}}{\Lambda_{ETC}}\right)^2.$$

To generate the mass hierarchy among ordinary quarks, one has to first put all those ordinary quarks in a single ETC multiplet and arrange to break the multiplet through different cascades, thus generating different scales. But the exchanges of the same ETC gauge fields also generate operators with four ordinary quarks, namely, $(\bar{q}q)^2/\Lambda_{ETC}^2$, which severely violate flavor and CP particularly because all those SM quarks are in the same multiplet. Data on K and B mixing as well as rare meson decays introduce a very strong constraint $\Lambda_{ETC} > 10^{3-5}$ TeV, which is at least two to four orders of magnitude larger than the value of Λ_{ETC}, required to predict the correct strange quark mass. How to resolve this tension between large enough quark mass vis-à-vis too large FCNC rates? Here comes the rôle of *walking technicolor* [27]. Without going into a detailed discussion, we just point out that the dimension of the operator $(\bar{q}q)(\bar{\Psi}_{TC}\Psi_{TC})$ could be $(6+\gamma)$, instead of the classical value 6,

where γ is the anomalous dimension generated by the TC group. The TC coupling g_{TC} may have a large fixed point value at $\mu \sim \Lambda_{TC}$ and its evolution above Λ_{TC} may be slow (hence, 'walking', instead of 'running'). The formula for the ordinary quark mass is then modified to

$$m_q \sim \Lambda_{TC} \left(\frac{\Lambda_{TC}}{\Lambda_{ETC}} \right)^{(2+\gamma)}.$$

If γ is large and negative, then for a given m_q, one can accommodate a larger Λ_{ETC} than when $\gamma = 0$, i.e. one can have a large Λ_{ETC} without suppressing the quark mass. On the other hand, the suppression of FCNC rates still goes as $1/\Lambda_{ETC}^2$ since the SM color group cannot generate any large anomalous dimension. This way the quark mass versus FCNC tension is considerably ameliorated in the walking technicolor scenario.

We conclude our discussion on technicolor by just mentioning the idea of top quark condensates. Although Nambu first postulated it, Bardeen, Hill and Lindner formulated the theory of dynamical breaking of electroweak theory in the SM by a top quark condensate [28]. Here the Higgs boson is a $\bar{t}t$ bound state. Essentially, one implements the BCS or Nambu–Jona-Lasinio mechanism in which a new interaction at a high scale Λ triggers a low energy condensate $\langle t\bar{t} \rangle$. Generally, top quark mass turns out to be somewhat larger than the presently known value. This minimal scheme was further extended by Hill in a specific *topcolor* scheme [29]. In a subsequent development it was shown that in an ETC theory, where it is hard to generate a large top quark mass without adversely affecting the ρ parameter, a substantial part of the top quark mass may be generated by additionally incorporating the topcolor dynamics [30].

1.3. Plan of the review

We shall start our discussion with a brief recapitulation of the idea of gauge invariance. In the subsequent sections, we shall briefly review the essential structure of the electroweak part of the SM, illustrate the Higgs mechanism and raise the issue of the quantum instability of the scalar potential. We shall then demonstrate how the quadratic divergence is tamed in a toy scenario reminiscent of a supersymmetric model. Then we go on to explore different avenues through which one can successfully realize electroweak phase transition. In the process, we shall discuss minimal supersymmetry (only the Higgs sector), and some relatively recent ideas like little Higgs, gauge–Higgs unification and Higgsless scenarios. The latter two scenarios explicitly rely on the existence of extra dimension with a TeV-size inverse radius of compactification. It should be noted that many of these non-supersymmetric scenarios are often reminiscent of the technicolor models from the standpoint of AdS/CFT correspondence, which we shall just mention in passing without actually going into details. For each of these modern non-supersymmetric scenarios, we shall first illustrate the basic concepts using simple toy models, and then discuss, without going into calculational details, their phenomenological features and the strategies for detecting their signatures at the LHC. Finally, we shall conclude with a brief stock-taking of different aspects that the model-builders

should keep in mind, followed by a short discussion on how to distinguish the different EWSB models at the LHC.

2. A short recap of the idea of gauge invariance

This is a brief survey of the idea of gauge invariance required to formulate the basic structure of the SM. Let us first consider QED, which is governed by a U(1) gauge symmetry. We start with the Lagrangian of the electron field $\psi(x)$ with a mass m:

$$\mathcal{L} = \bar{\psi}(i\gamma^\alpha \partial_\alpha - m)\psi, \tag{2.1}$$

where $\partial_\alpha \equiv \partial/\partial x^\alpha$. Observe that for $\psi(x) \rightarrow \psi'(x) = e^{i\Lambda}\psi(x)$, where $\Lambda =$ real constant, the Lagrangian remains unaltered: $L(\psi) = L(\psi')$. Various transformations of the group U(1) commute. Such groups are called 'abelian'. Since Λ is a constant, the group is also called 'global'.

Now suppose that the group is still U(1), but 'local', i.e. $\Lambda \equiv \Lambda(x)$. Then $\psi'(x) = e^{i\Lambda(x)}\psi(x) \equiv U(x)\psi(x)$. Let us see how the derivative $\partial_\alpha \psi(x)$ transform:

$$\partial_\alpha \psi(x) = \partial_\alpha U^{-1}(x)\psi'(x) = U^{-1}(x) \underbrace{U(x)\partial_\alpha U^{-1}(x)}_{\begin{cases} (1+i\Lambda)\partial_\alpha(1-i\Lambda)\psi'(x) \\ = (\partial_\alpha - i\partial_\alpha\Lambda(x))\psi'(x) \end{cases}} \psi'(x)$$

$$= U^{-1}(\partial_\alpha - i\partial_\alpha\Lambda(x))\psi'(x). \tag{2.2}$$

Although for illustration we used infinitesimal transformation, it is actually *not a necessary condition*. Note that in the first term on the rhs the derivative acts on everything on its right, but in the end where we obtain $(\partial_\alpha - \partial_\alpha\Lambda(x))\psi'(x)$, the second ∂_α acts only on $\Lambda(x)$ and not on $\psi'(x)$. The message is the following: although $\psi(x) = U^{-1}(x)\psi'(x)$, $\partial_\alpha\psi(x) \neq U^{-1}(x)\partial_\alpha\psi'(x)$, i.e. the field and its derivative do not transform the same way under a local transformation. For the global case, if we recall, they did transform in the same way, and the Lagrangian remained invariant. But now for the local case, $\mathcal{L}(\psi) \neq \mathcal{L}(\psi')$.

Now, we write the Lagrangian in equation (2.1) with $D_\alpha \equiv \partial_\alpha - ieA_\alpha(x)$ instead of ∂_α, where e is a coupling constant. D_α is called the *covariant derivative*. We now observe the following:

$$[\partial_\alpha - ieA_\alpha(x)]\psi(x) = U^{-1}U(x)[\partial_\alpha - ieA_\alpha(x)]U^{-1}(x)\psi'(x)$$
$$= U^{-1}(x)[U(x)\partial_\alpha U^{-1}(x) - ieU(x)A_\alpha(x)U^{-1}(x)]\psi'(x)$$
$$= U^{-1}(x)[\partial_\alpha - i\partial_\alpha\Lambda(x) - ieA_\alpha(x)]\psi'(x)$$
$$= U^{-1}(x)[\partial_\alpha - ieA'_\alpha(x)]\psi'(x), \tag{2.3}$$

where

$$A'_\alpha \equiv A_\alpha(x) + \frac{1}{e}\partial_\alpha\Lambda(x). \tag{2.4}$$

We observe that the covariant derivative transforms like the field itself: $D_\alpha\psi(x) = U^{-1}(x)D'_\alpha\psi'(x)$, where $D'_\alpha \equiv \partial_\alpha - ieA'_\alpha(x)$. This ensures that $\mathcal{L}$ of equation (2.1), after replacing ∂_α by D_α, is invariant under the gauge transformation.

The gauge field strength tensor is defined as $F_{\alpha\beta} \equiv \partial_\alpha A_\beta - \partial_\beta A_\alpha$. Under gauge transformation

$$F'_{\alpha\beta} = \partial_\alpha \left(A_\beta + \frac{1}{e}\partial_\beta\Lambda \right) - \partial_\beta \left(A_\alpha + \frac{1}{e}\partial_\alpha\Lambda \right) = F_{\alpha\beta}. \tag{2.5}$$

The kinetic term of gauge field is given by $L_{\text{kin}} = -\frac{1}{4} F_{\alpha\beta} F^{\alpha\beta}$. One can also write $F_{\alpha\beta} = (1/e)[D_\alpha, D_\beta]$. It is instructive to check, in terms of the electric and magnetic field components that

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{pmatrix}. \qquad (2.6)$$

It follows immediately that $-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2}(\vec{E}^2 - \vec{B}^2)$, which is the kinetic term.

Let us now concentrate on the non-abelian group SU(2). Consider a fermion field $\psi(x)$ which transforms as a doublet under SU(2): $\psi(x) = \binom{\psi^1(x)}{\psi^2(x)}$. Let us follow its local SU(2) transformation: $\psi \rightarrow \psi' = e^{i(\tau_a/2)\Lambda_a(x)}\psi(x)$, and $\bar{\psi} \rightarrow \bar{\psi}' = \bar{\psi}(x)e^{-i(\tau_a/2)\Lambda_a(x)}$, where $\tau_a (a = 1, 2, 3)$ are the Pauli matrices which satisfy $[\tau_a, \tau_b] = 2i\epsilon_{abc}\tau_c$. It is easy to check that $\partial_\alpha \psi(x) = U^{-1}(x)[\partial_\alpha - i(\tau_a/2)\partial_\alpha\Lambda_a(x)]\psi'(x)$, i.e. $\psi(x)$ and $\partial_\alpha \psi(x)$ do not transform identically, and hence the Lagrangian is not invariant under SU(2) transformation. To ensure gauge invariance we must start with the covariant derivative $D_\alpha \psi(x) \equiv [\partial_\alpha - ig(\tau_a/2)A_\alpha^a(x)]\psi(x)$, where g is the coupling constant (like the symbol e used for U(1)). We obtain

$$D_\alpha \psi(x) = U^{-1} \underbrace{U(x)\left(\partial_\alpha - ig\frac{\tau_a}{2}A_\alpha^a(x)\right)U^{-1}(x)}\psi'(x)$$
$$= \partial_\alpha - ig\frac{\tau_a}{2}A_\alpha'^a \equiv D_\alpha'$$
$$= U^{-1}D_\alpha'\psi'(x), \qquad (2.7)$$

where

$$A_\alpha'^a = A_\alpha^a + \frac{1}{g}\partial_\alpha\Lambda^a + \epsilon^{abc}A_{\alpha b}\Lambda_c. \qquad (2.8)$$

If we do a straightforward generalization of the abelian case and construct $G_{\alpha\beta}^a = \partial_\alpha A_\beta^a - \partial_\beta A_\alpha^a$, the product $G_{\alpha\beta}^a G_a^{\alpha\beta}$ is not gauge invariant. We must redefine field the strength tensor in the non-abelian case as

$$F_{\alpha\beta}^a \equiv (\partial_\alpha A_\beta^a - \partial_\beta A_\alpha^a) + g\epsilon^{abc}A_{\alpha b}A_{\beta c}. \qquad (2.9)$$

It is instructive to use the transformation properties of the gauge fields, discussed above, to check that $F_{\alpha\beta}^a F_a^{\alpha\beta}$ remains invariant under gauge transformation, and constitutes the gauge boson kinetic term in the Lagrangian.

3. The Standard Model Higgs mechanism

Now we will discuss the idea and implementation of spontaneous symmetry breaking (SSB). Whenever a system does not show *all* the symmetries by which it is governed, we say that the symmetry is 'spontaneously' broken. More explicitly, when there is a solution which does not exhibit a given symmetry which is encoded and respected in the Lagrangian, or Hamiltonian, or the equations of motion, the symmetry is said to be spontaneously broken. In the context of the SM, the SSB idea is used to generate gauge boson masses without spoiling the calculability (which we technically call *renormalizability*) of the theory. To gain insight into different aspects of SSB, we will consider different cases one by one.

3.1. SSB of discrete symmetry

Consider a real scalar field $\varphi(x)$. The Hamiltonian is given by

$$H = \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}\left(\vec{\nabla}\varphi\right)^2 + V(\varphi),$$
$$\text{where } V(\varphi) = \frac{1}{2}m^2\varphi^2 + \frac{1}{4}\lambda^2\varphi^4. \qquad (3.1)$$

Above, we have assumed a $\varphi \leftrightarrow -\varphi$ discrete symmetry which prohibits odd powers of φ. Clearly, the minimum of $V(\varphi)$ is at $\varphi = 0$. Now, as the next step, consider

$$V(\varphi) = -\frac{1}{2}m^2\varphi^2 + \frac{1}{4}\lambda^2\varphi^4, \qquad \text{where } m^2 > 0. \quad (3.2)$$

Since $V'(\varphi)|_{\varphi=0} = 0$, it follows that $\varphi = 0$ is indeed an extremum. Moreover, $V''(\varphi)|_{\varphi=0} = -m^2$ implies that $\varphi = 0$ is rather a *maximum* and not a minimum. Stable minima occur at two points $\varphi = \pm m/\lambda$, where $V(m/\lambda) = -m^4/4\lambda^2$. Recall, we can always add a constant term in $V(\varphi)$, which does not change the physics. Using this idea, we can write the potential as a complete square as such

$$V(\varphi) = \frac{1}{4}\lambda^2(\varphi^2 - v^2)^2, \qquad (3.3)$$

where $v = m/\lambda$. With this redefined potential, the system can be at either of the two minima ($\pm v$). Once one solution is chosen, the symmetry breaks spontaneously. Note, the potential $V(\varphi)$ attains its minimum value zero for a non-zero value of φ. The zero energy state, characterized by $V(\varphi) = 0$, is called the ground state or the minimum energy state, while $v \equiv \langle 0|\varphi|0\rangle$ is called the 'vacuum expectation value' (VEV).

We should remember two points:

- When we consider the VEV of a field, this field has to be a 'classical' field. Remember, a quantum field can always be expanded in terms of creation and annihilation operators whose vacuum expectation would always vanish.

- When we write $v \equiv \langle 0|\varphi(x)|0\rangle$, a naïve question comes to mind as to how the lhs is independent of x while the rhs is a function of x. It happens because the translational invariance of the vacuum can be used to write

$$\langle 0|\varphi(x)|0\rangle = \langle 0|e^{ipx}\varphi(0)e^{-ipx}|0\rangle = \langle 0|\varphi|0\rangle = v.$$

3.2. SSB of global U(1) symmetry

For U(1) symmetry, we must start with a *complex* scalar field φ. The scalar potential is given by

$$V(|\varphi|) = \frac{1}{2}\lambda^2(|\varphi|^2 - \frac{1}{2}v^2)^2. \qquad (3.4)$$

This potential has a global U(1) symmetry: $\varphi \rightarrow \varphi' = e^{i\alpha}\varphi$, where α is any real constant. The potential is minimum (which is zero) at all points on the orbit of radius $|\varphi| = v/\sqrt{2}$, different points corresponding to different values of $\text{Arg}(\varphi)$. The shape of the potential takes the form of a 'Mexican hat'—see figure 1. We write

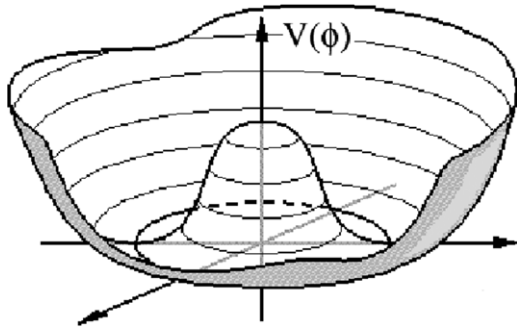$$\varphi(x) = \frac{1}{\sqrt{2}}(v + \chi(x) + i\psi(x)), \qquad (3.5)$$

Figure 1. The Mexican hat potential.

where $\chi(x)$ and $\psi(x)$ are the two components of the complex quantum field around the stable minima. The Lagrangian in terms of the χ and ψ fields can be expressed as

$$\mathcal{L} = \mathcal{K} - \tfrac{1}{2}\lambda^2 \left[\tfrac{1}{2}|v + \chi + i\psi|^2 - \tfrac{1}{2}v^2\right]^2$$
$$= \mathcal{K} - \tfrac{1}{8}\lambda^2 \left[\chi^2 + 2v\chi + \psi^2\right]^2, \qquad (3.6)$$

where $\mathcal{K} = \tfrac{1}{2}(\partial_\mu \chi)^2 + \tfrac{1}{2}(\partial_\mu \psi)^2$ is the kinetic term. It is clear from equation (3.6) that the component of excitation along the v-direction (χ) acquires a mass ($m_\chi = \lambda v$), while the component (ψ), which is in a direction tangential to the orbit, remains massless ($m_\psi = 0$). That ψ is massless is not surprising as traversing along the orbit does not cost any energy. What is important is that as a result of a spontaneous breaking of a continuous global symmetry, a massless scalar has been generated. Such a massless scalar field is called the 'Nambu–Goldstone boson' or often the 'Goldstone boson'.

3.3. SSB of global SU(2) symmetry

Here, the complex scalar field is a doublet of SU(2), given by $\Phi = \binom{\varphi_+}{\varphi_0}$. The Lagrangian is given by

$$\mathcal{L} = (\partial_\mu \Phi)^\dagger (\partial_\mu \Phi) - V(\Phi^\dagger \Phi), \qquad (3.7)$$

where $\Phi^\dagger \Phi = \varphi_+^* \varphi_+ + \varphi_0^* \varphi_0$. Here φ_+ and φ_0 have two real components each, i.e. there are in total four degrees of freedom (d.o.f.). At this level, the subscripts + and 0 are simply labels. We will see later on that these labels would correspond to electric charge +1 and 0, respectively. After SSB, three d.o.f. remain massless, one becomes massive. It can be proved that the number of Goldstone bosons is the number of broken generators. To appreciate this from a geometric point of view, note that a 4d sphere has three tangential directions, and clearly, quantum oscillations along these directions yield massless modes.

Of course, the next question is what happens when a global symmetry is gauged?

3.4. SSB with local U(1) symmetry

Now we deal with a local U(1) symmetry. The Lagrangian can be written as

$$\mathcal{L} = |D_\mu \Phi|^2 - \tfrac{1}{2}\lambda^2 \left(|\Phi|^2 - \frac{v^2}{2}\right)^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}. \qquad (3.8)$$

Here we have used a slightly different notation compared with the global U(1) case. The complex scalar will be denoted by Φ, which can be written as $\Phi(x) = \varphi(x)e^{i\theta(x)}$ where $\varphi(x)$ and $\theta(x)$ are the two real d.o.f. Recall that the covariant derivative and the gauge field strength tensors are given by

$$D_\mu = \partial_\mu - ieA_\mu, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

Now, under gauge transformation $\Phi \to \Phi' = e^{i\alpha(x)}\Phi$ and the Lagrangian still remains invariant. This phase $\alpha(x)$ is different at different space–time points, but it is not a physical parameter and at each and every such point one has the liberty to choose it in such a way that it precisely cancels the $\theta(x)$ at that point. This choice of gauge is called *unitary gauge*. In other words, $\Phi(x)$ can be chosen to be real everywhere, and can be written, without any loss of generality, as

$$\Phi(x) = \varphi(x) = \frac{1}{\sqrt{2}}(v + \chi(x)). \qquad (3.9)$$

The Lagrangian takes the following form:

$$\mathcal{L} = |(\partial_\mu - ieA_\mu)\varphi(x)|^2 - \frac{\lambda^2}{2}\left[\frac{1}{2}(v + \chi(x))^2 - \frac{1}{2}v^2\right]^2$$
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(\partial_\mu \chi(x))^2 + \frac{e^2}{2}A_\mu A_\nu(v^2 + 2v\chi(x)$$
$$+\chi^2(x)) - \frac{\lambda^2}{8}(2v + \chi(x))^2 \chi^2(x) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \qquad (3.10)$$

This describes a real scalar field $\chi(x)$ with mass λv and a massive vector field A_μ with a mass ev. Note that SSB resulted in a redistribution of fields: one of the two real fields forming the complex scalar has been *gauged away* but it has reappeared in the form of a longitudinal component of the vector field A_μ. The total number of d.o.f. thus remains unaltered: $2+2 = 3+1$. The Goldstone boson is *eaten up* by the gauge boson. This is called the *Higgs mechanism* and $\chi(x)$ is called the *Higgs field*[1].

3.5. SSB with local SU(2) symmetry

Denoting the complex scalar doublet as Φ, the Lagrangian can be written as

$$L = |D_\mu \Phi|^2 - \tfrac{1}{2}\lambda^2(|\Phi|^2 - \tfrac{1}{2}v^2)^2 - \tfrac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a, \qquad (3.11)$$

where

$$\Phi = \binom{\varphi_+}{\varphi_0} = \frac{1}{\sqrt{2}}\binom{\varphi_1 + i\varphi_2}{\varphi_3 + i\varphi_4}, \qquad (3.12)$$

$$D_\mu \Phi = \left(\partial_\mu - \frac{ig}{2}\tau^a W_\mu^a\right)\Phi,$$
$$F_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\epsilon_{abc}W_\mu^b W_\nu^c \quad (a, b = 1, 2, 3).$$
$$(3.13)$$

---

[1] The basic idea of the Higgs mechanism was borrowed from condensed matter physics. Similar things happen in the BCS theory of superconductivity. Electromagnetic gauge invariance is spontaneously broken and a photon becomes massive inside a superconductor from where magnetic fields are repelled due to the Meissner effect. For historical reasons, the mechanism is also known as the Anderson–Higgs mechanism and Higgs–Brout–Englert–Guralnik–Hagen–Kibble mechanism.

Note that the definition of the field strength tensor $F^a_{\mu\nu}$ follows from equation (2.9). Here, $|\Phi|^2 = \varphi_+^*\varphi_+ + \varphi_0^*\varphi_0 = \frac{1}{2}(\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2)$. The potential is minimum when $\sum_i \varphi_i^2 = v^2$, where v is the radius of the orbit. Without any loss of generality we can assume that the entire VEV is in the φ_3 direction, i.e. $\langle\Phi\rangle = \Phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$. The Higgs field $h(x)$ is the real excitation around the VEV. Thus, in the unitary gauge where the scalar field has only the real component, $\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h(x) \end{pmatrix}$.

The gauge boson masses arise from the expansion of $|D_\mu\Phi|^2$-piece of the Lagrangian. This gives

$$\frac{g^2}{8}\left|\tau^a W^a_\mu \begin{pmatrix} 0 \\ v \end{pmatrix}\right|^2 = \frac{g^2}{8}\left|\begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}\right|^2$$
$$= \left(\frac{gv}{2}\right)^2 W^+_\mu W^-_\mu + \frac{1}{2}\left(\frac{gv}{2}\right)^2 W^3_\mu W^3_\mu,$$

where $W^\pm \equiv (W^1_\mu \mp iW^2_\mu)\sqrt{2}$. This means that all the three gauge bosons have the same mass $(gv/2)$. The equality of $W^\pm$ and W^3 masses *does not follow from gauge symmetry but results from a global 'custodial' symmetry*. What is this custodial symmetry? Looking at the orbit structure, $\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2 = v^2$, we note that before the SSB the potential had a SO(4) symmetry, which is reduced to SO(3) once one direction is fixed for the VEV. The group SO(3) is isomorphic to SU(2). This SU(2) is global and should not be confused with the SU(2) we gauged. It is *this* SU(2) that we call the custodial SU(2). This remains unbroken even after the VEV is generated, and this unbroken symmetry enforces the equality of the gauge boson masses. The bottom line is that all the three Goldstone bosons related to the global SU(2) have now disappeared, and three massive (but degenerate) gauge bosons have emerged.

3.6. SSB with local SU(2) × U(1) symmetry (the electroweak part of the SM)

3.6.1. Why SU(2) × U(1)?

Obviously we need two gauge bosons to meet the observations already made. There has to be a massive charged gauge boson which would mediate beta decay. The smallest unitary group which provides an off-diagonal generator (corresponding to the charged gauge boson) is SU(2). The relevant generators are τ^1 and τ^2. We further need a massless gauge boson. Any association of a photon with the neutral generator τ^3 would lead to contradiction with respect to the charge assignment of particles. The gauge charges of fermions in a doublet coupling to W^3 are $\pm\frac{1}{2}$, clearly different from the electric charges. Moreover, W^3 couples to a neutrino, but a photon does not. All in all, just with SU(2) gauge theory we cannot explain both weak and electromagnetic interactions. The next simplest construction is to avoid taking a simple group, but consider SU(2) × U(1).

The covariant derivative will now contain gauge bosons of both SU(2) and U(1):

$$D_\mu = \partial_\mu - ig\frac{\tau^a}{2}W^a_\mu - ig'\frac{Y}{2}B_\mu, \qquad (3.14)$$

where the quantum number Y is the 'hypercharge' of the particle on which D_μ acts.

The SM contains five representations of fermions (quarks and leptons) for each generation—two doublets and three singlets:

$$L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L, \qquad e_R, \qquad Q \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L, \qquad u_R, \qquad d_R.$$

Ψ_L and Ψ_R are left- and right-chiral states of a fermion field Ψ, such that $\gamma_5\Psi_L = -\Psi_L$ and $\gamma_5\Psi_R = \Psi_R$.

3.6.2. Notion of hypercharge.

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad \begin{array}{l} \nearrow \; t_3 = \frac{1}{2}, \quad Q = 0 \quad \therefore Q - t_3 = -\frac{1}{2} \\[6pt] \searrow \; t_3 = -\frac{1}{2}, \quad Q = -1 \quad \therefore Q - t_3 = -\frac{1}{2} \end{array}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{array}{l} \nearrow \; t_3 = \frac{1}{2}, \quad Q = \frac{2}{3} \quad \therefore Q - t_3 = \frac{1}{6} \\[6pt] \searrow \; t_3 = -\frac{1}{2}, \quad Q = -\frac{1}{3} \quad \therefore Q - t_3 = \frac{1}{6}. \end{array}$$

Note that the $(Q-t_3)$ assignments are the same for all members inside a given multiplet, i.e. the generator corresponding to $(Q-t_3)$ commutes with all the SU(2) generator τ_a. Hence, either $(Q - t_3)$ or some multiple of it can serve as the hypercharge quantum number of $U(1)_Y$. We follow the convention

$$2(Q - t_3) = Y \Longrightarrow Q = t_3 + \frac{Y}{2}. \qquad (3.15)$$

It is instructive to check that the currents satisfy $J^Q_\mu = J^3_\mu + \frac{1}{2}J^Y_\mu$.

3.6.3. How is the symmetry broken?

If a generator $\hat{O}$ is such that the corresponding operator $e^{i\hat{O}}$ acting on the vacuum $|0\rangle$ cannot change it, i.e. $e^{i\hat{O}}|0\rangle = |0\rangle$, then obviously the operation corresponds to a symmetry of the vacuum and the corresponding generator kills the vacuum, i.e. $\hat{O}|0\rangle = 0$. In the context of gauge theory, when the vacuum is left unbroken by a generator, the gauge boson corresponding to that generator would remain massless. Let us now check how the neutral (diagonal) generators of SU(2) and U(1) act on the scalar VEV:

$$t_3\Phi_0 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 \\ -v \end{pmatrix} \neq 0,$$
$$\frac{Y}{2}\Phi_0 = \frac{1}{2\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \neq 0, \qquad (3.16)$$

but $(t_3 + \frac{Y}{2})\Phi_0 = 0$. This means that $Q_{em} = t_3 + \frac{Y}{2}$ is indeed the electromagnetic charge generator and consequently a photon is massless. This is the *only* combination that yields a massless gauge boson, and the massless state is neither a SU(2) nor a U(1) state, but a mixed state. In other words, the masslessness of a photon is a consequence of the vacuum being invariant under the operation by $e^{iQ_{em}}$.

An electrically charged field does not acquire any VEV, as otherwise charge will be spontaneously broken in the following way: If φ^+ is the charge $(+)$ field, then one can write $[Q, \varphi^+] = +\varphi^+$. This means that if $\langle 0|\varphi^+|0\rangle = v \neq 0$, then using the commutator relation one can show that $Q|0\rangle \neq 0$, i.e. electric charge is spontaneously broken!

3.6.4. Masses of the gauge bosons. There are four gauge bosons. One of them is the massless photon, but the other three are massive. Here we calculate their masses. To do this we look into the kinetic term with the covariant derivative:

$$|D_\mu \Phi|^2 \Longrightarrow \left| \left(-\frac{ig}{2}\tau^a W_\mu^a - \frac{ig'}{2}B_\mu \right) \Phi_0 \right|^2$$

$$= \frac{1}{8} \left| \begin{pmatrix} gW_\mu^3 + g'B_\mu & \sqrt{2}gW_\mu^- \\ \sqrt{2}gW_\mu^+ & -gW_\mu^3 + g'B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2$$

$$= \left(\frac{1}{2}gv \right)^2 W_\mu^+ W_\mu^- + \frac{1}{8}v^2 (W_\mu^3 \ B_\mu)$$

$$\times \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}. \tag{3.17}$$

Clearly, the charged $W^\pm$ gauge boson mass is given by $M_W = gv/2$. Recall, $W_\mu^\pm$ has been constructed out of W_μ^1 and W_μ^2, the gauge bosons corresponding to the off-diagonal generators τ^1 and τ^2.

We now look into the neutral part. The mass matrix in the (W_μ^3, B_μ) basis has zero determinant. This is not unexpected as one of the states has to be the massless photon (A). The other eigenstate is the Z boson. Thus the orthogonal neutral states and their masses are

$$A_\mu = \frac{gB_\mu + g'W_\mu^3}{\sqrt{g^2 + g'^2}} \ : \ M_A = 0,$$

$$Z_\mu = \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2 + g'^2}} \ : \ M_Z = \frac{v}{2}\sqrt{g^2 + g'^2}. \tag{3.18}$$

Introducing $\cos\theta_W \equiv \frac{g}{\sqrt{g^2+g'^2}}$, $\sin\theta_W \equiv \frac{g'}{\sqrt{g^2+g'^2}}$, where θ_W is called the weak angle, one can express

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3,$$

$$Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu,$$

$$\frac{M_W}{M_Z} = \frac{\frac{1}{2}gv}{\frac{v}{2}\sqrt{g^2 + g'^2}} = \cos\theta_W. \tag{3.19}$$

Observe that $M_Z > M_W$, i.e. the custodial symmetry associated with the SU(2) gauge group is broken, and it has been broken by hypercharge mixing, i.e. by expanding the gauge group to SU(2) × U(1). One can easily check that in the $g' \to 0$ limit, one recovers the custodial symmetry. Experimentally, $M_Z = 91.1875 \pm 0.0021$ GeV and $M_W = 80.399 \pm 0.025$ GeV, which are almost the same values as predicted by the SM. The weak mixing angle is given by $\sin^2\theta_W \simeq 0.23$.

We will here define an important parameter:

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W}. \tag{3.20}$$

With the SU(2) doublet scalar representation (and at tree level), one can easily check from the above relations that $\rho = 1$. Experimental measurements on the Z pole at LEP also indicate ρ to be very close to unity within a *per mille* precision.

If there are several representations of scalars whose electrically neutral members acquire VEVs v_i, then

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \frac{\sum_{i=1}^N v_i^2 \left[T_i(T_i + 1) - \frac{1}{4}Y_i^2 \right]}{\sum_{i=1}^N \frac{1}{2}v_i^2 Y_i^2}, \tag{3.21}$$

where T_i and Y_i are the weak isospin and hypercharge of the ith multiplet. It is easy to check that only those scalars are allowed to acquire VEVs which satisfy $(2T + 1)^2 - 3Y^2 = 1$, as otherwise $\rho = 1$ will not be satisfied at the tree level. The simplest choice is to have a scalar with $T = \frac{1}{2}$ and $Y = 1$, which corresponds to the SM doublet Φ. More complicated scalar multiplets, e.g. one with $T = 3$ and $Y = 4$, also satisfy this relation.

3.6.5. Couplings of a photon, Z and $W^\pm$ with fermions. The interaction of the gauge bosons with the fermions arise from $i\bar{\Psi}\gamma^\mu D_\mu \Psi$, where $D_\mu = \partial_\mu - ig\frac{\tau^a}{2}W_\mu^a - ig'\frac{Y}{2}B_\mu$. In the SM, a generic fermion field (Ψ) has a left-chiral SU(2)-doublet representation: $\Psi_L = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}_L$, and right-chiral SU(2)-singlet representations: ψ_{1R} and ψ_{2R}.

Now we look into the charged-current interaction. We write the relevant part of the Lagrangian as

$$\mathcal{L}_{CC} = \frac{g}{2}(J_\mu^1 W_\mu^1 + J_\mu^2 W_\mu^2),$$

$$\text{where } J_\mu^{1,2} = \bar{\Psi}\gamma_\mu P_L \tau^{1,2}\Psi, \tag{3.22}$$

using $P_{L,R} \equiv (1 \mp \gamma_5)/2$. Expressing $W_\mu^\pm = (W_\mu^1 \mp iW_\mu^2)/\sqrt{2}$, we rewrite the charged-current Lagrangian as

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}}[\bar{\psi}_1\gamma_\mu P_L \psi_2 W_\mu^+ + \bar{\psi}_2\gamma_\mu P_L \psi_1 W_\mu^-]. \tag{3.23}$$

Now we come to the neutral-current part. We can express the Lagrangian as

$$\mathcal{L}_{NC} = \frac{g}{2}J_\mu^3 W_\mu^3 + \frac{g'}{2}J_\mu^Y B_\mu,$$

where

$$J_\mu^3 = \bar{\Psi}\gamma_\mu P_L \tau^3 \Psi,$$

$$J_\mu^Y = \bar{\Psi}\gamma_\mu P_L Y_L \Psi + \bar{\psi}_1\gamma_\mu P_R Y_R^1 \psi_1 + \bar{\psi}_2\gamma_\mu P_R Y_R^2 \psi_2, \tag{3.24}$$

and where Y_L is the hypercharge of the left-handed doublet while Y_R^1 and Y_R^2 are hypercharges of the two right-handed singlets. Now rewriting W^3 and B in terms of the photon (A) and the Z boson, as $W_\mu^3 = \cos\theta_W Z_\mu + \sin\theta_W A_\mu$ and $B_\mu = -\sin\theta_W Z_\mu + \cos\theta_W A_\mu$, one can write the neutral current Lagrangian in the (A, Z) basis as

$$\mathcal{L}_{NC} = J_\mu^Q A_\mu + J_\mu^Z Z_\mu,$$

where

$$J_\mu^Q = eQ_i\bar{\psi}_i\gamma_\mu\psi_i, \qquad \text{with } e \equiv g\sin\theta_W,$$

$$\text{and the sum over } i \text{ is implied}, \tag{3.25}$$

$$J_\mu^Z = \frac{g}{\cos\theta_W}[a_L^i\bar{\psi}_i\gamma_\mu P_L\psi_i + a_R^i\bar{\psi}_i\gamma_\mu P_R\psi_i],$$

$$\text{with } a_L^i \equiv t_3^i - Q_i\sin^2\theta_W, \quad a_R^i \equiv -Q_i\sin^2\theta_W. \tag{3.26}$$

As we observe, the Z boson couples to the left- and right-handed fermions with different strengths. Quite often the Z boson's interaction with fermions are expressed in terms of vector and axial-vector couplings, which are simply linear combinations of a_L and a_R. Thus, for a given fermion f, the $Zf\bar{f}$ vertex is given by

$$\frac{g}{\cos\theta_W}\gamma_\mu(a_L^f P_L + a_R^f P_R) \equiv \frac{g}{2\cos\theta_W}\gamma_\mu(v^f - a^f\gamma_5),$$
$$\text{where } v^f \equiv t_3^f - 2Q_f\sin^2\theta_W, \quad a^f \equiv t_3^f, \quad (3.27)$$

are the tree level couplings of the Z boson to the fermion f.

3.6.6. The decay width of the Z boson. The Z boson decays into all $f\bar{f}$ pair, except the $t\bar{t}$ because $m_t \simeq 173$ GeV, while $M_Z \simeq 91$ GeV. The expression of the decay width of the Z boson in the $f\bar{f}$ channel is given by (the derivation can be found in text books)

$$\Gamma_f = \frac{G_F}{6\pi\sqrt{2}}M_Z^3(v_f^2 + a_f^2)f\left(\frac{m_f}{M_Z}\right), \quad (3.28)$$

where

$$f(x) = (1 - 4x^2)^{1/2}\left(1 - x^2 + 3x^2\frac{v_f^2 - a_f^2}{v_f^2 + a_f^2}\right).$$

One can easily verify some of the SM predictions of the Z boson properties: total decay width $\Gamma_Z \simeq 2.5$ GeV, hadronic decay width $\Gamma_{\text{had}} \simeq 1.74$ GeV, charged lepton decay width (average of e, μ, τ) $\Gamma_\ell \simeq 84.0$ MeV, invisible decay width (into all neutrinos) $\Gamma_{\text{inv}} \simeq 499.0$ MeV, hadronic cross section (peak) $\sigma_{\text{had}} \simeq 41.5$ nanobarn [31].

While doing the algebraic manipulation it will be useful to remember that the Fermi coupling G_F can be expressed in many ways:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{1}{2v^2} = \frac{g^2}{8M_Z^2\cos^2\theta_W}$$
$$= \frac{e^2}{8M_Z^2\sin^2\theta_W\cos^2\theta_W}. \quad (3.29)$$

4. The LEP legacy

4.1. Cross section and decay width

Let us consider the total cross section of $e^+e^- \rightarrow \mu^+\mu^-$ mediated by the photon and the Z boson. It is given by ($\sqrt{s}$ = c.m. energy)

$$\sigma = \frac{4\pi\alpha^2}{3s}(1 + a_1), \quad (4.1)$$

where

$$a_1 = 2v_\ell^2 f_Z + (v_\ell^2 + a_\ell^2)^2 f_Z^2, \quad \text{with}$$
$$f_Z = \frac{s}{s - M_Z^2}\left(\frac{1}{\sin^2 2\theta_W}\right).$$

Note that the effect of the Z mediation is encoded in a_1, whereas setting $a_1 = 0$ we get the contribution of the photon.

For the leptons $\ell = e, \mu, \tau, v_\ell \propto (1 - 4\sin^2\theta_W) \sim$ zeroish. Therefore,

$$\sigma(e^+e^- \xrightarrow{\gamma, Z} \mu^+\mu^-) \simeq \frac{4\pi\alpha^2}{3s}\left[1 + \frac{1}{16\sin^4 2\theta_W}\frac{s^2}{(s - M_Z^2)^2}\right]. \quad (4.2)$$

Thus, in the vicinity of $\sqrt{s} = M_Z$, we would expect a sharp increase of cross section. This is the sign of a resonance of the Z boson mediation. But, in reality, the cross section does not diverge at $s = M_Z^2$. The reason is that the Z boson has a decay width Γ_Z, which would lead to the following modification:

$$\frac{s^2}{(s - M_Z^2)^2} \rightarrow \frac{s^2}{\left[s - |M_Z - \frac{i}{2}\Gamma_Z|^2\right]^2}.$$

The factor $\frac{1}{2}$ in front of Γ_Z comes from the definition of the width as half-width at the maximum. Consequently,

$$\sigma_{\text{max}} \simeq \frac{4\pi\alpha^2}{3M_Z^2}\left[1 + \frac{1}{16\sin^4 2\theta_W}\frac{M_Z^2}{\Gamma_Z^2}\right] \simeq \frac{4}{27}\frac{\pi\alpha^2}{\Gamma_Z^2}. \quad (4.3)$$

In general, for $e^+e^- \rightarrow f\bar{f}$, $(v_\ell^2 + a_\ell^2)^2$ should be replaced by $(v_e^2 + a_e^2)(v_f^2 + a_f^2)$, i.e. f is not necessarily μ. Therefore,

$$\sigma(e^+e^- \rightarrow f\bar{f})|_{s=M_Z^2 - \frac{\Gamma_Z^2}{4}}$$
$$\simeq \frac{4\pi\alpha^2}{3M_Z^2}\left[1 + \frac{(v_e^2 + a_e^2)(v_f^2 + a_f^2)}{\sin^4 2\theta_W}\frac{M_Z^2}{\Gamma_Z^2}\right]. \quad (4.4)$$

Substituting $\Gamma_f = \alpha M_Z(v_f^2 + a_f^2)/(3\sin^2 2\theta_W)$, we obtain

$$\sigma_{\text{max}}(e^+e^- \rightarrow f\bar{f}) \simeq \frac{4\pi\alpha^2}{3M_Z^2}\left(1 + \frac{9}{\alpha^2}\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2}\right). \quad (4.5)$$

Numerically, $9\Gamma_e\Gamma_f \gg \alpha^2\Gamma_Z^2$. Thus we arrive at the *master formula*:

$$\sigma_{\text{max}}^f \simeq \frac{12\pi}{M_Z^2}\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2}. \quad (4.6)$$

Now, we make some important observations.

1. From the peak position of the Breit–Wigner resonance, we can measure M_Z for any final state f.

2. The half-width at the maximum gives us the *total* width Γ_Z for any final state f.

3. By measuring Bhabha scattering cross section (σ^e) at the Z pole, we can calculate Γ_e.

4. By measuring the peak cross section for any other final state ($f = e, \mu, \tau$, hadron), we can calculate the corresponding Γ_f.

5. Since neutrinos are invisible, we cannot directly measure the neutrino decay width. But the total invisible decay width $\Gamma_{\text{inv}} = \Gamma_Z - \Gamma_{\text{visible}} = \Gamma_Z - \Gamma_e - \Gamma_\mu - \Gamma_\tau - \Gamma_{\text{had}}$.

6. The number of light neutrinos is $N_\nu = \Gamma_{\text{inv}}/\Gamma_\nu^{\text{SM}} = 2.984 \pm 0.008$, which for all practical purposes is 3.

4.2. Forward–backward asymmetry

The differential cross section in the $\ell^+\ell^-$ channel ($\ell = \mu, \tau$) is given by

$$\frac{d\sigma}{d\Omega}(e^+e^- \xrightarrow{\gamma,Z} \ell^+\ell^-)$$
$$= \frac{e^4}{64\pi^2 s}\left[(1+a_1)(1+\cos^2\theta) + a_2\cos\theta\right], \quad (4.7)$$

where, a_1 has been defined in equation (4.1), and $a_2 = 8v_\ell^2 a_\ell^2 f_Z^2 + 4a_\ell^2 f_Z$.

The a_1 contribution has the same angular dependence—$(1+\cos^2\theta)$—as in QED. The a_2 contribution makes a vital qualitative and quantitative difference by introducing a term proportional to $\cos\theta$. This term arises due to interference between vector and axial-vector couplings. This gives rise to the forward–backward asymmetry, which is defined as

$$A_{\mathrm{FB}}^l = \frac{\int_0^{\pi/2} d\theta \sin\theta \frac{d\sigma}{d\Omega} - \int_{\pi/2}^{\pi} d\theta \sin\theta \frac{d\sigma}{d\Omega}}{\int_0^{\pi/2} d\theta \sin\theta \frac{d\sigma}{d\Omega} + \int_{\pi/2}^{\pi} d\theta \sin\theta \frac{d\sigma}{d\Omega}}$$
$$= \frac{3}{8}\left(\frac{a_2}{1+a_1}\right). \quad (4.8)$$

Even though the top quark could not be produced at LEP due to kinematic reasons, its existence was inferred from the measurement of $\Gamma_b \equiv \Gamma(Z \to b\bar{b})$ and the forward–backward asymmetry A_{FB}^b in the following way. Note

$$\Gamma_b^{\mathrm{SM}} = \frac{G_{\mathrm{F}} M_Z^3}{3\pi\sqrt{2}}[(a_{\mathrm{L}}^b)^2 + (a_{\mathrm{R}}^b)^2]$$
$$= \frac{G_{\mathrm{F}} M_Z^3}{3\pi\sqrt{2}}[(t_3^b - Q_b \sin^2\theta_W)^2 + (-Q_b \sin^2\theta_W)^2]$$
$$= \frac{1.166 \times 10^{-5}\ \mathrm{GeV}^{-2} \times (91.2\,\mathrm{GeV})^3}{3\pi\sqrt{2}}$$
$$\times \left[\left(-\frac{1}{2} + \frac{1}{3} \times 0.23\right)^2 + \left(\frac{1}{3} \times 0.23\right)^2\right]$$
$$\simeq 376\ \mathrm{MeV}. \quad (4.9)$$

If the top quark did not exist, i.e. the bottom quark were a SU(2) singlet, its isospin would have been zero. In that situation, by putting $t_3^b = 0$ in the above formula, we would get $\Gamma_b \simeq 23.5\,\mathrm{MeV}$. Even though the lighter quarks could not be well discriminated from one another, bottom tagging was quite efficient thanks to the micro-vertex detector at LEP. As a result, Γ_b could be measured with good accuracy and the measurement was very close to the SM value. The discrepancy (between 376 and 23.5 MeV) was too much to be put down to radiative corrections! The immediate conclusion was that the bottom quark should have a partner: the top quark. But is the bottom an isospin 'minus half' or a 'plus half' quark? The measured decay width is consistent with $t_3^b = -\frac{1}{2}$. One could reach the same conclusion from the measurement of A_{FB}^b. If the bottom quark were an SU(2) singlet, its coupling to the Z boson would have been vector-like and A_{FB}^b would have been identically zero. But LEP measured a statistically significant non-vanishing asymmetry. Moreover, A_{FB}^b is sensitive to

$a_b = t_3^b$ (*not* a_b^2). This way too it was settled that $t_3^b = -\frac{1}{2}$. Thus even before the top quark was discovered, not only its existence was confirmed but also all its gauge quantum numbers were comprehensively established by studying how the Z boson couples to the bottom quark. Measurements of electroweak radiative effects at LEP further provided some hint of what would be the expected value of the top mass. This will be discussed in the context of the quantum corrections to the tree level value of the ρ parameter.

4.3. Main radiative corrections

The main radiative corrections relevant at the Z-pole originate from one particle irreducible gauge boson two-point functions. A generic fermion-induced two point correlation function with gauge bosons in the two external lines has the following structure (λ and λ' can be +1 or −1):

$$X^{\mu\nu}(m_1, m_2, \lambda, \lambda') = (-)\int \frac{d^4k}{(2\pi)^4}$$
$$\mathrm{Tr}\left\{\gamma^\mu \frac{1-\lambda\gamma_5}{2}(\slashed{q}+\slashed{k}+m_1)\gamma^\nu \frac{1-\lambda'\gamma_5}{2}(\slashed{k}+m_2)\right\}$$
$$\times \frac{}{\{(q+k)^2 - m_1^2\}(k^2 - m_2^2)}$$
$$= \frac{i}{16\pi^2}\int_0^1 dx\left[\Delta - \ln\left\{\frac{-q^2 x(1-x) + m_1^2 x + m_2^2(1-x)}{\mu^2}\right\}\right]$$
$$\times \left[2(1+\lambda\lambda')x(1-x)(q_\mu q_\nu - q^2 g_{\mu\nu})\right.$$
$$\left. + (1+\lambda\lambda')(m_1^2 x + m_2^2(1-x))g_{\mu\nu} - (1-\lambda\lambda')m_1 m_2 g_{\mu\nu}\right]. \quad (4.10)$$

Above, m_1 and m_2 are the masses of the fermions inside the loop, and $\Delta(\equiv 2/(4-d) - \gamma + \ln 4\pi)$ is a measure of divergence in the dimensional regularization scheme. The terms of our interest are proportional to $g_{\mu\nu}$, which we will call X. Below, we will write the Π-functions, which are defined as $\Pi(q^2, m_1, m_2) = -iX(q^2, m_1, m_2)$. By putting $\lambda = 1$ and $\lambda' = 1$, we will get the left–left (LL) Π-function, given by

$$\Pi_{\mathrm{LL}}(q^2, m_1^2, m_2^2)$$
$$= -\frac{1}{4\pi^2}\int_0^1 dx\left[\Delta + \ln\frac{\mu^2}{-q^2 x(1-x) + M^2(x)}\right]$$
$$\times \left[q^2 x(1-x) - \frac{1}{2}M^2(x)\right],$$
where $M^2(x) = m_1^2 x + m_2^2(1-x)$. $\quad (4.11)$

As before, we denote the SU(2) currents by J_μ^i. Then

$$\Pi_{33}(q^2) = \langle J_\mu^3, J_\mu^3\rangle = t_{3L}^2 \Pi_{\mathrm{LL}}(q^2, m^2, m^2), \quad (4.12)$$

$$\Pi_{11}(q^2) = \langle J_\mu^+, J_\mu^-\rangle = \frac{1}{2}\Pi_{\mathrm{LL}}(q^2, m_1^2, m_2^2). \quad (4.13)$$

Now, supposing m_1 and m_2 are the masses of the two fermion states appearing in a SU(2) doublet, it immediately follows that

$$\Pi_{33}(q^2) = \frac{1}{4}[\Pi_{\mathrm{LL}}(q^2, m_1^2, m_1^2) + \Pi_{\mathrm{LL}}(q^2, m_2^2, m_2^2)],$$
$$\Pi_{11}(q^2) = \frac{1}{2}\Pi_{\mathrm{LL}}(q^2, m_1^2, m_2^2). \quad (4.14)$$

The ρ parameter, which is unity at tree level (discussed earlier), receives a one-loop radiative correction due to the mass splitting $m_1 \neq m_2$. This is a consequence of the breaking of custodial SU(2) due to weak isospin violation. The effect is captured by

$$\Delta\rho \equiv \alpha T = \alpha \frac{4\pi}{\sin^2\theta_W \cos^2\theta_W M_Z^2}[\Pi_{11}(0) - \Pi_{33}(0)].$$
$$(4.15)$$

The dominant effect of isospin violation indeed comes from top–bottom mass splitting, given by

$$\Delta\rho^{t-b} = \alpha \frac{4\pi}{\sin^2\theta_W \cos^2\theta_W M_Z^2} \frac{N_c}{32\pi^2}$$
$$\times \left[\frac{m_t^2 + m_b^2}{2} - \frac{m_t^2 m_b^2}{m_t^2 - m_b^2} \ln \frac{m_t^2}{m_b^2} \right] \simeq \frac{\alpha}{\pi} \frac{m_t^2}{M_Z^2}. \quad (4.16)$$

The last step follows from the approximation that the ratio (m_b^2/m_t^2) is very small. The dependence on the fermion mass is quadratic because the longitudinal gauge bosons are equivalent to the Goldstones whose coupling to fermions are proportional to the fermion mass. Also note that in the limit $m_t = m_b$, the contribution to $\Delta\rho$ vanishes, as expected.

The Higgs contribution is milder in the sense that the dependence on the Higgs mass is logarithmic. The contribution arises from ZZh and W^+W^-h interactions. It turns out that

$$\Delta\rho^h = -\frac{3G_F}{8\pi^2\sqrt{2}}(M_Z^2 - M_W^2)\ln\left(\frac{m_h^2}{M_Z^2}\right) \simeq -\frac{\alpha}{2\pi}\ln\frac{m_h}{M_Z}.$$
$$(4.17)$$

The Higgs contribution to $\Delta\rho$ follows from custodial SU(2) violation due to hypercharge mixing, i.e. the fact that the gauge group is not just SU(2) but SU(2) × U(1). Besides $T(\equiv\Delta\rho/\alpha)$, two more parameters S (isospin preserving) and U (isospin violating but less important than T) capture the radiative effects. The S parameter is particularly sensitive to non-decoupled types of physics (see definition below). The Higgs contribution to the S parameter is again logarithmic:

$$S \equiv \frac{16\pi}{M_Z^2}[\Pi_{3Y}(0) - \Pi_{3Y}(M_Z^2)] \xrightarrow{\text{Higgs}} \frac{1}{6\pi}\ln\left(\frac{m_h}{M_Z}\right). \quad (4.18)$$

Note that Bose symmetry does not admit Zhh coupling. The Z boson is a spin-1 particle. If it has to decay into two scalars, then the system of two scalars would be in an antisymmetric $l = 1$ state and there is no other quantum number to symmetrize the system of two identical Bose particles. One can also argue as follows: The Z boson couples in a gauge invariant manner through the corresponding $F_{\mu\nu}$, but $\partial_\mu h \partial_\nu h$ being symmetric in (μ, ν) would not couple to $F_{\mu\nu}$.

S, T, U: why just three? There are four two-point functions: $\Pi_{\gamma\gamma}(q^2)$, $\Pi_{\gamma Z}(q^2)$, $\Pi_{ZZ}(q^2)$, $\Pi_{WW}(q^2)$. Measurements have been made at two energy scales: $q^2 = 0, M_Z^2$. So there are eight two-point correlators (four types at two different scales). Of these eight, $\Pi_{\gamma\gamma}(0) = \Pi_{\gamma Z}(0) = 0$ due to the QED Ward identity. Of the remaining six, three linear combinations are absorbed in the redefinition of the experimental inputs: α, G_μ (Fermi coupling extracted from muon decay) and M_Z. The remaining three independent combinations are S, T and U. The parameters T and U capture the effects of

custodial and weak isospin violation, while S is custodially symmetric but weak isospin breaking [32][2].

Through the total and partial Z decay width measurements, LEP settled the number of light families to be just 3. What about heavier $(> M_Z/2)$ families, which cannot be produced at LEP due to kinematic inaccessibility? If the heavier generations are *chiral*, i.e. receive mass through the Higgs mechanism, then no matter how heavy they are, there is a (nondecoupled) contribution to the S parameter ($S = 2/3\pi$ for each degenerate chiral family) [32]. After maintaining consistency with precision electroweak data, a heavy *fourth* chiral family can be barely accommodated. This has a lot of interesting consequences, e.g. it broadens the allowed range of the Higgs mass [34].

4.4. Measurements of the radiative effects

The ρ parameter is essentially the wavefunction renormalization of the external Z boson line. Therefore, it is of paramount importance in the context of LEP physics. There are three places where radiative corrections enter in a sizable fashion: (*i*) the vector (v_f) and axial vector (a_f) couplings receive an overall $\sqrt{\rho}$ multiplication, (*ii*) the weak angle θ_W is modified to effective $\bar\theta_W$ and (*iii*) the $Zb\bar{b}$ vertex receives a large (m_t^2-dependent) radiative correction. We will not talk about the $Zb\bar{b}$ vertex any more. The other corrections are called 'oblique' corrections which are lumped inside the following parametrization:

$$v_f = \sqrt{\rho}(t_3^f - 2Q_f \sin^2\bar\theta_W), \qquad a_f = \sqrt{\rho}t_3^f. \quad (4.19)$$

Now note that the width $\Gamma_f \propto (v_f^2 + a_f^2)$, while the forward–backward asymmetry A_{FB}^f is a function of v_f/a_f. So, through a combined measurement of Γ_f and A_{FB}^f, one can measure v_f and a_f. It is then straightforward to compare the measured v_f and a_f with their radiatively corrected SM expectations. Noting, $\sin^2\bar\theta_W \simeq \sin^2\theta_W - \frac{3}{8}\Delta\rho$, it is intuitively clear that one can make a prediction on the Higgs mass, as the top quark mass is now known to a pretty good accuracy.

To appreciate why radiative corrections became necessary not long after LEP started running, let us look back into the situation in the summer 1992 [35]: the measured $v_\ell^{\text{exp}} = -0.0362^{+0.0035}_{-0.0032}$, when compared with its tree level SM prediction $v_\ell^{\text{(SM,tree)}} = -0.5 + 2\sin^2\theta_W = -0.076$ ($\sin^2\theta_W$ obtained from the muon decay data: $G_\mu = \pi\alpha(0)/\sqrt{2}M_Z^2\sin^2\theta_W\cos^2\theta_W$), showed a 13σ discrepancy, inevitably calling for the necessity of dressing the Born-level prediction with radiative corrections. However, just the consideration of running of the electromagnetic coupling $\alpha(0) \to \alpha(m_Z)$ and extracting $\sin^2\theta$ (to replace $\sin^2\theta_W$ in the expression of v_ℓ) from $\cos^2\theta\sin^2\theta = \pi\alpha(M_Z)/\sqrt{2}G_\mu M_Z^2$, enabled one to obtain $v_\ell = -0.037$, i.e. within 1σ of its experimental value at that period. The essential point is that it was possible to establish a significant consistency between data and predictions just by considering the running of α and it was only much later, with significantly more data, that the weak loop effects ($\mathcal{O}(G_F m_t^2)$) were felt. In fact, before the discovery

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[2] A generalization of the number of such parameters required to cover *all* electroweak results was done in [33].
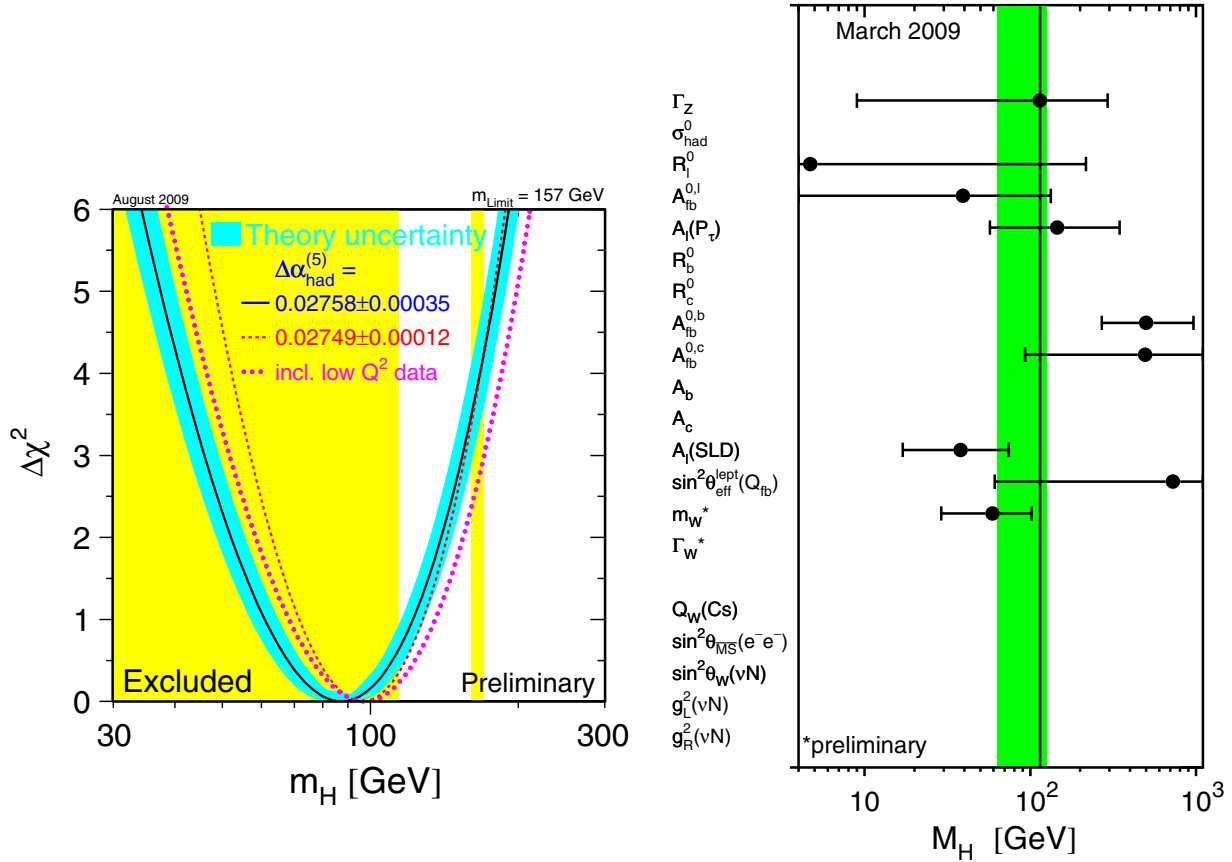
Figure 2. (*a*) Left panel: the blue-band plot showing the Higgs mass upper limit [36]. (*b*) The upper limits on the Higgs mass from different measurements. The central band corresponds to the 'average' [36].

of the top quark at Fermilab in 1995, the main indirect information on the top quark mass used to come from $\Delta\rho$.

5. Constraints on the Higgs mass

5.1. Electroweak fit

As emphasized in the previous section, the Higgs mass enters EWPT through $\Delta\rho$ and S. The quantum corrections, as we noticed in equations (4.16)–(4.18), exhibit a logarithmic sensitivity to the Higgs mass:

$$\Delta\rho^{SM} \simeq \frac{\alpha}{\pi} \frac{m_t^2}{M_Z^2} - \frac{\alpha}{2\pi} \ln\left(\frac{m_h}{M_Z}\right),$$

$$S^{h(SM)} \simeq \frac{1}{6\pi} \ln\left(\frac{m_h}{M_Z}\right). \tag{5.1}$$

At present, the CDF and D0 combined estimate is $m_t = 173.3 \pm 1.1$ GeV (updated July 2010 [36]). This translates into an upper limit on the Higgs mass: $m_h < 186$ GeV at 95% CL The lower limit $m_h > 114.4$ GeV on the Higgs mass is obtained from non-observation of the Higgs by direct search at LEP-2 via the Bjorken process $e^+e^- \rightarrow Zh$ [36]. Why the limit is so is not difficult to understand: simple kinematics tells us that the limit should roughly be $\sqrt{s} - M_Z \simeq 205 - 91 = 114$ GeV, where $\sqrt{s}$ is the maximum c.m. energy at LEP-2.

Figure 2(*a*) is the famous blue-band plot (August 2009 update shown) which is generated using electroweak data

obtained from LEP and by SLD, CDF and D0, as a function of the Higgs mass, assuming that Nature is completely described by the SM. The preferred value for the Higgs mass, corresponding to the minimum of the curve, is 87 GeV, with an experimental uncertainty of +35 and −26 GeV (68% CL which corresponds to $\Delta\chi^2 = 1$). This serves as a guideline in our attempt to find the Higgs boson. The 95% CL upper limit (corresponding to $\Delta\chi^2 = 2.7$) on the Higgs mass is 157 GeV, which is pushed up to 186 GeV when the LEP-2 direct search limit of 114 GeV is taken as a constraint in the fit. In a recent development, the Tevatron experiments CDF and D0 have excluded the Higgs mass in the range 160–170 GeV at 95% CL In figure 2(*b*) we see that the extraction of the Higgs mass from individual measurements indicates different ranges, though all are consistent within errors.

5.2. Theoretical limits

5.2.1. Perturbative unitarity. Unitarity [37] places an upper bound on m_h beyond which the theory becomes non-perturbative. Here, we shall call it a 'tree level unitarity' as we would require that the tree level contribution of the first partial wave in the expansion of different scattering amplitudes does not saturate unitarity (in other words, some probability should not exceed unity). The scattering amplitudes involving gauge bosons and Higgs can be decomposed into partial waves, using

the 'equivalence theorem', as (θ is the scattering angle)

$$A = \sum_{J=0}^{\infty} (2J+1) P_J(\cos\theta) a_J, \qquad (5.2)$$

where a_J is the Jth partial wave and P_J is the Jth Legendre polynomial (where $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = 3x^2/2 - 1/2, \ldots$). Using the orthogonality of the Legendre polynomials, the cross section can be written as

$$\sigma = \frac{16\pi}{s} \sum_{J=0}^{\infty} (2J+1)|a_J|^2 = \frac{16\pi}{s} \sum_{J=0}^{\infty} (2J+1) \, \text{Im} \, a_J. \qquad (5.3)$$

The second equality in equation (5.3) is obtained using optical theorem. Therefore,

$$|a_J|^2 = \text{Re} \, (a_J)^2 + \text{Im} \, (a_J)^2 = \text{Im} \, a_J. \qquad (5.4)$$

This translates to the bound

$$|\text{Re}(a_J)| \leqslant \tfrac{1}{2}. \qquad (5.5)$$

For the channel $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$, and for $s \gg m_h^2$, the $J = 0$ mode is given by (at tree level)

$$a_0 = -\frac{m_h^2}{8\pi v^2}. \qquad (5.6)$$

The requirement that $|a_0| \leqslant 0.5$ thus sets an upper limit $m_h < 2\sqrt{\pi} v = 870\,\text{GeV}$. The *most* divergent scattering amplitude arises from $2W_L^+ W_L^- + Z_L Z_L$ channel leading to $a_0 = -5m_h^2/64\pi v^2$, which yields $m_h < 780\,\text{GeV}$.

5.2.2. Triviality. The triviality argument provides an upper limit on the Higgs mass [38, 39]. First, recall that the SM scalar potential has the following form (be alert that the normalizations are different from those in equation (3.2)):

$$V(\Phi) = -|\mu^2|(\Phi^\dagger \Phi) + \lambda(\Phi^\dagger \Phi)^2, \qquad (5.7)$$

where

$$\Phi = \begin{pmatrix} \varphi_+ \\ \varphi_0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix} \xrightarrow{\text{unitary gauge}} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}.$$

Now consider only the scalar sector of the theory. The scalar quartic coupling evolves as

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} = \frac{3\lambda^2}{4\pi^2}, \qquad \text{where } t = \ln\left(\frac{Q^2}{Q_0^2}\right). \qquad (5.8)$$

Here Q_0 is some reference scale, which could as well be the VEV v. The solution of the above equation is

$$\lambda(Q) = \frac{\lambda(Q_0)}{1 - \frac{3\lambda(Q_0)}{4\pi^2} \ln\left(\frac{Q^2}{Q_0^2}\right)}. \qquad (5.9)$$

This means there is a pole at $Q_c = Q_0 e^{4\pi^2/3\lambda(Q_0)}$, which is called the 'Landau pole'. This pole has to be avoided during the course of RG running. The general triviality argument states

that in order to remain perturbative at all scales one needs to have $\lambda = 0$ (which means Higgs remains massless), thus rendering the theory 'trivial', i.e. non-interacting. However, one can have an alternative view: use the RG of quartic coupling λ to establish the energy domain in which the SM is valid, i.e. find out the energy cutoff Q_c below which λ remains finite. If we denote the cutoff by Λ, then

$$\frac{1}{\lambda(\Lambda)} = \frac{1}{\lambda(v)} - \frac{3}{4\pi^2} \ln \frac{\Lambda^2}{v^2} > 0. \qquad (5.10)$$

The above inequality follows from the requirement $\lambda(\Lambda) < \infty \Rightarrow \frac{1}{\lambda(\Lambda)} > 0$. This immediately leads to

$$\lambda(v) \leqslant \frac{4\pi^2}{3\ln\left(\frac{\Lambda^2}{v^2}\right)} \implies m_h^2 = 2\lambda v^2 < \frac{8\pi^2 v^2}{3\ln\left(\frac{\Lambda^2}{v^2}\right)}. \qquad (5.11)$$

Putting numbers, $m_h < 160\,\text{GeV}$, for a choice of the cutoff close to the typical GUT scale $\Lambda = 10^{16}\,\text{GeV}$.

Now let us include the full structure of fermions and gauge bosons in RG equations:

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} \simeq \frac{1}{16\pi^2}\left[12\lambda^2 + 12\lambda h_t^2 - 12h_t^4 - \frac{3}{2}\lambda(3g_2^2 + g_1^2) \right. \\ \left. + \frac{3}{16}\left\{ 2g_2^4 + (g_2^2 + g_1^2)^2 \right\} \right], \qquad (5.12)$$

where $h_t = \sqrt{2}m_t/v$ is the top quark Yukawa coupling. For a rather large $\lambda > h_t, g_1, g_2$, i.e. for a 'heavy' Higgs boson, the dominant contribution to running is

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} \simeq \frac{1}{16\pi^2}\left[12\lambda^2 + 12\lambda h_t^2 - \frac{3}{2}\lambda(3g_2^2 + g_1^2) \right]. \qquad (5.13)$$

Note that whenever the quartic coupling λ, calculated at the weak scale v, is equal to $\lambda_c \equiv \frac{1}{8}(3g_2^2 + g_1^2) - h_t^2$, which corresponds to the vanishing rhs of the above RG equation, the coupling reaches a critical limit. If one starts the evolution with a $\lambda(v) > \lambda_c(v)$, i.e. for $m_h > m_h^c \equiv \sqrt{2\lambda_c}\,v$, then during the course of RG running the quartic coupling hits the Landau pole, i.e. becomes infinite, at some scale and the theory ceases to be perturbative. From this requirement, one obtains an upper limit (at two-loop level):

$$m_h < m_h^c = 170 \text{ GeV} \qquad \text{for } \Lambda = 10^{16} \text{ GeV.} \qquad (5.14)$$

The limits for other choices of Λ can be read off from figures 3(a) and (b).

5.2.3. Vacuum stability. The argument of vacuum stability is based on the requirement that the potential is always bounded from below. This means $\lambda(Q)$ has to remain positive throughout the history of RG running. This gives rise to a lower bound on the Higgs mass [38–40]. If the Higgs mass is too small, i.e. λ is very small, then the top quark contribution dominates which can drive λ to a negative value. If it happens then the vacuum is not stable as it has no minimum. For small λ, equation (5.12) becomes

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} \simeq \frac{1}{16\pi^2}\left[-12h_t^4 + \frac{3}{16}\{2g_t^4 + (g_1^2 + g_2^2)^2\} \right]. \qquad (5.15)$$
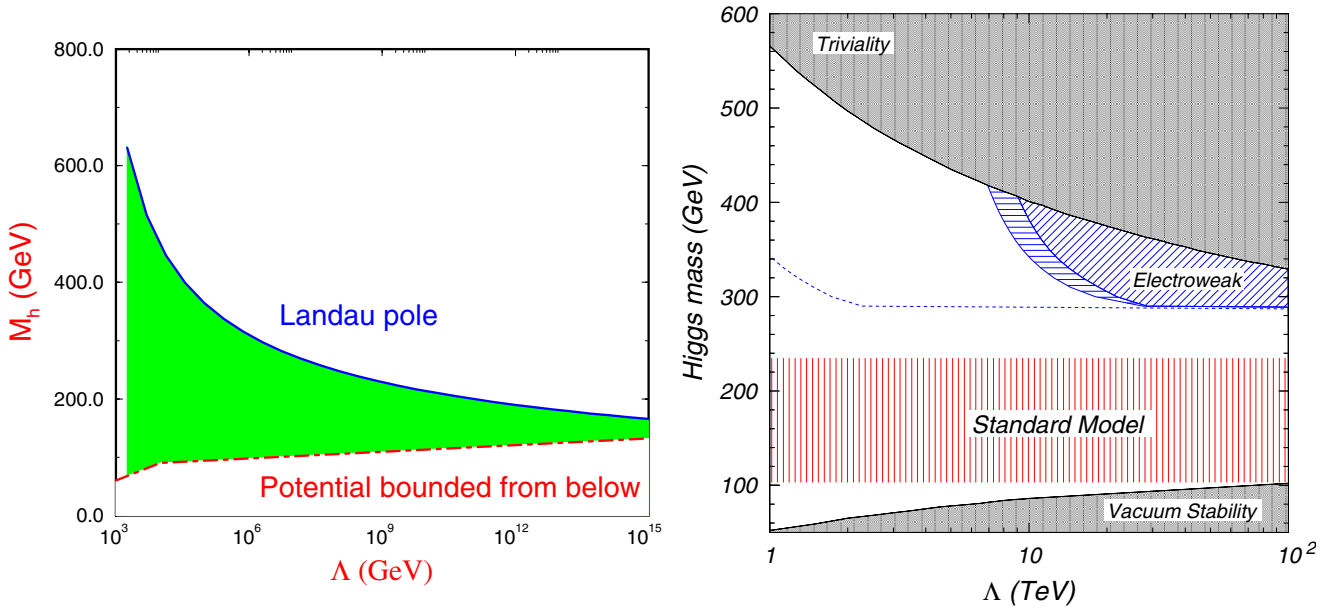
Figure 3. (*a*) Left panel: the triviality and vacuum stability limits (taken from [41]). (*b*) Right panel: the region up to $\Lambda = 100$ TeV is zoomed. In the hatched region 'Electroweak' is ruled out by electroweak precision data. Only the 'Standard Model' region is allowed at 95% CL [39].

To provide intuitive understanding through easy analytic implementation, we perform a one-step integration and obtain

$$\lambda(\Lambda) = \lambda(v) + \frac{1}{16\pi^2}\left[-12h_t^4 + \frac{3}{16}\{2g_2^4 + (g_1^2 + g_2^2)^2\}\right]$$
$$\times \ln\left(\frac{\Lambda^2}{v^2}\right). \tag{5.16}$$

To ensure that $\lambda(\Lambda)$ remains positive, the Higgs mass must satisfy

$$m_h^2 > \frac{v^2}{8\pi^2}\left[12h_t^4 - \frac{3}{16}\{2g_2^4 + (g_1^2 + g_2^2)^2\}\right]\ln\left(\frac{\Lambda^2}{v^2}\right). \tag{5.17}$$

Clearly the above steps are very simple-minded, yet provide the rationale behind the lower limit. By actually solving the RG equation at 2-loop level, one obtains

$$m_h > 134 \text{ GeV} \qquad \text{for } \Lambda = 10^{16} \text{ GeV}. \tag{5.18}$$

If the cutoff $\Lambda = 1$ TeV, then [40]

$$m_h > 50.8 + 0.64 \, (m_t - 173.1 \text{ GeV}),$$

which indicates that such a low cutoff is clearly disfavored by LEP (see also Quigg's paper in [5]). Again, the limits for other choices of the cutoff can be read off from figures 3(*a*) and (*b*).

6. Gauge hierarchy problem

6.1. Quadratic divergence

Let us illustrate the problem of quadratic divergence in the Higgs sector through an explicit calculation. Recall that in the

unitary gauge the doublet $\Phi(x) = \binom{0}{\varphi(x)} = \frac{1}{\sqrt{2}}\binom{0}{v + h(x)}$. We write the Yukawa interaction Lagrangian as

$$\mathscr{L} = -h_f \varphi \bar{f}_L f_R + \text{h.c.},$$

where $f_{L,R}$ are the left- and right-chiral projection of the fermion f. After SSB,

$$\mathscr{L} = -\frac{h_f}{\sqrt{2}}h\bar{f}_L f_R - \frac{h_f}{\sqrt{2}}v\bar{f}_L f_R + \text{h.c.} \tag{6.1}$$

The fermion mass is therefore given by $m_f = h_f \frac{v}{\sqrt{2}}$.

Let us compute the two-point function with zero momentum Higgs as the two external lines and fermions inside the loop. The corresponding diagram is in figure 4(*a*) and can be written as

$$i\Pi_{hh}^f(0)$$
$$= (-)\int \frac{\mathrm{d}^4k}{(2\pi)^4}\text{Tr}\left[\left(-i\frac{h_f}{\sqrt{2}}\right)\frac{i}{\not{k} - m_f}\left(-i\frac{h_f}{\sqrt{2}}\right)\frac{i}{\not{k} - m_f}\right]$$
$$= -2h_f^2\int \frac{\mathrm{d}^4k}{(2\pi)^4}\left[\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2}\right]. \tag{6.2}$$

The correction Δm_h^2 is proportional to $\Pi_{hh}^f(0)$. The first term on the rhs is quadratically divergent. The divergent correction to m_h^2 looks like

$$\Delta m_h^2(f) = \frac{\Lambda^2}{16\pi^2}(-2h_f^2). \tag{6.3}$$

Another divergent piece will appear from quartic Higgs vertex. The corresponding diagram is similar to what is displayed in figure (4*c*), except that the internal line is also h. The divergent contribution to m_h^2 is

$$\Delta m_h^2(h) = \frac{\Lambda^2}{16\pi^2}(\lambda). \tag{6.4}$$
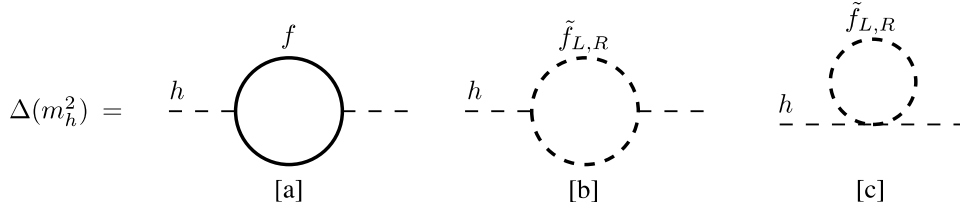
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Figure 4. One-loop quantum corrections to the Higgs mass, due to a Dirac fermion f (a), and scalars $\tilde{f}_{L,R}$ ((b) and (c)).

For the sake of simplicity, we neglect the gauge boson contributions to the quadratic divergence. Combining the above two divergent pieces, we obtain

$$\Delta m_h^2 = \frac{\Lambda^2}{16\pi^2}(-2h_f^2 + \lambda). \quad (6.5)$$

Now, we contemplate the following issues.

(i) The Yukawa coupling h_f and the quartic scalar coupling λ are totally unrelated. Suppose, we set $\lambda = 2h_f^2$. First of all, this is a huge fine-tuning. Second, at higher loops, this relation will not be able to prevent the appearance of divergence. It is also interesting to note that if we set $\lambda = h_f^2$, then we would require two scalars to cancel the quadratic divergence caused by one fermion.

(ii) Suppose we do not attempt to relate λ and h_f for canceling the quadratic divergence. Now, remember that we have a tree level bilinear mass term, which is the bare mass. We can absorb the quadratic divergent in a redefinition of the bare mass. Still, there is a residual finite part to the mass correction, given by $\sim \frac{h_f^2 m_f^2}{8\pi^2}$ (see equation (6.2)). What is the value of the loop mass m_f? If SM gives way to some GUT theory at high scale we can have fermions where $m_f \sim M_{\rm GUT} \sim 10^{16}$ GeV. In that case, even after removing the quadratic cutoff dependence, the leading contribution to Δm_h^2 would be order $M_{\rm GUT}^2/(8\pi^2)$. One would then have to do an unnatural fine-tuning $(1-10^{26})$ between the bare term $m_{h_0}^2$ and the correction term Δm_h^2 to maintain the renormalized mass ($m_h^2 = m_{h_0}^2 + \Delta m_h^2$) at around 100 GeV. Furthermore, this fine-tuning has to be done order-by-order in perturbation theory to prevent the Higgs mass from shooting up to the highest mass scale of the theory. This constitutes what is technically called the *gauge hierarchy problem* [42].

(iii) The primary problem is that the correction is independent of m_h. Setting $m_h = 0$ does not increase the symmetry of the theory. In QED, in the limit of vanishing electron mass we have exact *chiral symmetry*, and since the photon mass is zero we have exact *gauge symmetry*. But there is no symmetry that protects the Higgs mass.

One of the biggest challenges in the SM is to stabilize the scalar potential, i.e. to protect it from a run-away quantum behavior. Although we said that it is the Higgs mass which is not stable but, more precisely, it is the electroweak VEV (v) which is unstable. Since v feeds into all masses in the SM through SSB, none of them which is proportional to v is stable either. In fact, the argument of protection from gauge and chiral symmetry applicable to QED is strictly not applicable for the SM because all the SM particle masses are proportional to v.

6.2. Cancellation of quadratic divergence in a toy supersymmetric scenario

Supersymmetry, a theory with an intrinsic fermion $\leftrightarrow$ boson symmetry, unambiguously solves the gauge hierarchy problem and restores naturalness. For an early study of supersymmetric model building and demonstration of quadratic divergence cancellation, we refer to [43, 44]. The content of this subsection is adapted from the textbook by Drees *et al* [15].

We consider a toy model which contains $\varphi(x) = \frac{1}{\sqrt{2}}(v + h(x))$ plus two additional complex scalar fields $\tilde{f}_{L,R}(x)$. Suppose the interaction is encoded in the following effective Lagrangian:

$$\begin{aligned}\mathcal{L}_{\tilde{f}\tilde{f}\varphi} &= -\tilde{\lambda}_f|\varphi|^2\left(|\tilde{f}_L|^2 + |\tilde{f}_R|^2\right) + \left(h_f A_f \varphi \tilde{f}_L \tilde{f}_R^* + {\rm h.c.}\right)\\ &= -\frac{1}{2}\tilde{\lambda}_f h^2\left(|\tilde{f}_L|^2 + |\tilde{f}_R|^2\right) - \tilde{\lambda}_f h v\left(|\tilde{f}_L|^2 + |\tilde{f}_R|^2\right)\\ &\quad + \frac{h_f}{\sqrt{2}}A_f\left(h\tilde{f}_L\tilde{f}_R^* + {\rm h.c.}\right) + \cdots.\end{aligned} \quad (6.6)$$

Above, the dots correspond to Higgs independent terms which need not be spelt out. A_f has the dimension of mass and it measures the strength of triple scalar vertex. The Yukawa coupling h_f is multiplied to it by convention. The fermion loop, described before, is shown in figure $(4(a))$. The new loops involving scalars are displayed in figures $4(b)$ and (c). The contributions of the scalar loops are given by

$$\begin{aligned}i\Pi_{hh}^{\tilde{f}}(0) &= \tilde{\lambda}_f\int\frac{{\rm d}^4k}{(2\pi)^4}\left[\frac{1}{k^2 - m_{\tilde{f}_L}^2} + \frac{1}{k^2 - m_{\tilde{f}_R}^2}\right]\\ &\quad\Longleftarrow \text{figure } 4(c)\\ &\quad + (\tilde{\lambda}_f v)^2\int\frac{{\rm d}^4k}{(2\pi)^4}\left[\frac{1}{(k^2 - m_{\tilde{f}_L}^2)^2} + \frac{1}{(k^2 - m_{\tilde{f}_R}^2)^2}\right]\\ &\quad\Longleftarrow \text{figure } 4(b)\\ &\quad + |h_f A_f|^2\int\frac{{\rm d}^4k}{(2\pi)^4}\left[\frac{1}{k^2 - m_{\tilde{f}_L}^2}\frac{1}{k^2 - m_{\tilde{f}_R}^2}\right]\\ &\quad\Longleftarrow \text{figure } 4(b)\end{aligned} \quad (6.7)$$

Combining equations (6.2) (fermion loop) and (6.7) (scalar loops) we make the following observations:

- The fermion loop contribution (figure $4(a)$) and the scalar loop contribution (figure $4(c)$) give quadratic divergence. However, if one computes the net contribution to the two-point function, given by $\Pi_{hh}^f(0) + \Pi_{hh}^{\tilde{f}}(0)$, the quadratic divergence exactly cancels if one sets $\tilde{\lambda}_f = h_f^2$. This cancellation of quadratic divergence occurs regardless of the magnitude of any mass dimensional parameter, namely, $m_{\tilde{f}_{L,R}}$ or A_f.

- A log sensitivity to the cutoff (or, the unknown scalar mass) still remains. If we assume $m_{\tilde{f}_{\mathrm{L}}} = m_{\tilde{f}_{\mathrm{R}}} = m_{\tilde{f}}$, then

$$
\Pi_{hh}^{f}(0) + \Pi_{hh}^{\tilde{f}}(0) = \frac{h_f^2}{16\pi^2} \Bigg[-2m_f^2 \left\{ 1 - \ln\left(\frac{m_f^2}{\mu^2}\right) \right\}
$$
$$
+ 4m_f^2 \ln\left(\frac{m_f^2}{\mu^2}\right) + 2m_{\tilde{f}}^2 \left\{ 1 - \ln\left(\frac{m_{\tilde{f}}^2}{\mu^2}\right) \right\}
$$
$$
- 4m_f^2 \ln\left(\frac{m_{\tilde{f}}^2}{\mu^2}\right) - |A_f|^2 \ln\left(\frac{m_{\tilde{f}}^2}{\mu^2}\right) \Bigg]. \tag{6.8}
$$

- Now, if we further assume that (i) $m_f = m_{\tilde{f}}$ and (ii) $A_f = 0$, then we have $(\Pi_{hh}^{f}(0) + \Pi_{hh}^{\tilde{f}}(0)) = 0$ i.e. even the finite contribution vanishes.

All these points are shared by supersymmetric extension of the Standard Model. Quadratic divergence cancels due to the equality of two types of dimensionless couplings. If supersymmetry is broken in masses, e.g. $m_f \neq m_{\tilde{f}}$, i.e. gives rise to the 'soft' terms (mass dimension < 4) of the Lagrangian, the quadratic divergence still cancels. Also, in the limit of exact supersymmetry, i.e. (i) $m_f = m_{\tilde{f}}$ and (ii) $A_f = 0$, the correction to the Higgs mass exactly vanishes. This toy scenario is reminiscent of supersymmetric models.

6.3. The Higgs bosons of the minimal supersymmetric Standard Model

We need two complex scalar doublets of opposite hypercharge to ensure EWSB:

$$
H_1 = \begin{pmatrix} h_1^0 \\ h_1^- \end{pmatrix}_{Y=-1}, \qquad H_2 = \begin{pmatrix} h_2^+ \\ h_2^0 \end{pmatrix}_{Y=1}. \tag{6.9}
$$

There are three reasons behind the need for at least two doublets.

- Chiral or ABJ (Adler–Bardeen–Jackiw) anomaly cancellation requires $\sum Y_f = 0 = \sum Q_f$, where the sum is on fermions only. If we use only one Higgs doublet, its spin-1/2 (Higgsino) components will spoil the cancellation. We therefore need two Higgs doublets with opposite hypercharge. (This anomaly arises from triangular fermionic loops involving axial vector couplings. The theory ceases to be renormalizable if it has an ABJ anomaly.)

- Recall that in the SM we use the scalar doublet Φ and $\tilde{\Phi} = i\tau_2\Phi^*$ for giving masses to up- and down-type fermions. In supersymmetry, Φ is a chiral superfield, and we cannot use a chiral superfield and its complex conjugate in the same superpotential. Therefore, we need two chiral superfields.

- Unless we introduce both H_1 and H_2, we cannot provide the right number of degrees of freedom necessary to make the charginos massive. In this sense, introducing at least two complex doublets is an experimental compulsion.

In the MSSM, the scalar potential V_H receives contributions from three sources:

(a) *the D term*;

$$
V_D = \frac{1}{2} \sum_{a=1}^{3} \left(\sum_i g_a S_i^* T^a S_i \right)^2 : \quad \begin{array}{l} a \text{ runs over groups} \\ i \text{ runs over particles} \end{array}
$$
(S_i is a generic scalar).

Keeping only the Higgs contributions, i.e. neglecting slepton/squark contributions, we obtain,

for U(1)$_Y$: $V_D^{(1)} = \frac{1}{2}[\frac{g_1}{2}(|H_2|^2 - |H_1|^2)]^2$,

for SU(2)$_\mathrm{L}$: $V_D^{(2)} = \frac{1}{2}[\frac{g_2}{2}(H_1^{i*}\tau_{ij}^a H_1^j + H_2^{i*}\tau_{ij}^a H_2^j)]^2$.

Here, $g_1 \equiv g'$ and $g_2 \equiv g$.

Using $\tau_{ij}^a \tau_{kl}^a = 2\delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl}$, one obtains

$$
V_D = V_D^{(1)} + V_D^{(2)} = \frac{g_2^2}{8}[4|H_1^\dagger H_2|^2 - 2|H_1|^2|H_2|^2
$$
$$
+ |H_1|^4 + |H_2|^4] + \frac{g_1^2}{8}(|H_2|^2 - |H_1|^2)^2.
$$

(b) *the F term*; $V_\mathrm{F} = \sum_i |\frac{\partial W(\varphi_j)}{\partial \varphi_i}|^2$. The superpotential $W = \mu \hat{H}_1 \hat{H}_2$ ('hat' denotes superfields) leads to

$$
V_\mathrm{F} = \mu^2(|H_1|^2 + |H_2|^2).
$$

(c) *the soft supersymmetry breaking terms*;

$$
V_\mathrm{soft} = m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 + (B_\mu H_2 H_1 + \text{h.c.}).
$$

We now introduce the notation: $\bar{m}_1^2 \equiv |\mu|^2 + m_{H_1}^2$, $\bar{m}_2^2 \equiv |\mu|^2 + m_{H_2}^2$, $\bar{m}_3^2 \equiv B_\mu$. Using the charged and neutral components of the doublet scalars, we can write the full scalar potential as

$$
V_H = \bar{m}_1^2(|h_1^0|^2 + |h_1^-|^2) + \bar{m}_2^2(|h_2^0|^2 + |h_2^+|^2)
$$
$$
+ \bar{m}_3^2(h_1^- h_2^+ - h_1^0 h_2^0 + \text{h.c.})
$$
$$
+ \left(\frac{g_2^2 + g_1^2}{8}\right)(|h_1^0|^2 + |h_1^-|^2 - |h_2^0|^2 - |h_2^+|^2)^2
$$
$$
+ \frac{g_2^2}{2}|h_1^{-*}h_1^0 + h_2^{0*}h_2^+|^2. \tag{6.10}
$$

We then require that the minimum of V_H breaks SU(2)$_\mathrm{L} \times$ U(1)$_Y$ to U(1)$_Q$. One can always choose $\langle h_1^- \rangle = \langle h_2^+ \rangle = 0$ to avoid breakdown of electromagnetism without any loss of generality. Note two important features at this stage:

- only B_μ can be complex. However, the phase can be absorbed into the phases of H_1 and H_2. Hence, the MSSM tree level scalar potential has no source of CP violation;

- the quartic scalar couplings are fixed in terms of the SU(2) and U(1) gauge couplings.

Note that it is sufficient to write the potential keeping only the (neutral) fields which can acquire VEVs.

$$
V_H^0 = \frac{1}{8}(g_1^2 + g_2^2)(|h_1^0|^2 - |h_2^0|^2)^2 + \bar{m}_1^2|h_1^0|^2
$$
$$
+ \bar{m}_2^2|h_2^0|^2 - \bar{m}_3^2(h_1^0 h_2^0 + \text{h.c.}) \tag{6.11}
$$

Again, note the following points:

- V_H^0 will be bounded from below if $\bar{m}_1^2 + \bar{m}_2^2 > 2\bar{m}_3^2$. This relation has to be valid at all scales. (Note, there is no quartic term in the direction $|h_1^0| = |h_2^0|$);

- V_H^0 (quadratic part) $= (h_1^{0*} h_2^0) \begin{pmatrix} \bar{m}_{12}^2 & -\bar{m}_3^2 \\ -\bar{m}_3^2 & \bar{m}_2^2 \end{pmatrix} \begin{pmatrix} h_1^0 \\ h_2^{0*} \end{pmatrix}$
  SSB requires $\bar{m}_3^4 > \bar{m}_1^2 \bar{m}_2^2$. This has to be necessarily valid at the weak scale where SSB occurs;
- the above two conditions cannot be satisfied simultaneously if $\bar{m}_1^2 = \bar{m}_2^2$. Hence, $\bar{m}_1^2 \neq \bar{m}_2^2 \Rightarrow m_{H_1}^2 \neq m_{H_2}^2$, which indicates a connection between supersymmetry breaking and EWSB.

Putting $\langle h_1^0 \rangle = \frac{v_1}{\sqrt{2}}$ and $\langle h_2^0 \rangle = \frac{v_2}{\sqrt{2}}$,

$$V_H^0(\min) = \tfrac{1}{32}(g_1^2 + g_2^2)(v_1^2 - v_2^2)^2 + \tfrac{1}{2}\bar{m}_1^2 v_1^2$$
$$+ \tfrac{1}{2}\bar{m}_2^2 v_2^2 - \bar{m}_3^2 v_1 v_2. \tag{6.12}$$

The minimization conditions $\frac{\partial V_H^0(\min)}{\partial v_i} = 0$, for $i = 1, 2$ yield

$$\bar{m}_1^2 = \bar{m}_3^2 \frac{v_2}{v_1} - \tfrac{1}{8}(g_1^2 + g_2^2)(v_1^2 - v_2^2) \qquad \text{and}$$

$$\bar{m}_2^2 = \bar{m}_3^2 \frac{v_1}{v_2} + \tfrac{1}{8}(g_1^2 + g_2^2)(v_1^2 - v_2^2). \tag{6.13}$$

Now using the above equations and putting back $\bar{m}_1^2 \equiv m_{H_1}^2 + |\mu|^2$, $\bar{m}_2^2 \equiv m_{H_2}^2 + |\mu|^2$, we obtain the two conditions of EWSB:

$$\tfrac{1}{2}M_Z^2 = \left(\frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} \right) - |\mu|^2,$$

$$\text{where} \quad \tan \beta \equiv \frac{v_2}{v_1}, \tag{6.14}$$

$$-2B_\mu = (m_{H_1}^2 - m_{H_2}^2) \tan 2\beta + M_Z^2 \sin 2\beta. \tag{6.15}$$

Our next task is to extract the different masses from the quadratic part of the potential: $V_H^{\text{quad}} = \tfrac{1}{2} m_{ij}^2 \varphi_i \varphi_j$.

6.3.1. Charged Higgs and Goldstone.
The mass matrix is given by

$$V_{h^\pm} = \left(\frac{\bar{m}_3^2}{v_1 v_2} + \frac{1}{4} g_2^2 \right) (h_1^+ \; h_2^+) \begin{pmatrix} v_2^2 & v_1 v_2 \\ v_1 v_2 & v_1^2 \end{pmatrix} \begin{pmatrix} h_1^- \\ h_2^- \end{pmatrix}. \tag{6.16}$$

Note that the determinant of the mass matrix is zero, which is a consequence of the masslessness of the Goldstones ($m_{G^\pm}^2 = 0$). The charged Higgs mass is given by

$$m_{h^\pm}^2 = \left(\frac{\bar{m}_3^2}{v_1 v_2} + \frac{1}{4} g_2^2 \right) (v_1^2 + v_2^2). \tag{6.17}$$

The mass eigenstates are given by
$$H^\pm = \sin \beta \, h_1^\pm + \cos \beta \, h_2^\pm,$$
$$G^\pm = -\cos \beta \, h_1^\pm + \sin \beta \, h_2^\pm. \tag{6.18}$$

6.3.2. Neutral CP-odd Higgs and Goldstone.
The Goldstone is massless, while the mass of the CP odd scalar depends on $\bar{m}_3^2 = B_\mu$:

$$m_{G^0}^2 = 0, \qquad m_A^2 = \frac{2\bar{m}_3^2}{\sin 2\beta}. \tag{6.19}$$

The physical states are given by

$$\frac{A}{\sqrt{2}} = \sin \beta \, \text{Im} \, h_1^0 + \cos \beta \, \text{Im} \, h_2^0,$$

$$\frac{G^0}{\sqrt{2}} = -\cos \beta \, \text{Im} \, h_1^0 + \sin \beta \, \text{Im} \, h_2^0. \tag{6.20}$$

6.3.3. Neutral CP-even Higgses.
The 2×2 mass-squared matrix for the neutral CP-even sector in the (Re h_1^0, Re h_2^0) basis is given by

$$M_{\text{Re } h^0}^2 = \frac{1}{2} \begin{pmatrix} 2\bar{m}_1^2 + \frac{1}{4}(g_2^2 + g_1^2)(3v_1^2 - v_2^2) \\ -2\bar{m}_3^2 - \frac{1}{2}(g_1^2 + g_2^2)v_1 v_2 \\ -2\bar{m}_3^2 - \frac{1}{2}(g_1^2 + g_2^2)v_1 v_2 \\ 2\bar{m}_2^2 + \frac{1}{4}(g_1^2 + g_2^2)(3v_2^2 - v_1^2) \end{pmatrix}$$
$$= \begin{pmatrix} m_A^2 \sin^2 \beta + M_Z^2 \cos^2 \beta \\ -(m_A^2 + M_Z^2) \sin \beta \cos \beta \\ -(m_A^2 + M_Z^2) \sin \beta \cos \beta \\ m_A^2 \cos^2 \beta + M_Z^2 \sin^2 \beta \end{pmatrix}. \tag{6.21}$$

The mass-squared eigenvalues are then given by (h is lighter, H heavier)

$$m_{h,H}^2 = \tfrac{1}{2}[m_A^2 + M_Z^2 \mp \{(m_A^2 + M_Z^2)^2 - 4M_Z^2 m_A^2 \cos^2 2\beta\}^{1/2}]. \tag{6.22}$$

6.3.4. Important equalities and inequalities.
The following are some of the important relations:

$$m_h < \min \, (m_A, M_Z)|\cos 2\beta| < \min \, (m_A, M_Z),$$
$$m_h^2 + m_H^2 = m_A^2 + M_Z^2,$$
$$m_H > \max \, (m_A, M_Z), \qquad m_{H^\pm}^2 = m_A^2 + M_W^2. \tag{6.23}$$

The tree level inequality $m_h < M_Z$ is an important prediction of the MSSM. This is a consequence of the fact that the quartic couplings in MSSM are related to the gauge couplings.

6.3.5. Radiative correction to the lightest Higgs mass.
The lightest neutral Higgs mass (m_h) receives large quantum corrections. The correction is dominated by the top quark Yukawa coupling (h_t) and the masses of the stop squarks ($\tilde{t}_1$, $\tilde{t}_2$). The corrected Higgs mass-squared is given by (original references can be found in [15, 17])

$$m_h^2 \simeq M_Z^2 \cos^2 2\beta + \frac{3m_t^4}{2\pi^2 v^2} \ln \left(\frac{m_{\tilde{t}}^2}{m_t^2} \right), \tag{6.24}$$

where $m_{\tilde{t}} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ is an average stop mass, This is a one-loop expression. Including two-loop calculations pushes the upper limit on the Higgs mass to around 135 GeV. If a neutral Higgs is not found at LHC approximately within this limit, the *two-Higgs doublet* version of supersymmetric model will be strongly disfavored. In the next-to-minimal supersymmetric model (NMSSM) [45], which contains an additional gauge singlet scalar (N) coupled to H_1 and H_2 through the superpotential $\lambda N H_1 H_2$, there is an additional tree level contribution to m_h^2. It turns out that [46] $m_h^2(\text{tree, NMSSM}) = M_Z^2[\cos^2 2\beta + 2\lambda^2(g^2 + g'^2)^{-1} \sin^2 2\beta]$. Including radiative corrections, the upper limit on m_h is relaxed to about 150 GeV [47].

6.4. Radiative electroweak symmetry breaking in MSSM

One of the most attractive features of supersymmetry is that the electroweak symmetry is broken radiatively. Recall that in the SM we had to put a *negative* sign by hand in front of μ^2 to ensure EWSB, which was *ad hoc*. In supersymmetry this

happens dynamically thanks to the large top quark Yukawa coupling. We will demonstrate how one of the Higgs mass-squared, more precisely $m^2_{H_2}$, starting from a positive value at a high scale is driven to a negative value at low scale by RG running. To appreciate the salient features, we will take into consideration only the effect of h_t in RG evolution and ignore the gauge and other Yukawa couplings' contributions (for details, see text books). This estimate may be crude, but it brings out the essential features. First we write down the RG evolution of $m^2_{H_2}$, $m^2_{\tilde{Q}_3}$ and $m^2_{\tilde{u}_3}$:

$$\frac{\mathrm{d}m^2_{H_2}}{\mathrm{d}t} = -3h_t^2(m^2 + A_t^2), \qquad \frac{\mathrm{d}m^2_{\tilde{Q}_3}}{\mathrm{d}t} = -h_t^2(m^2 + A_t^2),$$

$$\frac{\mathrm{d}m^2_{\tilde{u}_3}}{\mathrm{d}t} = -2h_t^2(m^2 + A_t^2), \tag{6.25}$$

where $\tilde{Q}_3$ and $\tilde{u}_3$ are the third generation squark doublet and singlet, respectively, $t \equiv \ln(M^2_{\mathrm{GUT}}/Q^2)/16\pi^2$, h_t is the top quark Yukawa coupling, A_t is the scalar trilinear coupling involving the top squark, and $m^2 \equiv m^2_{H_2} + m^2_{\tilde{Q}_3} + m^2_{\tilde{u}_3}$. Now recall that Bernoulli's equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + yP(x) = Q(x)$$

has a solution

$$y \exp\left(\int \mathrm{d}x\, P(x)\right) = \int \mathrm{d}x\, Q(x) \exp\left(\int \mathrm{d}x\, P(x)\right)$$
$$+\text{constant.}$$

Therefore, the equation (obtained by summing the individual RGs in equation (6.25))

$$\frac{\mathrm{d}m^2}{\mathrm{d}t} + 6h_t^2 m^2 = -6h_t^2 A_t^2 \tag{6.26}$$

has a solution

$$m^2 \exp\left(6\int_0^t \mathrm{d}t'\, h_t^2\right) = \int_0^t \mathrm{d}t'(-6h_t^2 A_t^2) \exp\left(6\int_0^{t'} \mathrm{d}t''\, h_t^2\right)$$
$$+\text{constant.} \tag{6.27}$$

Now, ignore the running of h_t and A_t to avoid complications, i.e. treat them as fixed values. This eases calculational hassles but preserves the important features of radiative EWSB. Then

$$m^2 \exp(6h_t^2 t) = -6h_t^2 A_t^2 \int_0^t \mathrm{d}t' \exp(6h_t^2 t') + C$$
$$= -A_t^2 \exp(6h_t^2 t) + C. \tag{6.28}$$

At $t = 0$ (i.e. $Q = M_{\mathrm{GUT}}$), assume universal boundary conditions, i.e. $m_0^2 \equiv m^2_{H_2} = m^2_{\tilde{Q}_3} = m^2_{\tilde{u}_3}$. Therefore, $m^2(t = 0) = 3m_0^2$, hence $C = 3m_0^2 + A_0^2$ (where $A_t = A_0$, since we ignored the running of A_t). Using these relations, it is simple to obtain the solution

$$m^2 = -A_t^2[1 - \exp(-6h_t^2 t)] + 3m_0^2 \exp(-6h_t^2 t). \tag{6.29}$$

Now we solve the individual equations in (6.25). The mathematical steps are easy, hence we do not display them

here. The solutions are

$$m^2_{H_2} = \tfrac{1}{2}(3m_0^2 + A_t^2) \exp(-6h_t^2 t) - \tfrac{1}{2}m_0^2 - \tfrac{1}{2}A_t^2$$
$$\xrightarrow{t\to\infty, A_t=0} -\tfrac{1}{2}m_0^2,$$

$$m^2_{\tilde{Q}_3} = \tfrac{1}{6}(3m_0^2 + A_t^2) \exp(-6h_t^2 t) + \tfrac{1}{2}m_0^2 - \tfrac{1}{6}A_t^2$$
$$\xrightarrow{t\to\infty, A_t=0} \tfrac{1}{2}m_0^2,$$

$$m^2_{\tilde{u}_3} = \tfrac{1}{3}(3m_0^2 + A_t^2) \exp(-6h_t^2 t) - \tfrac{1}{3}A_t^2 \xrightarrow{t\to\infty, A_t=0} 0. \tag{6.30}$$

The limit $t \to \infty$ refers to the electroweak scale ($v \simeq 246\,\mathrm{GeV}$). We observe that at low energy the up-type Higgs mass-squared is driven to a negative value due to strong h_t-effect. The above assumptions are indeed too simplistic. Addition of gauge loops yield additional positive contributions proportional to the gaugino mass-square (M_i^2). Moreover, running of h_t and A_t should also be considered which make the solutions more complicated. All in all, RG evolution enforces a sign-flip in $m^2_{H_2}$ only at the low scale, thus triggering EWSB.

7. Little Higgs

The contents of this section (key ideas and illustration) have been developed together with Romesh K Kaul. See also the discussion on little Higgs models in [6].

We first discuss the basic ideas. Pions are spin-0 objects. The Higgs is also a spin-0 particle. Pions are composite objects. The Higgs is perhaps elementary (as indicated by electroweak precision measurements), but it can very well turn out to be composite. The important thing is that the pions are light, and there are reasons. Can Higgs be light too for similar reasons? The lightness of the pions owes its origin to their pseudo-Goldstone nature. These are Goldstone bosons which arise when the chiral symmetry group $SU(2)_L \times SU(2)_R$ spontaneously breaks to the isospin group $SU(2)_I$. The Goldstone scalar ϕ has a shift symmetry $\phi \to \phi + c$, where c is some constant. Therefore, any interaction which couples ϕ not as $\partial_\mu \phi$ breaks the Goldstone symmetry and attributes mass to the previously massless Goldstone. Quark masses and electromagnetic interaction *explicitly* break the chiral symmetry. Electromagnetism attributes a mass to π^+ (more precisely, to the mass difference between π^+ and π^0) of order $m^2_{\pi^+} \sim (\alpha_{\mathrm{em}}/4\pi)\Lambda^2_{\mathrm{QCD}}$. Can we think of the Higgs mass generation in the same way? We know that Yukawa interaction has a non-derivative Higgs coupling, so it must break the Goldstone symmetry. Then, if we replace α_{em} by $\alpha_t \equiv h_t^2/4\pi$ and Λ_{QCD} by some cutoff Λ, we obtain

$$m_h^2 \sim \left(\frac{\alpha_t}{4\pi}\right)\Lambda^2. \tag{7.1}$$

Is this picture phenomenologically acceptable? The answer is a big 'no', since a $100\,\mathrm{GeV}$ Higgs would imply $\Lambda \sim 1\,\mathrm{TeV}$. This is what happens in *technicolor* models. Such a low cutoff is strongly disfavored by EWPT. Suppose that we arrange the prefactor in front of Λ^2 to be not $(\frac{\alpha_t}{4\pi})$ but $(\frac{\alpha_t}{4\pi})^2$, i.e. the leading cutoff sensitivity appears not at one-loop but parametrically at
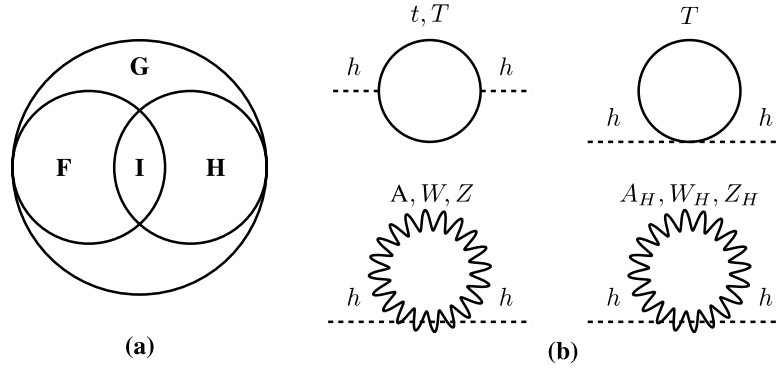
Figure 5. (*a*) Left panel: little Higgs cartoon. (*b*) Right panel: Feynman diagrams among which the same statistics cancellation takes place. Note that T is a (new) heavy quark and A_H, W_H, Z_H are (new) heavy gauge bosons—see text.

two-loop order, then the problem might be *temporarily* solved. Let us see how. The Higgs mass will then be given by

$$m_h^2 \sim \left(\frac{\alpha_t}{4\pi}\right)^2 \Lambda^2. \qquad (7.2)$$

For $m_h \sim 100$ GeV, the cutoff would now be $\Lambda \sim 10$ TeV. In a sense, this is nothing but a *postponement* of the problem as the cutoff of the theory is now pushed by one order of magnitude. The idea of a little Higgs is all about achieving this extra prefactor of $(\alpha_t/4\pi)$—see reviews [48] and [49, 50]. There are indeed other concerns, which we will discuss later.

To appreciate the little Higgs trick we look into figure 5(*a*). A global group G spontaneously breaks to H at a scale f. The origin of this symmetry breaking is irrelevant below the cutoff scale $\Lambda \sim 4\pi f$. H must contain SU(2) × U(1) as a subgroup so that when a part of G, labeled F, is weakly gauged the unbroken SM group (more precisely, the electroweak part of the SM) $I =$ SU(2) × U(1) comes out. The Higgs doublet (under SU(2) of I), which would ultimately trigger electroweak breaking, is a part of the Goldstone multiplet that parametrizes the coset space G/H. Choosing G, H and F is an open game. There are many choices. We will give some examples in a while. In fact, the little Higgs idea would work if the Higgs is a Goldstone boson under two different shift symmetries, i.e. $h \rightarrow h + c_1$ and $h \rightarrow h + c_2$. Both symmetries have to be broken. This is the idea of 'collective symmetry breaking'. It is important to note that the generators of the gauged part of G do not commute with the generators corresponding to the Higgs, and thus gauge interaction breaks the Goldstone symmetry. Yukawa interaction also breaks the Goldstone symmetry. Thus both gauge and Yukawa interactions induce Higgs mass at one-loop level (the cutoff dependence would appear parametrically at two-loop order, as we will see towards the end of this section).

7.1. A simple example with $G = SU(3) \times SU(3)$

For the purpose of illustration in this review, let us consider a global group SU(3)$_V$ × SU(3)$_A$. Assume that there are two scalars Φ_1 and Φ_2 which transform as (3, 3) and (3, $\bar{3}$) respectively. Now, imagine that each SU(3) spontaneously breaks to SU(2). So we start with $8 + 8 = 16$ generators from the two SU(3), and end up with $3 + 3 = 6$ unbroken

generators corresponding to the two SU(2) groups. This means that $16 - 6 = 10$ generators are broken, thus yielding 10 massless Goldstone bosons.

Now, we gauge SU(3)$_V$, but keep SU(3)$_A$ global. Hence, 5 out of 10 broken generators are *eaten up* as the gauged SU(3)$_V$ is broken to SU(2), but 5 Goldstone bosons still remain. This happens at a scale higher than that of EWSB, i.e. the corresponding order parameter f is larger than the electroweak VEV v. Note that since both Φ_1 and Φ_2 transform as 3 under SU(3)$_V$, both couple to the same set of gauge bosons with identical couplings. We can write Φ_1 and Φ_2 as

$$\Phi_1 = e^{i\frac{\theta_E}{f}} e^{i\frac{\theta_A}{f}} \begin{pmatrix} 0 \\ 0 \\ f + \rho_1(x) \end{pmatrix},$$

$$\Phi_2 = e^{i\frac{\theta_E}{f}} e^{-i\frac{\theta_A}{f}} \begin{pmatrix} 0 \\ 0 \\ f + \rho_2(x) \end{pmatrix}. \qquad (7.3)$$

Above, ρ_1 and ρ_2 are real scalar fields which acquire masses $\sim f$. The phase θ_E (where E stands for 'eaten') contains the d.o.f which are eaten up (i.e. gauged away), while θ_A contains five Goldstone bosons: $\theta_A = \sum_{a=4}^{8} \theta_A^a T_a$, where $T_4, .., T_8$ are broken generators. One can express

$$\theta_A = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & h^+ \\ 0 & 0 & h^0 \\ h^- & h^{0*} & 0 \end{pmatrix} + \frac{\eta}{4} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \qquad (7.4)$$

The complex scalar $H = \binom{h^+}{h^0}$ doublet under the *yet* unbroken SU(2) is *our* Higgs doublet, i.e. the one with which we will implement the electroweak SSB. But, until this point, H (in fact, both the charged and neutral components contained in H) and η are both massless.

Now recall that in the case of pions, the original SU(2)$_L$ × SU(2)$_R$ symmetry was *not* there to start with, as it was explicitly violated by electromagnetic interaction and quark masses. In the present case, the gauge and Yukawa interactions *explicitly* violate SU(3)$_A$. This is the reason as to why we will be able to finally write down a potential involving H.

7.1.1. How does gauge interaction violate $SU(3)_A$? With SU(3)$_V$ as the gauge group, the gauge interaction can be
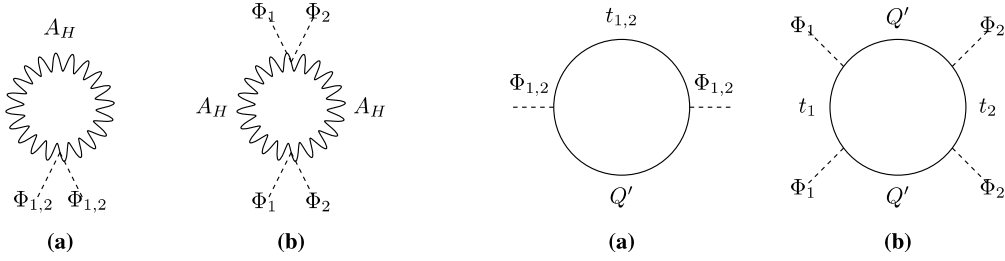
Figure 6. (i) Left panel: heavy gauge boson loops on the SU(3) triplet Φ lines; (*a*) yields quadratic cutoff dependence which does not contribute to the Higgs potential; (*b*) yields a log-divergent contribution to the Higgs mass. (ii) Right panel: heavy fermion loops on the SU(3) triplet Φ lines; (*a*) yields quadratic cutoff sensitivity but does not contribute to the Higgs potential; (*b*) contributes to the Higgs potential with a log sensitivity to the cutoff.

expressed as

$$(D_\mu \Phi_1)^\dagger (D_\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D_\mu \Phi_2),$$
$$D_\mu = \partial_\mu + i g A_\mu^a T_a \qquad (a = 1, 2, \ldots, 8), \qquad (7.5)$$

where
$$\Phi_1 = e^{i \frac{\theta_A}{f}} \begin{pmatrix} 0 \\ 0 \\ f + \rho_1 \end{pmatrix}, \qquad \Phi_2 = e^{-i \frac{\theta_A}{f}} \begin{pmatrix} 0 \\ 0 \\ f + \rho_2 \end{pmatrix}. \qquad (7.6)$$

After integrating out the heavy ($\sim g f$) gauge bosons—see figure 6(*a*) (left panel)—we obtain the following term in the effective Lagrangian

$$-\frac{g^2}{16\pi^2} \Lambda^2 (\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2). \qquad (7.7)$$

Now, we observe two important things:

- θ_A-dependence goes away in the above expression. Since the Higgs doublet $H = \frac{1}{\sqrt{2}} \binom{h^+}{h^0}$ is contained inside θ_A, it is rotated away in $\Phi_i^\dagger \Phi_i$ and is hence insensitive to the quadratic cutoff dependence of equation (7.7). This is not unexpected as the above piece of the Lagrangian is SU(3)$_A$ invariant, and hence is blind to θ_A or H.

- The scalar excitations ρ_1 and ρ_2 can sense the quadratic cutoff, and therefore their masses (naïvely of order $\sim f$) are not protected. This implies that the VEV f is also not protected from quadratic cutoff dependence[3].

We reiterate that all the shift symmetries of the Goldstone boson have to be broken, as any unbroken symmetry would keep the Goldstone massless. Quadratic divergence appears in those diagrams which involve only a single coupling operator, and such an operator cannot sense the breaking of all the symmetries. For Higgs mass generation, the responsible pieces of the Lagrangian involve *all* the symmetry breaking operators. Thus, the relevant Feynman diagrams involve more internal propagators, which is why there is no quadratic divergence.

Let us look at the diagram in figure 6(*b*) (left panel). After the heavy gauge bosons are integrated out, one obtains the following piece of the effective Lagrangian, which breaks the SU(3)$_A$ symmetry and hence can contribute to the Higgs potential. The Lagrangian term has the following form:

$$-\frac{g^4}{16\pi^2} \ln \left(\frac{\Lambda^2}{f^2} \right) |\Phi_1^\dagger \Phi_2|^2. \qquad (7.8)$$

---

[3] This is reminiscent of the quadratic cutoff sensitivity of the electroweak VEV v in the SM. The lack of 'protection' is identical in the two cases.

We now calculate $|\Phi_1^\dagger \Phi_2|^2$:

$$\Phi_1 = e^{i\theta_A/f} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} = \left(1 + i\frac{\theta_A}{f} - \frac{\theta_A^2}{2f^2} \right)_{3\times 3} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}_{3\times 1},$$

$$\theta_A = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & h^+ \\ 0 & 0 & h^0 \\ h^- & h^{0*} & 0 \end{pmatrix},$$

therefore, $\theta_A^2 = \frac{1}{2} \begin{pmatrix} 0 & 0 & h^+ \\ 0 & 0 & h^0 \\ h^- & h^{0*} & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & h^+ \\ 0 & 0 & h^0 \\ h^- & h^{0*} & 0 \end{pmatrix},$

therefore, $\theta_A^2|_{\text{3rd col}} = \frac{1}{2} \begin{pmatrix} 0 \\ 0 \\ h^- h^+ + h^{0*} h^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ H^\dagger H \end{pmatrix},$

where $H = \frac{1}{\sqrt{2}} \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}.$

Hence, $\Phi_1 = \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} + \frac{i}{\sqrt{2}} \begin{pmatrix} h^+ \\ h^0 \\ 0 \end{pmatrix}$

$$+ \begin{pmatrix} 0 \\ 0 \\ -\frac{H^+ H}{2f^2} \end{pmatrix} f = \begin{pmatrix} i H_{2\times 1} \\ f \left(1 - \frac{H^\dagger H}{2f^2} \right)_{1\times 1} \end{pmatrix},$$

therefore, $\Phi_1^\dagger = \left(-i H_{1\times 2}^\dagger \quad f \left(1 - \frac{H^\dagger H}{2f^2} \right)_{1\times 1} \right).$

Recall, $\Phi_2 = \begin{pmatrix} -i H_{2\times 1} \\ f \left(1 - \frac{H^\dagger H}{2f^2} \right)_{1\times 1} \end{pmatrix},$

therefore, $\Phi_1^\dagger \Phi_2 = -(H^\dagger H) + f^2 \left(1 - \frac{H^\dagger H}{2f^2} \right)^2$

$$= f^2 - 2(H^\dagger H) + \frac{(H^\dagger H)^2}{4f^2},$$

and hence $|\Phi_1^\dagger \Phi_2|^2 = -4f^2 (H^\dagger H) + \frac{9}{2} (H^\dagger H)^2 + \cdots.$

Note that a potential of H is generated with a bilinear and a quartic term. Interestingly, the bilinear term has the negative sign required for SSB, and the sign of the quartic term is positive as required by the stability of the potential. After SSB, the Higgs mass is given by

$$m_h^2 \simeq \frac{g^4}{16\pi^2} f^2 \ln \left(\frac{\Lambda^2}{f^2} \right). \qquad (7.9)$$

It appears somewhat miraculous that unlike in SM, here the one-loop generated m_h^2 is not proportional to $\Lambda^2/16\pi^2$, but $f^2/16\pi^2$. The cancellation of quadratic divergence takes place between two sets of diagrams, one that contains the massless SU(2) gauge bosons and the other that contains the massive gauge bosons (see figure 5(b)). This is an example of *same statistics cancellation*.

7.1.2. *How does Yukawa interaction violate* $SU(3)_A$*?*

Consider a left-handed SU(3) triplet $Q'_L \equiv \begin{pmatrix} t \\ b \\ T \end{pmatrix}_L$ and three right-handed singlets t_R, b_R and T_R, i.e. the 'new' states are $T_{L,R}$. When the gauged $SU(3)_V$ breaks to SU(2) by the scalar VEVs, the part $Q_L \equiv \begin{pmatrix} t \\ b \end{pmatrix}_L$ inside Q'_L transforms as a doublet under the SU(2).

Now, start with the following SU(3) invariant Yukawa interaction Lagrangian:

$$\mathscr{L}_Y = \frac{h_t}{\sqrt{2}}[t_1^c \Phi_1^\dagger Q'_L + t_2^c \Phi_2^\dagger Q'_L], \qquad \text{where}$$

$$h_t \equiv h_t^{(1)} = h_t^{(2)}, \qquad t_{1,2} \equiv \frac{1}{\sqrt{2}}(T_R \pm it_R). \qquad (7.10)$$

We now make the following algebraic steps:

$$\Phi_1^\dagger Q'_L = \left(-iH_{1\times2}^\dagger \; f\left(1 - \frac{H^\dagger H}{2f^2}\right)_{1\times1}\right)\begin{pmatrix} Q_{L(2\times1)} \\ T_{L(1\times1)} \end{pmatrix}$$

$$= -iH^\dagger Q_L + f\left(1 - \frac{H^\dagger H}{2f^2}\right) T_L,$$

$$\Phi_2^\dagger Q'_L = iH^\dagger Q_L + f\left(1 - \frac{H^\dagger H}{2f^2}\right) T_L,$$

therefore

$$\begin{aligned}
\mathscr{L}_Y &= \frac{h_t}{\sqrt{2}}\left[t_1^c\left\{-iH^\dagger Q_L + f\left(1 - \frac{H^\dagger H}{2f^2}\right)T_L\right\}\right. \\
&\quad \left. + t_2^c\left\{iH^\dagger Q_L + f\left(1 - \frac{H^\dagger H}{2f^2}\right)T_L\right\}\right] \\
&= h_t\left[\frac{-i}{\sqrt{2}}(t_1^c - t_2^c)Q_L H^\dagger\right. \\
&\quad \left. + \frac{f}{\sqrt{2}}(t_1^c + t_2^c)\left(1 - \frac{H^\dagger H}{2f^2}\right)T_L\right] \\
&= h_t \bar{t}_R Q_L H^\dagger + h_t f\left(1 - \frac{H^\dagger H}{2f^2}\right)\bar{T}_R T_L. \qquad (7.11)
\end{aligned}$$

The first term in the above expression contains the SM top quark Yukawa coupling, and the second term indicates that the T quark is heavy ($\sim f$).

Figure 6(a) (right panel) yields an one-loop effective Lagrangian as such

$$-\frac{h_t^2}{16\pi^2}\Lambda^2(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2). \qquad (7.12)$$

This is exactly the same as equation (7.7) with $g \leftrightarrow h_t$. Again, this Lagrangian preserves $SU(3)_A$, and hence is not relevant to the Higgs potential. We then turn to figure 6(b) (right panel), which yields

$$-\frac{h_t^4}{16\pi^2}\ln\left(\frac{\Lambda^2}{f^2}\right)|\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1|^2. \qquad (7.13)$$

This Lagrangian is similar to equation (7.8) with $g \leftrightarrow h_t$. This piece of the Lagrangian is of interest to us as it yields the bilinear and quartic terms involving H with the right sign of the coefficients. After SSB the Higgs mass is generated as

$$m_h^2 \simeq \frac{h_t^4}{16\pi^2}f^2\ln\left(\frac{\Lambda^2}{f^2}\right), \qquad (7.14)$$

which is similar to equation (7.9) with $g \leftrightarrow h_t$. Again, the apparently miraculous cancellation of quadratic divergence can be diagrammatically understood by the cancellation occurring between the t and T loops (see figure 5(b)), which is yet another example of *same statistics cancellation*.

7.2. *Salient features of little Higgs models*

7.2.1. *Quadratic cutoff sensitivity.*
Although *same statistics cancellations* enable us to express m_h^2 as proportional to $f^2/16\pi^2$ (i.e. *not* as $\Lambda^2/16\pi^2$) with only a logarithmic cutoff sensitivity at one-loop, as reflected in equations (7.9) and (7.14), the quadratic cutoff sensitivity comes back parametrically at two-loop order. To appreciate this, first recall that in the SM the one-loop correction to the Higgs mass goes as

$$\Delta m_h^2\,(\text{SM}) \sim \frac{\Lambda^2}{16\pi^2}. \qquad (7.15)$$

This means that the electroweak VEV (v) receives a quadratic (Λ^2) correction

$$v^2 \to v^2 + \frac{\Lambda^2}{16\pi^2}. \qquad (7.16)$$

We now consider the gauging of $SU(3)_V$ as discussed in the previous subsection. The corresponding order parameter is f, but note that f is as unprotected as the electroweak VEV v is in the SM [6]. Hence

$$\begin{aligned}
f^2 \to F^2 &= f^2 + \frac{a}{16\pi^2}\Lambda^2 = (1+a)f^2 \\
&(\text{since } \Lambda = 4\pi f), \qquad (7.17)
\end{aligned}$$

where $a \sim \mathcal{O}(1)$. Then, what did we gain *vis-à-vis* the SM? For little Higgs models

$$\begin{aligned}
m_h^2\,(\text{LH}) &\sim \left(\frac{1}{16\pi^2}\right)F^2\ln\left(\frac{\Lambda^2}{F^2}\right) \Longrightarrow \Delta m_h^2\,(\text{LH}) \\
&\sim \left(\frac{1}{16\pi^2}\right)^2\Lambda^2. \qquad (7.18)
\end{aligned}$$

Note that the quadratic cutoff sensitivity of the Higgs mass-square exists not only in the SM but also in the little Higgs models. Then, what purpose did little Higgs serve? In the little Higgs case there is an extra loop suppression factor—compare equation (7.15) with equation (7.18). The appearance of the cutoff in the little Higgs models is thus postponed by one decade in energy scale compared to the SM. One important thing should be kept in mind. A Goldstone boson becoming massive in little Higgs models is not a surprise. The global symmetry is *explicitly* broken to start with by the gauge and Yukawa interactions, and precisely for this reason the loosely mentioned Goldstone boson is actually a pseudo-Goldstone boson (pGB). Up to this point there is no difference with the

theory of pions where electromagnetic interaction and quark masses explicitly break the Goldstone symmetry. What is new here, i.e. the reason for which we consider the little Higgs construction as an important achievement over the SM, is the appearance of the quadratic cutoff dependence of the Higgs mass at the next order in perturbation theory, i.e. at the two-loop level.

If we want $m_h \sim (f/4\pi) \sim 100\,\text{GeV}$, it immediately follows that $f \sim F \sim 1\,\text{TeV}$, and the cutoff of the theory is $\Lambda \sim 4\pi f \sim 10\,\text{TeV}$, as against the SM cutoff of $4\pi v \sim 1\,\text{TeV}$. The ultraviolet completion beyond $10\,\text{TeV}$ in little Higgs models is a detailed model-dependent issue [51].

7.2.2. Large quartic coupling. A clever construction of a little Higgs theory should yield the following electroweak Higgs potential:

$$V = -\frac{(g \text{ or } h_t)^4}{16\pi^2} f^2 \ln\left(\frac{\Lambda^2}{f^2}\right)(H^\dagger H) + \lambda(H^\dagger H)^2, \quad (7.19)$$

i.e. the bilinear term should have a one-loop suppression but, crucially, the quartic interaction should be *unsuppressed*, i.e. $\lambda \sim g^2$ (or h_t^2). If both quadratic and quartic terms are suppressed, it is not possible to simultaneously obtain the correct W boson mass and a phenomenologically acceptable Higgs mass. In the simple scenario used for our illustration, both the quadratic and quartic terms are generated by loops, so the phenomenological problem survives. In more realistic scenarios, as we will see shortly, this problem can be avoided. We will discuss only some of these scenarios below.

7.3. Realistic little Higgs scenarios—a brief description

7.3.1. Different choices of groups. The 'littlest Higgs' [49] construction is based on a choice of a global group $G = SU(5)$ which breaks to $H = SO(5)$ by the VEV (Σ_0) of a scalar field, expanded as $\Sigma = e^{2i\Pi/f}\Sigma_0$, where $\Pi = \Pi^a X_a$ contains the Goldstone bosons, X_a being the broken generators. The 5×5 VEV matrix is given by $\Sigma_0 = $ anti-diagonal $(1_{2\times2}, 1, 1_{2\times2})$. The subgroup of $SU(5)$ that is gauged is $[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2$ which breaks to $SU(2)_D \times U(1)_Y$. Out of the $14 (=24 - 10)$ pGBs generated during $G \rightarrow H$, four are absorbed as the longitudinal components of the massive gauge bosons A_H, Z_H and $W_H^\pm$ corresponding to the broken $SU(2) \times U(1)$ generators. The other 10 scalar degrees of freedom arrange themselves as a complex $SU(2)$ scalar doublet H with the right quantum numbers required to make a $SU(2)$ Higgs doublet with hypercharge $(=1/2)$ and a complex scalar $SU(2)$ triplet Φ with hypercharge $(=1)$. In the limit when any pair of gauge couplings (g_1, g_1') or (g_2, g_2') goes to zero, the Higgs field becomes exactly massless. Therefore, any loop diagram contributing to the Higgs mass must involve a product $g_1 g_2$ (or, $g_1' g_2'$). Due to this *collective* symmetry breaking, all such diagrams are logarithmically sensitive to the cutoff at one-loop.

The type of little Higgs models discussed earlier for the purpose of illustration, i.e. where the global group is $G = SU(3) \times SU(3)$ and the gauged subgroup is the simple group $SU(3)$, is called the 'simplest' [50]. The difficulty

of achieving a large quartic coupling was overcome by considering $G = [SU(4)]^4$ which breaks to $H = [SU(3)]^4$, while the gauged subgroup is $SU(4) \times U(1)$ which breaks down to $SU(2) \times U(1)$. Out of the 28 pGBs, 12 are eaten up by the massive gauge bosons. The 16 degrees of freedom are distributed as two complex doublets, three complex singlets and two real singlet scalars. The scalar quartic coupling is generated at tree level.

The authors of [52] have considered $G = SU(6)$ and $H = Sp(6)$. The gauged subgroup is $[SU(2) \times U(1)]^2$ which breaks to $SU(2)_D \times U(1)_Y$. So, out of the $35 - 21 = 14$ pGBs four are absorbed by the massive gauge bosons, and the remaining 10 degrees of freedom are decomposed into two complex doublet scalars and one complex singlet scalar. A distinct advantage here is that there is no triplet scalar which could have caused some trouble in EWPT (see discussions later).

The moose models are, on the other hand, based on the concept of *deconstruction* (a term borrowed from economics). The electroweak sector is described by a product global symmetry G^N which is broken by the condensates transforming as bi-fundamentals under $G_i \times G_j$, where i, j are the sites. In [53], the global group considered is $G^N = [SU(3)]^8$, and a subgroup of it is gauged which eventually breaks to $SU(2) \times U(1)$. The scalar spectrum contains two complex $SU(2)$ doublets, a complex $SU(2)$ triplet and a complex singlet. To ensure custodial $SU(2)$ symmetry, i.e. to maintain consistency with the oblique $\Delta\rho$ (or, T) parameter, the global group was enlarged in [54] to $[SO(5)]^8$ with the gauge group $SO(5) \times SU(2) \times U(1)$. To further minimize the scalar contribution to $\Delta\rho$, a coset space $SO(9)/[SO(5) \times SO(4)]$ was constructed with the gauge symmetry $SU(2)_L \times SU(2)_R \times SU(2) \times U(1)$ [55]. A review of these and many other models can be found in [48].

7.3.2. Bounds from EWPT. In an effective field theory description [56], two dimension-6 operators $\mathcal{O}_T \propto |H^\dagger D_\mu H|^2$ and $\mathcal{O}_S \propto H^\dagger \sigma^a H W_{\mu\nu}^a B_{\mu\nu}$ serve as the primary filters before certifying whether a model passes EWPT or not. Recall that an $SU(2)$ global custodial symmetry in the SM guarantees the tree level relation $M_W = M_Z \cos\theta_W$. The operator $\mathcal{O}_T$ violates that symmetry, which is not difficult to conceive: when H goes to the vacuum, $\mathcal{O}_T \propto Z_\mu Z^\mu$ but there is no similar contribution for $W_\mu W^\mu$, i.e. there is a contribution to M_Z but not to M_W, and this mismatch violates custodial symmetry. Similarly, the operator $\mathcal{O}_S$ induces kinetic mixing between W_μ^3 and B_μ. The coefficients of $\mathcal{O}_T$ and $\mathcal{O}_S$ will, therefore, indicate the contributions to the T and S parameters, respectively.

Unless special care is taken, a general class of little Higgs models gives a large contribution to T, and hence receives a strong constraint: $f > (2-5)\,\text{TeV}$ [57]. A large f means that to obtain the Higgs mass in the $100\,\text{GeV}$ range one has to fine tune the parameters. The constraints arise primarily from the tree level mixing of the SM particles with the new particles. In the littlest Higgs model, the T parameter receives a large contribution from the custodial symmetry breaking trilinear operator $H^T \Phi H$, which mixes the doublet H with the triplet Φ. Also, the $W_L W_H H H$ term (W_L is the SM gauge boson and

W_H is the heavy one) yields a sizable contribution to T. To circumvent these constraints, the authors of [58] introduced, more in the spirit of R-parity in supersymmetry, what is called T-parity under which all (*but one*) new particles are odd and the SM particles are even. It is a discrete Z_2 symmetry, which is an automorphism of the gauge groups that exchanges the gauge bosons of $[SU(2) \times U(1)]_1$ and $[SU(2) \times U(1)]_2$. It also means $g_1 = g_2$ and $g_1' = g_2'$. Under this symmetry $H \to H$, but $\Phi \to -\Phi$, so the problematic $H^T \Phi H$ coupling is absent. Contributions to T and S from heavy particles arise only at the loop level. As a result, f as low as 500 GeV can be accommodated without facing any inconsistency with EWPT [59]. It should be noted that there is one new, yet T-even, state in this scenario, the so-called 'top partner' which cancels the standard top induced quadratic divergence to the Higgs mass. This state has a positive contribution to the T parameter, and to compensate that one may need a Higgs mass as large as 800 GeV [59]. Chen's review in [48] covers the EWPT and naturalness constraints on quite a few such scenarios. In a recent development, the authors of [60] have considered a $SO(6) \times SO(6)/SO(6)$ model, called it the 'bestest' little Higgs, and claimed that quartic coupling can be generated without violating custodial symmetry (S and T vanish at tree level) and at the same time keeping the fine-tuning within 10% in the top sector.

7.3.3. Collider signals of little Higgs models.　　Since each little Higgs model involves a G/H coset space and an extended electroweak gauge sector, there are invariably new weak gauge bosons, new fermions and new scalars. To confirm little Higgs models, those new particles have to be looked for in the colliders (see the study made by the ATLAS collaboration at the LHC [61]).

New gauge bosons.　　In the littlest Higgs model, the couplings of the heavy gauge bosons Z_H and W_H with the fermions are universal which, beside a mixing angle factor, depend only on the weak isospin t_3 of the fermions (i.e. purely left-handed) and not on the electric charge Q. It has been shown that about 30000 Z_H can be produced annually at the LHC with 100fb^{-1} luminosity. These heavy gauge bosons would decay into the SM fermions ($V_H \to f\bar{f}'$), or into the SM gauge bosons ($Z_H \to W_L^+ W_L^-$, $W_H \to W_L Z_L$, where $V_L \equiv V_{SM}$), or into the Higgs and SM gauge boson ($V_H \to V_L h$). The branching ratios would follow a definite pattern, which would serve as 'smoking gun signals' [62, 63].

New fermions.　　A colored vector-like T quark features in almost all little Higgs models. It may be produced singly by $bW \to T$ at the LHC. Typically, $\Gamma(T \to th) \approx \Gamma(T \to tZ) \approx \frac{1}{2}\Gamma(T \to bW)$. This branching ratio relation would constitute a characteristic signature for T quark discovery [62, 64]. When T-parity is conserved, one has a T-odd state t_- and a T-even state t_+ (which has been referred to above as the T quark, and which also cancels the SM top induced quadratic divergence to the Higgs mass), and $m_{t_+} > m_{t_-}$. The QCD production cross section $\sigma(gg \to t_- t_-) \approx 0.3$ pb for $m_{t_-} = 800$ GeV, and almost all time t_- would decay as

$t_- \to A_H t$, where A_H is the lightest T-odd gauge boson which, being stable, would escape the detector carrying missing energy [59].

New scalars.　　The presence of a doubly charged scalar ϕ^{++}, as a component of a complex triplet scalar, is a hallmark signature of a large class of little Higgs models. Its decay into like-sign dileptons ($\phi^{++} \to \ell^+ \ell^+$) which would lead to an unmistakable signal with a separable SM background [62]. The other spectacular signal of the doubly charged scalar would be a resonant enhancement of $W_L W_L \to W_L W_L$ proceeding via ϕ^{++} exchange. An analysis of $M(W^+ W^+)$ invariant mass distribution was carried out in [62] with the claim that with 300 fb^{-1} luminosity at the LHC about 100 events would pop up over the SM background for $m_{\phi^{++}} = 1.5$ TeV, assuming a triplet to doublet VEV ratio $v'/v = 0.05$. One can go a little further by employing the triplet scalar in generating neutrino mass via type-II see-saw. The maximal mixing in the $\mu - \tau$ sector would predict equal branching ratios of ϕ^{++} in the $\mu^+ \mu^+$, $\mu^+ \tau^+$ and $\tau^+ \tau^+$ channels, which can be tested at the LHC. Employing this correlation, a discovery limit of $m_{\phi^{++}} = 700$ GeV has been claimed with only 30 fb^{-1} luminosity at the LHC, where the authors take into consideration particle reconstruction efficiencies as well as Gaussian distortion functions for the momenta and missing energy of final state particles [65].

We conclude this section with the statement that little Higgs models with T-parity and supersymmetry with R-parity would be hard to distinguish at the LHC. Universal extra dimension (UED) with KK-parity would also give similar signals. The best way to study them is to consider their production via strong interaction and their decay via weak interaction. The authors of [66] have concentrated on final states containing an unspecified number of jets, three or four leptons and missing transverse momentum. They have asserted that the jet multiplicity distributions are the crucial discriminating factors among the scenarios and they have constructed several discriminating variables. This is still an open issue and constitutes a challenging *inverse problem*.

8. Gauge–Higgs unification

The basic idea of gauge–Higgs unification (GHU) is that the Higgs boson would arise from the internal components of a higher dimensional gauge field. As a result, higher dimensional gauge invariance would protect the Higgs mass from quadratic divergence. When the extra space coordinate is not simply connected (e.g. S^1), there are Wilson line phases associated with the extra-dimensional component of the gauge field (this is conceptually similar to Aharonov-Bohm phase in quantum mechanics). Their 4d quantum fluctuation is identified with the Higgs field. Higher dimensional gauge invariance does not allow any scalar potential at the tree level. The scalar potential is generated through radiative corrections. The Higgs boson acquires a mass through this radiatively generated potential. One of the earliest realizations of GHU was provided by Antoniadis in a work on extra dimension in the supersymmetric context where the Higgs was coming from an $N = 4$ supermultiplet, i.e. from a higher dimensional

gauge field [67]. But for the purpose of illustration we do not bring in any supersymmetric aspect. We rather focus on the underlying dynamics of the GHU mechanism in the non-supersymmetric extra-dimensional context, for which we proceed step by step [68].

8.1. 5d QED as an illustrative example

The 5d Lagrangian, a function of the usual 4d coordinates (x_μ) and the 5th space coordinate (y), is given by

$$\mathscr{L}(x, y) = -\tfrac{1}{4} F_{MN}(x, y) F^{MN}(x, y) + \mathscr{L}_{GF}(x, y), \qquad (8.1)$$

where

$$F_{MN}(x, y) = \partial_M A_N(x, y) - \partial_N A_M(x, y).$$

The indices $M, N = (\mu, 5)$; with $\mu = 0, 1, 2, 3$. The symbol 'GF' means gauge-fixing.

The 5d gauge field A_M transforms as a vector under the Lorentz group SO(1,4). In the absence of gauge fixing, the 5d QED Lagrangian is invariant under a U(1) gauge transformation

$$A_M(x, y) \to A_M(x, y) + \partial_M \Theta(x, y).$$

The compactification is on an orbifold S^1/Z_2, i.e. with $y \to (-y)$ identification. In order not to spoil gauge symmetry the following conditions need to be satisfied, which allow a massless photon in 4d:

$$A_M(x, y) = A_M(x, y + 2\pi R),$$
$$A_\mu(x, y) = A_\mu(x, -y), \qquad A_5(x, y) = -A_5(x, -y),$$
$$\Theta(x, y) = \Theta(x, y + 2\pi R), \qquad \Theta(x, y) = \Theta(x, -y).$$
$$(8.2)$$

The above conditions guarantee that the theory remains gauge invariant even after compactification. The Fourier mode expansions of different 5d fields are given by (R is the radius of compactification)

$$A_\mu(x, y) = \frac{1}{\sqrt{2\pi R}} A_\mu^{(0)}(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} A_\mu^{(n)}(x) \cos\left(\frac{ny}{R}\right),$$

$$A_5(x, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} A_5^{(n)}(x) \sin\left(\frac{ny}{R}\right),$$

$$\Theta(x, y) = \frac{1}{\sqrt{2\pi R}} \Theta^{(0)}(x) + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \Theta^{(n)}(x) \cos\left(\frac{ny}{R}\right).$$
$$(8.3)$$

Above, $A_\mu^{(0)}(x)$ and $\Theta^{(0)}(x)$ are zero modes, which are the relevant fields for ordinary 4d QED. As expected, there is no zero mode for A_5.

The 4d effective Lagrangian is obtained by integrating out the fifth coordinate, and is given by

$$\mathscr{L}(x) = \int_0^{2\pi R} dy \, \mathscr{L}(x, y).$$

The higher dimensional physics is reflected by the infinite tower of Kaluza–Klein (KK) modes. A simple calculation yields the following 4d Lagrangian

$$\mathscr{L}(x) = -\frac{1}{4} F_{\mu\nu}^{(0)} F^{\mu\nu(0)}$$
$$+ \sum_{n=1}^{\infty} \left[-\frac{1}{4} F_{\mu\nu}^{(n)} F^{\mu\nu(n)} + \frac{1}{2} \left(\frac{n}{R} A_\mu^{(n)} + \partial_\mu A_5^{(n)} \right)^2 \right]$$
$$+ \mathscr{L}_{GF}(x). \qquad (8.4)$$

The following steps lead to the above equation:

$$F_{MN} F^{MN} = F_{\mu\nu} F^{\mu\nu} + F_{\mu 5} F^{\mu 5} + F_{5\mu} F^{5\mu}$$
$$= F_{\mu\nu} F^{\mu\nu} + 2 F_{\mu 5} F^{\mu 5},$$
$$F_{\mu 5} F^{\mu 5} = (\partial_5 A_\mu - \partial_\mu A_5)^2 = (\partial_5 A_\mu)^2 + (\partial_\mu A_5)^2$$
$$- 2(\partial_5 A_\mu)(\partial_\mu A_5),$$
$$\int_0^{2\pi R} dy (\partial_5 A_\mu)^2 = \frac{n^2}{R^2} \frac{1}{\pi R} (A_\mu^{(n)}(x))^2 \int_0^{2\pi R} dy \sin^2 \frac{ny}{R}$$
$$= \frac{n^2}{R^2} (A_\mu^{(n)}(x))^2,$$
$$\int_0^{2\pi R} dy (-)\partial_5 A_\mu \partial_\mu A_5 = \int_0^{2\pi R} dy \left(\frac{n}{R}\right) \frac{1}{\sqrt{\pi R}} A_\mu^{(n)}(x)$$
$$\times \sin \frac{ny}{R} \frac{1}{\sqrt{\pi R}} \partial_\mu A_5^{(n)}(x) \sin \frac{ny}{R}$$
$$= \left(\frac{n}{R}\right) A_\mu^{(n)}(x) \partial_\mu A_5^{(n)}(x).$$

Now we shall show that the modes $A_5^{(n)}$, which are scalars with respect to 4d Lorentz group, play the rôle of 'would-be' Goldstone modes to be 'eaten up' by the massive $A_\mu^{(n)}$. In fact, in a sense, the mass generation of heavy KK gauge modes by compactification can be viewed as a kind of geometric Higgs mechanism.

We should keep in mind that the Lagrangian $\mathscr{L}(x)$ is still manifestly gauge invariant by the joint actions of two transformations at each KK level:

$$A_\mu^{(n)}(x) \to A_\mu^{(n)}(x) + \partial_\mu \Theta^{(n)}(x),$$
$$A_5^{(n)}(x) \to A_5^{(n)}(x) - \frac{n}{R} \Theta^{(n)}(x). \qquad (8.5)$$

Now we use 't Hooft's gauge fixing condition by which the terms that mix $A_\mu^{(n)}$ and $A_5^{(n)}$ are removed from the 4d effective Lagrangian. We write

$$\mathscr{L}_{GF}(x, y) = -\frac{1}{2\xi} (\partial_\mu A_\mu(x, y) - \xi \partial_5 A_5(x, y))^2. \qquad (8.6)$$

Note that in the last equation the requirement of covariance of the gauge fixing Lagrangian with respect to the y-direction has been sacrificed, which is nothing serious as compactification (S^1/Z_2) breaks SO(1,4) invariance under ordinary 4d Lorentz transformation any way.

Now we calculate $\mathscr{L}(x) = \int_0^{2\pi R} dy \mathscr{L}(x, y)$ where $\mathscr{L}(x, y)$ contains the above $\mathscr{L}_{GF}(x, y)$. All mixing terms involving $A_\mu^{(n)}$ and $A_5^{(n)}$ are now reduced to total derivatives which are irrelevant. Then the gauge-fixed 4d Lagrangian

looks like

$$\mathcal{L}(x) = -\frac{1}{4} F^{(0)}_{\mu\nu} F^{\mu\nu(0)} - \frac{1}{2\xi} (\partial_\mu A^{(0)}_\mu)^2$$

$$+ \sum_{n=1}^{\infty} \left[-\frac{1}{4} F^{(n)}_{\mu\nu} F^{\mu\nu(n)} - \frac{1}{2\xi} \left(\partial_\mu A^{(n)}_\mu \right)^2 \right.$$

$$\left. + \frac{1}{2} \left(\frac{n}{R} \right)^2 A^{(n)}_\mu A^{(n)}_\mu \right]$$

$$+ \sum_{n=1}^{\infty} \left[\frac{1}{2} \left(\partial_\mu A^{(n)}_5 \right)^2 - \frac{1}{2} \xi \left(\frac{n}{R} \right)^2 \left(A^{(n)}_5 \right)^2 \right]. \quad (8.7)$$

The scalars $A^{(n)}_5$ with 'gauge dependent masses' resemble the would-be Goldstone bosons of an ordinary 4d Abelian theory in R_ξ gauge, so we have

$$A^{(n)}_\mu \text{ propagator} \Rightarrow \frac{1}{k^2 - \frac{n^2}{R^2}} \left[-g^{\mu\nu} + \frac{(1-\xi)k^\mu k^\nu}{k^2 - \xi \left(\frac{n}{R} \right)^2} \right],$$

$$A^{(n)}_5 \text{ propagator} \Rightarrow \frac{1}{k^2 - \xi \left(\frac{n}{R} \right)^2}. \quad (8.8)$$

Clearly, the $A^{(n)}_5$ modes are unphysical, and they provide the longitudinal components of the massive $A^{(n)}_\mu$ states.

8.2. 5d SU(2) model as an illustration

The gauge group is SU(2), the compactification is on S^1/Z_2, and we impose a non-trivial Z_2 parity:

$$P = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \begin{array}{c} A_\mu \xrightarrow{Z_2} P A_\mu P^\dagger, \\ A_5 \xrightarrow{Z_2} -P A_5 P^\dagger, \end{array} \quad (8.9)$$

where $A_\mu = A^a_\mu \tau_a$ is the Lie-algebra valued 5d gauge field. In component form

$$A_\mu = A^a_\mu \tau_a = \begin{pmatrix} A^3_\mu & A^1_\mu - iA^2_\mu \\ A^1_\mu + iA^2_\mu & -A^3_\mu \end{pmatrix}. \quad (8.10)$$

Therefore

$$P A_\mu P^\dagger = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} A^3_\mu & A^1_\mu - iA^2_\mu \\ A^1_\mu + iA^2_\mu & -A^3_\mu \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} A^3_\mu & (-)(A^1_\mu - iA^2_\mu) \\ (-)(A^1_\mu + iA^2_\mu) & -A^3_\mu \end{pmatrix}. \quad (8.11)$$

Clearly

$$A^3_\mu \xrightarrow{Z_2} A^3_\mu, \qquad (A^1_\mu, A^2_\mu) \xrightarrow{Z_2} (-)(A^1_\mu, A^2_\mu). \quad (8.12)$$

Hence, $A^3_\mu(x, y)$ has zero mode $A^{3(0)}_\mu(x)$, but $A^1_\mu(x, y)$ and $A^2_\mu(x, y)$ do not have zero modes. Since $A_5 \xrightarrow{Z_2} -P A_5 P^\dagger$, it is easy to show that $A^1_5(x, y)$ and $A^2_5(x, y)$ (and not $A^3_5(x, y)$) have zero modes which can acquire VEVs. Thus we witness an *explicit* breaking

$$\begin{array}{c} G \xrightarrow{P} H \\ \text{SU(2)} \xrightarrow{} \text{U(1)}. \end{array}$$

We can therefore write

$$\langle A^a_5 \rangle = (\langle A^{1(0)}_5 \rangle, \langle A^{2(0)}_5 \rangle, 0).$$

Using the unbroken U(1) symmetry, we can assign the entire VEV in one component and hence without any loss of generality we can write $\langle A^a_5 \rangle = (B, 0, 0)$, where B is the VEV.

The gauge boson masses originate from $F^a_{\mu5} F^{\mu5}_a = (\partial_\mu A^a_5 - \partial_5 A^a_\mu + g\epsilon_{abc} A^b_\mu A^c_5)^2$. The relevant term of the Lagrangian leading to the mass matrix is $A^a_\mu (D_5 D_5)_{ab} A^b_\mu$, where

$$(D_5 D_5)_{ab} = \begin{pmatrix} \partial_5 \partial_5 & 0 & 0 \\ 0 & \partial_5 \partial_5 - g^2 B^2 & -2gB\partial_5 \\ 0 & 2gB\partial_5 & \partial_5 \partial_5 - g^2 B^2 \end{pmatrix},$$

with $a, b = 1, 2, 3$ as adjoint representation indices. There is no KK-number mixing and this mass matrix holds for each n. The derivatives in the above matrix would act on the KK states. For $n \neq 0$, $A^{3(n)}_\mu \sim \frac{1}{\sqrt{\pi R}} \cos \frac{ny}{R}$, $A^{1,2(n)}_\mu \sim \frac{1}{\sqrt{\pi R}} \sin \frac{ny}{R}$, which is a consequence of our choice of $P = \text{diag}(1, -1)$. Each derivative then picks up a factor n/R. The KK gauge boson mass-squared matrix turns out to be (for $n \neq 0$)

$$\begin{pmatrix} \frac{n^2}{R^2} & 0 & 0 \\ 0 & \frac{n^2}{R^2} + \frac{\alpha^2}{R^2} & \frac{2\alpha n}{R^2} \\ 0 & \frac{2\alpha n}{R^2} & \frac{n^2}{R^2} + \frac{\alpha^2}{R^2} \end{pmatrix}, \quad (8.13)$$

where $\alpha \equiv gBR$. The eigenvalues are $\frac{n^2}{R^2}, \frac{(n+\alpha)^2}{R^2}, \frac{(n-\alpha)^2}{R^2}$. We have thus seen a two-stage symmetry breaking: (*i*) SU(2) breaks to U(1) explicitly by the action of P, as a result only A^3_μ has zero mode, and then (*ii*) U(1) breaks to *nothing* by the VEV B, when $A^{3(0)}_\mu$ picks up a mass $\frac{\alpha}{R}$.

Why is the example of SU(2) better than U(1)? In the U(1) example, the scalar turned out to be unphysical. From SU(2) we got a physical scalar, which can acquire a non-zero VEV. However, we want a scalar which is a doublet under SU(2), and the scalar we got in the above example is not a doublet of SU(2). To achieve this, we move to SU(3).

8.3. 5d SU(3) as a toy model

Now consider that the 5d gauge group is SU(3), which is compactified on S^1/Z_2. The Lie-algebra valued gauge fields are $A_M = A^a_M \frac{\lambda^a}{2}$. Here, λ^a are Gell-Mann matrices, given by

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \qquad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

We impose Z_2-projection by requiring

$$P A_\mu P^\dagger = A_\mu \quad \text{and} \quad P A_5 P^\dagger = -A_5,$$

$$\text{where} \quad P = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = e^{i\pi\lambda_3}. \quad (8.14)$$

The explicit transformations of the gauge boson fields are

$$\begin{pmatrix} A_\mu^3 + \frac{1}{\sqrt{3}}A_\mu^8 & A_\mu^1 - iA_\mu^2 & A_\mu^4 - iA_\mu^5 \\ A_\mu^1 + iA_\mu^2 & -A_\mu^3 + \frac{1}{\sqrt{3}}A_\mu^8 & A_\mu^6 - iA_\mu^7 \\ A_\mu^4 + iA_\mu^5 & A_\mu^6 + iA_\mu^7 & -\frac{2}{\sqrt{3}}A_\mu^8 \end{pmatrix}$$

$$\xrightarrow{P} \begin{pmatrix} \oplus & \oplus & \ominus \\ \oplus & \oplus & \ominus \\ \ominus & \ominus & \oplus \end{pmatrix}, \quad (8.15)$$

where $\oplus$ and $\ominus$ represent the relative signs upon transformation under the given projection. *For the A_5 scalars, $\oplus$ and $\ominus$ should be replaced by $\ominus$ and $\oplus$, respectively.* The fields which are projected with $\oplus$ sign contain zero modes, but those with the $\ominus$ sign do not have zero modes.

As a consequence of the above projection,

$$\begin{array}{ccc} G & \xrightarrow{P} & H \\ \mathrm{SU(3)} & & \mathrm{SU(2) \times U(1)} \end{array}.$$

Now, the **8** generators of SU(3) are decomposed as **3 + 2 + 2 + 1** under the unbroken SU(2). From equation (8.15), it is clear that only the triplet **3** (A_μ^1, A_μ^2, A_μ^3) and the singlet **1** (A_μ^8) gauge bosons have zero modes. Also, the components of the doublet **2** scalar ($A_5^4 - iA_5^5$, $A_5^6 - iA_5^7$)^T have zero modes. We identify the *zero mode doublet scalar* with our Higgs doublet, which is expressed as

$$H_5^{(0)} = \begin{pmatrix} A_5^{4(0)} - iA_5^{5(0)} \\ A_5^{6(0)} - iA_5^{7(0)} \end{pmatrix}. \quad (8.16)$$

In other words, when $G \xrightarrow{P} H$, the generators of the *massless gauge bosons* belong to H, while those of the *massless scalars* belong to the coset G/H.

We now turn our attention to the gauge transformations in bulk:

$$A_\mu \to A_\mu + \partial_\mu \Theta(x, y) + i[\Theta(x, y), A_\mu],$$

$$A_5 \to A_5 + \partial_5 \Theta(x, y) + i[\Theta(x, y), A_5].$$

For the scalars A_5, which correspond to the broken generators, $\Theta(x, 0) = \Theta(x, \pi R) = 0$, but still $A_5 \to A_5 + \partial_5 \Theta$. Because of this shift symmetry, there cannot be any tree level potential for A_5. Just like gauge invariance forbids $A_\mu A_\mu$ term in the ordinary 4d QED Lagrangian, the higher dimensional gauge invariance forbids $A_5 A_5$ term in the 5d Lagrangian as well. But this is true only at tree level, as quantum corrections generate the potential.

The quadratic $(A_5)^2$ and the quartic $(A_5)^4$ terms are generated at one-loop level via two- and four-point diagrams with A_5 in external lines and with KK fermions and bosons in internal lines. Such loops generate the effective potential whose minimization yields the VEV of A_5. The gauge loops

tend to push $\langle A_5^0 \rangle$ to zero while minimizing the potential, while the fermionic loops tends to shift $\langle A_5^0 \rangle$ away from zero in the minimum of the potential. In fact, the KK fermions are instrumental for generating the correct VEV. This way of breaking SU(2) × U(1) symmetry to U(1)$_\mathrm{em}$ is called the *Hosotani mechanism* [69]. The one-loop generated Higgs mass is given by

$$m_h^2 \simeq \frac{g^4}{128\pi^6} \frac{1}{R^2} \sum_{\mathrm{KK}} V''(\alpha), \quad (8.17)$$

where α is a dimensionless parameter arising from bulk interactions, which corresponds to the minimum of the potential where the double-derivative is calculated. The summation is over all KK particles. Clearly, 5d gauge symmetry is recovered in the limit $1/R \to 0$.

In fact, this A_5 is a symbolic representation of $H_5^{(0)}$. A VEV in $H_5^{(0)}$ induces SSB of H = SU(2)× U(1) to E = U(1)$_Q$. The composition of photon in this scenario is $\gamma_\mu \propto (A_\mu^3 + \frac{1}{\sqrt{3}}A_\mu^8)$. Recalling that the composition of photon in the SM, as given in equation (3.19), is

$$\gamma_\mu = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu,$$

we obtain the following relations for the GHU scenario under consideration:

$$\cot\theta_W = \frac{1}{\sqrt{3}} = \cot\frac{\pi}{3} \Rightarrow \theta_W = \pi/3 \Rightarrow \sin^2\theta_W = \frac{3}{4} \quad \text{and}$$

$$\frac{M_W^2}{M_Z^2} = \cos^2\theta_W = \frac{1}{4}, \qquad \text{therefore } M_Z = 2M_W.$$

This is clearly experimentally ruled out! But this scenario provides the basic intuitive picture of how a GHU scenario works through a simple illustration. In this scenario

$$M_W^{(n)} = \frac{n + \alpha}{R}, \qquad M_Z^{(n)} = \frac{n + 2\alpha}{R}, \qquad m_\gamma^{(n)} = \frac{n}{R}.$$

The periodicity property demands that the spectrum will remain invariant under $\alpha \to \alpha + 1$. This restricts α in the range [0, 1]. Orbifolding further reduces it to $\alpha = [0, \frac{1}{2}]$. In principle, α can be fixed from the W mass.

8.4. Realistic gauge–Higgs unification scenarios—a brief description

There are quite a few obstacles that one faces in constructing a realistic scenario. Since the Yukawa coupling arises from higher dimensional gauge coupling, it turns out to be too small to produce the correct top quark mass. In particular, one has to also worry about generating hierarchical Yukawa interaction starting from higher dimensional gauge interaction which is, after all, universal. The scalar potential is generated at one-loop, which tends to yield rather low Higgs boson mass. The compactification scale (R^{-1}) required for this purpose turns out to be smaller than its experimental lower limit. We briefly describe below some of the attempts made in removing these

obstacles.

(*i*) It has been argued in [70] in the context of a 5d S^1/Z_2 scenario that a large brane localized kinetic term can help jack up the Higgs mass to an acceptable range. Another option is to break the 5d Lorentz symmetry in the bulk [71, 72]. The key observation is that the stability of the loop generated scalar potential relies essentially on 5d gauge symmetry and not so much on the SO(1,4) Lorentz symmetry. If one breaks either explicitly or by some dynamics this Lorentz symmetry keeping the SO(1,3) Lorentz symmetry in the ordinary space–time dimension intact, then one can enhance the Higgs coupling to fermions. Such breaking can be parametrized by the following pieces of the Lagrangian:

$$\mathscr{L}_{\text{gauge}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\boldsymbol{a}}{4}F_{\mu 5}F^{\mu 5};$$
$$\mathscr{L}_{Yuk} = \bar{\Psi}(\mathrm{i}\gamma_\mu D^\mu - \boldsymbol{k}D_5\gamma^5)\Psi, \tag{8.18}$$

where the prefactors $\boldsymbol{a}$ and $\boldsymbol{k}$ need to be phenomenologically tuned to match the data.

(*ii*) If one goes to an even higher dimensional model, e.g. a 6d GHU scenario, the gauge kinetic term contains a quartic interaction for the internal components of the gauge fields, i.e. it yields a quartic term in the Higgs potential at the tree level. Its strength of course depends on the gauge coupling. The appearance of this tree level quartic coupling can solve the 'low Higgs mass' problem. But, in these scenarios, gauge symmetry allows some orbifold localized operator which gives Higgs mass terms at the tree level, and this brings back the quadratic cutoff sensitivity as encountered in the SM. The question is, therefore, how to tame this quadratic cutoff sensitivity. This was pursued in [73] with the SU(3) gauge group on T^2/Z_N orbifold (with $N = 2, 3, 4, 6$). It was shown that only for $N = 2$, under the assumption of successful EWSB, a condition $m_h = 2M_W$ has to be satisfied to keep the scalar potential free from quadratic divergence.

(*iii*) If one goes to the warped scenario [74], additional features emerge [75]. AdS/CFT correspondence [11] tells us that a weakly coupled theory in 5d AdS is equivalent to a strongly coupled 4d theory. In this case, the Higgs is a composite particle, a pseudo-Goldstone boson of the strongly coupled CFT sector. There is a global symmetry in the CFT sector that protects the Higgs mass. Gauge and Yukawa interactions are introduced in the dual 5d AdS theory, which explicitly break the global symmetry but do not induce quadratic divergence to the Higgs mass at any loop. The Higgs mass can be large enough thanks to the quartic interaction which can be generated dynamically at tree level. The quadratic term is, as expected, loop generated. The all order finiteness of the Higgs mass can be intuitively understood as follows. The Higgs is at the TeV brane and a scalar which breaks the gauge symmetry is at the Planck brane and the information of this breaking reaches from Planck to TeV brane by bulk propagators. This is a non-local effect which is the reason behind the finiteness of the Higgs mass. This type of model was further consolidated in [76] by considering a

SO(5) × U(1)$_{\text{B-L}}$ symmetry in the bulk, which eventually gives SO(3) custodial symmetry that prohibits any large correction to the oblique T parameter. The electroweak symmetry is dynamically broken by the top quark.

One distinct advantage of working in the warped space over the flat space is noteworthy. Recall that in the GHU context the Yukawa coupling of the Higgs arises from higher dimensional gauge coupling. In the context of the Hosotani mechanism [69] in flat space without any large brane kinetic term, the 5d gauge coupling $g_5\sqrt{R^{-1}} = g_4 \equiv g \sim 0.65$ is rather small to yield the Higgs quartic coupling. On the other hand, in the warped case the AdS dynamics gives a rather large 5d gauge coupling $g_5\sqrt{k} \geqslant 4$ [76], which is why the Higgs quartic coupling can be sufficiently large to yield the correct Higgs mass.

There are other GHU constructions in flat and warped space with different features, which we are not going to cover here. We refer the readers to the papers in [77] and to two excellent reviews on composite Higgs scenarios [24, 78].

8.5. Comparison between gauge–Higgs/composite scenario and little Higgs models

Conceptually, gauge–Higgs models and little Higgs models are related [75, 76]. More precisely, through the AdS/CFT correspondence GHU in a 5d warped scenario (Randall–Sundrum model) replicates a little Higgs model in 4d. In the conventional (i.e. the way we developed the idea in this review) little Higgs models the sensitivity of the Higgs mass to the UV cutoff is logarithmic at one-loop and quadratic at two-loop. In the composite picture, which is dual to 5d gauge theory where the 5th component of the gauge boson makes the Higgs boson, the Higgs mass is finite at all orders. The little Higgs models are calculable below the cutoff scale ($\sim$10 TeV), while the QCD-like composite models are calculable in the large N limit allowing $1/N$-expansion. There is another difference between the composite models and the little Higgs models. The global symmetry that protects the Higgs mass in a composite model is a symmetry of the strong CFT sector and not of the SM. Hence the new TeV scale resonances form a complete multiplet of the global group of the strong sector, unlike in the little Higgs models where the new states are the partners of the SM particles. Another distinguishing feature is the presence of a KK gluon in the extra-dimensional models that is absent in the conventional little Higgs constructions.

8.6. Collider signals of gauge–Higgs unification models

Are there *smoking gun* signals of the GHU models? These models generally contain fermions with exotic electric charge, e.g. (5/3). But the exact value of the charge is a model-dependent question. In most cases, the lightest nonstandard particle turns out to be a colored fermion and not any exotic (KK) gauge boson. This has got something to do with the fact that large contributions from the exotic fermions are crucial in triggering correct amount of EWSB. Also, the gauge boson coupling to the right-handed top quark in such scenarios is about 10–15% different from its SM value. In a study [79], the

indirect effects of the KK particles on the Higgs production via gluon fusion and Higgs decay to two photons were analyzed in the context of a toy 5d scenario with SU(3) gauge group on an S^1/Z_2 orbifold. If the KK states weigh around 1 TeV, the loop effects provide about 10% deviation from the SM results. Moreover, the overall sign of the gluon–gluon-Higgs coupling was claimed to be opposite to the one in the SM or the UED model, but consistent with the corresponding sign in the little Higgs or the supersymmetric models. In a warped scenario with SO(5) $\times$ U(1)$_X$ gauge symmetry in the bulk (chosen for preserving custodial symmetry), the authors of [80] have studied the LHC detection of a KK top quark which is strongly coupled to a KK gluon. In the composite Higgs context, one of the crucial tests is to measure the scattering of the longitudinal gauge bosons ($V_L V_L \rightarrow V_L V_L$) and find an excess event (see [24] for a pedagogical illustration).

9. Higgsless scenarios

The idea is to trigger EWSB without actually having a physical Higgs. The mechanism relies on imposing different boundary conditions (BCs) on gauge fields in an extra-dimensional set-up. The BCs can be carefully chosen such that the rank of a gauge group can be lowered. For the purpose of illustration outlined in this review, we heavily rely on the discussions given in [81, 82]. To start with, we consider a 5d gauge theory. The extra dimension is compactified on a circle of radius R with a $y \leftrightarrow (-y)$ identification, i.e. on a S^1/Z_2 orbifold. The fixed points are $y = 0, \pi R$. We can use different BCs at the two fixed points.

9.1. Types of boundary conditions

Let us consider a 5d scalar field $\phi(x, y)$ in the interval $[0, \pi R]$. The minimization of action requires *either* or *both* of the following:

- $\phi|_{y=0,\pi R} = $ constant. When the constant $= 0$, it is called the *Dirichlet BC*.
- $(\partial_5\phi + V\phi)|_{y=0,\pi R} = 0$, where V is some boundary mass parameter. When $V = 0$, $\partial_5\phi = 0$, which is called the *Neumann BC*. When $V \neq 0$, it corresponds to a mixed BC.

Although we took a scalar field for demonstration, the BCs can be applied to any other field as well. We now perform some warm-up exercises to appreciate the essential features of Higgsless scenarios.

9.2. Breaking $SU(2) \rightarrow U(1)$ by BCs

This is a simple example to demonstrate that by appropriate choices of BCs we can indeed get a massless gauge boson state (to be identified with the photon) and massive states (to be identified with the W and Z boson). Consider a SU(2) gauge symmetry in 5d. The gauge bosons are $A_M^a(x, y)$, where $a = 1, 2, 3$ and $M = \mu, 5$. Now we apply the BCs at the two

fixed points:

- $\partial_5 A_\mu^a|_{y=0} = 0$ for $a = 1, 2, 3$, i.e. at the $y = 0$ fixed point, we apply the Neumann BC for all the three gauge bosons.
- $A_\mu^{1,2}|_{y=\pi R} = 0$, $\partial_5 A_\mu^3|_{y=\pi R} = 0$, i.e. at the $y = \pi R$ fixed point, we apply the Dirichlet BC for the first two components of the gauge bosons and the Neumann BC for the third component.

The y-dependent parts of the various KK mode gauge fields are then

$$A_\mu^3(y) \Longrightarrow \cos\left(\frac{ny}{R}\right) \qquad (n = 0, 1, 2, \ldots),$$

$$A_\mu^{1,2}(y) \Longrightarrow \cos\left(\frac{(2m+1)y}{2R}\right) \qquad (m = 0, 1, 2, \ldots).$$

(9.1)

Their mass spectra are therefore given by

$$A_\mu^3 \Longrightarrow M_n = 0, \frac{1}{R}, \frac{2}{R}, \ldots,$$

$$A_\mu^{1,2} \Longrightarrow M_m = \frac{1}{2R}, \frac{3}{2R}, \frac{5}{2R}, \ldots. \quad (9.2)$$

Thus we identify

$$M_\gamma = 0, \qquad M_W = \frac{1}{2R}, \qquad M_Z = \frac{1}{R}. \quad (9.3)$$

Clearly, this is not a phenomenologically acceptable situation as $M_Z = 2M_W$. The main problem here is that the gauge boson masses are independent of the gauge couplings. Somehow, we have to bring that dependence in.

9.3. Breaking $SU(2) \rightarrow$ 'nothing' by BCs

Let us impose the following BCs:

- $\partial_5 A_\mu^a|_{y=0} = 0$ for $a = 1, 2, 3$. This is just like the previous example.
- $\partial_5 A_\mu^a|_{y=\pi R} = V A_\mu^a|_{y=\pi R}$. This is a mixed BC. The $V \rightarrow 0$ limit corresponds to the Neumann BC and the $V \rightarrow \infty$ limit corresponds to the Dirichlet BC. Note that in the previous example, we took the $V \rightarrow \infty$ limit for $a = 1, 2$ and $V \rightarrow 0$ limit for $a = 3$.

A general solution that satisfies the above BCs is

$$A_\mu^a(x, y) = \sum_{n=1}^{\infty} A_\mu^{a(n)}(x) f_n(y),$$

with

$$f_n(y) = \alpha_n \frac{\cos(M_n y)}{\sin(M_n \pi R)}. \quad (9.4)$$

Since $f_n(y)$ is a cosine expansion, the BC at $y = 0$ is trivially satisfied. The BC at $y = \pi R$ leads to

$$\sum_{n=1}^{\infty} A_\mu^{(n)}(x) \frac{(-)\alpha_n M_n \sin(M_n y)}{\sin(M_n \pi R)}\bigg|_{y=\pi R}$$

$$= V \sum_{n=1}^{\infty} A_\mu^{(n)}(x) \frac{\alpha_n \cos(M_n y)}{\sin(M_n \pi R)}\bigg|_{y=\pi R}, \quad (9.5)$$

which leads to the following transcendental equation from where the mass spectrum is obtained:

$$M_n \tan (M_n \pi R) = -V. \qquad (9.6)$$

Now observe the following:

(i) when $V = 0$, which corresponds to the Neumann BC for all $a = 1, 2, 3$, gauge symmetry is unbroken;

(ii) when $V \neq 0$, the SU(2) gauge symmetry is fully broken. The amount of breaking is controlled by V. The mass spectrum is given by the solution of the above transcendental equation.

The normalization factor α_n is determined by requiring that the KK modes are canonically normalized, i.e.

$$\int_0^{\pi R} \mathrm{d}y f_n^2(y) = 1. \qquad (9.7)$$

Therefore, using equation (9.3),

$$\alpha_n = \frac{\sqrt{2}}{\sqrt{\pi R \, \mathrm{cosec}^2(M_n \pi R) + \dfrac{\cot(M_n \pi R)}{M_n}}}. \qquad (9.8)$$

Now using the transcendental equation (9.6), one can express

$$\alpha_n = \frac{\sqrt{2}}{\sqrt{\pi R \left(1 + \dfrac{M_n^2}{V^2}\right) - \dfrac{1}{V}}}. \qquad (9.9)$$

Now we are all set to calculate the mass spectrum. Let us consider the following two cases:

(i) $V = 0$: no breaking of gauge symmetry. All A_μ^a ($a = 1, 2, 3$) have a cosine expansion.

(ii) $V \neq 0$: we assume $V \gg \frac{1}{R}$, then the transcendental equation (9.6) implies that to the zeroth approximation $\cot(M_n \pi R) = 0$. This means $M_n \pi R = (2n + 1)\frac{\pi}{2}$, i.e.

$$M_n = \frac{2n + 1}{2R} \qquad (n = 0, 1, 2, \ldots). \qquad (9.10)$$

Then to the next level of approximation, we take $M_n \pi R = (2n + 1)\frac{\pi}{2} + \epsilon$, where ϵ is a small number. Then $\cot(M_n \pi R) = \cot\{(2n + 1)\frac{\pi}{2} + \epsilon\} = \cot(2n + 1)\frac{\pi}{2} + \epsilon\{-\mathrm{cosec}^2(2n + 1)\frac{\pi}{2}\} = -\epsilon$. Putting back the above relation into the transcendental equation, we obtain $\epsilon = \frac{M_n}{V}$. Therefore, $M_n \pi R = (2n + 1)\frac{\pi}{2} + \frac{M_n}{V}$, i.e.

$$M_n \simeq \frac{2n + 1}{2R} \left(1 + \frac{1}{\pi R V} + \cdots\right) \qquad (n = 0, 1, 2, \ldots). \qquad (9.11)$$

Clearly, there is no zero mode. SU(2) gauge symmetry is thus completely broken.

9.4. A model of EWSB by BCs: Higgsless scenario in flat space

Right at the beginning, we set two goals:

(i) the gauge boson masses have to be related to the gauge couplings;

(ii) there should be a custodial symmetry in the bulk so as to be consistent with EWPT.

We therefore start with the gauge symmetry $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ in the bulk. The notation of gauge bosons and gauge couplings are as follows (*the dimension of a 5d gauge coupling is $M^{-1/2}$*):

• group: $SU(2)_L$, gauge coupling: g, gauge bosons: A_M^{La} where $a = 1, 2, 3$;

• group: $SU(2)_R$, gauge coupling: g, gauge bosons: A_M^{Ra} where $a = 1, 2, 3$;

• group: $U(1)_{B-L}$, gauge coupling: g', gauge bosons: B_M.

We denote the gauge bosons of the $SU(2)_D$ group, which is the diagonal subgroup of $SU(2)_L \times SU(2)_R$, as A_M^{+a}, where $A_M^{\pm a} = \frac{1}{\sqrt{2}}(A_M^{La} \pm A_M^{Ra})$. We should remember that in order to have a zero mode of a generic gauge boson A_μ, i.e. to preserve the gauge symmetry, one should use the Neumann BC: $\partial_5 A_\mu = 0$. Although we display below the BCs of the gauge fields A_μ and B_μ only, the conditions for A_5 and B_5 are not hard to obtain. We just have to remember that the conditions have to be swapped between the μ and y components. In other words, the Dirichlet BCs for gauge bosons mean Neumann BCs for the corresponding scalars and vice versa. We now apply the following BCs at the two fixed points:

• $y = 0$ *fixed point*:

  (i) $\partial_5 A_\mu^{+a} = 0$ and $\partial_5 B_\mu = 0$ (i.e. $SU(2)_L \times SU(2)_R$ broken down to $SU(2)_D$, also $U(1)_{B-L}$ unbroken).

  (ii) $A_\mu^{-a} = 0$ (i.e. the SU(2) orthogonal to $SU(2)_D$ is broken).

• $y = \pi R$ *fixed point*:

  (i) $\partial_5 A_\mu^{La} = 0$ (i.e. $SU(2)_L$ unbroken).

  (ii) $\partial_5 A_\mu^{R\,1,2} = V A_\mu^{R\,1,2}$, where $V = -\frac{1}{4}g^2 v_R^2$. At the $y = \pi R$ brane we localize a scalar doublet under $SU(2)_R$, which acquires a VEV v_R leading to $SU(2)_R \times U(1)_{B-L}$ breaking down to $U(1)_Y$. Eventually, we take the $v_R \to \infty$ limit and the scalar will decouple without spoiling unitarity.

  (iii) $\partial_5 A_\mu^{R3} = \frac{V}{g}(g A_\mu^{R3} - g' B_\mu)$.

  (iv) $\partial_5 B_\mu = -\frac{V g'}{g^2}(g A_\mu^{R3} - g' B_\mu)$.

  The last three BCs ensure that both $SU(2)_R$ and $U(1)_{B-L}$ are broken when $V \neq 0$. Note additionally that $\partial_5(g' A_\mu^{R3} + g B_\mu) = 0$. Finally, the only symmetry left unbroken is $U(1)_Q$.

The BCs originate from the following consideration: the orbifold projection around $y = 0$ fixed point has a $SU(2)_L \rightleftharpoons SU(2)_R$ outer automorphism, while around $y = \pi R$ fixed point the orbifold projections are $SU(2)_L \leftrightarrow SU(2)_L$ and

$SU(2)_R \leftrightarrow SU(2)_R$. Define $\hat{y} = y + \pi R$. The BCs can be derived from

$$A_\mu^{La}(x, -y) = A_\mu^{Ra}(x, y), \qquad B_\mu(x, -y) = B_\mu(x, y),$$
$$A_\mu^{La}(x, -\hat{y}) = A_\mu^{La}(x, \hat{y}), \qquad A_\mu^{Ra}(x, -\hat{y}) = A_\mu^{Ra}(x, \hat{y}),$$
$$B_\mu(x, -\hat{y}) = B_\mu(x, \hat{y}). \tag{9.12}$$

Once the BCs are enforced, a given 4d gauge field is shared among many 5d fields. We now take the $V \to \infty$ limit. Then the 5d gauge fields in the $(A_\mu^L, A_\mu^R, B_\mu)$ basis can be expressed in terms of the 4d fields, namely γ_μ (photon), $Z_\mu^{(n)}$ and $W_\mu^{\pm(n)}$, in the following way $(A_\mu^{L,R\pm} = (A_\mu^{L,R1} \mp iA_\mu^{L,R2})/\sqrt{2})$:

$$B_\mu(x, y) = \frac{1}{\sqrt{\pi R(g^2 + 2g'^2)}}$$
$$\times \left[g\gamma_\mu(x) + \sqrt{2}g' \sum_{n=1}^{\infty} Z_\mu^{(n)}(x) \cos(M_Z^{(n)} y) \right],$$

$$A_\mu^{L3}(x, y) = \frac{1}{\sqrt{\pi R(g^2 + 2g'^2)}}$$
$$\times \left[g'\gamma_\mu(x) - \sqrt{2}g \sum_{n=1}^{\infty} Z_\mu^{(n)}(x) \frac{\cos(M_Z^{(n)}(y - \pi R))}{2\cos(M_Z^{(n)} \pi R)} \right],$$

$$A_\mu^{R3}(x, y) = \frac{1}{\sqrt{\pi R(g^2 + 2g'^2)}}$$
$$\times \left[g'\gamma_\mu(x) - \sqrt{2}g \sum_{n=1}^{\infty} Z_\mu^{(n)}(x) \frac{\cos(M_Z^{(n)}(y + \pi R))}{2\cos(M_Z^{(n)} \pi R)} \right],$$

$$A_\mu^{L\pm}(x, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} W_\mu^{n\pm}(x) \cos(M_W^{(n)}(y - \pi R)),$$

$$A_\mu^{R\pm}(x, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} W_\mu^{n\pm}(x) \cos(M_W^{(n)}(y + \pi R)).$$
$$\tag{9.13}$$

Thus, we obtain the massless photon γ, corresponding to the unbroken $U(1)_Q$, and some KK towers of massive $W^{(n)\pm}$ and $Z^{(n)}$ gauge bosons. The $Z^{(1)}$ and $W^{(1)\pm}$ are to be identified with the observed Z and $W^\pm$ bosons, respectively.

9.4.1. The charged $W^{(n)\pm}$ tower. The solutions would be similar to the one as obtained from the transcendental equation for the $SU(2) \to$ 'nothing' case.

$$M_W^{(n)} \tan(2M_W^{(n)} \pi R) = -V = \frac{1}{4}g^2 v_R^2, \tag{9.14}$$

which leads to the solution

$$M_W^{(n)} = \left(\frac{2n-1}{4R} \right) \left(1 - \frac{2}{\pi R g^2 v_R^2} + \cdots \right)$$
$$(n = 1, 2, \ldots). \tag{9.15}$$

9.4.2. The neutral $Z^{(n)}$ tower. We enforce the BC at $y = \pi R$:

$$\partial_5 A_\mu^{R3} = \frac{V}{g}(gA_\mu^{R3} - g'B_\mu). \tag{9.16}$$

The lhs of equation (9.16) is

$$\partial_5 A_\mu^{R3} \bigg|_{y=\pi R} = \frac{\sqrt{2}g}{\sqrt{\pi R(g^2 + 2g'^2)}}$$
$$\times \sum_{n=1}^{\infty} M_Z^{(n)} \sin(M_Z^{(n)} \pi R) Z_\mu^{(n)}(x).$$

The rhs of equation (9.16) can be written as

$$\frac{\sqrt{2}g}{\sqrt{\pi R(g^2 + 2g'^2)}} \frac{v_R^2}{4} \sum_{n=1}^{\infty}$$
$$\times \left[g'^2 \cos(M_Z^{(n)} \pi R) + g^2 \frac{\cos(2M_Z^{(n)} \pi R)}{2\cos(M_Z^{(n)} \pi R)} \right] Z_\mu^{(n)}(x).$$

Therefore

$$M_Z^{(n)} \sin(M_Z^{(n)} \pi R)$$
$$= \frac{v_R^2}{4} \left[g'^2 \cos(M_Z^{(n)} \pi R) + g^2 \frac{\cos(2M_Z^{(n)} \pi R)}{2\cos(M_Z^{(n)} \pi R)} \right],$$

which leads to the simplified form of the eigenvalue equation as

$$M_Z^{(n)} \tan(M_Z^{(n)} \pi R) = \frac{v_R^2}{8}(g^2 + 2g'^2) - \frac{v_R^2 g^2}{8} \tan^2(M_Z^{(n)} \pi R). \tag{9.17}$$

9.4.3. Solution of equation (9.17). We rewrite the equation as

$$M_Z^{(n)} \pi R \tan(M_Z^{(n)} \pi R)$$
$$= \frac{\pi R g^2 v_R^2}{8}[\tan^2(M_0 \pi R) - \tan^2(M_Z^{(n)} \pi R)],$$
$$\text{where } \tan^2(M_0 \pi R) = \left(1 + \frac{2g'^2}{g^2} \right).$$

Now we take the limit $v_R \to \infty$. Then, $[\tan^2(M_0 \pi R) - \tan^2(M_Z^{(n)} \pi R)] = 0$ is our zeroth approximation, so that the lhs of equation (9.17) is finite. The solution is

$$\tan(M_Z^{(n)} \pi R) = \pm \tan(M_0 \pi R).$$

Let us first take the (+) sign solution and proceed. Then

$$\tan(M_Z^{(n)} \pi R) = + \tan(M_0 \pi R) = \tan(M_0 \pi R + (n-1)\pi),$$
$$(n = 1, 2, \cdots),$$

which means

$$M_Z^{(n)} = M_0 + \frac{n-1}{R}.$$

Now, instead of taking $v_R \to \infty$, if we take v_R to be large and expand in its inverse powers, we obtain

$$M_Z^{(n)} = \left(M_0 + \frac{n-1}{R} \right) \left[1 - \frac{2}{(g^2 + g'^2)v_R^2 \pi R} + \cdots \right]$$
$$(n = 1, 2, \ldots). \tag{9.18}$$

If we take the (−) sign solution in the zeroth order approximation, then through similar steps, we obtain

$$M_{Z'}^{(n)} = \left(-M_0 + \frac{n}{R} \right) \left[1 - \frac{2}{(g^2 + g'^2)v_R^2 \pi R} + \cdots \right]$$
$$(n = 1, 2, \ldots). \tag{9.19}$$

Thus we see there are two towers of neutral bosons: the Z tower has a spectrum given by equation (9.18) and the Z' tower spectrum is given by equation (9.19). It is not unexpected to have two towers, as the solutions come from a quadratic equation.

9.4.4. Range of M_0. Let us recall that

$$M_0 = \frac{1}{\pi R} \tan^{-1} \sqrt{1 + \frac{2g'^2}{g^2}}.$$

The maximum value of any $\tan^{-1}$ is $\frac{\pi}{2}$. The minimum value of $\tan^{-1}\sqrt{1 + \frac{2g'^2}{g^2}}$ is $\tan^{-1}(1) = \frac{\pi}{4}$. These limits set the range of M_0:

$$\frac{1}{4R} < M_0 < \frac{1}{2R}. \tag{9.20}$$

For $v_R \to \infty$, we get the following range of the masses of the lightest ($n = 1$) KK state of the Z and Z' towers:

$$M_Z \equiv M_Z^{(1)} = M_0 = \left[\frac{1}{4R}, \frac{1}{2R} \right],$$

$$M_{Z'}^{(1)} = -M_0 + \frac{1}{R} = \left[\frac{1}{2R}, \frac{3}{4R} \right]. \tag{9.21}$$

In fact, the Z' boson is heavier than the Z boson level by level, i.e. $M_{Z'}^{(n)} > M_Z^{(n)}$. The mass of the $W^{(1)}$ boson (which is in fact the W boson of the SM) putting $n = 1$ in equation (9.15) and letting $v_R \to \infty$, is given by

$$M_W \equiv M_W^{(1)} = \frac{1}{4R}, \qquad \text{i.e.} \quad M_Z > M_W \tag{9.22}$$

as expected.

9.4.5. The 4d gauge couplings (g_4, g_4') and the ρ parameter. From equation (9.13) we take the expression for B_μ and look at its expansion for $y = 0$:

$$B_\mu(x, 0) = \frac{1}{\sqrt{\pi R(g^2 + 2g'^2)}}$$
$$\times \left[g\gamma_\mu(x) + \sqrt{2}g'Z_\mu^{(1)}(x) + \text{higher } Z_\mu^{(n)} \text{ terms} \right]. \tag{9.23}$$

Note that the mass dimension of the 5d B_μ is $\frac{3}{2}$ while that of the 4d B_μ is 1. Now we compare equation (9.23) with the SM expression of B_μ in terms of the photon and Z boson fields, namely,

$$B_\mu = \frac{1}{\sqrt{g_4^2 + g_4'^2}} \left[g_4\gamma_\mu + g_4'Z_\mu \right]. \tag{9.24}$$

It immediately follows that $(g_4'/g_4) = (\sqrt{2}g'/g)$. We are now all set to calculate the ρ-parameter in this scenario:

$$\frac{M_W^2}{M_Z^2} \equiv \frac{(M_W^{(1)})^2}{(M_Z^{(1)})^2} = \frac{1}{16R^2 M_0^2} = \frac{\pi^2}{16} \left(\tan^{-1} \sqrt{1 + \frac{2g'^2}{g^2}} \right)^{-2}$$

$$= \frac{\pi^2}{16} \left(\tan^{-1} \sqrt{1 + \frac{g_4'^2}{g_4^2}} \right)^{-2} \sim 0.85. \tag{9.25}$$

Hence,

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} \sim 1.10. \tag{9.26}$$

We summarize now what we have learned from this scenario.

(*i*) A big achievement is that the W and Z boson masses depend on the gauge couplings. Without actually having a Higgs boson, just by applying BCs on the boundaries, one can obtain the correct W and Z masses.

(*ii*) In this scenario $\Delta\rho \sim 10\%$ is far too large. This scenario is thus disfavored by EWPT. A slightly more acceptable value of ρ can be obtained by keeping a finite v_R at the $y = \pi R$ fixed point. Then unitarizing the theory would be a problem. The reason for such a large $\Delta\rho$ is the following. Although the bulk and the $y = 0$ brane respect custodial SU(2), the $y = \pi R$ brane does not. Since the KK wave functions have significant presence around the $y = \pi R$ brane, a large $\Delta\rho$ results. The remedy lies in expelling the higher ($n > 1$) KK modes from the custodial symmetry breaking brane.

9.5. Features of realistic Higgsless scenarios

9.5.1. Warped models and oblique parameters. One of the advantages of going to the warped extra dimension is that the contributions to the S and T parameters can be kept under control. Following [83], we consider a conformally flat metric

$$ds^2 = h(y)^2(\eta_{\mu\nu}dx^\mu dx^\nu - dy^2), \tag{9.27}$$

where the extra spatial dimension is in the interval $[R, R']$. A flat extra dimension scenario can be recovered if $h(y) = \text{constant}$, while the AdS limit is obtained when $h(y) = R/y$. Typically, $R^{-1} \approx M_{Pl}$ and $(R')^{-1} \approx \text{TeV}$ scale. The gauge symmetry in the bulk corresponds to SU(2)$_L \times$ SU(2)$_R \times$ U(1)$_{B-L}$, and the choice of the left-right gauge symmetry is motivated from the requirement of a custodial symmetry for EWPT consistency. The W and Z boson masses in this scenario are given by (with the approximation $R' \gg R$)

$$M_W^2 \approx \frac{1}{R'^2 \ln\left(\frac{R'}{R}\right)}, \qquad M_Z^2 \approx \frac{g^2 + 2g'^2}{g^2 + g'^2} \frac{1}{R'^2 \ln\left(\frac{R'}{R}\right)}, \tag{9.28}$$

where $g(= g_L = g_R)$ and g' are 5d SU(2) and U(1) gauge couplings, respectively. To leading order, T (or equivalently $\Delta\rho$) and S are both vanishing—this is the limit when the warp factor is infinitely large, i.e. when the Planck brane is moved to the AdS boundary. For a finite warped factor, T and S will be suppressed by $\ln(R'/R)$. Since a 5d warped model is dual, in the AdS/CFT sense, to a 4d theory involving a strongly coupled sector which is conformally invariant between the Planck scale and the TeV scale, a lot of insight can be gained about the origin of T and S suppression from this correspondence. Weakly charged left-right gauged symmetry in the 5d bulk ensures a global custodial symmetry in the strongly coupled CFT side which keeps T and S under control [84].

An important question that naturally arises in the Higgsless context is how to generate the fermion masses. In the absence of a Higgs, one cannot write a Yukawa coupling. However, just like in the case of gauge bosons, appropriate BCs for fermions would generate their masses. But where to

localize the fermions? They cannot be localized at the UV (Planck) brane where the gauge symmetry is that of the SM. The reason is that the theory at the UV brane is chiral and there is no way a zero mode chiral fermion mass can be generated. On the other hand in the IR (TeV) brane the unbroken gauge symmetry corresponds to $SU(2)_D$ which preserves isospin and would yield equal up- and down-type masses. So the SM fermions have to be placed inside vector-like multiplets residing in the 5d bulk which should feel different gauge symmetry breaking at the two boundaries [85]. Needless to mention that orbifold projection, or equivalently a set of appropriate BCs, removes half of the vector-like spectrum yielding a chiral fermion structure at the lowest KK level. Also, when the fermions are delocalized from the boundaries and judiciously placed at different locations in the bulk, their couplings with the KK gauge bosons can be made to vanish, which minimizes the S parameter [84].

The third family continues to give some headache. The requirement of a large top quark mass necessitates the localization of the t_L (and hence b_L) field(s) near the TeV brane. At this brane, because the unbroken gauge symmetry is $SU(2)_D \times U(1)_{B-L}$, the Zb_Lb_L coupling is different from its SM value, which leads to a contradiction with the precision measurement of the Zbb vertex through R_b. This problem can be solved, but at the price of making the model more complicated, e.g. by invoking a separate mechanism of the top quark mass generation (analogous to the concept of topcolor in technicolor models). To sum up, the localization of the third family is a major thorn in the construction of realistic Higgsless models.

Moose models. Several features of Higgsless models have been investisgated in the context of *deconstructed* gauge theories by discretizing the extra dimension. By doing it we get a finite set of 4d gauge theories, each corresponding to a particular lattice site [86]. The fifth component of the gauge field, A_5, which is the connection field, goes into the definition of the 'link variable' $\Sigma_i \equiv \exp(-iaA_5^{(i-1)})$ realizing the parallel transport between two lattice sites, where a is the lattice spacing. The link variables can be identified with 'chiral fields' which satisfy the condition $\Sigma\Sigma^\dagger = 1$ [87]. In this way, the 5d gauge theory is replaced by a collection of 4d gauge theories with chiral fields Σ_i having gauge interactions—this is described by 'moose diagram'. A moose diagram is like a Feynman diagram where *lines* correspond to *links* and *vertices* to *gauge groups*. If there is no loop, then one can show that $G = E - 1$, where G is the number of remaining Goldstone multiplets and E is the number of external links. Clearly, we need at least two external links to construct a minimal model (which has only one Goldstone multiplet). It has been shown that in this scenario the S parameter can be made vanishingly small either by ideal fermion delocalization [88], or by introducing a dynamical non-local field connecting the two ends of a moose [89].

9.5.2. Tension between unitarity and EWPT. This is a major issue that decides the fate of a Higgsless model. We follow the discussions in [90]. First, we ask the obvious question:

what unitarizes the theory in the absence of the Higgs? If we consider the elastic scattering process $W_L^\pm Z_L \to W_L^\pm Z_L$, then in the absence of the Higgs boson the amplitude will go like

$$(g_{WWZZ} - g_{WWZ}^2)[aE^4 + bE^2 + \cdots], \tag{9.29}$$

where the notation for the three- and four-point gauge couplings are self-explanatory. In the Higgsless models, there are additional KK gauge bosons. The charged vector boson KK states $V_i^\pm$, which the same V_iWZ Lorentz structure as the SM WWZ, would contribute to the above amplitude. However, once we take into account the contributions from all the states $i = 1, 2, \cdots, \infty$ and two sum rules involving the trilinear gauge couplings are satisfied, the new contributions completely cancel the E^4 and E^2 growths. This is a consequence of higher dimensional gauge symmetry. However, the residual growth would make the Higgsless theories break down at a few TeV scale. In fact, at higher energies an increasing number of inelastic channels leads to unitarity violation by inducing a linear growth. This is not unexpected from a 5d point of view, as the dimensionless 5d loop factor grows with energy as $g^2E/24\pi^3$, where g is the 5d gauge coupling. In the warped Higgsless scenarios, the naïve dimensional analysis (NDA) cutoff would boil down to

$$\Lambda_{NDA} \sim \frac{12\pi^4 M_W^2}{g^2 M_{W^{(1)}}}, \tag{9.30}$$

which is around 12 TeV, putting $M_{W^{(1)}} \sim 1.2$ TeV. Explicit calculation shows that this simple estimate is valid up to a factor of $1/4$ [91]. What we thus learned is that in the Higgsless scenario, because of the appearance of the new weakly coupled states in the TeV scale, the unitarity saturation is postponed by roughly a factor of 10 beyond the SM NDA cutoff scale $\Lambda_{NDA}^{SM} \sim 4\pi M_W/g \sim 1.8$ TeV. Clearly, the heavier the KK W boson the lower is the scale at which perturbative unitarity is lost. Now, $M_{W^{(1)}}^2/M_W^2 = \mathcal{O}(\ln(R'/R))$. If we increase R, i.e. lower the UV cutoff from the Planck scale, then the first KK W boson mass decreases from 1.2 TeV to sub-TeV and the NDA cutoff scale goes up. But one cannot arbitrarily increase R, as this would increase the T parameter which varies as $1/\ln(R'/R)$—see figure 4 of [90]—even though S can be kept under control via fermion delocalization. The tension between extending the domain of perturbative unitarity and at the same time fitting precision electroweak data have also been discussed in scenarios [92].

Three- and four-site Higgsless models. In the language of deconstruction, delocalization of fermions corresponds to allowing them to derive their electroweak properties from more than one lattice site or gauge group. It has been shown in [93] that a linear moose model, with several SU(2) gauge fields along the string and $SU(2)_L$ and $U(1)_Y$ as the two end-points, can reconcile EWPT constraints and increased unitarity bound at the expense of some fine tuning. It has been demonstrated in [94] that several properties of the Higgsless models, like ideal fermion delocalization, EWPT consistency, fermion masses, etc, can be illustrated in a highly deconstructed model with only three sites. The electroweak part of the gauge group

corresponds to SU(2) × SU(2) × U(1), i.e. it contains only one 'interior' SU(2) group. It therefore contains only one set of (W', Z') states, which can be arranged to be fermiophobic to minimize precision electroweak corrections. If one extends this three-site model by one more site, i.e. with one more interior SU(2) gauge group making it a four-site Higgsless model, the gauge boson resonances need not be fermiophobic to satisfy EWPT constraints [95].

9.5.3. Collider signatures of Higgsless models. The strongly coupled physics in the Higgsless scenario at a scale which is roughly 10 times $\Lambda_{\rm NDA}^{\rm SM}$ as a result of delayed unitarity violation, is too large to be observed at the LHC. But it will be possible to pin down those weakly coupled states which are responsible for unitarity postponement. In this context we follow the analysis in [96]. Different Higgsless models vary in different aspects, like fermion placements and how the SM particles interact with the KK states, but the mechanism by which Λ is raised is common to all. Weakly coupled TeV-size new massive vector bosons V_i (where i is the KK label), whose couplings to the SM gauge bosons are dictated by the *unitarity sum rules*, enforce the cancellation of the E^2 and E^4 terms in the amplitudes of longitudinal gauge boson scattering thereby postponing unitarity violation. What are the experimental signatures of V_i bosons? It is advantageous to consider the production of these vector bosons by the SM gauge boson fusions as the couplings between the V_i and the SM W/Z bosons are almost model independent, dictated by the unitarity sum rules. The sum rule implies the following inequality:

$$g_{WZV}^{(1)} \leqslant \frac{g_{WWZ}M_Z^2}{\sqrt{3}M_1^{\pm}M_W}. \qquad (9.31)$$

Putting $M_1^{\pm} = 700$ GeV gives $g_{WZV}^{(1)} \leqslant 0.04$, which means that the heavier the mass of V_1 the less the chance to produce it at the LHC. If we study the scattering channel $W^{\pm}Z \to W^{\pm}Z$, a process which can be mediated by $V_1^{\pm}$ in the s-channel, there will be a sharp resonance as soon as the V_1 threshold is crossed. Recall that a t-channel Higgs exchange unitarizes this amplitude in the SM which therefore does not give any resonance. Conventional theories of strong EWSB dynamics may give a somewhat heavier (∼2 TeV) resonance but that would be broad due to strong coupling. But in the Higgsless theories, V_1 can be as light as 700 GeV, and the resonance will be narrow because the V_1 decay width is very small. The reason is the following: the decay of $V_1^{\pm}$ takes place only in a single channel, and the width is given by

$$\Gamma(V_1^{\pm} \to W^{\pm}Z) \approx \frac{\alpha(M_1)^3}{144\sin^2\theta_W M_W^2}, \qquad (9.32)$$

under the assumption that the unitarity sum rule is saturated by the first set of KK vector boson states (i.e. with just $i = 1$ of V_i). Putting $M_1 = 700$ GeV, the width turns out to be only about 13 GeV. We must remember that $V_1^{\pm}$ do not have any significant fermionic couplings as otherwise EWPT consistency will be jeopardized. Further details of $V_1^{\pm}$ search strategies are beyond the scope of this review.

10. Conclusions and outlook

1. We take a snapshot of all the limits on the SM Higgs mass:
   - direct search: $m_h > 114.4$ GeV (at LEP-2, from non-observation in the $e^+e^- \to Zh$ channel);
   - EWPT: $m_h < 186$ GeV (at 95% CL, with direct search non-observation as a constraint in the fit);
   - perturbative unitarity: $m_h < 780$ GeV (in the $2W_L^+W_L^- + Z_LZ_L$ channel);
   - triviality: $m_h < 170$ GeV for $\Lambda = 10^{16}$ GeV (scalar quartic coupling should not hit the Landau pole);
   - vacuum stability: $m_h > 134$ GeV for $\Lambda = 10^{16}$ GeV (quartic coupling should always stay positive).

In the MSSM, there is a firm prediction on the upper limit of the lightest Higgs mass. If the top squarks weigh around a TeV, then $m_h \lesssim 135$ GeV.

2. All the BSM models we have considered are based on *calculability* and *symmetry* arguments. In all cases, the electroweak scale M_Z can be expressed in terms of some high scale parameters a_i, i.e. $M_Z = \Lambda_{\rm NP} f(a_i)$, where $\Lambda_{\rm NP}$ is the new physics scale and $f(a_i)$ are calculable functions of physical parameters. In all these models

   - the new physics scales originate from different dynamics: $\Lambda_{\rm SUSY} \sim M_S$ (the supersymmetry breaking scale), $\Lambda_{\rm LH} \sim f \sim F$ (the VEV associated with $G \to H$ breaking), $\Lambda_{\rm GHU} \sim R^{-1}$ (the inverse radius of compactification);
   - the dynamical sign-flip of a scalar mass-square happens not only in supersymmetry, but also in little Higgs models and in GHU scenarios. In all cases, the large top quark Yukawa coupling plays a crucial rôle. A positive scalar quartic coupling can be arranged in all these scenarios.

3. In supersymmetry, the cutoff can be as high as the GUT or the Planck scale. In little Higgs as well as in many variants of extra dimensional scenarios the cutoff is significantly lower. The ultraviolet completion in little Higgs models is an open question, though some attempts have already been made in this direction.

4. In supersymmetry the cancellation of quadratic divergence takes place between a particle loop and a sparticle loop. Since a particle *cannot* mix with a sparticle, the oblique electroweak corrections and the $Zb\bar{b}$ vertex correction can be kept under control. In the non-supersymmetric scenarios (recall what happens in little Higgs models), the cancellation occurs between loops with the same spin states. Such states *can* mix among themselves, leading to dangerous tree level contributions to electroweak observables. This is the reason why a decoupling theory like supersymmetry is comfortable with EWPT, while a technicolor-like non-decoupling theory faces a stiff challenge from EWPT.

5. How do we distinguish between the different models in colliders? We have already discussed some of the smoking gun signals of different scenarios. Here we highlight a few features that are the trademark signals of some specific models. We first compare supersymmetry with little Higgs models. The

chances are very high that one may mistake supersymmetry with R-parity for little Higgs with T-parity or vice versa in the LHC environment, since the pattern of cascade decays in the two models are very similar and particle spin measurements are, in general, difficult. A dictionary between superparticles and little Higgs heavy states is the following: (i) electroweak gauginos $\leftrightarrow$ T-odd gauge bosons, (ii) sfermions $\leftrightarrow$ T-odd fermion doublets, (iii) second Higgs doublet $\leftrightarrow$ scalar triplet, (iv) higgsinos and gluino $\leftrightarrow$ none, (v) none $\leftrightarrow$ T-even top partner. What is interesting to observe is that there is no analog of the gluino in little Higgs models, and no analog of t_+ in supersymmetry.

In a general class of composite Higgs models (e.g. GHU or little Higgs), the strengths of VVh and $VVhh$ couplings are different from their SM predictions. A recent work suggests that double Higgs production via $W_L W_L \to hh$ can be an interesting probe for verifying the compositeness of the Higgs since the rate of this process is much larger (than in the SM) if h is a pseudo-Goldstone boson [97].

The presence of a KK gluon in a GHU model differentiates it from a little Higgs model. Moreover, the gauge–Higgs models have a special feature that their lightest non-standard particle is a colored fermion and not a KK gauge boson. Such models also contain fermions with exotic electric charge, whose value is different in different models. The Higgsless models are characterized by the presence of the V_i vector boson states that delay the unitarity saturation. The lightest of such states may pop up in the scattering of $W^\pm Z \to W^\pm Z$ as an s-channel narrow resonance.

6. The model-builders have three-fold goals: (i) unitarize the theory, (ii) successfully confront the EWPT and (iii) maintain naturalness to the extent possible. The tension arises as naturalness criteria requires the spectrum to be compressed, while *EWPT compatibility* pushes the new states away from the SM states.

7. A dark matter candidate is badly needed to justify observational evidence. Besides the neutrino mass, dark matter provides the only other concrete experimental motivation to go beyond the SM. The SM fails to provide it. A favorite supersymmetric candidate is the lightest neutralino if R-parity is conserved. The little Higgs models provide a heavy stable gauge boson if T-parity (which can be defined in the 'littlest' Higgs model) is conserved. In extra-dimensional models, the lightest KK particle is a stable dark matter candidate if KK-parity is conserved.

8. After all is said and done, the LHC is a *win–win* machine in terms of discovery. If we discover the Higgs, we would expect to also discover the new states that tame the unruly quantum correction to its mass. If the Higgs is not there, the new resonances which would restore unitarity in gauge boson scattering would be crying out for verification. In order to identify the latter, we need the super-LHC (the high luminosity option) to cover the entire spectrum. However, once we observe some new states at the LHC, we definitely need a linear collider to know what these states actually are.

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