Hunting the Higgs

Tilman Plehn
SUPA, School of Physics, University of Edinburgh, Scotland

This is another of my frequent attempts to bridge the gap between the usual lectures on experimental or theoretical high–energy physics and the basic knowledge needed to understand research seminars on issues related to the upcoming LHC. During the LHC years we expect to hear many exciting talks about Higgs physics, and it might come in handy to know some basics about how the Higgs can be produced at the LHC, how it typically decays, which search channels are valid discovery channels or where we can measure properties of the Higgs boson. I hope this little writeup will be useful to cover some of the very basics of Higgs physics at the LHC and allow the more or less experienced reader to step into the middle of the endless literature on Higgs physics at the LHC...

Contents

I. THEORY CONSTRAINTS ON HIGGS SECTOR

Before we start looking for the famous Higgs Boson, mostly at the LHC, let us briefly review the Higgs mechanism. To make it a little more interesting we include higher dimensional operators on top of the usual renormalizable (dimension–four) operators in the Higgs potential. Such operators generally occur in effective weak–scale models which involve an ultraviolet completion of the Standard Model, but their effects are often small or assumed to be small.

A. Higgs and higher-dimensional operators

In the renormalizable Standard Model all terms in the Lagrangian are of mass dimension four, like $m_f\bar{\psi}\psi$ or $\bar{\psi}\partial_\mu\psi$ or $F_{\mu\nu}F^{\mu\nu}$. This mass dimension is easy to read off if we remember that for example, scalar fields or vector–boson fields contribute mass dimension one, while fermion spinors carry mass dimension 3/2. The renormalizability assumption we usually also make for all terms in the Higgs potential, leaving us with only two possible terms:

$$V_{\rm SM} = \mu^2|\phi|^2 + \lambda|\phi|^4 + \text{const.} \tag{1}$$

To make this brief review of the Higgs mechanism a little less boring to everyone who has seen it before, and to emphasize that renormalizability is a strong (and not necessarily very justified) theoretical assumption in LHC Higgs physics, we simply allow for a few more operators in the Higgs potential. If we expand the possible mass dimensions and the operator basis, there are exactly two gauge-invariant operators of dimension six we can write down in terms of the Higgs doublet $|\phi|^2$, *i.e.* before electroweak symmetry breaking:

$$\mathcal{O}_1 = \frac{1}{2}\partial_\mu(\phi^\dagger\phi)\,\partial^\mu(\phi^\dagger\phi) \qquad\qquad \mathcal{O}_2 = -\frac{1}{3}(\phi^\dagger\phi)^3 \tag{2}$$

The prefactors in the Lagrangian are conventional, because to construct a Lagrangian we have to multiply these operators with coefficients of mass dimension (−2):

$$\mathcal{L}_{\rm D6} = \sum_{i=1}^{2}\frac{f_i}{\Lambda^2}\mathcal{O}_i \tag{3}$$

The mass scale Λ suppresses these additional operators. In other words, as long as the typical energy scale E in the numerator in our matrix element is small ($E \ll \Lambda$), the corrections from the additional operators are small as well.

There is in principle one additional operator $(\mathcal{D}_\mu\phi)^\dagger(\phi^\dagger\phi)(\mathcal{D}^\mu\phi)$, but it violates the custodial symmetry and leads to a very large contribution to the Peskin–Takeuchi parameter $\Delta S = -f_3v^2/(2\Lambda^2)$. Such large contributions to S are firmly ruled out by electroweak precision data, so we ignore this operator in our analysis. With the additional dimension-6 operator $\mathcal{O}_2$ we can write the Higgs potential as

$$\boxed{V = \mu^2|\phi|^2 + \lambda|\phi|^4 + \frac{f_2}{3\Lambda^2}|\phi|^6} \tag{4}$$

Note the positive sign in the last term of the potential V, which corresponds to a negative sign in the Lagrangian and ensures that for $f_2 > 0$ the potential be bounded from below for large field values ϕ. The non–trivial minimum for $\phi \neq 0$ is given by

$$\begin{aligned}\frac{\partial V}{\partial\,|\phi|^2} = \mu^2 + \lambda|\phi|^2 + \frac{f_2}{3\Lambda^2}|\phi|^4 &\stackrel{!}{=} 0\\ |\phi|^4 + \frac{2\lambda\Lambda^2}{f_2}|\phi|^2 + \frac{\mu^2\Lambda^2}{f_2} &= 0\end{aligned} \tag{5}$$

which translates into

$$
\begin{aligned}
|\phi|^2 &= -\frac{\lambda\Lambda^2}{f_2} \pm \left[\left(\frac{\lambda\Lambda^2}{f_2} \right)^2 - \frac{\mu^2\Lambda^2}{f_2} \right]^{\frac{1}{2}} \;=\; \frac{\lambda\Lambda^2}{f_2} \left[-1 \;\pm\; \sqrt{1 - \frac{\mu^2 f_2}{\Lambda^2\lambda^2}} \right] \\
&\simeq \frac{\lambda\Lambda^2}{f_2} \left[-1 \;\pm\; \left(1 - \frac{f_2\mu^2}{2\lambda^2\Lambda^2} + \frac{f_2^2\mu^4}{8\lambda^4\Lambda^4} + \mathcal{O}(\Lambda^{-6}) \right) \right] \\
&= \begin{cases} -\frac{\mu^2}{2\lambda} + \frac{f_2\mu^4}{8\lambda^3\Lambda^2} + \mathcal{O}(\Lambda^{-4}) = -\frac{\mu^2}{2\lambda} \left(1 - \frac{f_2\mu^2}{4\lambda^2\Lambda^2} + \mathcal{O}(\Lambda^{-4}) \right) = \frac{v_0^2}{2} \left(1 - \frac{f_2 v_0^2}{4\lambda\Lambda^2} + \mathcal{O}(\Lambda^{-4}) \right) \\ -\frac{2\lambda\Lambda^2}{f_2^2} \end{cases}
\end{aligned}
\tag{6}
$$

The first solution we have expanded around the Standard Model minimum, $v_0^2 = -\mu^2/\lambda$. Note that from the W, Z masses we know that $v = 246$ GeV so in a way v is our first observable in the Higgs sector, sensitive to the higher–dimensional operators.

Before we compute the corrections to the Higgs mass and self coupling due to $\mathcal{O}_2$ we will first look at the effects of the dimension–6 operator $\mathcal{O}_1$. It contributes to the kinetic term the Higgs field in the Lagrangian (before or after symmetry breaking)

$$
\begin{aligned}
\mathcal{O}_1 &= \frac{1}{2}\partial_\mu(\phi^\dagger\phi)\;\partial^\mu(\phi^\dagger\phi) \\
&= \frac{1}{2}\partial_\mu \left(\frac{1}{2}(\tilde{H}+v)^2 \right) \partial^\mu \left(\frac{1}{2}(\tilde{H}+v)^2 \right) \\
&= \frac{1}{8}(\partial_\mu\tilde{H}^2 + 2v\partial_\mu\tilde{H})(\partial^\mu\tilde{H}^2 + 2v\partial^\mu\tilde{H}) \\
&= \frac{1}{8}(2\tilde{H}\partial_\mu\tilde{H} + 2v\partial_\mu\tilde{H})(2\tilde{H}\partial^\mu\tilde{H} + 2v\partial^\mu\tilde{H}) \\
&= \frac{1}{2}\partial_\mu\tilde{H}\;\partial^\mu\tilde{H}\;(\tilde{H}+v)^2
\end{aligned}
\tag{7}
$$

Note that I have used $\tilde{H}$ for the Higgs field as part of ϕ, because from the formula above it is clear that there will be a difference between $\tilde{H}$ and the physics Higgs field at the end of the day. The contribution of $\mathcal{O}_1$ leaves us with a combined kinetic term in the Lagrangian

$$
\mathcal{L}_{\text{kin}} \;\sim\; \frac{1}{2}\partial_\mu\tilde{H}\partial^\mu\tilde{H} \left(1 + \frac{f_1 v^2}{\Lambda^2} \right) \stackrel{!}{=} \frac{1}{2}\partial_\mu H\;\partial^\mu H \qquad \Leftrightarrow \qquad H = \sqrt{1 + \frac{f_1 v^2}{\Lambda^2}}\,\tilde{H} \equiv N \cdot \tilde{H}
\tag{8}
$$

This is a simple rescaling to define the <u>canonical kinetic term</u> in the Lagrangian, but it also means we have to make sure we use H in the complete Higgs sector.

The additional potential operator in terms of H (or $\tilde{H}$) reads

$$
\begin{aligned}
\mathcal{O}_2 &= -\frac{1}{3}(\phi^\dagger\phi)^3 = -\frac{1}{3}\left(\frac{1}{2}(\tilde{H}+v)^2 \right)^3 = -\frac{1}{24}(\tilde{H}+v)^6 \\
&= -\frac{1}{24}(\tilde{H}^6 + 6\tilde{H}^5 v + 15\tilde{H}^4 v^2 + 20\tilde{H}^3 v^3 + 15\tilde{H}^2 v^4 + 6\tilde{H} v^5 + v^6)
\end{aligned}
\tag{9}
$$

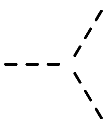
which gives us a combined mass term:

$$\begin{aligned}
\mathcal{L}_{\text{mass}} = -m_H^2 H^2 &= -\mu^2 \tilde{H}^2 - \frac{6}{2}\lambda v^2 \tilde{H}^2 - \frac{f_2}{\Lambda^2}\frac{15}{24} v^4 \tilde{H}^2 \\
&= -\left(\mu^2 + 3\lambda v^2 + \frac{5}{8}\frac{f_2 v^4}{\Lambda^2}\right)\tilde{H}^2 \\
&= -\left(-\lambda v^2\left(1 + \frac{f_2 v^2}{4\lambda\Lambda^2}\right) + 3\lambda v^2 + \frac{5}{8}\frac{f_2 v^4}{\Lambda^2}\right)\tilde{H}^2 \\
&= -\left(2\lambda v^2 - \frac{f_2 v^4}{4\Lambda^2} + \frac{5}{8}\frac{f_2 v^4}{\Lambda^2}\right)\left(1 + \frac{f_1 v^2}{\Lambda^2}\right)^{-1} H^2 \\
&= -2\lambda v^2 \left(1 + \frac{f_2 v^2}{8\Lambda^2\lambda}\left(-1 + \frac{5}{2}\right)\right)\left(1 - \frac{f_1 v^2}{\Lambda^2} + \mathcal{O}(\Lambda^{-4})\right) H^2 \\
&= -2\lambda v^2 \left(1 - \frac{f_1 v^2}{\Lambda^2} + \frac{3 f_2 v^2}{16\Lambda^2\lambda} + \mathcal{O}(\Lambda^{-4})\right) H^2
\end{aligned} \tag{10}$$

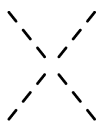
In other words, the relation between the vacuum expectation value, the Higgs masses and the factor in front of the $|\phi|^4$ term in the potential has changed. Because we do not know the Higgs mass, this does not change much. However, we can now compute the trilinear and quadrilinear Higgs self couplings (just as the Higgs-mass term) and find

$$\begin{aligned}
\mathcal{L}_{\text{self}} = &- \frac{m_H^2}{2v}\left[\left(1 - \frac{f_1 v^2}{2\Lambda^2} + \frac{2 f_2 v^4}{3\Lambda^2 m_H^2}\right) H^3 - \frac{2 f_1 v^2}{\Lambda^2 m_H^2} H\,\partial_\mu H\,\partial^\mu H\right] \\
&- \frac{m_H^2}{8v^2}\left[\left(1 - \frac{f_1 v^2}{\Lambda^2} + \frac{4 f_2 v^4}{\Lambda^2 m_H^2}\right) H^4 - \frac{4 f_2 v^2}{\Lambda^2 m_H^2} H^2\,\partial_\mu\,H \partial^\mu H\right]
\end{aligned} \tag{11}$$

This gives the Feynman rules



$$-i\frac{3m_H^2}{v}\left(1 - \frac{f_1 v^2}{2\Lambda^2} + \frac{2 f_2 v^4}{3\Lambda^2 m_H^2} + \frac{2 f_1 v^2}{3\Lambda^2 m_H^2}\sum_{j<k}^{4}(p_j \cdot p_k)\right) \tag{12}$$



$$-i\frac{3m_H^2}{v^2}\left(1 - \frac{f_1 v^2}{\Lambda^2} + \frac{4 f_2 v^4}{\Lambda^2 m_H^2} + \frac{2 f_2 v^2}{3\Lambda^2 m_H^2}\sum_{j<k}^{4}(p_j \cdot p_k)\right) \tag{13}$$

From this discussion we see that in the Higgs sector the Higgs self couplings as well as the Higgs mass are fixed by the operators Higgs potential and depend on the operator basis we take into account. As mentioned before, in the Standard Model we usually use only the dimension-4 operators which appear in the renormalizable Lagrangian

$$\boxed{\mathcal{L}_{\text{self}} = -\frac{m_H^2}{2v} H^3 - \frac{m_H^2}{8v^2} H^4} \qquad \text{with} \qquad m_H^2 = 2\lambda v^2 = -2\mu^2 \tag{14}$$

but we should keep in mind that when the Higgs sector becomes more complicated, not the existence but the form of such relations between masses and couplings will change.

Before we move on and discuss unitarity as another source of theoretical constraints on the Higgs sector we discuss the Goldstone theorem:
If we spontaneously break a continuous symmetry the Lagrangian will include massless scalar degrees of freedom, called Goldstone bosons. Their number is equal to the number of broken generators. For example, if we break $SU(2)_L \times U(1)_Y \to U(1)_Q$ we expect 3 Goldstone bosons (plus a fundamental massive Higgs boson). We know that these Goldstones do not appear in the Standard Model; they become part of the weak gauge bosons and promote

those from massless gauge bosons (with 2 degrees of freedom) to massive gauge bosons (with 3 degrees of freedom). For example in a general $R-\xi$ gauge we can see these Goldstone modes appear in the gauge-boson propagators

$$\begin{aligned}\Delta^{\mu\nu}_{VV}(q) =& \frac{-i}{q^2-m_V^2+i\epsilon}\left[g^{\mu\nu}+(\xi-1)\frac{q^\mu q^\nu}{q^2-\xi m_V^2}\right] \\ =& \frac{-i}{q^2-m_V^2+i\epsilon}\left[g^{\mu\nu}-\frac{q^\mu q^\nu}{m_V^2}\right] && \text{(unitary gauge } \xi\to\infty) \\ & \frac{-i}{q^2-m_V^2+i\epsilon}\,g^{\mu\nu} && \text{(Feynman gauge } \xi=1) \\ & \frac{-i}{q^2-m_V^2+i\epsilon}\left[g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right] && \text{(Landau gauge } \xi=0)\end{aligned} \tag{15}$$

Obviously, these gauge choices are physically equivalent. However, something has to compensate, for example, for the fact that in the Feynman gauge this whole Goldstone term vanishes and the polarization sum looks like that for massless gauge bosons, while in the unitary gauge we can see the effect of these modes directly. The key is the actual Goldstone propagator, *i.e.* additional propagating scalar degrees of freedom

$$\Delta_{VV}(q^2) = \frac{-i}{q^2-\xi m_V^2+i\epsilon} \tag{16}$$

The Goldstone mass is something like ξm_V^2, dependent on the gauge ($V=Z,W^+$). In unitary gauge the infinitely heavy Goldstones do not propagate ($\Delta_{VV}(q^2)=\text{const}$), while in Feynman gauge and in Landau gauge we have to include them as particles. From the form of the Goldstone propagators we can guess that they will indeed cancel the second term of the gauge-boson propagators.

B. Unitarity constraints

If we want to compute transition amplitudes at very high energies, the Goldstone picture becomes very useful. In the V rest frame we can write the three polarization vectors of the gauge bosons as

$$\epsilon^\mu_{T,1}=\begin{pmatrix}0\\1\\0\\0\end{pmatrix}\qquad \epsilon^\mu_{T,2}=\begin{pmatrix}0\\0\\1\\0\end{pmatrix}\qquad \epsilon^\mu_{L}=\begin{pmatrix}0\\0\\0\\1\end{pmatrix} \tag{17}$$

If we boost V into the z direction, giving it a 4-momentum $p^\mu=(E,0,0,|\vec{p}|)$, the polarization vectors become

$$\epsilon^\mu_{T,1}=\begin{pmatrix}0\\1\\0\\0\end{pmatrix}\qquad \epsilon^\mu_{T,2}=\begin{pmatrix}0\\0\\1\\0\end{pmatrix}\qquad \epsilon^\mu_{L}=\frac{1}{m_V}\begin{pmatrix}|\vec{p}|\\0\\0\\E\end{pmatrix}\stackrel{E\gg m_V}{\longrightarrow}\frac{1}{m_V}\begin{pmatrix}|\vec{p}|\\0\\0\\|\vec{p}|\end{pmatrix}\equiv\frac{1}{m_V}\,p^\mu \tag{18}$$

Or in other words, very relativistic gauge bosons are dominated by their longitudinal polarization $|\epsilon^\mu_L|\sim E/m_V\gg 1$. However, this longitudinal degree of freedom is precisely the Goldstone boson. This means that at high energies we can approximate the complicated vector bosons $Z,W^\pm$ as scalar Goldstone bosons $\theta^0,\theta^\pm$. This comes in handy for example when we next talk about unitarity, another theory constraint on the Higgs sector. This relation between Goldstones and gauge bosons at very high energies is, by the way, called the equivalence theorem.
If we want to compute cross sections using the equivalence theorem we need some basic Feynman rules, for example the Goldstone coupling to the Higgs boson. If we write down the Higgs doublet, now including the Goldstone modes

$$\begin{aligned}\phi=\begin{pmatrix}\theta_2+i\theta_1\\ \dfrac{v+H}{\sqrt{2}}-i\theta_3\end{pmatrix}\Rightarrow\qquad \phi^\dagger\phi &= \theta_1^2+\theta_2^2+\theta_3^2+\frac{(v+H)^2}{2} \\ \Rightarrow\qquad (\phi^\dagger\phi)^2 &= \left(\sum_i\theta_i^2\right)^2+\left(v^2+H^2\right)\sum_i\theta_i^2+\frac{(v+H)^4}{4} \\ &= \left(\sum_i\theta_i^2\right)^2+2vH\sum_i\theta_i^2+\cdots\end{aligned} \tag{19}$$

we find for the potential — only keeping the relevant terms at dimension four

$$\begin{aligned} V &= \mu^2|\phi|^2 - \frac{\mu^2}{v^2}|\phi|^4 \\ &= -\frac{m_H^2}{2}|\phi|^2 + \frac{m_H^2}{2v^2}|\phi|^4 \\ &\supset \frac{m_H^2}{2v^2}\left[\left(\sum_i \theta_i^2\right)^2 + 2vH\sum_i \theta_i^2 + \cdots\right] \\ &= \frac{m_H^2}{2v^2}\left(\sum_i \theta_i^2\right)^2 + \frac{m_H^2}{v} H\sum_i \theta_i^2 + \cdots \end{aligned} \tag{20}$$

with $\theta_\pm = (\theta_1 \pm i\theta_2)/\sqrt{2}$ or $\theta_+\theta_- = (\theta_1^2+\theta_2^2)/2$ we find for example

$$\begin{aligned} V &= \frac{m_H^2}{2v^2}\,4\,(\theta_+\theta_-)^2 + \frac{m_H^2}{v}H\,2\,\theta_+\theta_- + \cdots \\ &= \frac{2m_H^2}{v^2}\theta_+\theta_-\theta_+\theta_- + \frac{2m_H^2}{v^2}H\theta_+\theta_- + \cdots \end{aligned} \tag{21}$$

which fixes for example the charged Goldstone couplings to the Higgs..
In terms of scalar Goldstones bosons we can compute the amplitude for $WW \to WW$ scattering at very large energies ($E \gg m_W$) and write it in terms of the particle waves as

$$A = -\left[2\frac{m_H^2}{v^2} + \left(\frac{m_H^2}{v}\right)^2 \frac{1}{s-m_H^2} + \left(\frac{m_H^2}{v}\right)^2 \frac{1}{t-m_H^2}\right] \; = \; 16\pi\sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)a_l \tag{22}$$

for orbital angular momentum l. P_l are the Legendre polynomials of the scattering angle θ, which obey the orthogonality condition

$$\int_{-1}^{1} dx\, P_l(x)P_{l'}(x) = \frac{2}{2l+1}\delta_{ll'} \tag{23}$$

The scattering cross section is then given by

$$\begin{aligned} \sigma &= \int d\Omega\,\frac{|A|^2}{64\pi^2 s} \\ &= \frac{(16\pi)^2 2\pi}{64\pi^2 s}\int_{-1}^{1} d\cos\theta \sum_l\sum_{l'}(2l+1)(2l'+1)\; a_l a_{l'}^* \; P_l(\cos\theta)P_{l'}(\cos\theta) \\ &= \frac{8\pi}{s}\sum_l 2(2l+1)\;|a_l|^2 = \frac{16\pi}{s}\sum_l(2l+1)\;|a_l|^2 \end{aligned} \tag{24}$$

The optical theorem tells us that for any cross section (of asymptotically free fields)

$$\sigma = \frac{1}{s}\;\mathrm{Im}A(\theta=0) \quad \Leftrightarrow \quad \frac{16\pi}{s}(2l+1)\;|a_l|^2 \;=\; \frac{1}{s}\;16\pi(2l+1)\;a_l \tag{25}$$

using $P_l(\cos\theta = 1) = 1$. This condition we can rewrite as

$$\begin{aligned} (\mathrm{Re}\;a_l)^2 + (\mathrm{Im}\;a_l)^2 = \mathrm{Im}\;a_l \quad &\Leftrightarrow \quad (\mathrm{Re}\;a_l)^2 + \left(\mathrm{Im}\;a_l - \frac{1}{2}\right)^2 = \frac{1}{4} \\ &\Leftrightarrow \quad \boxed{|\mathrm{Re}\,a_l| \;<\; \frac{1}{2}} \end{aligned} \tag{26}$$

The last step is obvious once we recognize that the condition on $\mathrm{Im}\,a_l$ an on $\mathrm{Re}\,a_l$ is a unit cycle around $a_l = (0, 1/2)$ with radius $1/2$. Applying the perturbative unitary constraint to a_0 we find

$$\begin{aligned} a_0 = \frac{1}{16\pi s}\int_s^0 dt\,|A| \;&=\; -\frac{1}{16\pi s}\int_s^0 dt\,A \\ &=\; -\frac{m_H^2}{16\pi v^2}\left[2 + \frac{m_H^2}{s-m_H^2} - \frac{m_H^2}{s}\log\left(1+\frac{s}{m_H^2}\right)\right] \sim -\frac{m_H^2}{16\pi v^2}\left(2 + \mathcal{O}\left(\frac{m_H^2}{s}\right)\right) \end{aligned} \tag{27}$$

which in the limit of $s \gg m_H^2$ means

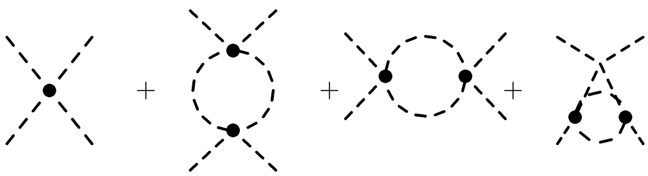
$$\frac{m_H^2}{8\pi v^2} < \frac{1}{2} \qquad \Leftrightarrow \qquad \boxed{m_H^2 < 4\pi v^2 \leq (870 \text{ GeV})^2} \tag{28}$$

This is the maximum value of m_H which we can choose to maintain perturbative unitarity for $WW \to WW$ scattering. Of course, if we limit s to a finite value this bound changes, and we can even compute a maximum scale s_{max} which leaves $WW \to WW$ perturbatively unitary for fixed m_H: for $m_H \lesssim v$ this typically becomes $\sqrt{s_{\text{max}}} \sim 1.2$ TeV.

One last word: this unitarity argument only works if the WWH coupling is exactly what it should be. Or in other words, while perturbative unitarity only gives us an upper limit on m_H it uniquely fixes g_{WWH}. Looking at processes like $WW \to f\bar{f}$ or $WW \to WWH$ or $WW \to HHH$ we can exactly the same way fix all Higgs couplings in the Standard Model, including g_{Hff}, g_{HHH}, g_{HHHH}. In other words, the most important effect of unitarity might not be the upper bound on the Higgs mass, but that the entire argument only works if all Higgs couplings to Standard-Model particle are the exactly as given in the Standard Model.

C. Constraints from the renormalization group

Two theoretical constraints we can derive from the renormalization group equation of the Higgs potential, specifically from the renormalization scale dependence of the self coupling $\lambda(Q^2)$. Such a scale dependence arises automatically when we encounter ultraviolet divergences of parameters and absorb the $1/\epsilon$ divergences into a counter term. From the quartic Higgs coupling λ alone the relevant s, t and u-channel diagrams are



(29)

Skipping the calculation we quote the complete renormalization group equation

$$\frac{d\,\lambda}{d\,\log Q^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda\lambda_t - 3\lambda_t^4 - \frac{3}{2}\lambda\left(3g_2^2 + g_1^2\right) + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right] \tag{30}$$

with $\lambda_t = \sqrt{2}m_t/v$. The first regime we can study is where this coupling λ becomes strong

$$\frac{d\,\lambda}{d\,\log Q^2} = \frac{1}{16\pi^2}\,12\lambda^2 + \cdots = \frac{3}{4\pi^2}\lambda^2 + \cdots \tag{31}$$

This equation we can solve by replacing $\lambda = g^{-1}$

$$\begin{aligned}
\frac{d\,\lambda}{d\log Q^2} &= \frac{d}{d\log Q^2}\frac{1}{g} = -\frac{1}{g^2}\frac{d\,g}{d\log Q^2} \stackrel{!}{=} \frac{3}{4\pi^2}\frac{1}{g^2} \\
\Leftrightarrow \quad \frac{d\,g}{d\log Q^2} &= -\frac{3}{4\pi^2} \quad \Leftrightarrow \quad g(Q^2) = -\frac{3}{4\pi^2}\log Q^2 + C
\end{aligned} \tag{32}$$

If we define a boundary condition $\lambda(Q^2 = v^2) = \lambda_0$ we find:

$$\begin{aligned}
g_0 = \frac{1}{\lambda_0} &= -\frac{3}{4\pi^2}\log v^2 + C \quad \Leftrightarrow \quad C = g_0 + \frac{3}{4\pi^2}\log v^2 \\
\Rightarrow \quad g(Q^2) &= -\frac{3}{4\pi^2}\log Q^2 + g_0 + \frac{3}{4\pi^2}\log v^2 = -\frac{3}{4\pi^2}\log\frac{Q^2}{v^2} + g_0 \\
\Leftrightarrow \quad & \boxed{\lambda(Q^2) = \left[\frac{3}{4\pi^2}\log\frac{Q^2}{v^2} + \frac{1}{\lambda_0}\right]^{-1} = \lambda_0\left[1 - \frac{3}{4\pi^2}\lambda_0\log\frac{Q^2}{v^2}\right]^{-1}}
\end{aligned} \tag{33}$$

For very large values of Q we know from the original differential equation that $d\lambda/d\log Q^2 > 0$, which means λ is divergent eventually. It will have a pole at

$$\begin{aligned} 1 - \frac{3}{4\pi^2}\lambda_0 \log\frac{Q_c^2}{v^2} = 0 \qquad &\Leftrightarrow \quad \frac{3}{4\pi^2}\lambda_0 \log\frac{Q_c^2}{v^2} = 1 \\ &\Leftrightarrow \quad \log\frac{Q_c^2}{v^2} = \frac{4\pi^2}{3\lambda_0} \\ &\Leftrightarrow \quad Q_c = v\ \exp\frac{2\pi^2}{3\lambda_0} = v\ \exp\frac{4\pi^2 v^2}{3m_H^2} \end{aligned} \tag{34}$$

Such a pole is called a Landau pole and gives us a maximum scale beyond which we cannot rely on our perturbative theory to work. This limit is often referred to as the triviality bound. For a given value of Q_c it translates into a maximum value of m_H for which the theory is well defined. The name originates from the behavior in the opposite limit $Q \ll v^2$

$$\lambda(Q^2) \sim \left[\frac{3}{4\pi^2}\left|\log\frac{Q^2}{v^2}\right|\right]^{-1} \ \to\ 0 \tag{35}$$

It means that if we want our Higgs potential to be perturbative at all scales the coupling λ has to stay zero for all scales, otherwise it will hit a Landau pole at some scale. A theory with zero interaction is called trivial.

We can look at the same renormalization group equation again and ask the question: how long will $\lambda > 0$ ensure that our Higgs potential is bounded from below. This bound is called the stability bound. In the equation for $d\lambda/d\log Q^2$ there are two forms with a negative sign which in principle drive λ through zero, one of which did not vanish for $\lambda \sim 0$, so we approximately have competing terms around $\lambda = 0$

$$\begin{aligned} \frac{d\,\lambda}{d\log Q^2} &\sim \frac{1}{16\pi^2}\left[-3\frac{4m_t^4}{v^2} + \frac{3}{16}\left(2g_2^4 + \left(g_2^2+g_1^2\right)^2\right)\right] \\ \Leftrightarrow \quad \lambda(Q^2) &\sim \lambda(v^2) + \frac{1}{16\pi^2}\left[-\frac{12m_t^4}{v^4} + \frac{3}{16}\left(2g_2^4 + \left(g_2^2+g_1^2\right)^2\right)\right]\ \log\frac{Q^2}{v^2} \end{aligned} \tag{36}$$

with the usual boundary condition in terms of $\lambda(v^2)$. Requiring $\lambda > 0$ means

$$\begin{aligned} \lambda(v^2) = \frac{m_H^2}{2v^2} &> \frac{-1}{16\pi^2}\left[-\frac{12m_t^4}{v^4} + \frac{3}{16}\left(2g_2^4 + \left(g_2^2+g_1^2\right)^2\right)\right]\ \log\frac{Q^2}{v^2} \\ \Leftrightarrow \quad m_H^2 &> \frac{v^2}{8\pi^2}\left[-\frac{12m_t^4}{v^4} + \frac{3}{16}\left(2g_2^4 + \left(g_2^2+g_1^2\right)^2\right)\right]\ \log\frac{Q^2}{v^2} \\ \Leftrightarrow \quad m_H &= \begin{cases} 70\ \text{GeV} & \text{for} \quad Q < 10^3\ \text{GeV} \\ 130\ \text{GeV}) & \text{for} \quad Q < 10^{16}\ \text{GeV} \end{cases} \end{aligned} \tag{37}$$

This means that from the renormalization group we have two constraints on the Higgs mass in the renormalizable Standard model: the Landau pole (or triviality bound) gives an upper limit on m_H as a function of the cutoff scale, while the vacuum stability gives a lower bound on m_H as a function of the cutoff scale. Running this scale towards the Planck mass $Q \to 10^19$ GeV, we find that only values around $m_H = 180 \cdots 190$ GeV are allowed. As a matter of fact, in the Standard Model electroweak precision data also experimentally points to $m_H \leq 250$ GeV, which means we are probably looking for a Higgs boson well below 1 TeV at the LHC.

II. HIGGS DECAYS

All decay rates are at tree level determined by the Higgs coupling to Standard Model particles, which are fixed by unitarity. The rule for Higgs decays is simple: because the Higgs couples to all particles (including itself) proportional to their masses it tends to decay to the heaviest states allowed by phase space. We can see this in Fig. 1. Starting at low masses this is true for the decays to $\tau\tau$ and $b\bar{b}$. When the (off-shell) decays to WW open, they very soon dominate. Because of the small mass difference between the W and Z bosons the decay to ZZ is not as dominant, compared to the WW decay which has two degrees of freedom (W^+W^- and W^-W^+) in the final state. Above the top threshold the $t\bar{t}$ decay becomes sizeable.

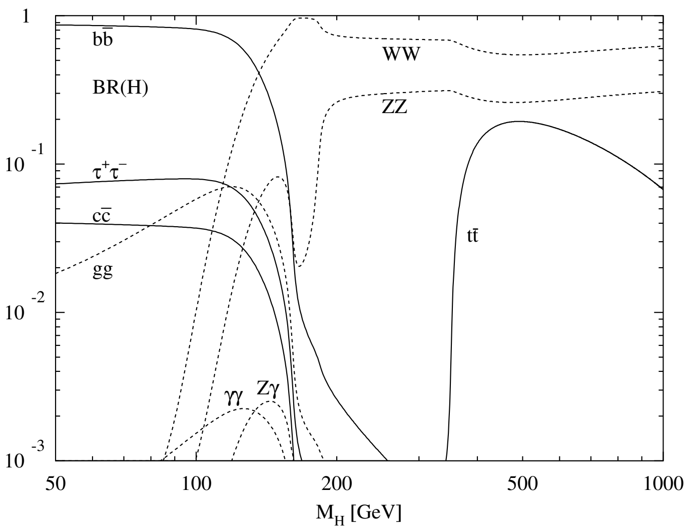


FIG. 1: Branching ratios of the Standard–Model Higgs boson as a function of its mass. Off-shell effects in the decays to WW and ZZ are taken into account.

At low masses there is also a loop-induced decay to two photons which plays an important role in LHC phenomenology. It proceeds via a top and a W triangle, both of which entering with opposite signs in the amplitude. Its structure is similar to the production process $gg \to H$ which we will discuss in the following section. The reason for considering this decay channel are the LHC detectors. To extract a Higgs signal from the backgrounds we usually try to measure the 4-momenta of the Higgs decay products and reconstruct the invariant mass of them. The signal should then peak around m_H, while the backgrounds we expect to be more or less flat. The LHC detectors, and in particular CMS are designed to measure the photon momentum and energy particularly well. The resolution on $m_{\gamma\gamma}$ will at least be a factor of 10 better than for any other decay channel, except for four muons. Moreover, photons do not decay, so we can use all photon events in the Higgs search, while for example hadronically decaying $W \to 2$ jets are not particularly useful at the LHC. These enhancement factors make the Higgs decay to two photons a promising signature, in spite of its small branching ratio.

Because the Higgs sector could easily deviate from the minimal Standard Model, the LHC should study the different Higgs decays and (as a function of m_H) determine:

– are gauge-boson couplings proportional to $m_{W,Z}$?

– are fermion Yukawa couplings proportional to m_f?

– is there a self coupling, *i.e.* a Higgs potential?

– do λ_{HHH} and λ_{HHHH} show signs of additional operators?

We unfortunately know already that in the Standard Model the quadrilinear couplings will not be observed at any planned collider. But in any case, before we decay the Higgs we need to produce it!

III. HIGGS PRODUCTION IN GLUON FUSION

Numerically, the production of Higgs bosons in gluon fusion is the dominant process at the LHC, as shown in Fig. 2. It turns out to have the largest production cross section and can be combined with many decay channels to form LHC signatures.

A. Effective ggH coupling

The dominant contribution to the gluon-fusion Higgs production will obviously come from the triangular top quark loop.

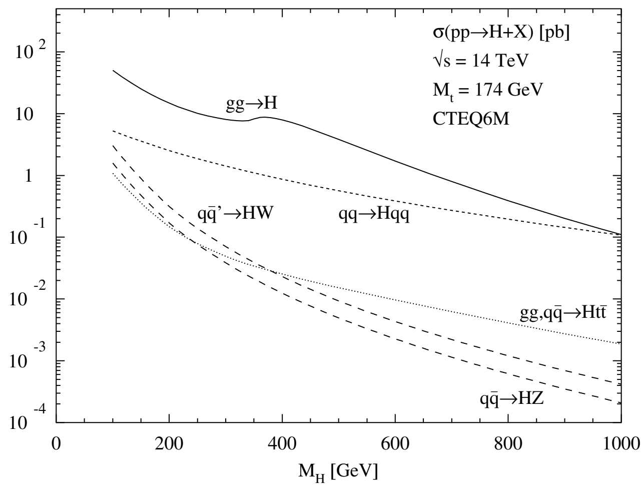
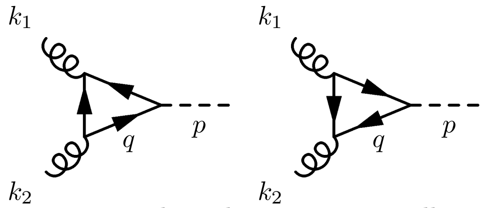


FIG. 2: Production cross section for a Standard–Model Higgs boson at the LHC, as a function of the Higgs mass. We will only discuss the two leading production mechanisms, gluon fusion and weak–boson fusion.



Let us first evaluate the Dirac trace occurring in these diagrams, using all incoming momenta with $k_1^2 = k_2^2 = 0$ and $p^2 = m_H$. The Dirac indices of the two gluons are μ, ν and the loop momentum is q, so we need to compute

$$\begin{aligned} \text{Tr}^{\mu\nu} = \text{Tr}\,\big[&(\not{q} + m_t)\,\gamma^\mu\,(\not{q} + \not{k}_1 + m_t)\,\gamma^\nu\,(\not{q} + \not{k}_1 + \not{k}_2 + m_t) \\ &+(\not{q} + m_t)\,\gamma^\nu\,(\not{q} + \not{k}_2 + m_t)\,\gamma^\mu\,(\not{q} + \not{k}_1 + \not{k}_2 + m_t)\big] \end{aligned} \tag{38}$$

The problem is the tensor structure of this trace. Because of gauge invariance we can neglect terms proportional to k_1^μ and k_2^ν, because they would not survive the multiplication with the transverse gluon polarization ($k \cdot \epsilon = 0$). In a so-called axial gauge we can also get rid of the remaining terms proportional to k_1^ν and k_2^μ. However, we *cannot* get rid of the remaining $q^\mu q^\nu$ terms. To save some time and nevertheless show how the calculation works we will remove these terms by computing only the $g_{\mu\nu}$ term in $\text{Tr}^{\mu\nu}$ and correct for them later. The $g_{\mu\nu}$ we can compute by projecting

$$g_{\mu\nu}\,\text{Tr}^{\mu\nu} = -8m_t(m_H^2 - 4m_t^2) \qquad \Rightarrow \qquad \text{Tr}^{\mu\nu} = -2m_t(m_H^2 - 4m_t^2)g^{\mu\nu} + \ldots \tag{39}$$

Next, we have to compute the integral over the loop momentum q, which in our $G_{\mu\nu}$ terms luckily does not occur in the trace. The scalar integral

$$C(k_1^2, k_2^2, m_H^2; m_t, m_t, m_t) \equiv C(m_H, m_t) = \int \frac{d^4q}{i\pi^2}\,\frac{1}{[q^2 - m_t^2][(q+k_1)^2 - m_t^2][(q+k_1+k_2)^2 - m_t^2]} \tag{40}$$

is given in the literature as a special case of

$$\begin{aligned}
C(m_H, m_t) =& \frac{1}{m_H^2} F(m_H^2, m_t^2, 0) \\
F(a,b,y) =& \int_0^1 \frac{dx}{x-y} \; \log\left(\frac{ax(1-x)-b}{ay(1-y-b)}\right) \\
\stackrel{y=0}{=}& \int_0^1 \frac{dx}{x} \; \log\left(\frac{m_H^2 x(1-x)-m_t^2}{(-m_t^2)}\right) \\
=& \frac{1}{2} \; \log^2 \left[-\frac{1+\sqrt{1-4m_t^2/m_H^2}}{1-\sqrt{1-4m_t^2/m_H^2}} \right] \\
\Rightarrow \quad C(m_H, m_t) =& \frac{1}{2m_H^2} \; \log^2 \left[-\frac{1+\sqrt{1-4m_t^2/m_H^2}}{1-\sqrt{1-4m_t^2/m_H^2}} \right] \qquad \text{for} \quad \frac{4m_t^2}{m_H^2} < 1
\end{aligned} \tag{41}$$

We can now collect the factors of i and 2π and write for our (still wrong) effective ggH coupling

$$\begin{aligned}
& i^3 \, (-ig_s)^2 \; \mathrm{Tr}(T^a T^b) \frac{im_t}{v} \; \frac{i\pi^2}{16\pi^4} \; (-2) \; m_t \; \left(m_H^2 - 4m_t^2\right) \; C(m_H, m_t) \\
=& - ig_s^2 \; \frac{1}{8\pi^2} \; \mathrm{Tr}(T^a T^b)(-1) \; \frac{m_t^2}{v} \; \left(m_H^2 - 4m_t^2\right) \; C(m_H, m_t) \qquad \mathrm{Tr}(T^a T^b) = \frac{1}{2} \; \delta^{ab} \\
=& + ig_s^2 \; \frac{1}{16\pi^2} \; \delta^{ab} \; \frac{m_t^2}{v} \; \left(m_H^2 - 4m_t^2\right) \; \frac{1}{2m_H^2} \; \log^2[- \ldots] \qquad \tau = \frac{4m_t^2}{m_H^2} \quad \Leftrightarrow \quad m_t^2 = \tau \frac{m_H^2}{4} \\
=& + ig_s^2 \; \frac{1}{16\pi^2} \; \delta^{ab} \; \frac{1}{v} \; \frac{\tau m_H^2}{4} \; \frac{1}{2} (1-\tau) \; \log^2 \left[-\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} \right]
\end{aligned} \tag{42}$$

This result we can now correct for the missing $q^\mu q^\nu$ terms by replacing

$$(1-\tau) \log^2[\ldots] \; \mapsto \; (1-\tau) \log^2[\ldots] - 4 = -4 \left[1 + (1-\tau) \frac{(-1)}{4} \log^2[\ldots] \right] \tag{43}$$

Another short cut was that we only solved $C(m_H, m_t)$ for $\tau < 1$. We can in general write (with prefactors like in the usual literature).

$$\begin{aligned}
(1-\tau) \log^2[\ldots] \; \mapsto& \; -4 \left[1 - (1-\tau) \; f(\tau) \right] \\
\text{with} \quad f(\tau) =& \begin{cases} \left[\sin^{-1} \sqrt{\frac{1}{\tau}} \right]^2 & \tau > 1 \\ -\frac{1}{4} \left[\log\left(\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} \right) - i\pi \right]^2 & \tau < 1 \end{cases}
\end{aligned} \tag{44}$$

including imaginary (absorptive) terms for $\tau < 1$. We then write for the effective ggH coupling

$$-i \, g_s^2 \; \frac{1}{32\pi^2} \; \frac{m_h^2}{v} \; \tau \left[1 + (1-\tau) f(\tau) \right] \tag{45}$$

This expression is valid for all values of m_t and m_H. It corresponds to an effective Lagrangian with a term $HG^{\mu\nu}G_{\mu\nu}$ which has mass dimension five and is therefore not renormalizable. Sine we started the calculation of the ggH coupling in axial gauge we have to write $G^{\mu\nu}G_{\mu\nu}$ consistently in the same gauge:

$$\begin{aligned}
HG^{\mu\nu}G_{\mu\nu} \; \sim& \; H \; \left({\epsilon_1}_\mu {k_1}_\nu - {k_1}_\mu {\epsilon_1}_\nu \right) \left({\epsilon_2}^\mu {k_2}^\nu - {k_2}^\mu {\epsilon_2}^\nu \right) \\
=& \; 2H \; \left[(k_1 \cdot k_2)(\epsilon_1 \cdot \epsilon_2) - (k_1 \cdot \epsilon_2)(k_2 \cdot \epsilon_1) \right]
\end{aligned} \tag{46}$$

Usually, we would use the transverse tensor $g^{\mu\nu} - k_1^\mu k_2^\nu/(k_1 k_2)$ to write down the ggH coupling. However, in axial gauge the second term is gauged away, so we only need to consider

$$H \, G^{\mu\nu}G_{\mu\nu} \; \sim \; 2 \, H \, (k_1 \cdot k_2)(\epsilon_1 \cdot \epsilon_2) = m_H^2 \, H \, (\epsilon_1 \cdot \epsilon_2) \tag{47}$$

This means that the actual ggH coupling is given by

$$\boxed{\mathcal{L}_{ggH} \supset \frac{1}{v}\, g_{ggH}\; H\, G^{\mu\nu} G_{\mu\nu}} \qquad\qquad \frac{1}{v}\, g_{ggH} = -i\,\frac{\alpha_s}{8\pi}\,\frac{1}{v}\,\tau\,[1-(1-\tau)f(\tau)] \tag{48}$$

with the appropriate factors of $(-)$ and i removed in the Feynman rule. The tensor structure in the usual unitary gauge is $g^{\mu\nu} - k_1^\mu k_2^\nu/(k_1 \cdot k_2)$. Note that the necessary factor $\frac{1}{\Lambda}$ in front of the dimension-5 operator is $\frac{1}{v}$ and not $\frac{1}{m_t}$.

Of course, just like we have 3-gluon and 4-gluon couplings in QCD we can compute the $gggH$ and the $ggggH$ couplings from the ggH coupling simply using gauge invariance. This set of n-gluon couplings to the Higgs boson is *not an approximate result e.g.* in the top mass. Gauge invariance completely fixes the n-gluon coupling to the Higgs via one exact dimension-5 operator in the Lagrangian.

B. Low–energy theorem

The expression for g_{ggH} is not particularly handy, so we can for light Higgs bosons write it in a more compact form. We start with a Taylor series for $f(\tau)$ in the heavy–top limit $\tau \gg 1$

$$f(\tau) = \left[\sin^{-1}\sqrt{\frac{1}{\tau}}\right]^2 = \frac{1}{\tau} + \frac{1}{3\tau^2} + \mathcal{O}(\tau^{-3}) \to 0 \tag{49}$$

and combine it with all other τ-dependent terms in the full expression

$$\begin{aligned}
\tau\;[1+(1-\tau)f(\tau)] =& \tau\;\left[1+(1-\tau)\left(\frac{1}{\tau}+\frac{1}{3\tau^2}+\mathcal{O}(\tau^{-3})\right)\right] \\
=& \tau\;\left[1+\frac{1}{\tau}-1-\frac{1}{3\tau}+\mathcal{O}(\tau^{-2})\right] \\
=& \tau\;\left[\frac{2}{3\tau}+\mathcal{O}(\tau^{-2})\right] \\
=& \frac{2}{3}+\mathcal{O}(\tau^{-1})
\end{aligned} \tag{50}$$

In this low-energy or heavy–top limit where we have decoupled the top quark from the list of propagating particles. The ggH coupling does not depend on m_t anymore and gives a finite result. Note that for this finite result we had to introduce the top Yukawa coupling in the numerator. We emphasize again that while this low–energy approximation is very compact to analytically write down the effective ggH coupling, it is not necessary to numerically compute processes involving ggH coupling!

In the low-energy limit we can add more Higgs bosons to the loop. Attaching an external Higgs leg to the gluon self–energy diagram simply means replacing one of the two top propagators with two top propagators and adding a Yukawa coupling

$$\frac{1}{\not{q}-m_t} \quad\mapsto\quad \frac{1}{\not{q}-m_t}\,\frac{m_t}{v}\,\frac{1}{\not{q}-m_t} \tag{51}$$

This we can compare to a differentiation with respect to m_t

$$\frac{m_t}{v}\,\frac{\partial}{\partial m_t}\,\frac{1}{\not{q}-m_t} \sim \frac{m_t}{v}\,\frac{-1}{(\not{q}-m_t)(\not{q}-m_t)}\,(-1) \sim \frac{1}{\not{q}-m_t}\,\frac{m_t}{v}\,\frac{1}{\not{q}-m_t} \tag{52}$$

While this treatment of the gamma matrix is, strictly speaking, nonsense, it still gives us the idea how we can in the limit of heavy top mass derive ggH^n couplings from the gluon self energy or the ggH coupling:

$$\boxed{\mathcal{L}_{ggH} = G^{\mu\nu}G_{\mu\nu}\,\frac{\alpha_s}{\pi}\,\left(\frac{H}{12v}-\frac{H^2}{24v^2}+\dots\right) = \frac{\alpha_s}{12\pi}\,G^{\mu\nu}G_{\mu\nu}\;\log\left(1+\frac{H}{v}\right)} \tag{53}$$

Such a form is very convenient for simple calculations but note that for example for gg+jets production it only holds in the limit that *all* jet momenta are much smaller than m_t. It also becomes problematic for example in the $gg \to HH$

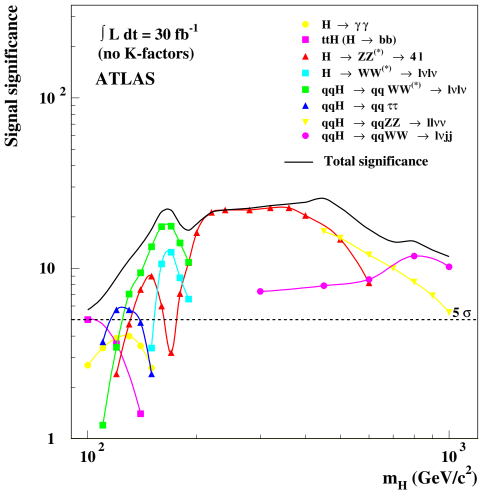


FIG. 3: Simulated statistical significance for different Higgs production and decay channels for an integrated luminosity of 30 fb^{-1}, by Atlas. Five standard deviations σ over the backgrounds are required by the high–energy physics community as the condition for any discovery. The phrase 'discovery channel' we use only for the best signature given a certain Higgs mass value.

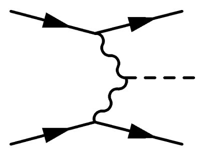
process close to threshold, where the momenta of slow–moving Higgs bosons lead to an additional scale in the process. We will come to this process later in a little more detail.

Now that we can compute Higgs production at the LHC, we can go through all decay signatures from the gluon–fusion production process and look at their strengths and challenges. Simulation result for all Higgs signatures (as simulated by Atlas) we show in Fig. 3.

$gg \to H \to b\bar{b}$	hopeless, because of sheer size of the QCD continuum background $gg \to b\bar{b}$. Note that there is little to cut on except for the invariant mass of the $b\bar{b}$ pair.
$gg \to H \to \tau^+\tau^-$	problematic, because of very small $p_{T\,H}$ which makes the reconstruction of $m_{\tau\tau} \sim m_H$ hard. We will discuss this decay and its reconstruction in detail later.
$gg \to H \to \gamma\gamma$	in spite of the small rate great to measure the Higgs mass, because $m_{\gamma\gamma}$ can be reconstructed to the order of a per-cent. A major problem is a detector background from pions mistaken for photons.
$gg \to H \to Z\gamma$	like $\gamma\gamma$, but smaller rate and small branching ratio of $Z \to \mu^+\mu^-$ or $Z \to e^+e^-$. Unlikely to be seen.
$gg \to H \to W^+W^-$	large rate, but hard to reconstruct m_H in the most promising leptonic channel due to escaping neutrinos. Analysis relying on angular correlation of the two leptons, which makes it hard below threshold.
$gg \to H \to ZZ$	great for ZZ to four leptons, again because of reconstructed $m_{4\ell} \sim m_{ZZ} \sim m_H$. Therefore usually referred to as 'golden channel'.
$gg \to H \to$ invisible	not predicted in Standard Model, hopeless if only the Higgs is produced.

IV. HIGGS PRODUCTION IN WEAK–BOSON FUSION

Since the Higgs has sizeable couplings only to the W, Z bosons and to the top quark we can alternatively produce it from two incoming quarks which each radiate a W or Z boson: $qq \to qqH$. Because the LHC is a pp collider and because the proton mostly contains the valence quarks (uud) and low-x gluons it is important that this process can proceed as $ud \to dutt$, where the u radiates a W^+ and the d radiates a W^-. The Feynman diagram for this process is:



If the Higgs were a Z boson, it could also bremsstrahlung off the quarks, but for Higgs signals at colliders we safely assume $m_q = 0$ for the first two generations. In a way, this process looks like deep inelastic scattering one from each of the protons — at least from a QCD point of view.

A. Final–state kinematics

Of course, we could describe the W bosons in the protons just like partons, assuming it to be essentially massless. This approximation is called effective W approximation and is bad at the LHC. The reason is that the Higgs mass is of the order of the W mass, as are the transverse momenta of the W or of the final state jets. We can see this from the typical momentum dependence of the intermediate W boson, which peaks at

$$\begin{aligned} 0 &\stackrel{!}{=} \frac{\partial}{\partial p_T} \frac{p_T}{-p_T^2 - m_W^2} = \frac{\partial}{\partial p_T} p_T (-p_T^2 - m_W^2)^{-1} \\ &= \left(-p_T^2 - m_W^2\right)^{-1} + p_T \, (-1) \, (-p_T^2 - m_W^2)^{-2} \, (-2p_T) \\ &= \frac{-p_T^2 - m_W^2 + 2p_T^2}{(p_T^2 + m_W^2)^2} \\ &= \frac{p_T^2 - m_W^2}{(p_T^2 + m_W^2)^2} \\ \Rightarrow \quad & \boxed{p_T^2 = m_W^2} \end{aligned} \tag{54}$$

Note the $(-)$ in the denominator which arises because of $p^2 = E^2 - p_T^2 - p_z^2$.

From a jets perspective this WBF signal has a particular geometry: because of the collinear enhancement which means that the cross section would diverge like $\log p_T$ in the absence of the m_W regulators, both jets prefer to be forward in the detector. The exchange of a gluon between the two quark legs multiplied with the Born diagram is proportional to the color factor $\mathrm{Tr}\, T^a \, \mathrm{Tr}\, T^b \delta^{ab} = 0$. Additional jet activity is therefore limited to collinear jets from the initial-state and final-state quarks, *i.e.* again forward in the detector. If we want, we can even try to veto additional hard jets in the central region, above $p_{Tj} \geq 20$ GeV or 30 GeV, to suppress backgrounds like $t\bar{t}$+jets.
In contrast, the Higgs and its decay products are expected in the central detector. So we are looking for two forward jets and for example two τ leptons in the central detector. Moreover, the Higgs is produced with a finite p_{TH}, which is largely given by the acceptance cuts on the forward jets.

Compared to the Higgs production in gluon fusion we buy this distinctive signature, *i.e.* the improved extraction of the Higgs signal out of the background, at the expense of the rate. The one-loop amplitude for $gg \to H$ is proportional to $\alpha_s y_t/(4\pi) \sim (1/10)\,(2/3)\,(1/12) = 1/180$. The cross section for WBF is proportional to g^6, but with two additional jets in the final state. Including the additional phase-space for two jets this very roughly means $(2/3)^6 \, 1/(16\pi)^2 = (64/729)\,(1/2500) \sim 1/25000$. This number we can compare with $(1/180)^2 \sim 1/40000$ for the gluon fusion process and find that the one–loop amplitude and the additional jets and weak couplings roughly balance each other. The difference comes from the quark and gluon luminosities. In the WBF signature, the 2 forward jets always combine to a large partonic $\hat{s} = x_1 x_2 s > (p_{j,1} + p_{j,2})^2 = 2(p_{j,1} \cdot p_{j,2})$, while a single Higgs probes the already large gluon parton density at typical x values of 10^{-3}. This means that each of the two production processes probes its most favorable parton x values: low-x for gluon fusion and high-x for quark-quark scattering. Looking at typical LHC energies, the gluon parton density grows very steeply for $x \lesssim 10^{-2}$. So for a 150 GeV Higgs the gluon-fusion rate of ~ 30 pb clearly exceeds the WBF rate of ~ 4 pb. On the other hand, these numbers mean little, for example when we battle an 800 pb $t\bar{t}$ background and for this have to rely on cuts either on forward jets or on Higgs decay products.

As one final remark — we also see that for large m_H the WBF rate starts to exceed the gluon-fusion rate. One reason is that for larger x values the rate for $gg \to H$ decreases steeply with the gluon density, while for the already large partonic $\hat{s}$ due to the tagging jets the increase in m_H makes no difference to the quark parton densities. The more important aspects are large logarithms: if we neglect $m_W \sim 0$ in the WBF process the $p_{T,j}$ distributions will diverge for small $p_{T,j}$ like $1/p_{T,j}$. This yields a $\log p_{T,j}^{\max}/p_{T,j}^{\min}$ in the total rate. With the W mass a a typical hard scale given by m_H this logarithm becomes (on each side)

$$\sigma_{\rm WBF} \propto \left(\log \frac{p_{T,j}^{\max}}{p_{T,j}^{\min}} \right)^2 \sim \left(\log \frac{m_W}{m_H} \right)^2 \tag{55}$$

For $m_H \to 1$ TeV this logarithm can give us the enhancement factors of up to 10 which WBF needs to make it the dominant Higgs production process. This large logarithms is, by the way, resummed in the effective W approximation, but with very bad results for the final–state kinematics.

B. Approximate Higgs mass reconstruction

The p_{TH} in the WBF process allows us to even reconstruct the invariant mass of a $\tau\tau$ system in the collinear approximation. If we assume that a τ with momentum p decays into a lepton with the momentum xp which moves into the same direction as $\vec{p}$ we can write the transverse directions of the decay lepton as:

$$\frac{\vec{k_1}}{x_1} + \frac{\vec{k_2}}{x_2} = \vec{p_1} + \vec{p_2} = \vec{k_1} + \vec{k_2} + \vec{\not{k}} \tag{56}$$

This works because the missing–energy vector at the LHC can be measured in the transverse plane, *i.e.* it has two components. Given the measured momenta $\vec{k_i}$ we can solve these two equations for x_1 and x_2 and compute the invariant $\tau\tau$ mass

$$\boxed{m_{\tau\tau} = 2\,(p_1 \cdot p_2) = 2\,x_1\,x_2\,(k_1 \cdot k_2)} \tag{57}$$

From the formula above it is obvious that this approximation does not only require a sizeable $p_i \gg m_\tau$, but also that back-to-back taus will not work — the vector $\vec{\not{k}}$ then largely cancels and in the Higgs center-of-mass frame and the computation fails. This is what happens for the other production channel $gg \to H \to \tau\tau$.

Again, we can make a list of signatures which work more or less well in connection to WBF production. The WBF channels are also included in the summary plot by Atlas, shown in Fig. 3.

WBF: $H \to b\bar{b}$	problematic because of large QCD 4-jet backgrounds and trigger in Atlas. Likely not to work in high-luminosity environment, because of structure in underlying event, due to multi-proton scattering.
WBF: $H \to \tau\tau$	new prime discovery channel for a light Higgs with $m_H \lesssim 130$ GeV. Mass reconstruction good to ~ 5 GeV?
WBF: $H \to \gamma\gamma$	compatible with $gg \to H \to \gamma\gamma$ with its smaller rate but improved background suppression. Mostly included in inclusive $H \to \gamma\gamma$ analysis.
WBF: $H \to Z\gamma$	difficult due to simply too small event rate $\sigma \times BR$
WBF: $H \to WW$	discovery channel for $m_H \geq 135$ GeV. In contrast to $gg \to H \to WW$ it works for off-shell W decays, because of a multitude of background–rejection cuts.
WBF: $H \to ZZ$	likely to work in spite of the smaller rate. Maybe even possible with one hadronic Z decay, but not many detailed studies.
WBF: $H \to$ invisible	only discovery channel for an invisible Higgs which works at the LHC. It relies on a pure tagging-jet signature, which means it is seriously hard.

Just a side remark: WBF was essentially unknown as a production mode for light Higgses until ~ 1998. Which means the Higgs chapter in the Atlas TDR had and still has to be completely re-written. It is simply not true that there is nothing new and yet simple to discover in LHC physics!

V. BEYOND HIGGS DISCOVERY

At the end of this lecture we will cover a few more advanced and interesting topics, somewhat unrelated with each other, except that they are all clearly relevant to LHC Higgs physics.

A. Minijet veto

Following the general WBF argument concerning the reduced jet activity in the central detector we need to estimate the probability of not observing additional central jets for different signal and background processes. One way to quantify this effect is a variable cut-off in the p_T of the additional jets, defined by the matching point

$$\sigma_{n+1}(p_T^{\text{crit}}) \equiv \sigma_n \tag{58}$$

where the indices $n+1$ and n are the numbers of final-state jets. This condition implicitly defines a critical p_T, below which our perturbation theory in α_s, *i.e.* in counting the number of external partons, breaks down. For WBF Higgs production we find $p_T^{\text{crit}} \sim 10$ GeV, while for QCD processes like $t\bar{t}$ production it becomes $p_T^{\text{crit}} = 40$ GeV. In other words, jets down to p_T =10 GeV are perturbatively well defined for WBF signatures, while in QCD processes jets below 40 GeV are much more frequent than they should be looking at the perturbative series in α_s.

Hence, we can suppress QCD backgrounds by applying a veto of the kind

$$p_{T,j} > 20 \text{ GeV} \qquad \text{and} \qquad \eta_j^{\text{tag1}} < \eta_j < \eta_j^{\text{tag2}} \tag{59}$$

If we naively assume that the radiation of additional jets proceeds with a constant probability, independent of the number of jets already radiated, we can even compute the survival probability after this veto starting from the computable probability

$$P_{\text{rad}} = \frac{\sigma_3(p_T^{\text{crit}})}{\sigma_2(p_T^{\text{crit}})} \stackrel{!}{=} \frac{\sigma_{n+1}(p_T^{\text{crit}})}{\sigma_n(p_T^{\text{crit}})} \tag{60}$$

Using Poisson statistics the probability of observing n jets now is

$$f(n; P_{\text{rad}}) = \frac{P_{\text{rad}}^n e^{-P_{\text{rad}}}}{n!} \qquad \Rightarrow \qquad \boxed{f(0; P_{\text{rad}}) = e^{-P_{\text{rad}}}} \tag{61}$$

which means the probability of surviving the veto comes out as $\exp(-P_{\text{rad}})$. This comes out roughly as 88% for the WBF signal and as 24% for the $t\bar{t}$ backgrounds. Notice, however, that this number does not incorporate the underlying event, *i.e.* additional energy–dependent but process–independent jet activity in the detectors from many not entirely understood sources.

B. CP Properties

Once we see a Higgs boson at the LHC, how do we test its quantum numbers? For example, the CP-even Standard-Model Higgs bosons couples like $g^{\mu\nu}$ to the W and Z bosons. However, for general CP-even and CP-odd Higgs bosons there are two more gauge–invariant ways to couple to W bosons, proportional to

$$g^{\mu\nu} - \frac{p_1^\mu p_2^\nu}{p_1 \cdot p_2} \qquad \text{and} \qquad \epsilon_{\mu\nu\rho\sigma}\, p_1^\rho p_2^\sigma \tag{62}$$

Note that the first tensor is not orthogonal to $g^{\mu\nu}$, so we could in principle replace it with any linear combination with $g^{\mu\nu}$. However, these two coupling structures nicely correspond to the gauge–invariant combinations $W^{\mu\nu}W_{\mu\nu}$ and $W^{\mu\nu}\widetilde{W}_{\mu\nu}$ in the electroweak Lagrangian.

One way to tell apart the $g^{\mu\nu}$ structure and the CP-odd coupling is to look at the $H \to ZZ$ decay. Each of the decaying Z bosons defines its own decay plane opened by the two lepton momenta. If the angle between these two planes in the Higgs rest frame is ϕ, the ϕ distribution can generally be written as

$$\frac{d\sigma}{d\phi} \propto 1 + a\cos(\phi) + b\cos(2\phi) \tag{63}$$

For the CP-odd Higgs coupling to $W^{\mu\nu}\widetilde{W}_{\mu\nu}$ we find $a = 0$ and $b = 1/4$, while for the CP-even Higgs coupling $g^{\mu\nu}$ we find $a > 1/4$ depending on m_H.Note, however, that this method only works if we observe the decay $H \to ZZ \to 4\ell$ with a good signal-background ratio S/B.

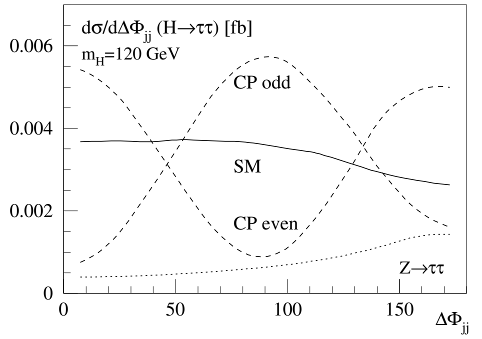
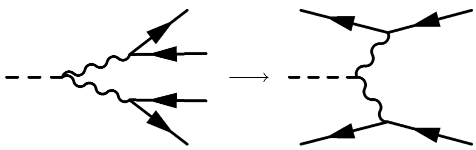


FIG. 4: Distribution for the azimuthal angle between the two forward jets in WBF production with a subsequent decay $H \to \tau\tau$. The signal and the leading Z background are simulated on the parton level. All signal rates are normalized to the Standard–Model value.

Starting from the Feynman diagram for $H \to ZZ \to 4\ell$ we can turn around two fermion legs


(64)

and read the diagram right-to-left. In other words, exactly the same information as in the decay–angle correlation is included in the angular correlation of the tagging jets in WBF production, independent of the Higgs decay! Indeed, for the three gauge–invariant coupling structures we find three distinctly different normalized distributions for the azimuthal angle between the tagging jets. One feature is obvious: the CP-odd $\epsilon^{\mu\nu\rho\sigma}$ coupling vanishes once two momenta become equal, because of the antisymmetric structure. Which explains the behavior of the ϕ distribution in that case for $\phi = 0,\ 180°$. The difference between the two CP-even couplings can be explained by the behavior of the fermion currents involved, but we have to move on and defer this discussion to the literature.

C. Higgs Self Coupling

The only way to probe the trilinear coupling at the LHC is to study HH production, for example in gluon fusion as shown in Fig. 5. In the low-energy limit we can compute the leading form factors associated with the triangle and box diagrams, multiplying the $g^{\mu\nu} - k_1^\mu k_2^\nu/(k_1 \cdot k_2)$ Dirac structure

$$F_\triangle = -F_\Box = \frac{2}{3} + \mathcal{O}\left(\frac{m_H^2}{4m_t^2}\right) \tag{65}$$

The loop-induced production cross section for $gg \to HH$ in terms of these form factors at threshold $\hat{s} \sim (2m_H)^2$ is proportional to

$$\left[3m_H^2\,\frac{F_\triangle}{\hat{s}-m_H^2} + F_\Box\right]^2 = \left[3m_H^2\,\frac{F_\triangle}{\hat{s}-m_H^2} - F_\triangle\right]^2 \sim \left[3m_H^2\,\frac{F_\triangle}{3m_H^2} - F_\triangle\right]^2 \to 0 \tag{66}$$

Note that in this relation we have used the fact that the Higgs self coupling is proportional to m_H. To see effects of the self coupling proportional to the first term above we should be looking at something like the $\hat{s}$ distribution or the m_{HH} distribution and measure its threshold behavior. In the absence of a self coupling this threshold behavior should be completely spoiled. This shape analysis of the threshold behavior would allow us to exclude the case of $\lambda_{HHH} = 0$ because of the expected large enhancement of the production cross section at threshold. It is still under study if such a measurement will work at the LHC.

The last two examples of measuring Higgs properties at the LHC are only a small fraction of the work which has gone into LHC Higgs physics over recent years. Also based on the success of the WBF signatures we now believe that the LHC will indeed be able to study in detail the Higgs sector and determine if it is really only the minimal Standard–Model version or maybe a supersymmetric two-Higgs-doublet model or something we have not even thought about....

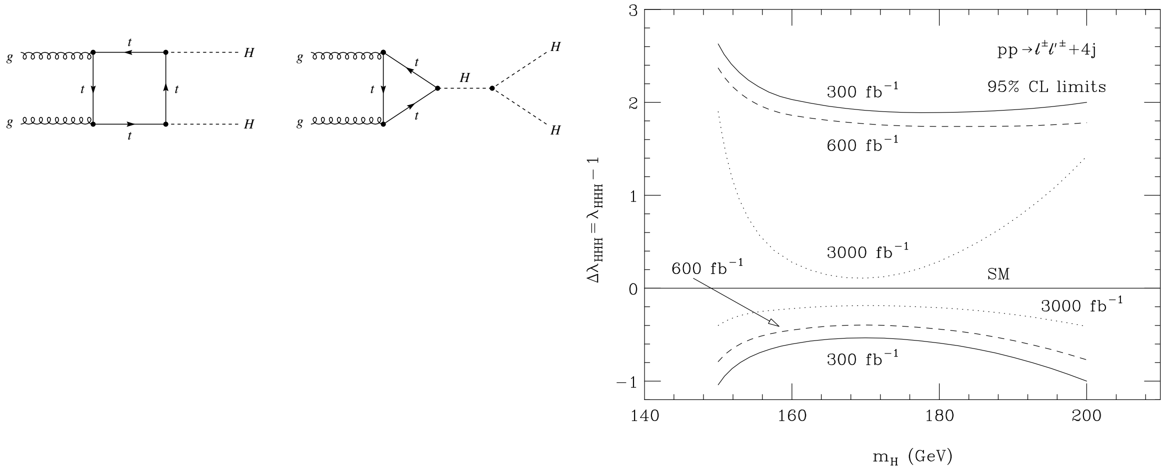


FIG. 5: Feynman graphs and (parton-level) sensitivity limits for Higgs pair production and a measurement of the Higgs self coupling. The analysis is based on the decay $HH \to WW$.

Acknowledgments: I would like to thank everybody who has tought me Higgs physics over the last ten years. Peter Zerwas and Michael Spira really triggered my interest in theoretical high-energy physics, starting with Higgs physics and for example including low–energy theorems in this writeup. Dieter Zeppenfeld and Dave Rainwater then convinced me that weak–boson fusion was the way to go for the LHC. And finally Tao Han, who taught me how to not give up even when things look sad, which happens all the time in Higgs physics. These pretty lecture notes are due to Erik Gerwick and Eoin Kerrane.

Literature:
For once, in LHC Higgs physics there are several very complete and useful reviews to

– Abdel Djouadi's hep-ph/0503172 includes about everything you ever want to know about Standard–Model Higgs phenomenology, at LHC and at other colliders.

– for more experimental details, have a look at Karl Jakobs' and Volker Büscher's hep-ph/0504099.

– as usual, there is a TASI lecture available on the topic, Dave Rainwater's hep-ph/0702124. It is very complete and includes a lot of information on Higgs physics beyond a discovery. If you want to know more about the WBF signatures and the minijet veto I also recommend Dave's thesis hep-ph/9908378.

– in hep-ph/9705337 Michael Spira probably includes the most complete theoretical discussion of the gluon–fusion production process, the low-energy theorems and related topics.