LECTURE 10: Standard Model (Part 2)

Overview:

- Construction of the Standard Model
- SM Higgs Mechanism

(I used Quigg and Novaes as references)

Standard Model Lagrangian

Last lecture, we ended with:

$$\mathcal{L}_{lep.} = \mathcal{L}_{lep} + \underbrace{\bar{L}\, i\gamma^\mu \left(i\frac{g}{2}\tau^i W_\mu^i + i\frac{g'}{2} Y B_\mu \right) L} + \bar{R}\, i\gamma^\mu \left(i\frac{g'}{2} B_\mu \right) R$$

expanding:

$$-g\bar{L}\gamma^\mu \left(\frac{\tau^1}{2} W_\mu^1 + \frac{\tau^2}{2} W_\mu^2 \right) L \qquad ①$$

$$-g\bar{L}\gamma^\mu \frac{\tau^3}{2} L W_\mu^3 - \frac{g'}{2} Y \bar{L}\gamma^\mu L B_\mu \qquad ②$$

We saw that Term ① involves charged current. We can write it as:

$$-\frac{g}{2}\bar{L}\gamma^\mu \begin{pmatrix} 0 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & 0 \end{pmatrix} L$$

We can define the charged bosons as: $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp W_\mu^2)$

Standard Model Lagrangian (cont.)

Term (1) becomes:

$$-\frac{g}{2\sqrt{2}}\left[\bar{\nu}\gamma^\mu(1-\gamma^5)\,\ell\, W_\mu^+ + \bar{\ell}\gamma^\mu(1-\gamma^5)\,\nu\, W_\mu^-\right]$$

note that $\quad \frac{g}{2\sqrt{2}} = \left(\frac{M_W^2 G_F}{\sqrt{2}}\right)^{1/2}$

We now concentrate on the neutral current terms which involves both L and R components:

$$-g\,\bar{L}\,\gamma^\mu \frac{\tau^3}{2} L\, W_\mu^3 - \frac{g'}{2}\left(\bar{L}\gamma^\mu Y L + \bar{R}\gamma^\mu Y R\right) B_\mu$$

$$= -g\, J_3^\mu W_\mu^3 - \frac{g'}{2} J_Y^\mu B_\mu$$

$\longrightarrow$ from last lecture: $J_3^\mu = \frac{1}{2}\left(\bar{\nu}_L\gamma^\mu\nu_L - \bar{\ell}\gamma^\mu\ell\right)$

$$J_Y^\mu = -\left(\bar{\nu}_L\gamma^\mu\nu_L + \bar{\ell}_L\gamma^\mu\ell_L + 2\,\bar{\ell}_R\gamma^\mu\ell_R\right)$$

Standard Model Lagrangian (cont.)

Remember that $J_{em} = J_3 + \frac{1}{2} J_Y$

We want the right combination of fields that couple to J_{em}.

We can do this by rotating the fields:

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}$$

$$W_\mu^3 = \sin\theta_W A_\mu + \cos\theta_W Z_\mu$$

$$B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu$$

with $\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$ $\qquad$ $\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}$

$\frac{g'}{g} = \tan\theta_W$, $\quad g \sin\theta_W = g' \cos\theta_W$

Standard Model Lagrangian (cont.)

With the rotated fields, we get:

$$\left[-\left(g \sin\theta_W J_3^\mu + \tfrac{1}{2} g' \cos\theta_W J_Y^\mu\right) A_\mu \right]$$ correct form for em interaction

$$\left[+ \left(-g\cos\theta_W J_3^\mu + \tfrac{1}{2} g' \sin\theta_W J_Y^\mu\right) Z_\mu \right]$$ something new ...

First term

$$= \underline{-g \sin\theta_W} \left(\bar{\ell}\gamma^\mu \ell\right) A_\mu \longrightarrow e = g\sin\theta_W$$

Second term: $$\frac{-g}{2\cos\theta_W}\left(2\cos^2\theta_W J_3^\mu - \frac{g'}{g}\cos\theta_W \sin\theta_W J_Y^\mu\right) Z_\mu$$

$$= \frac{-g}{2\cos\theta_W}\left(2\cos^2\theta_W J_3^\mu - \frac{g}{g}\sin^2\theta_W J_Y^\mu\right) Z_\mu$$

$$= \frac{-g}{2\cos\theta_W}\left(2(1-\sin^2\theta_W) J_3^\mu - \sin^2\theta_W J_Y^\mu\right) Z_\mu$$

$$J_Y^\mu = -\left(\bar{\nu}_L \gamma_\mu \nu_L + \bar{\ell}_L \gamma_\mu \ell_L + 2\bar{\ell}_R \gamma_\mu \ell_R\right), \quad J_3^\mu = \frac{1}{2}\left(\bar{\nu}_L \gamma_\mu \nu_L - \bar{\ell}_L \gamma_\mu \ell_L\right)$$

note that $\sin^2\theta_W$ terms cancel for neutrinos

Standard Model Lagrangian (cont.)

For electrons we have for $\sin^2\theta_W$ terms:

$$+\bar{l}_L\gamma^\mu l_L + \bar{l}_L\gamma^\mu l_L + 2\bar{l}_R\gamma^\mu l_R = 2(\bar{l}_L\gamma^\mu l_L + \bar{l}_R\gamma^\mu l_R)$$

other terms for neutrinos: $\bar{\nu}_L\gamma^\mu\nu_L$

" " " electrons: $-\bar{l}_L\gamma^\mu l_L$

Which can be summarized as:

$$-\frac{g}{2\cos\theta_W}\sum_{\psi_i=\nu,l}\bar{\psi}_i\gamma^\mu(\underbrace{g_V^i}_{T_3^i - 2Q_i\sin^2\theta_W} - \underbrace{g_A^i}_{T_3^i}\gamma^5)\psi_i Z_\mu$$

→ note that SM predicted neutral weak interactions which were observed ~5 years after model was proposed

more on that later...

Fermions and bosons are massless

Standard Model Lagrangian (cont.)

Back to the Higgs mechanism. We introduce the scalar doublet:

$$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \qquad \text{with } Y = 1$$
$$T = 1/2 \,, \quad T^3 = -1/2$$

$$\mathcal{L}_{scalar}: \partial_\mu \Phi^\dagger \partial^\mu \Phi - V(\Phi^\dagger \Phi)$$

$$V(\Phi^\dagger \Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

We introduce the gauge-covariant derivative:

$$\partial_\mu \to D_\mu = \partial_\mu + i g \frac{\tau^i}{2} W^i_\mu + i g' Y B_\mu$$

chose VEV: $\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$, $v = \sqrt{\frac{-\mu^2}{\lambda}}$

→ we need to keep $U(1)$ a symmetry of the vacuum (keep photon massless)

$$Q \langle \Phi \rangle_0 = (T_3 + \tfrac{1}{2} Y) \langle \Phi \rangle_0 = \frac{1}{2} \left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0$$

Standard Model Lagrangian (cont.)

The ρ parameter : $\rho = \dfrac{M_W^2}{\cos^2\theta_W M_Z}$

represents the relative strengths of the neutral and charged effective currents:

$$J^{0\mu} J^0_\mu \;/\; J^{+\mu} J^-_\mu$$

$\rho = 1$ in the SM (at tree level)

In model with arbitrary number of Higgs multiplets φ_i with isospin T_i and 3rd component T^3_i and VEV v_i,

ρ is given by : $\dfrac{\sum_i [T_i(T_i+1) - (T_{3i})^2] v_i^2}{2\sum_i (T_{3i})^2 v_i^2}$

will give $\rho = 1$ for doublets

$\rightarrow$ Tests isospin structure of Higgs sector

Standard Model Lagrangian (cont.)

Lepton masses:

We saw that adding explicit mass terms in the Lagrangian breaks gauge invariance. We therefore add another term:

$$\mathcal{L}_{Yukawa} = -G_\ell \left[\bar{R} (\varphi^\dagger L) + (\bar{L} \varphi) R \right]$$

$$= -G_\ell \frac{(v+\eta)}{\sqrt{2}} \left[\bar{\ell}_R (0 \;\; 1) \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} + (\bar{\nu}_L \;\; \bar{\ell}_L) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \ell_R \right]$$

$$= -\frac{G_\ell v}{\sqrt{2}} \bar{\ell} \ell - \frac{G_\ell}{\sqrt{2}} \bar{\ell} \ell \eta$$

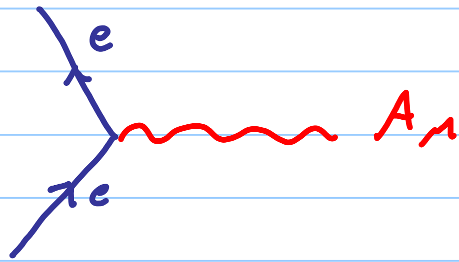
we get $m_\ell = \dfrac{G_\ell v}{\sqrt{2}}$ $\rightarrow$ no prediction for G_ℓ

$\rightarrow G_{Top} = 1$ (within 1%)

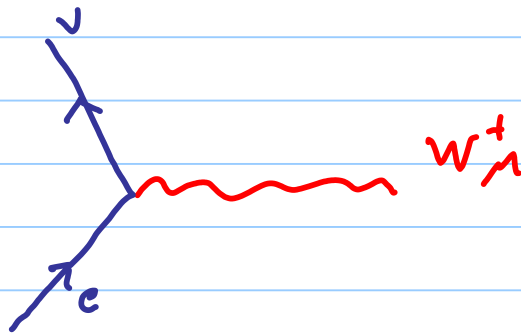
coupling strength $\bar{\ell} \ell H = \dfrac{m_\ell}{v}$ $\rightarrow$ $\boxed{\text{a Testable prediction}}$

The Standard Model (cont.)

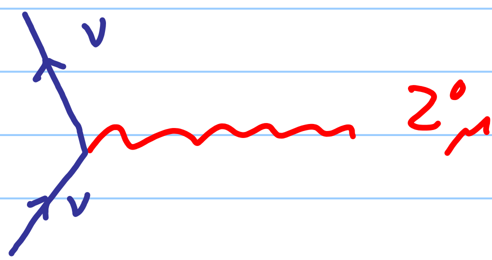
From the previous pieces of the SM Lagrangian we can deduce the following Feynman rules:



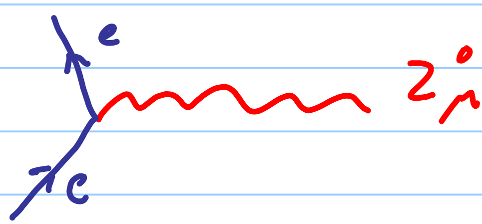
$= ie \, \bar{e} \gamma_\mu e$



$$-i \left(\frac{G_F M_W^2}{\sqrt{2}} \right)^{1/2} \bar{\nu} \gamma_\mu (1-\gamma_5) e$$



$$\frac{-i}{\sqrt{2}} \left(\frac{G_F M_Z^2}{\sqrt{2}} \right)^{1/2} \bar{\nu} \gamma_\mu (1-\gamma_5) \nu$$



$$\frac{-i}{\sqrt{2}} \left(\frac{G_F M_Z^2}{\sqrt{2}} \right)^{1/2} \bar{e} \gamma_\mu \left[2 s_W^2 (1+\gamma_5) + (2 s_W^2 - 1)(1-\gamma_5) \right] e$$

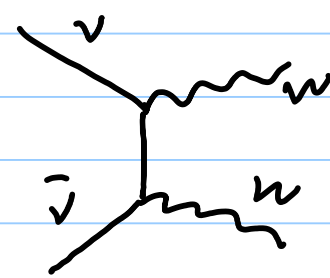
$s_W = \sin\theta_W$

The Standard Model (cont.)

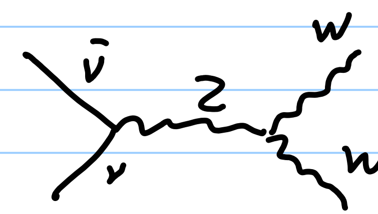
$$ie \cot\theta_W \left[g_{\alpha\beta}(k_+ - k_-)_\mu - g_{\alpha\mu}(p + k_+)_\beta + g_{\beta\mu}(p + k_-)_\alpha \right]$$

In lecture 9 we saw that the $\nu\bar{\nu} \to WW$ cross section was not well behaved at high energies:

$$\sigma \sim \frac{G_F^2 s}{3\pi}$$



We know we were missing one diagram:



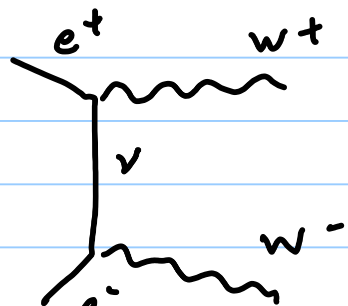
$$M_Z = \frac{-ig^2}{4(s - M_Z^2)} \bar{v}\gamma_\mu(1-\gamma_5)u \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{M_Z^2} \right) \times \varepsilon_+^{*\alpha} \varepsilon_-^{*\beta}$$
$$\left[g_{\alpha\beta}(k_- - k_+)_\nu + g_{\alpha\nu}(k_+ + p)_\beta - g_{\beta\nu}(k_- + p)_\alpha \right]$$

PROBLEM SET 2, PROBLEM #1:
show that extra diagram cures our problem.

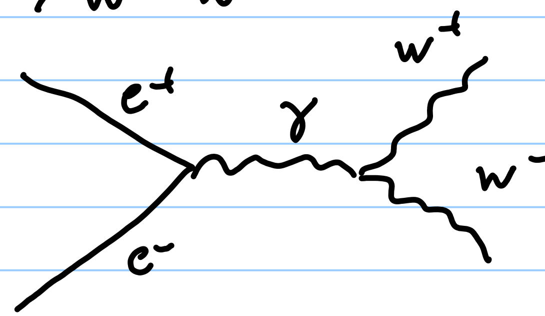
The Standard Model (cont.)

Another interesting case is provided by the reaction

$$e^+ e^- \longrightarrow W^+ W^-$$



(a)



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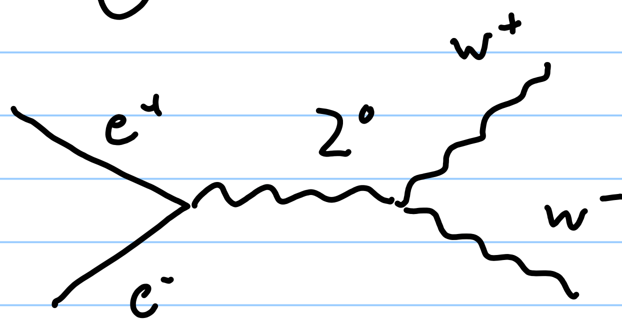
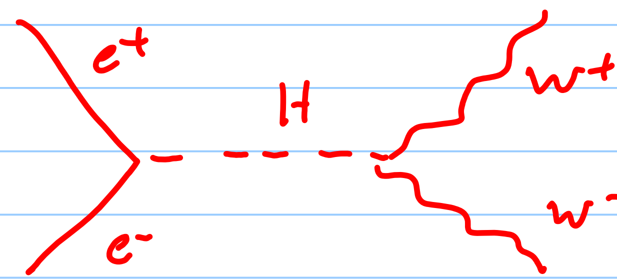


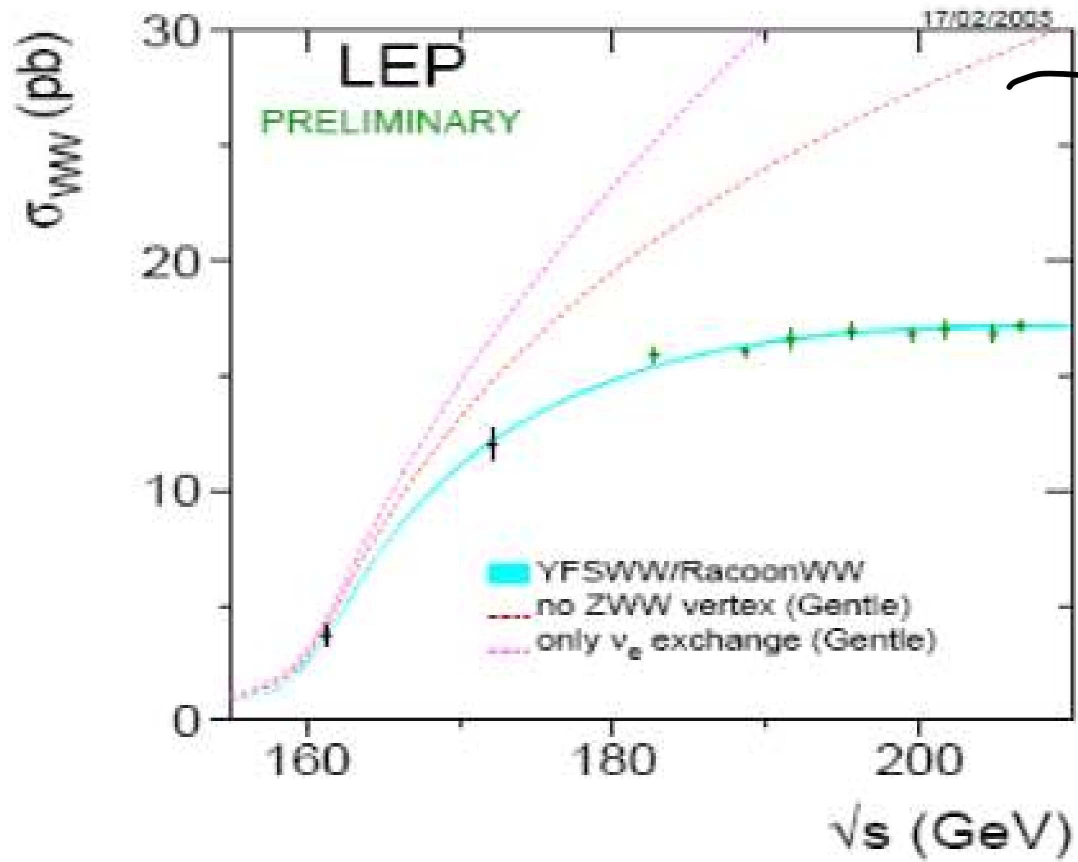
(c)



(d)

PROBLEM SET 2, PROBLEM #2 : show that you need diagram (d) to cancel the divergence in the cross section (you'll need to keep the electron mass).

$e^+e^- \to WW$ cross section from LEP II :



adding photon