LECTURE 12: Z Boson Physics and Neutral Currents

Overview:

- Higgs Boson (cont.)
- Z width
- Problem set #2 (last problem)

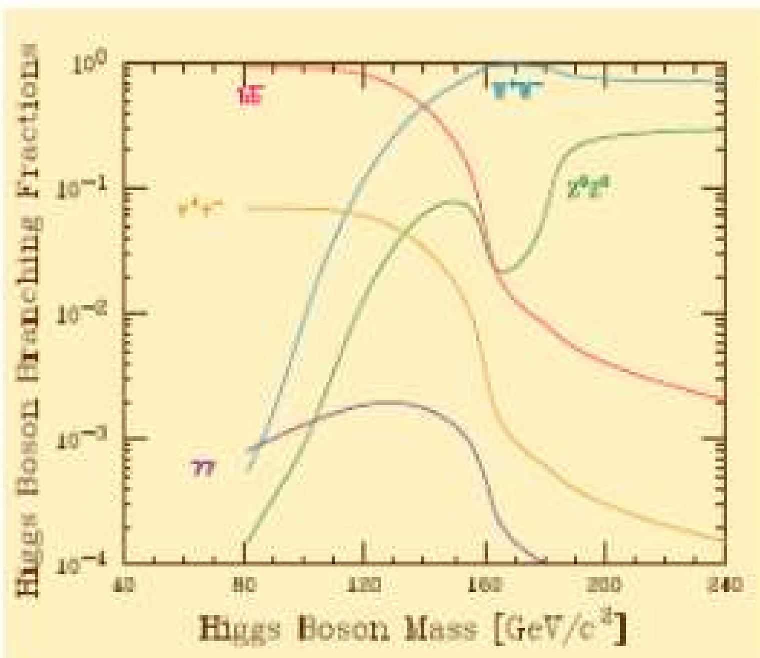
(I used Quigg, Griffiths, my thesis as references)

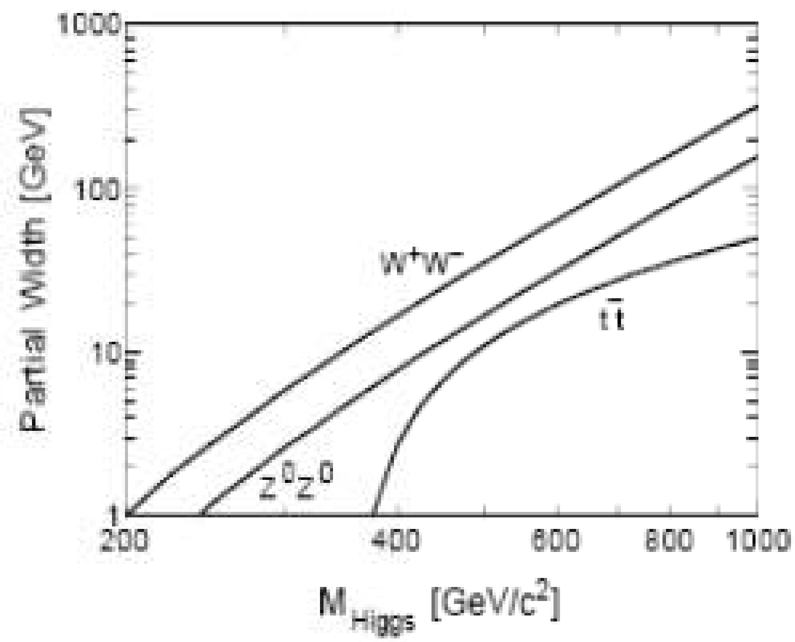
THE HIGGS BOSON (cont)

NOTES:

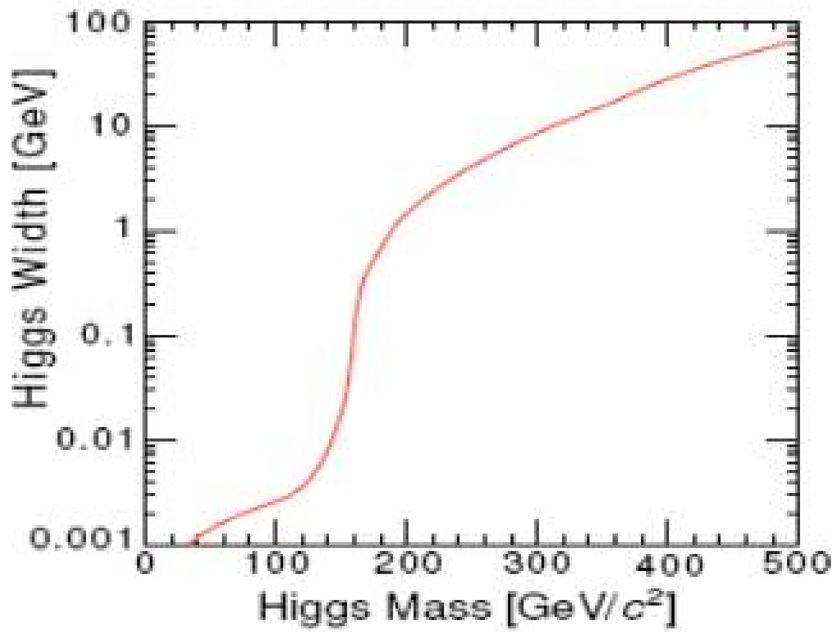
- in the limit where $M_H \gg M_W$, Γ becomes $\propto M_H^2 \cdot M_H$ instead of $M_P^2 \cdot M_H$ why?

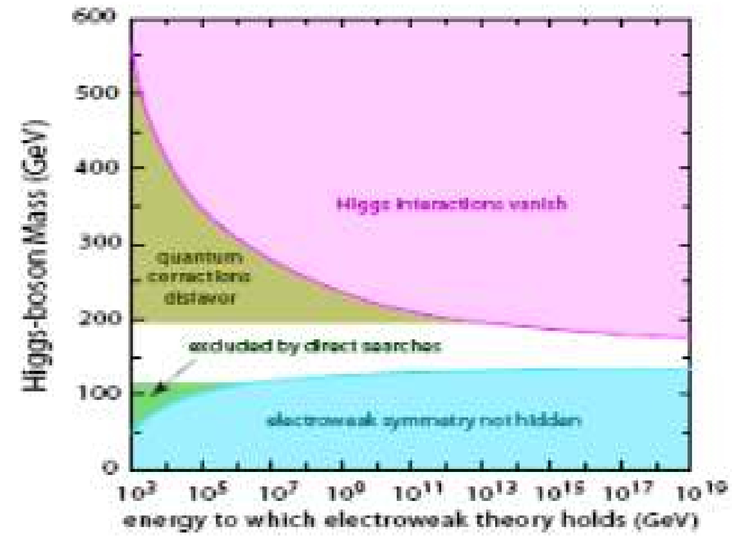
- why do we get a factor of 1/2 for Z bosons? (the obvious answer does not tell the full story)





THE HIGGS BOSON (cont.)





$\boxed{\text{Theory limits on the Higgs}}$

Unitarity in $W^+W^- \to W^+W^-$ cross section imply that m_H should be $\lesssim 900$ GeV

$\to$ Higgs or new physics at 1 TeV ...

THE HIGGS BOSON (cont.)

TRIVIALITY: evolution of renormalized coupling given by:

$$\lambda_R(Q) = \frac{\lambda(v)}{1 - \frac{3}{2\pi^2}\lambda(v)\log\left(\frac{Q}{v}\right)}$$

$\lambda(v) \to 0$ when $Q \to \infty$ i.e. Trivial Theory
free field theory

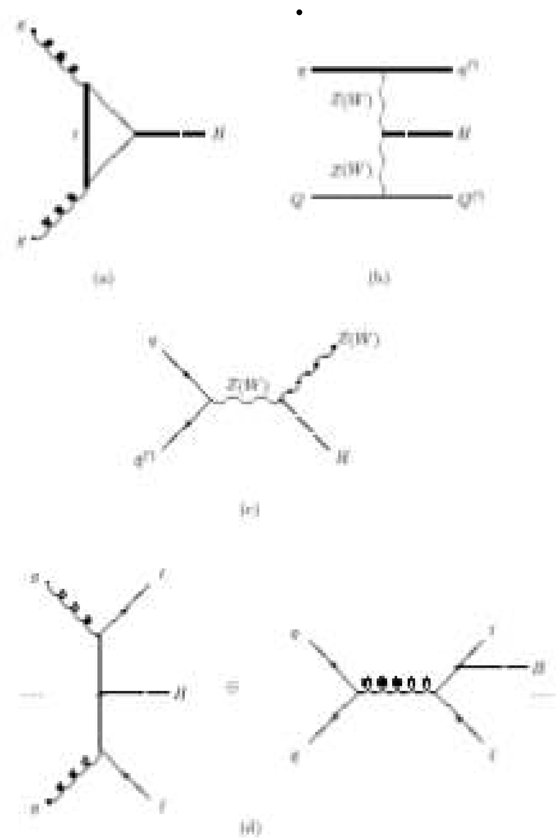
if Higgs potential valid up to 10^{16} GeV
then $m_H \lesssim 170$ GeV
1 TeV $m_H \lesssim 640$ GeV

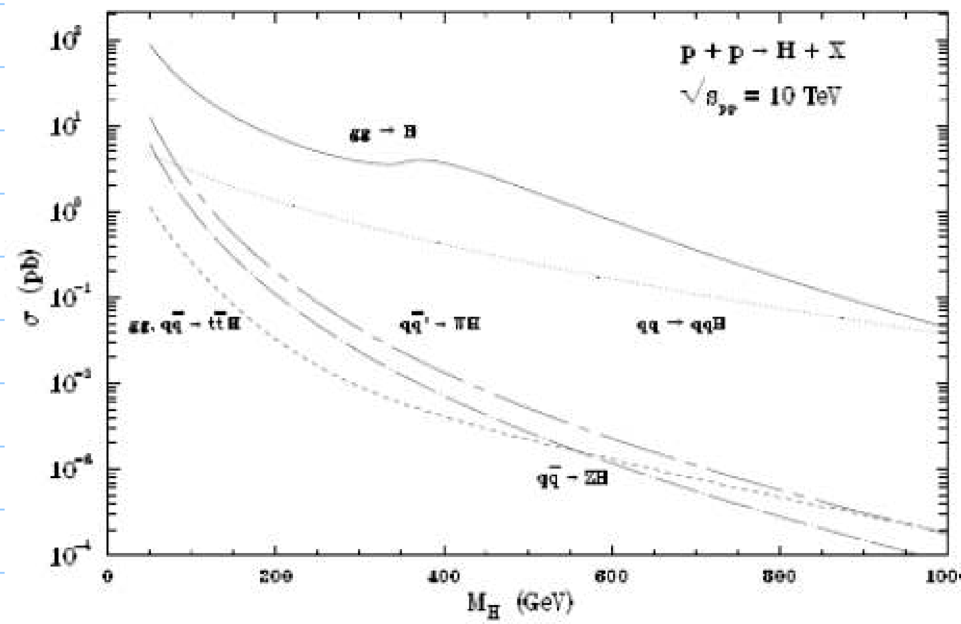
Vacuum stability:

$$V(\varphi) \sim \mu^2 \varphi^\dagger \varphi + \lambda(Q_0)(\varphi^\dagger\varphi)^2 + B_\lambda (\varphi^\dagger\varphi)^2 \log\left(\frac{Q^2}{Q_0^2}\right)$$

SM valid up to Planck scale $\rightarrow m_H \gtrsim 130$ GeV

HIGGS PRODUCTION AT THE LHC

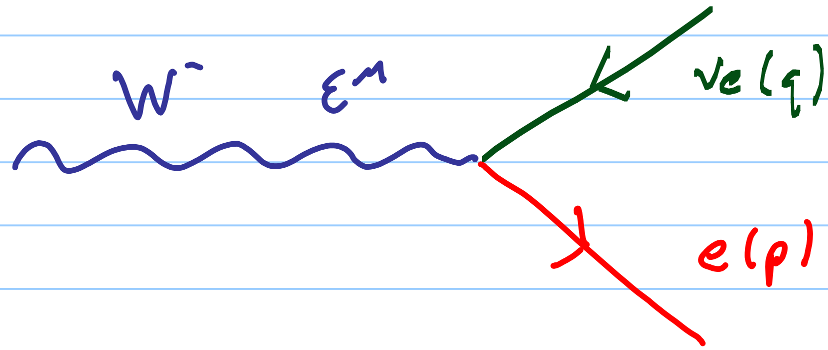




Let's look at slides from Chris Quigg, academic lectures part 5, 6

Z Boson Decay

A reminder from lecture 8 where we studied $W \to e\nu$:



$\to \bar{\nu}_e$ going forward, ν_e going backward

$$\mathcal{M} = -i \left(\frac{G_F M_W^2}{\sqrt{2}} \right)^{1/2} \bar{u}(p) \gamma_\mu (1-\gamma_5) v(q) \varepsilon^\mu$$

$\varepsilon^\mu \equiv (0, \hat{\varepsilon})$ is the polarization vector of the W

→ we neglect the electron mass

$$|\mathcal{M}|^2 = \frac{G_F M_W^2}{\sqrt{2}} \mathrm{Tr}\left[\not{\varepsilon} (1-\gamma_5) \not{q} (1+\gamma_5) \not{\varepsilon}^* \not{p} \right]$$

$$= \frac{G_F M_W^2}{\sqrt{2}} \; 2\mathrm{Tr}\left[(1+\gamma_5) \not{\varepsilon} \not{q} \not{\varepsilon}^* \not{p} \right]$$

W decay

$$|M|^2 = \frac{8 G_F M_W^2}{\sqrt{2}} \Big((\varepsilon \cdot q)(\varepsilon^* \cdot p) - (\varepsilon \cdot \varepsilon^*)(p \cdot q) + (\varepsilon \cdot p)(\varepsilon^* \cdot q)$$
$$+ i \varepsilon_{\mu\nu\rho\sigma} \varepsilon^\mu q^\nu \varepsilon^{*\rho} p^\sigma \Big]$$

let's pick the longitudinal polarization for the W : $\varepsilon^\mu = (0;0,0,1) = \varepsilon^{\mu *}$ (helicity 0)

$\rightarrow$ the $\varepsilon_{\mu\nu\rho\sigma}$ term vanishes

$$p = \frac{M_W}{2}(1, \sin\theta, 0, \cos\theta)$$
$$q = \frac{M_W}{2}(1, -\sin\theta, 0, -\cos\theta)$$

$$|M|^2 = \frac{8 G_F M_W^2}{\sqrt{2}} \cdot \frac{M_W^2}{4} \big(-\cos^2\theta - -1 \cdot [1 + \sin^2\theta + \cos^2\theta] - \cos^2\theta$$
$$= \frac{4 G_F M_W^4}{\sqrt{2}} \sin^2\theta$$

W Decay

$$\frac{d\Gamma}{d\Omega} = \frac{|M|^2}{64\pi^2 M_W} = \frac{G_F M_W^3}{16\pi^2\sqrt{2}} \sin^2\theta$$

$$d\Gamma = \cdot \frac{G_F M_W^3}{16\pi^2\sqrt{2}} \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta \sin^2\theta \, d\theta$$

$$\text{int} = -\cos\theta + \frac{\cos^3\theta}{3} \Big|_0^{\pi} = 1 - \frac{1}{3} - \left(-1 + 1/3\right)$$

$$= \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$

$$\boxed{\Gamma = \frac{G_F M_W^3}{16\pi^2\sqrt{2}} \cdot 2\pi \cdot \frac{4}{3} = \frac{G_F M_W^3}{6\pi\sqrt{2}}}$$

$= 227$ MeV (for $M_W = 80.4$)

$G_F = 1.166 \times 10^{-5}$ GeV^{-2}

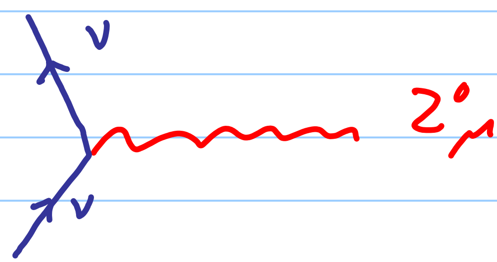
Total width: 2.06 GeV

Z DECAY

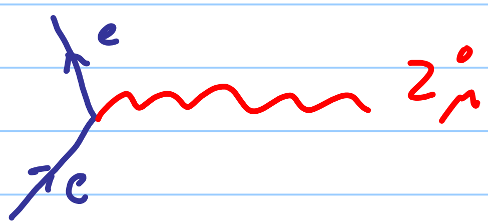
The W decay amplitude

$$\mathcal{M} = -i\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{1/2} \bar{u}(p)\gamma_\mu(1-\gamma_5)v(q)\,\varepsilon^\mu :$$

Needs to be modified for the Z given Feynman rules:



$$\frac{-i}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{1/2} \bar{\nu}\gamma_\mu(1-\gamma_5)\nu$$



$$\frac{-i}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{1/2} \bar{e}\gamma_\mu\left[2s_w^2(1+\gamma_5) + (2s_w^2-1)(1-\gamma_5)\right]e$$

$s_w = \sin\theta_w$

So $Z \to \nu_e\bar{\nu}_e$ is straightforward:

$$|\mathcal{M}(Z\to\nu_e\bar{\nu}_e)|^2 = \frac{1}{2}\,\frac{M_Z^2}{M_W^2}\,|\mathcal{M}(W\to e\nu_e)|^2$$

$$\to \Gamma(Z\to\nu\bar{\nu}) = \frac{G_F M_Z^3}{12\pi\sqrt{2}}$$

REMINDER:

$$-\frac{g}{\cos\theta_W}\,\bar{\psi}_i\,\gamma^\mu\,\frac{1}{2}\big(\underbrace{g_V^i}_{T_3^i - 2Q_i\sin^2\theta_W} - \underbrace{g_A^i}_{T_3^i}\,\gamma^5\big)\,\psi_i\,Z_\mu$$

i	Q^i	g_A^i	g_V^i
$\nu_e\,\nu_\mu\,\nu_\tau$	0	$1/2$	$1/2$
$e\,\mu\,\tau$	-1	$-1/2$	$-1/2 + 2\sin^2\theta_W \simeq -0.03$
$u\,c\,t$	$2/3$	$1/2$	$1/2 - 4/3\sin^2\theta_W \simeq 0.19$
$d\,s\,b$	$-1/3$	$-1/2$	$-1/2 + 2/3\sin^2\theta_W \simeq -0.34$

$$J_\mu^{em} = J_\mu^3 + 1/2\, J_\mu^Y$$

$$J_\mu^{NC} = J_\mu^3 - \sin^2\theta_W\, J_\mu^{em}$$

$$g_W = \frac{g_e}{\sin\theta_W}$$

REMINDER (CONT.)

Standard Model Lagrangian (cont.)

As you've done in problem set 1, we parametrize the Higgs doublet

$$\Phi = \exp\left(\frac{i\tau^i \chi_i}{2v}\right)\begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix}.$$

we make a gauge Trans. $\Phi \to \Phi' = \exp\left(-i\frac{\tau^i}{2v}\chi_i\right)\Phi$

$$= \frac{(v+\eta)}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathcal{L}_{scalar}: \left|\left(\partial_\mu + ig\frac{\tau^i}{2}W_\mu^i + i\frac{g'}{2}yB_\mu\right)\frac{(v+\eta)}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right|^2$$

$$-\frac{\mu^2}{2}(v+\eta)^2 - \frac{\lambda}{4}(v+\eta)^4$$

$$-gW_\mu^3 + g'B_\mu$$

$$\left|\begin{pmatrix} 0 \\ \partial_\mu\eta/\sqrt{2} \end{pmatrix} + \frac{ig}{2}(v+\eta)\begin{pmatrix} W_\mu^+ \\ \frac{-1}{\sqrt{2}\cos\theta_W}Z_\mu \end{pmatrix}\right|^2$$

$$-gW_\mu^3 + g'B_\mu$$

$$= \frac{-g}{\cos\theta_W}Z_\mu$$

Reminder (cont.)

Why do we write W_μ^+ as $2(W_\mu^1 - iW_\mu^2)$?

→ weak current for lepton ℓ:

$$J_\mu^+ = \bar{\ell}\gamma_\mu(1-\gamma_5)\nu = 2\bar{\ell}_L\gamma_\mu\nu_L$$

The charged weak current can be written as:

$$J_\mu^i = \bar{L}\gamma_\mu\frac{\tau^i}{2}L$$

Explicitly:

$$J_\mu^1 = \frac{1}{2}(\bar{\nu}_L\ \bar{\ell}_L)\gamma_\mu\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}\nu_L\\\ell_L\end{pmatrix} = \frac{1}{2}(\bar{\ell}_L\gamma_\mu\nu_L + \bar{\nu}_L\gamma_\mu\ell_L)$$

$$J_\mu^2 = \frac{1}{2}(\bar{\nu}_L\ \bar{\ell}_L)\gamma_\mu\begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}\nu_L\\\ell_L\end{pmatrix} = \frac{i}{2}(\bar{\ell}_L\gamma_\mu\nu_L - \bar{\nu}_L\gamma_\mu\ell_L)$$

$$J_\mu^3 = \frac{1}{2}(\bar{\nu}_L\ \bar{\ell}_L)\gamma_\mu\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}\nu_L\\\ell_L\end{pmatrix} = \frac{1}{2}(\bar{\nu}_L\gamma_\mu\nu_L - \bar{\ell}_L\gamma_\mu\ell_L)$$

Z DECAY (cont.)

$$\Gamma(Z \to e^+e^-) = \Gamma(Z \to \nu\bar{\nu})\left[(2\sin^2\theta_w - 1)^2 + (2\sin^2\theta_w)^2\right]$$

$$\boxed{\begin{aligned} &\left[2s_w^2 + 2s_w^2\gamma_5 + 2s_w^2 - 2s_w^2\gamma_5 - 1 + \gamma_5\right] \\ &= \left[2s_w^2 + (2s_w^2 - 1) - \gamma_5\right] \\ &\qquad \to \gamma_5 \text{ contribution will vanish as in } W \text{ case} \end{aligned}}$$

We have $M_W \simeq 80.4$ GeV

$\Gamma(W \to e\nu) \simeq 225$ GeV

$M_Z \simeq 91.2$ GeV

$\Gamma(Z \to e^+e^-) \simeq 84$ GeV

$\Gamma(Z \to \bar{\nu}_e\nu_e) \simeq 166$ GeV — note

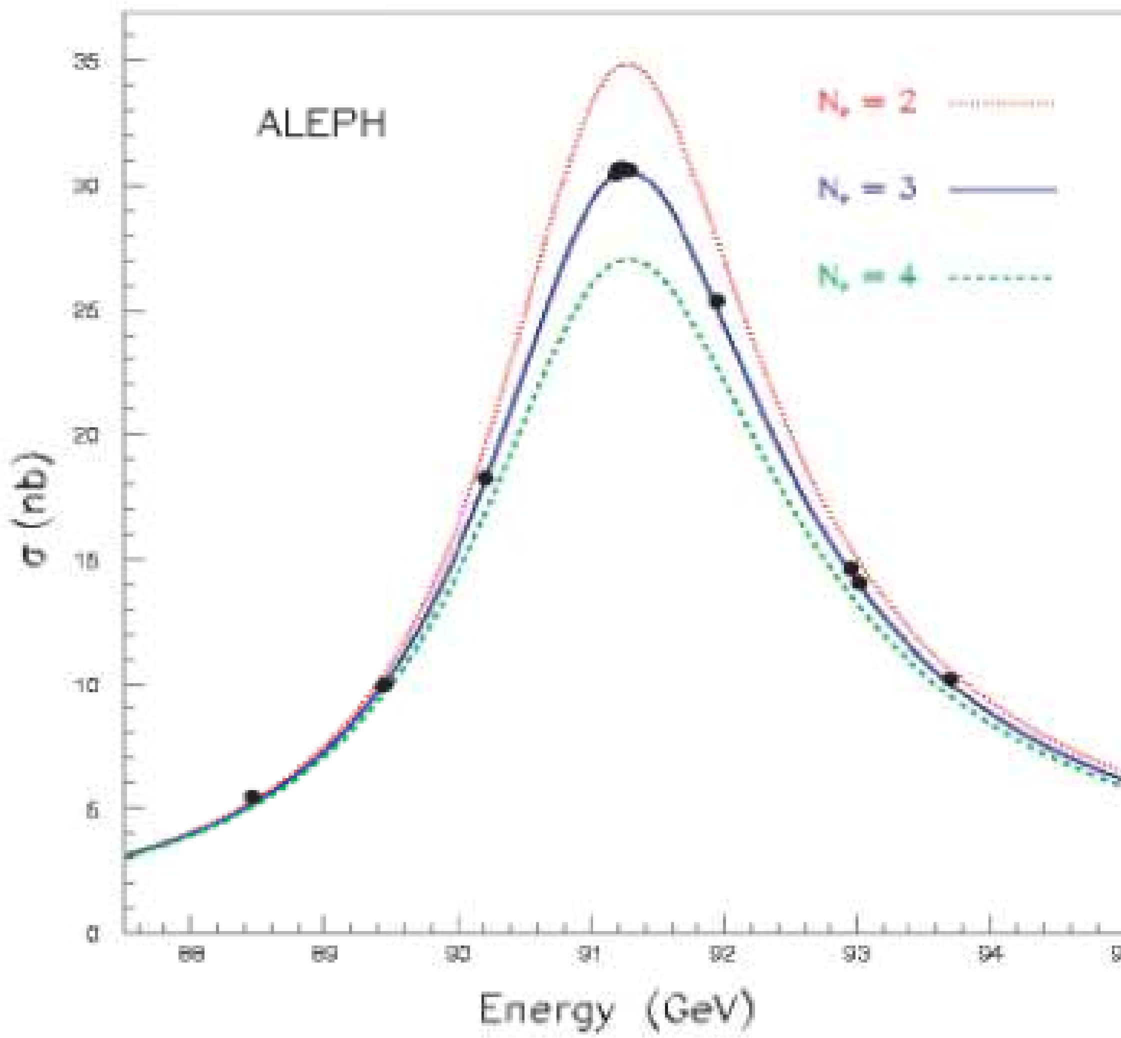
How do we measure $Z \to \nu\bar{\nu}$?

Z decay (cont.)

Most precise method is "indirect": look at Z lineshape:

Obtain Γ_{TOT} from lineshape. Subract Γ_{had}, $\Gamma_{e,\mu,\tau}$ and get $\Gamma_{inv} \sim 500$ MeV

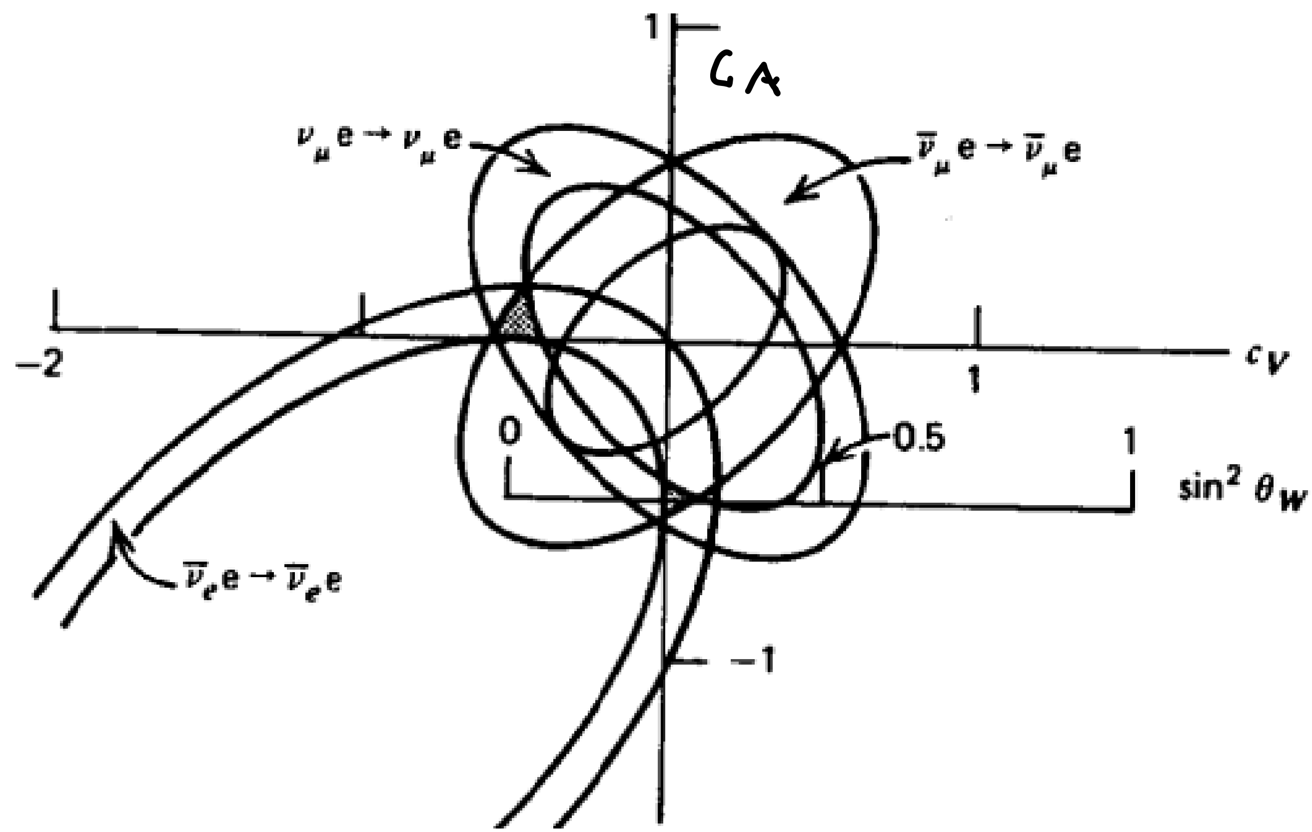
$\Rightarrow$ 3 neutrino families below $\frac{M_Z}{2}$



Z Boson Physics

Problem set #2 problem #3

→ Calculate $\sigma\ (e^+e^- \to Z \to \mu^+\mu^-)$

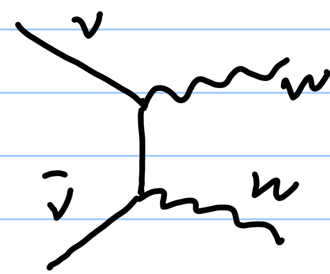


PROBLEM SET #2 due MARCH 11^{th} 1pm

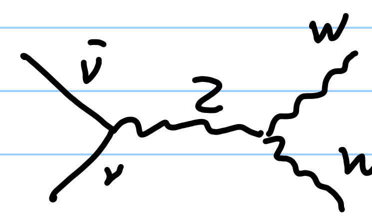
PROBLEM 1

In lecture 9 we saw that the $\nu\bar{\nu} \to WW$ cross section was not well behaved at high energies:

$$\sigma \sim \frac{G_F^2 s}{3\pi}$$



We know we were missing one diagram:



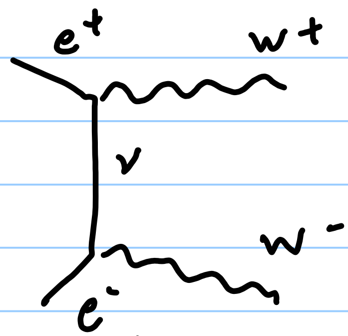
$$M_Z = \frac{-ig^2}{4(s-M_Z^2)} \bar{v}\gamma_\mu(1-\gamma_5)u\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{M_Z^2}\right) \times \varepsilon_+^{*\alpha}\varepsilon_-^{*\beta}$$
$$\left[g_{\alpha\beta}(k_- - k_+)_\nu + g_{\alpha\nu}(k_+ + p)_\beta - g_{\beta\nu}(k_- + p)_\alpha\right]$$

PROBLEM SET 2, PROBLEM #1:
show that extra diagram cures our problem.

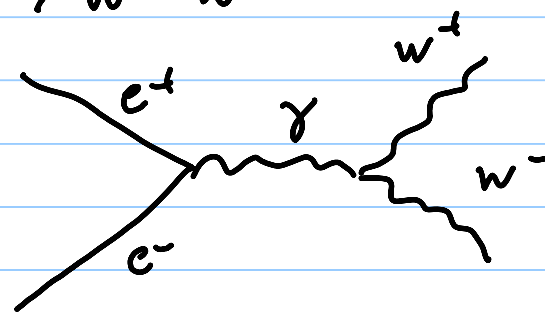
PROBLEM #2

Another interesting case is provided by the reaction

$$e^+ e^- \longrightarrow W^+ W^-$$



(a)



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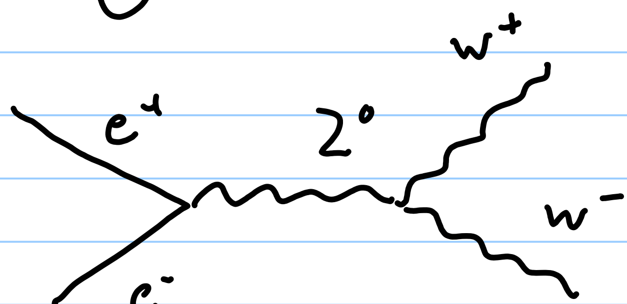
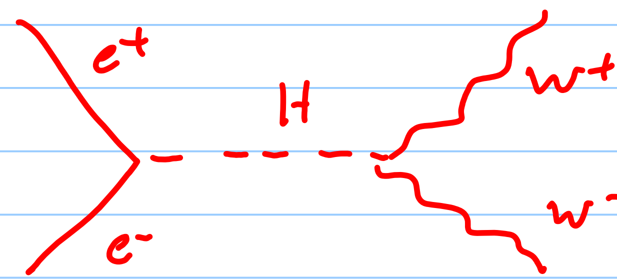


(c)



(d)

PROBLEM SET 2, PROBLEM #2 : show that you need diagram (d) To cancel the divergence in the cross section (you'll need To keep the electron mass).

problem #3

Calculate $\sigma(e^+e^- \to Z \to \mu^+\mu^-)$