QCD

- Describing Color
- The Feynman Rules
- Some Simple Examples
- Hadron collider physics

Quark Color

- Recall that we have inferred the existence of 3 quark colors from the hadron production rate

$$R = \frac{\sigma(e^+ \, e^- \to \text{hadrons})}{\sigma(e^+ \, e^- \to \mu^+ \, \mu^-)}$$

- Denoting the three colors as "red", "blue", and "green", we will need to append a *color vector*, c, to every external quark wavefunction (u or v):

$$c_r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad c_b = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad c_g = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

8 Gluon Colors

- When a gluon interacts with a quark, the quark color might change. This means that the gluon carries one unit of color and one unit of anticolor
- QCD is based on an $SU(3)$ color symmetry, so there are 8 gluons
- Explicitly, $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$ means that we have a *color octet*

$$
\begin{aligned}
|1\rangle &= (r\bar{b} + b\bar{r})/\sqrt{2} & |5\rangle &= -i(r\bar{g} - g\bar{r})/\sqrt{2} \\
|2\rangle &= -i(r\bar{b} - b\bar{r})/\sqrt{2} & |6\rangle &= (b\bar{g} + g\bar{b})/\sqrt{2} \\
|3\rangle &= (r\bar{r} - b\bar{b})/\sqrt{2} & |7\rangle &= -i(b\bar{g} - g\bar{b})/\sqrt{2} \\
|4\rangle &= (r\bar{g} + g\bar{r})/\sqrt{2} & |8\rangle &= (r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6}
\end{aligned}
$$

and a *colour singlet*

$$|9\rangle = (r\bar{r} + b\bar{b} + g\bar{g})/\sqrt{3}$$

The Gell-Mann Matrices

- The 8 gluon states can be regarded as 3×3 matrices in "color-space". These are the *Gell-Mann matrices*:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

- The Gell-Mann matrices will appear in the quark-gluon vertex factor for QCD.
- The Gell-Mann matrices λ^{α} are the $SU(3)$ counterparts of the Pauli matrices σ^i for $SU(2)$.

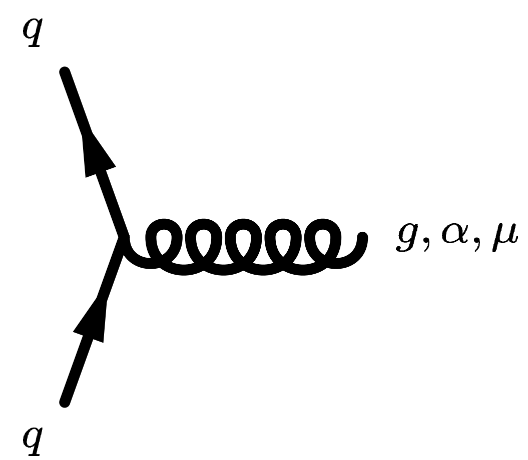
Gluon Wavefunctions

$$|1\rangle \Rightarrow a = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The gluon wavefunctions will consist of a polarization vector (ϵ^μ) and an 8-component color vector a^α:

Vertex Factors

- The basic QCD vertex,



  contributes a factor of

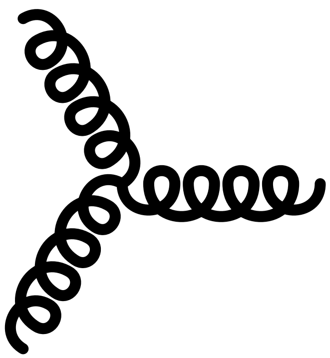
$$-\frac{ig_s}{2}\lambda^\alpha\gamma^\mu$$

- g_s is a dimensionless coupling constant and is related to the QCD version of the fine-structure constant by

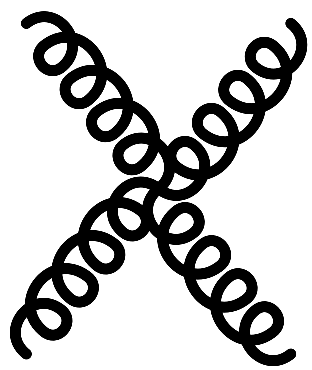
$$\alpha_s = \frac{g_s^2}{4\pi}$$

Gluon Self-Couplings

- Just as photons couple to particles with non-zero electric charge, gluons couple to particles with color.
- Photons are electrically neutral, so they do not couple (directly) to other photons. Gluons have color, therefore they can couple directly to each other :



$\mathcal{O}(g_s)$



$\mathcal{O}(g_s^2)$

Propagators

- Each internal gluon connects two vertices of the form $\lambda^\alpha \gamma^\mu$ and $\lambda^\beta \gamma^\nu$, so we should expect the gluon propagator to contract the indices α with β and μ with ν.

$$\text{Gluon propagator: } \frac{-i g_{\mu\nu} \delta^{\alpha\beta}}{q^2}$$

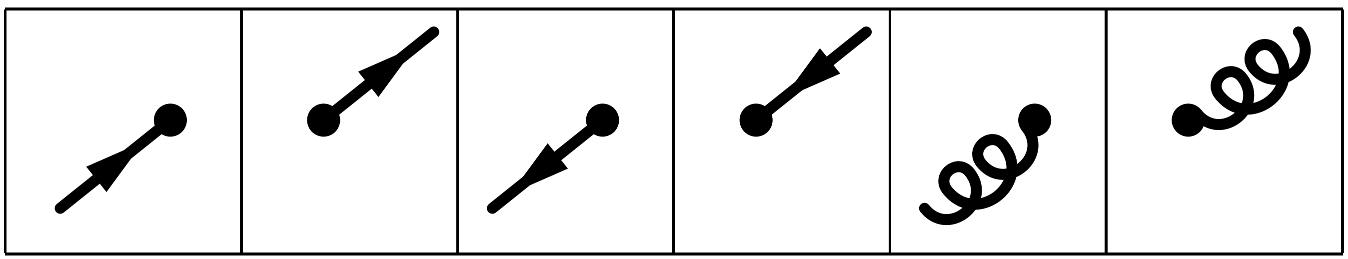
- Internal quarks have the familiar fermion propagator,

$$\text{Quark propagator: } \frac{i(\not{q} + m)}{q^2 - m^2}$$

The sign of q matters here — we take it to be in the same direction as the fermion arrow.

External Lines

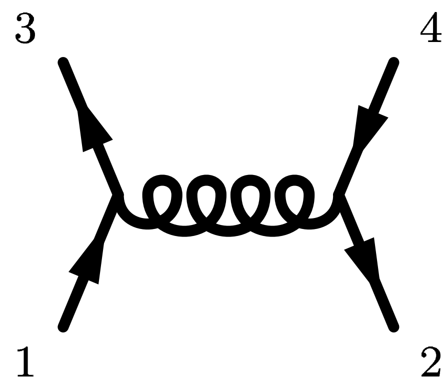
- As in QED, the external line factors must sit on the outside in order to make a number out of the amplitude.
- Work *backwards* along every quark line using:



q in	q out	$\bar{q}$ in	$\bar{q}$ out	g in	g out
$u\ c$	$\bar{u}\ c^\dagger$	$\bar{v}c^\dagger$	$v\ c$	$\epsilon_\mu\ a^\alpha$	$\epsilon^*_\mu\ a^{\alpha *}$

Example: The Quark-Antiquark Interaction

- Assuming that the quark flavors are different, we have just one diagram:



- From the Feynman rules, we have

$$\begin{aligned} \mathcal{M} &= i \left[\bar{u}_3 c_3^\dagger\right] \left(-\frac{ig_s}{2}\lambda^\alpha \gamma^\mu\right) [u_1 c_1] \left(\frac{-ig_{\mu\nu}\delta^{\alpha\beta}}{(p_1 - p_3)^2}\right) \\ &\quad \times \left[\bar{v}_2 c_2^\dagger\right] \left(-\frac{ig_s}{2}\lambda^\beta \gamma^\nu\right) [v_4 c_2] \end{aligned}$$

The Origin of Color Factors

$$\mathcal{M} = -\frac{g_s^2\, f}{q^2}\left[\bar{u}_3\gamma^\mu u_1\right]\left[\bar{v}_2\gamma_\mu v_4\right]$$

$$f = \frac{1}{4}\left(c_3^\dagger \lambda^\alpha c_1\right)\left(c_2^\dagger \lambda^\alpha c_4\right)$$

- This QCD amplitude looks just like the QED amplitude for $e^-\ \mu^+$ scattering except that we now have a *color factor* of f.
- This means that, insofar that α_s is sufficiently small to justify perturbative QCD, we will have a Coulomb-like potential

$$V_{q\bar{q}}(r) = -\frac{f\alpha_s}{r}$$

Colour factors

- Octet Configuration

$$f = \frac{1}{4}\left(c_3^\dagger \lambda^\alpha c_1\right)\left(c_2^\dagger \lambda^\alpha c_4\right) = -\frac{1}{6}$$

- Singlet Configuration

$$f = \frac{1}{4}\left(c_3^\dagger \lambda^\alpha c_1\right)\left(c_2^\dagger \lambda^\alpha c_4\right) = \frac{4}{3}$$

- With the color factors we have calculated, the $q\bar{q}$ potentials are:

$$V_{q\bar{q}}(r) = \begin{cases} -4\alpha_s/3r & \text{(color singlet)} \\ +\alpha_s/6r & \text{(color octet)} \end{cases}$$

- This means that the quark and antiquark are *attracted* to each other in the singlet configuration but *repelled* in the octet configuration.
  $\Rightarrow$ This explains why mesons are color singlets.

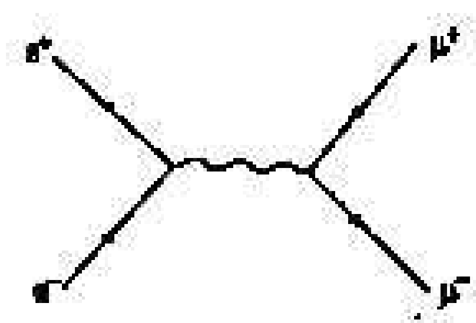
Feynman diagrams

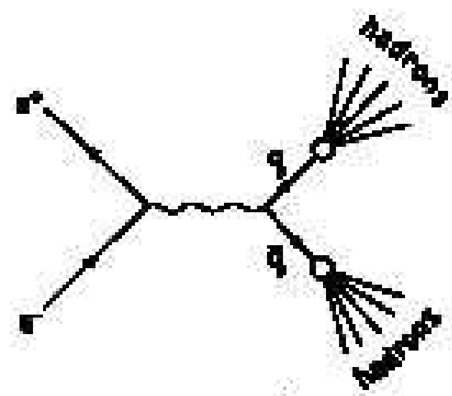
The calculation of QCD processes that can be compared to experimental results cannot be done with the Feynamnn rules alone.

We can calculate the $e^+ e^- \to q\,\overline{q}$ process but quarks do not exist outside the proton.

In high-energy collisions, quarks *hadronize* into other hadrons (e.g. pions, kaons, ...).

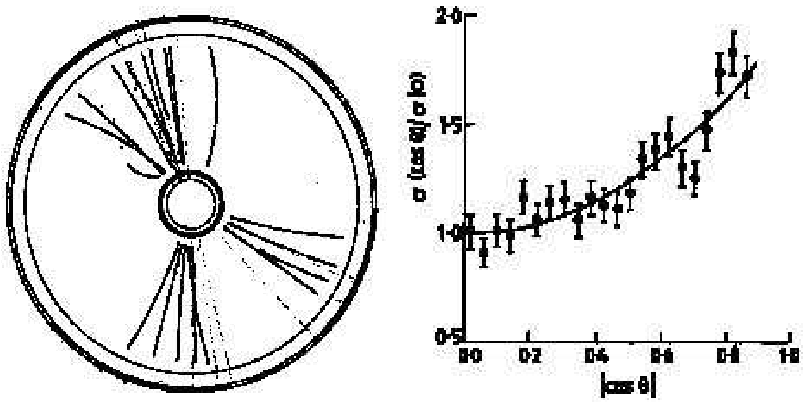
If the quarks are relativisitic, then the associated hadrons are seen to be part of *jets*.





Jets

In $e^+ e^-$ collisions, it was observed that the particles tended to be in collimated streams or jets.



The jets are postulated to be correlated with a hadronizing quark.

The $e^+ e^- \to q\,\overline{q}$ should follow a $(1 + cos^2\theta)$ angular distribution.

3-jet events were seen as evidence for the gluon

Hadron Collider projects

CERN	1980-1990	SPS Collider $p\bar{p}$ (discovery of W and Z)
Fermilab	1990-2000	Teavtron $p\bar{p}$
CERN	2008-2020	Large Hadron Collider (LHC) pp

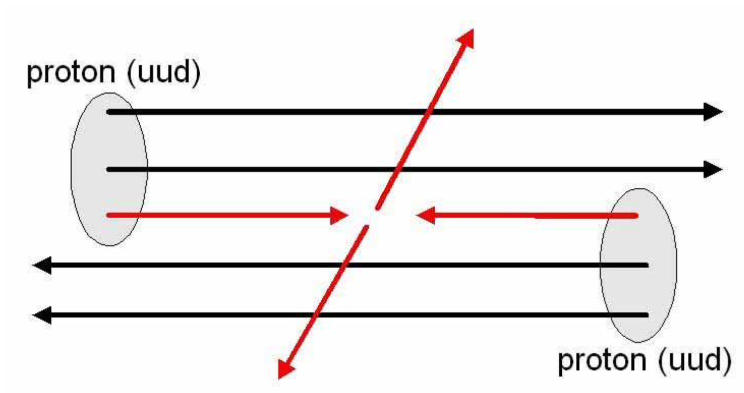
UVic participated in UA1 and UA2

Large UVic group in the ATLAS collaboration (LHC)

Quark-quark collisions I

How can one see the actual collision between the two quarks when there are four other *spectator* quarks?

The spectator quarks hadronize in the forward regions of the detectors, producing many mesons (primarily pions) that are parallel to the beam axis.



Quark-quark collisions II

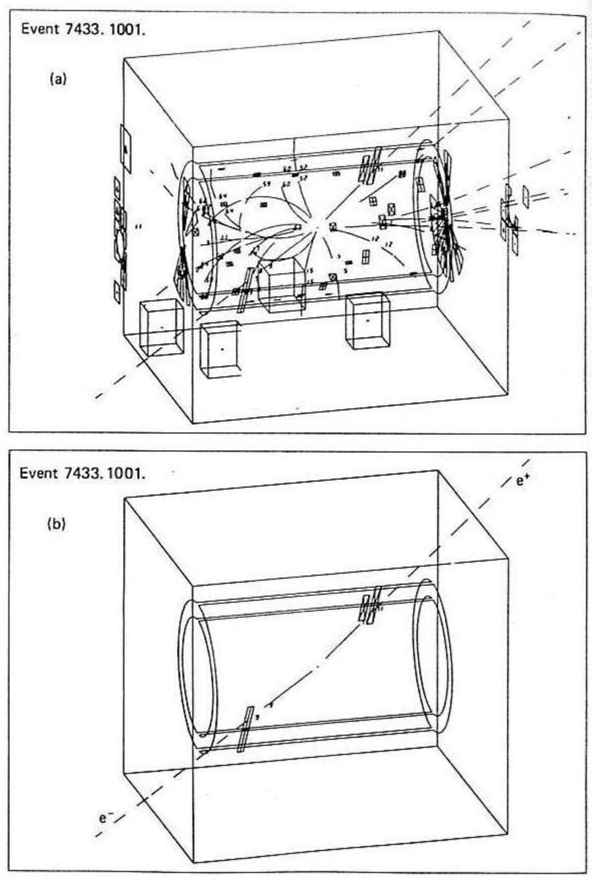


Fig. 11.21. An e^+e^-, Z^0 event from UA1. (a) shows all reconstructed vertex associated tracks while (b) shows the same event but including only tracks with $p_T > 2$ GeV/c and calorimeter hits with $E_T > 2$ GeV. Only the e^+e^- pair survives the cuts.

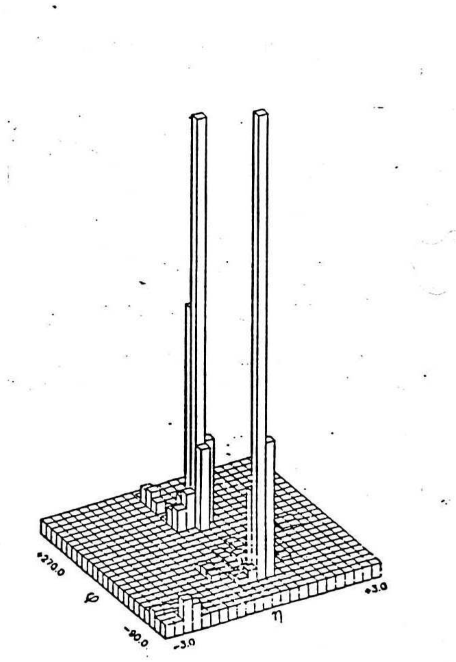
The trick learned by the UA1 and UA2 experiments in the 1980's is to look for tracks with a large amount of momentum in the direction transverse to the beams (transvere momentum).

Quark-quark collisions III

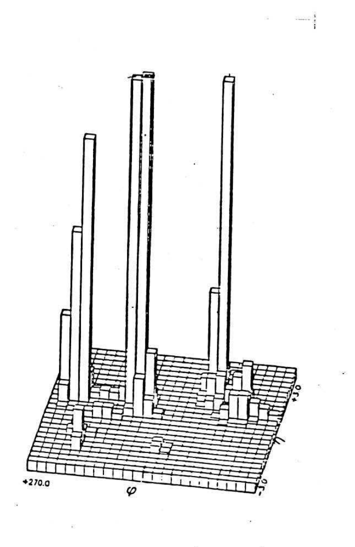
Observation of 2 and 3 jet events in the UA1 experiment.

Calorimeter energy deposits (θ versus ϕ)

A TYPICAL TWO-JET EVENT LEGO-PLOT



A TYPICAL 3-JET EVENT



5. MEASUREMENT OF 3-JET/2-JET RATIO

THEORETICAL CALCULATION:

INTEGRATE THE LEADING ORDER QCD SUBPROCESS CROSS-SECTIONS FOR 2→2 AND 2→3 OVER THE REGION DEFINED BY THE CUTS.

3-JET CROSS-SECTIONS	2-JET CROSS-SECTIONS	3-JET/2-JET RATIO
$\sigma_{gg\to ggg} = 111.55\frac{\alpha_s^3}{\hat{s}}$	$\sigma_{gg\to gg} = 110.4\frac{\alpha_s^2}{\hat{s}}$	$1.01\alpha_s$
$\sigma_{qg\to qgg} = 50.34\frac{\alpha_s^3}{\hat{s}}$	$\sigma_{qg\to qg} = 48.2\frac{\alpha_s^2}{\hat{s}}$	0.79 ~~1.04~~ α_s
* $\sigma_{q\bar{q}\to q\bar{q}g} = 28.14\frac{\alpha_s^3}{\hat{s}}$	* $\sigma_{q\bar{q}\to q\bar{q}} = 21.4\frac{\alpha_s^2}{\hat{s}}$	0.76 ~~1.31~~ α_s

* IDENTICAL FLAVOURS ONLY

- 2-JET CROSS-SECTION OBEYS THE "4/9'S RULE"

$$\sigma_{gg} : \sigma_{qg} : \sigma_{q\bar{q}} = (1) : (4/9) : (4/9)^2$$

- THE 3-JET CROSS-SECTION ALSO OBEYS THIS RUL

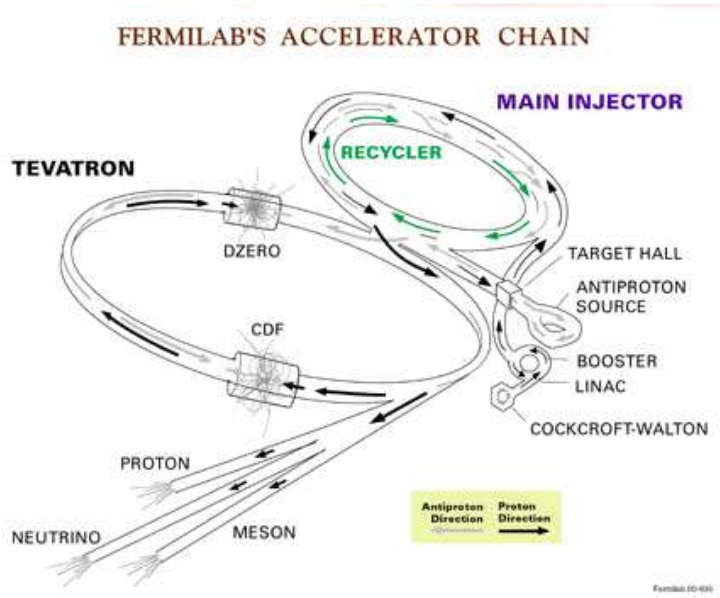
⇒ 3-JET/2-JET RATIO IS INDEPENDENT OF THE PARTON COMBINATION

→ ALSO, FOR THIS CHOICE OF CUTS,

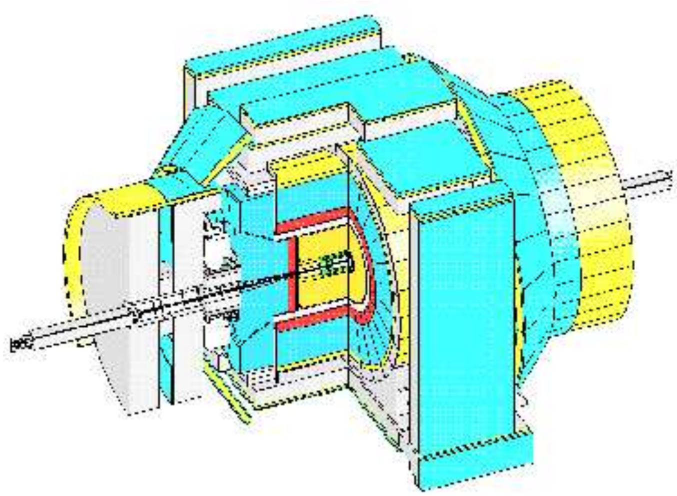
$$\frac{\#\ 3\text{-JET}}{\#\ 2\text{-JET}} \approx 1.0\,\alpha_s$$

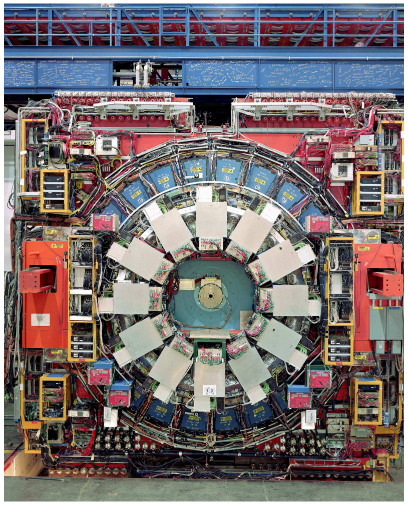
Quark-antiquark scattering at Fermilab

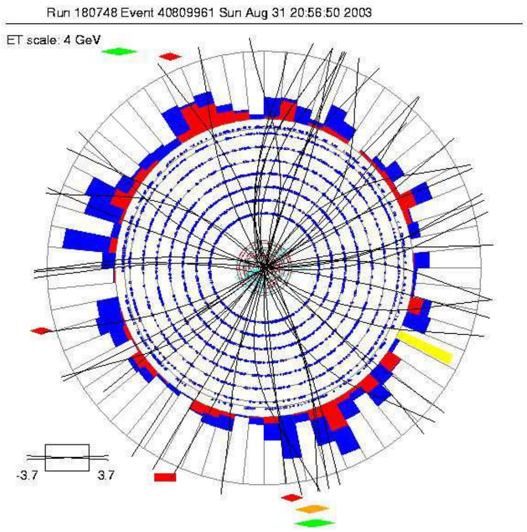
Fermilab collides protons and antiprotons.



CDF detector



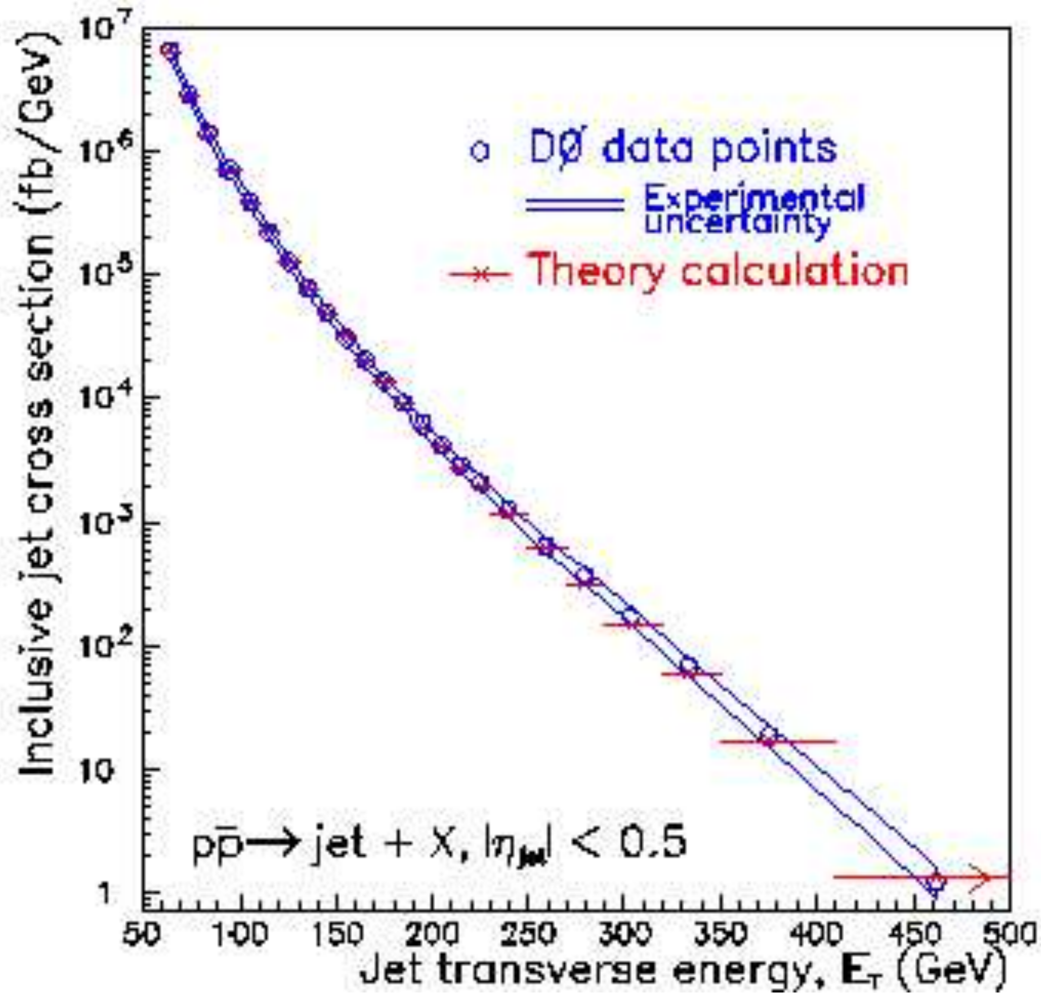




D0 event

Inclusive jet cross section

Fermilab collides protons and antiprotons.



Summary

- Color is an intrinsic characteristic of both quarks and gluons.
- Group theory figures prominently into the mathematics of QCD.
- The Feynman rules for QCD incorporate color and include 3- and 4-gluon self-interactions.
- QCD amplitudes factor into a QED-like part and a color factor.
- Perturbative QCD provides suggestions as to why only color singlets are observed as free particles.