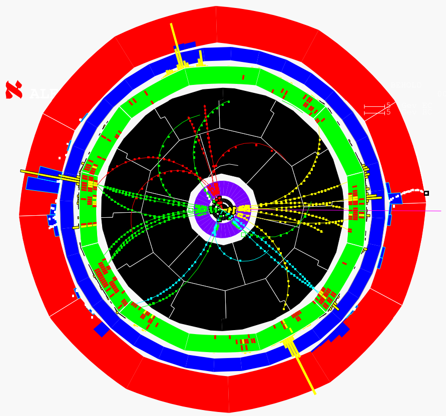
Phenomenology

Lecture 3
(QCD basics, jets)

Quarks ⟶ jets of hadrons



Aleph Higgs event:

- Claim: it corresponds to $ZH \to q\bar{q}b\bar{b}$.
- But actually just bunches ('jets') of hadrons.
- Can they be related? How?
  NB: not just '*are* they related?'

Need understanding of QCD
(and not just for this!)

QCD is not solved

Degrees of freedom of Lagrangian (quarks, gluons) $\neq$ physical particles (π, p, n, ...).

Lattice is not powerful enough to reach high energies; perturbative QCD only good for talking about *unphysical particles* (quarks, gluons).

So: phenomenology with QCD objects (jets, incoming protons) has to

work around these problems

- Choose the right observables (to let us ignore our ignorance).
- Learn from experiments what we cannot (yet) calculate.
- Know how to *quantify remaining ignorance...*

Lagrangian + colour

Quarks — 3 colours: $\psi_a = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$

Quark part of Lagrangian:

$$\mathcal{L}_q = \bar{\psi}_a (i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t^C_{ab} \mathcal{A}^C_\mu - m)\psi_b$$

$SU(3)$ local gauge symmetry $\leftrightarrow$ 8 $(= 3^2 - 1)$ generators $t^1_{ab} \dots t^8_{ab}$ corresponding to 8 gluons $\mathcal{A}^1_\mu \dots \mathcal{A}^8_\mu$.

A representation is: $t^A = \frac{1}{2}\lambda^A$,

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda^8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{-2}{\sqrt{3}} \end{pmatrix}$$

Lagrangian + colour (cont.)

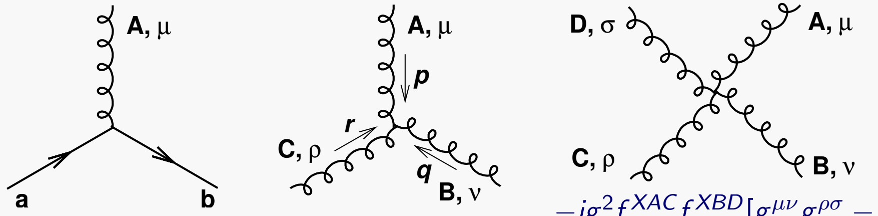
Field tensor:

$F^A_{\mu\nu} = \partial_\mu \mathcal{A}^A_\nu - \partial_\nu \mathcal{A}^A_\mu - g_s\, f_{ABC} \mathcal{A}^B_\mu \mathcal{A}^C_\nu \qquad [t^A, t^B] = i f_{ABC} t^C$

f_{ABC} are structure constants of $SU(3)$ (antisymmetric in all indices — $SU(2)$ equivalent was ϵ^{ABC}). Needed for gauge invariance of gluon part of Lagrangian:

$$\mathcal{L}_G = -\frac{1}{4} F^{\mu\nu}_A F^{A\,\mu\nu}$$

Interaction vertices of Feynman rules:



$-ig_W t^A_{ba} \gamma^\mu$

$-g_s f^{ABC}[(p-q)^\rho g^{\mu\nu} + (q-r)^\mu g^{\nu\rho} + (r-p)^\nu g^{\rho\mu}]$

$-ig_s^2 f^{XAC} f^{XBD}[g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\gamma}] + (C,\gamma) \leftrightarrow (D,\rho) + (B,\nu) \leftrightarrow (C,\gamma)$

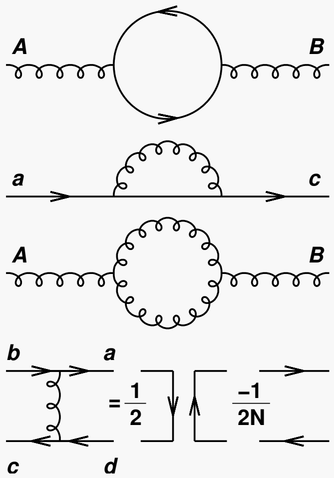
Quick guide to colour algebra

$$\mathrm{Tr}(t^A t^B) = T_R \delta^{AB}\,, \qquad T_R = \tfrac{1}{2}$$

$$\textstyle\sum_A t^A_{ab} t^A_{bc} = C_F \delta_{ac}\,, \qquad C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$$

$$\textstyle\sum_{C,D} f^{ACD} f^{BCD} = C_A \delta^{AB}\,, \qquad C_A = N_c = 3$$

$$t^A_{ab} t^A_{cd} = \frac{1}{2}\delta_{bc}\delta_{ad} - \frac{1}{2N_c}\delta_{ab}\delta_{cd} \quad \text{(Fierz)}$$



Running coupling

The strong coupling, α_s, *runs:*

$$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = \beta(\alpha_s)\,, \qquad \beta(\alpha_s) = -\alpha_s^2 (b_0 + b_1 \alpha_s + b_2 \alpha_s^2 + \ldots)\,,$$

$$b_0 = \frac{11 C_A - 2n_f}{12\pi}\,, \qquad b_1 = \frac{17 C_A^2 - 5 C_A n_f - 3 C_F n_f}{24\pi^2} = \frac{153 - 19 n_f}{24\pi^2}$$

Note sign: Asymptotic Freedom, due to gluon to self-interaction

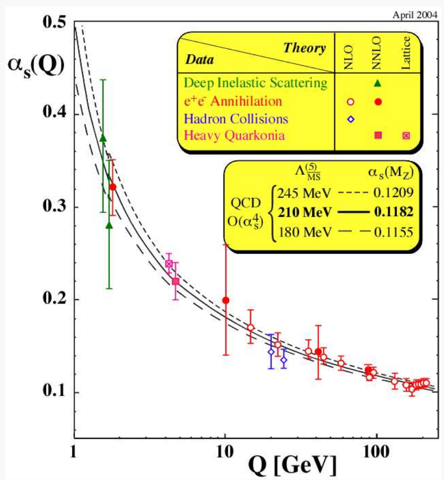
- At high scales Q, coupling is weak
  ➥quarks and gluons are almost free, interactions are just a perturbation
- At low scales, coupling is strong
  ➥quarks and gluons interact strongly — they are confined into hadrons. Perturbation theory fails.

Running coupling (cont.)

Solve $Q^2 \frac{\partial \alpha_s}{\partial Q^2} = -b_0 \alpha_s^2 \;\Rightarrow\; \alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 - b\alpha_s(Q_0^2) \ln \frac{Q^2}{Q_0^2}} = \frac{1}{b_0 \ln \frac{Q^2}{\Lambda^2}}$

Λ (aka Λ_{QCD}) is the fundamental scale of QCD, at which coupling blows up.

- Λ sets the scale for hadron masses
  (NB: Λ not unambiguously defined wrt higher orders)
- Perturbative calculations valid for scales $Q \gg \Lambda$.



What is the right scale?

Say we have some observable V in e^+e^- collisions at centre of mass energy $Q = \sqrt{s}$. After *renormalisation* at scale μ

$$V = C_0 + C_1 \cdot \alpha_s(\mu^2) + \left(C_2 + C_1 b_0 \ln \frac{\mu^2}{Q^2}\right) \cdot \alpha_s^2(\mu^2) + \ldots$$

- Coupling depends on μ^2; so do higher order coefficients.
- Sum of full series should be independent of μ.

But sum of truncated series *does* depend on μ. What do we take? Various scales in problem:

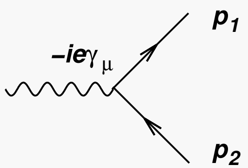
- centre of mass energy $Q \to$ result is perturbative
- masses of produced hadrons $\to$ result is non-perturbative

We'd *like* to say Q ('hard scale') is right one — but how do we know?

Soft gluon amplitude

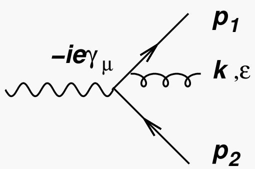
Start with $\gamma^* \to q\bar{q}$:

$$\mathcal{M}_{q\bar{q}} = -\bar{u}(p_1) i e_q \gamma_\mu v(p_2)$$



Emit a gluon:

$$\mathcal{M}_{q\bar{q}g} = \bar{u}(p_1) i g_s \not{\epsilon} t^A \frac{i}{\not{p}_1 + \not{k}} i e_q \gamma_\mu v(p_2) - \bar{u}(p_1) i e_q \gamma_\mu \frac{i}{\not{p}_2 + \not{k}} i g_s \not{\epsilon} t^A v(p_2)$$



Make gluon *soft* $\equiv k \ll p_{1,2}$; ignore terms suppressed by powers of k:

$$\mathcal{M}_{q\bar{q}g} \simeq \bar{u}(p_1) i e_q \gamma_\mu t^A v(p_2)\, g_s \left(\frac{p_1.\epsilon}{p_1.k} - \frac{p_2.\epsilon}{p_2.k}\right)$$

$\not{p} v(p) = 0$,
$\not{p}\not{k} + \not{k}\not{p} = 2p.k$

Squared amplitude

$$|M_{q\bar{q}g}^2| \simeq \sum_{A,\text{pol}} \left|\bar{u}(p_1) i e_q \gamma_\mu t^A v(p_2)\, g_s \left(\frac{p_1.\epsilon}{p_1.k} - \frac{p_2.\epsilon}{p_2.k}\right)\right|^2$$

$$= -|M_{q\bar{q}}^2| C_F g_s^2 \left(\frac{p_1}{p_1.k} - \frac{p_2}{p_2.k}\right)^2 = |M_{q\bar{q}}^2| C_F g_s^2 \frac{2p_1.p_2}{(p_1.k)(p_2.k)}$$

Include phase space:

$$d\Phi_{q\bar{q}g} |M_{q\bar{q}g}^2| \simeq (d\Phi_{q\bar{q}} |M_{q\bar{q}}^2|) \frac{d^3\vec{k}}{2\omega_k (2\pi)^3} C_F g_s^2 \frac{2p_1.p_2}{(p_1.k)(p_2.k)}$$

Note property of factorisation into hard $q\bar{q}$ piece and soft-gluon emission piece, $d\mathcal{S}$.

$$d\mathcal{S} = \omega_k d\omega_k d\cos\theta \frac{d\phi}{2\pi} \cdot \frac{2\alpha_s C_F}{\pi} \frac{2p_1.p_2}{(2p_1.k)(2p_2.k)}$$

$\theta \equiv \theta_{p_1 k}$
$\phi =$ azimuth

Soft & collinear gluon emission

Take squared matrix element and rewrite in terms of ω, θ,

$$\frac{2p_1.p_2}{(2p_1.k)(2p_2.k)} = \frac{1}{\omega^2(1 - \cos^2\theta)}$$

So final expression for soft gluon emission is

$$d\mathcal{S} = \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$

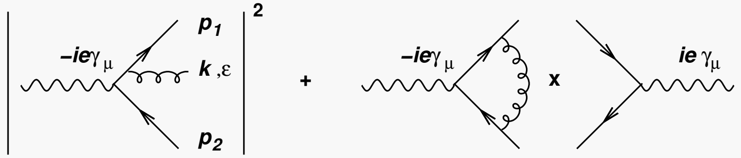
NB:

- It *diverges* for $\omega \to 0$ — *infrared (or soft) divergence*
- It *diverges* for $\theta \to 0$ and $\theta \to \pi$ — *collinear divergence*

Earlier question of what renormalisation scale to use, is closely connected with question of what kind of gluons are most relevant — hard ones, or soft and collinear ones...

Real-virtual cancellations: total X-sctn

Total cross section: sum of all real and virtual diagrams



Total cross section must be *finite*. If real part has divergent integration, so must virtual part. (Unitarity, conservation of probability)

$$\sigma_{tot} = \sigma_{q\bar{q}} \left(1 + \frac{2\alpha_s C_F}{\pi} \int \frac{d\omega}{\omega} \int \frac{d\theta}{\sin\theta} \int \frac{d\phi}{2\pi} R(\omega/Q, \theta) - \frac{2\alpha_s C_F}{\pi} \int \frac{d\omega}{\omega} \int \frac{d\theta}{\sin\theta} \int \frac{d\phi}{2\pi} V(\omega/Q, \theta)\right)$$

- $R(\omega/Q, \theta)$ parametrises real matrix element for hard emissions, $\omega \sim Q$.
- $V(\omega/Q, \theta)$ parametrises virtual corrections for all momenta.

Total X-section (cont.)

$$\sigma_{tot} = \sigma_{q\bar{q}} \left(1 + \frac{2\alpha_s C_F}{\pi} \int \frac{d\omega}{\omega} \int \frac{d\theta}{\sin\theta} \int \frac{d\phi}{2\pi} \left(R(\omega/Q, \theta) - V(\omega/Q, \theta)\right)\right)$$

- From calculation: $\lim_{\omega\to 0} R(\omega/Q, \theta) = 1$.
- For every divergence $R(\omega/Q, \theta)$ and $V(\omega/Q, \theta)$ should cancel:

$$\lim_{\omega\to 0}(R - V) = 0\,, \qquad \lim_{\theta\to 0,\pi}(R - V) = 0$$

Result:

- corrections to σ_{tot} come from hard ($\omega \sim Q$), large-angle gluons
- Soft gluons don't matter:
  - Physics reason: soft gluons emitted on long time scale ($\sim 1/\omega$) relative to collision ($1/Q$) — cannot influence cross section.
  - Transition to hadrons also occurs on long time scale ($\sim 1/\Lambda$) — and can also be ignored.
- Correct renorm. scale for α_s: $\mu \sim Q$ — perturbation theory valid.

total X-section (cont.)

Dependence of total cross section on only *hard* gluons is reflected in 'good behaviour' of perturbation series:

$$\sigma_{tot} = \sigma_{q\bar{q}} \left(1 + 1.045\frac{\alpha_s(Q)}{\pi} + 0.94\left(\frac{\alpha_s(Q)}{\pi}\right)^2 - 15\left(\frac{\alpha_s(Q)}{\pi}\right)^3 + \cdots\right)$$

(Coefficients given for $Q = M_Z$)

Estimate uncertainties

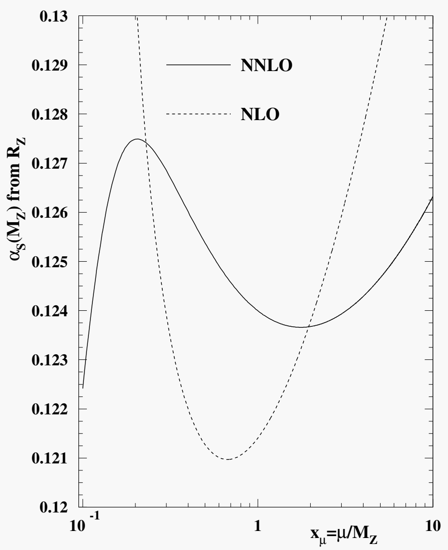
Arguments say $\mu \sim Q$.

- $\mu = Q$?
- $\mu = Q/2$?
- $\mu = 2Q$?

No way to say — but at very high orders of perturbation theory it should not matter...

Impact illustrated in extraction of $\alpha_s(M_Z)$ from data on $\sigma_{e^+e^-\to \text{hadrons}}$.

Inevitable residual uncertainty



Naive jet X-section

In lecture 2 we associated each parton with a ‘jet’ ($HZ \to q\bar{q}b\bar{b}$).
So let’s calculate X-section for 3 jets, as being that for 3 partons:

$$\sigma_{3-jet} = \sigma_{q\bar{q}} \left(\frac{2\alpha_s C_F}{\pi} \int \frac{d\omega}{\omega} \int \frac{d\theta}{\sin\theta} \int \frac{d\phi}{2\pi} R(\omega/Q, \theta) \right)$$

Virtual piece absent: it only has 2 ‘jets’.

Result diverges (for $\omega \to 0, \theta \to 0$):

- perturbatively infinite cross section for producing an extra gluon
- relevance of long time-scales ($1/\omega \sim 1/\Lambda \gg 1/Q$) implies strong sensitivity to hadronisation

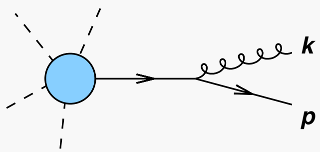
➥ identifying jets as partons is a *bad idea*.

So what *do* we mean by a ‘jet’?
Soft or collinear gluon should *not* be a separate jet.
Hard well-separated gluon should.

Infrared and Collinear Safety

To understand principles for defining a jet, first examine origin of divergence in general terms.

Take an n-parton amplitude and emit a soft collinear gluon k from parton p.



Combination of propagator and vertex give: $g_s t_p^A \frac{\epsilon.p}{k.p} \to^2 C_p \frac{g_s^2}{\omega_k^2 \theta^2}$

There are soft and collinear divergences (real & virtual)
for emission of a gluon off *any* coloured parton

If an observable is to be calculable in perturbative QCD, soft-collinear divergent contributions from real branching and the virtual (loop) correction must cancel *at all orders.*

➥The observable should be unaffected by any soft or collinear branching.

Infrared and Collinear Safety (definition)

For an observable’s distribution to be calculable in perturbation theory, the observable should be infra-red safe, i.e. insensitive to the emission of soft or collinear gluons. In particular if $\vec{p}_i$ is any momentum occurring in its definition, it must be invariant under the branching

$$\vec{p}_i \to \vec{p}_j + \vec{p}_k$$

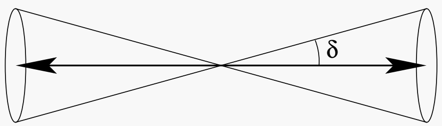
whenever $\vec{p}_j$ and $\vec{p}_k$ are parallel [collinear] or one of them is small [infrared].

[QCD and Collider Physics (Ellis, Stirling & Webber)]

Sterman-Weinberg jets

The *original* (finite) jet definition

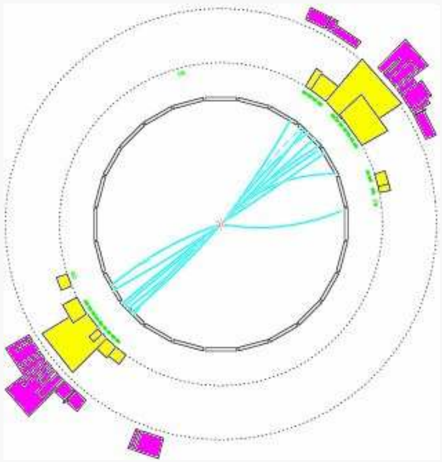
An event has 2 jets if at least a fraction $(1-\epsilon)$ of event energy is contained in two cones of half-angle δ.



$$\sigma_{2-jet} = \sigma_{q\bar{q}} \left(1 + \frac{2\alpha_s C_F}{\pi} \int \frac{d\omega}{\omega} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi} \left(R\left(\frac{\omega}{Q}, \theta\right) \times \right.\right.$$
$$\left.\left. \times \left(1 - \Theta\left(\frac{\omega}{Q} - \epsilon\right) \Theta(\theta - \delta) \right) - V\left(\frac{\omega}{Q}, \theta\right) \right) \right)$$

- For small ω or small θ this is just like total cross section — full cancellation of divergences between real and virtual terms.
- For large ω and large θ a *finite piece* of real emission cross section is *cut out.*
- Overall final contribution dominated by scales $\sim Q$ — cross section is perturbatively calculation.

Real event (a)



Near 'perfect' 2-jet event

2 well-collimated jets of particles.

All energy in two cones.

NB: picture of two quarks and a soft gluon does not reflect reality of event structure.

Origin of event structure?

- Multiple QCD radiation has *nested* soft and collinear divergences.

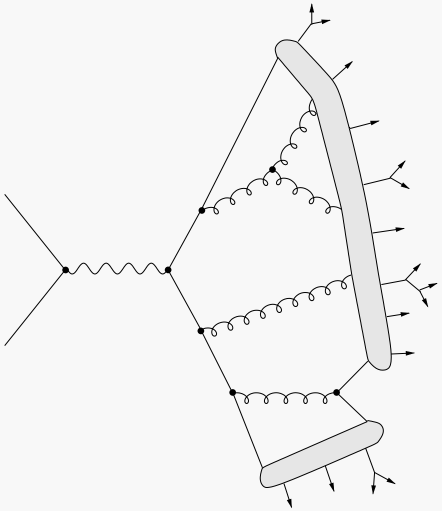
  Much of structure is calculable to all orders!

- Produce many soft and collinear gluons, $q\bar{q}$ pairs
- *Somehow* there is a transition from *partons* $\rightarrow$ *hadrons*

  Can only be modelled

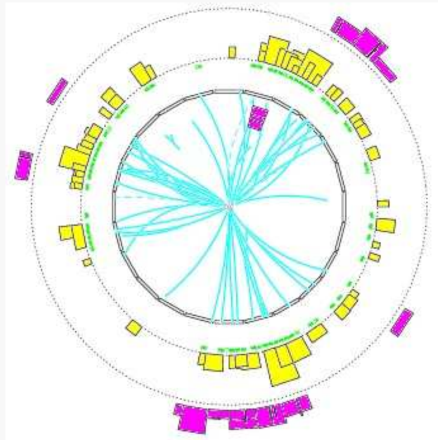
- These elements are encoded in Monte Carlo simulation programs

  Extremely successful, ubiquitous e.g. Pythia, Herwig, Sherpa



(picture from B.R. Webber)

Real event (b)



multi-jet event

How can we define jets for more complex events?

- Sterman-Weinberg ('cone') definition gets messy
- Jets may be broader than chosen cone
- Some of energy-momentum is outside jet cones ($\sum$ jet energy $\neq$ total energy)

Need a more sophisticated tool to relate real events to an *idealised* hard event.

Modern jet algorithms

Based on idea of successive clusterings and *resolution parameter* (y_{cut}):
Idea: try to *undo multiple QCD branching and 'hadronisation'*.

1. Calculate the *distance* y_{ij} (according to some measure) between all current pairs of particles/pseudo-jets i, j:

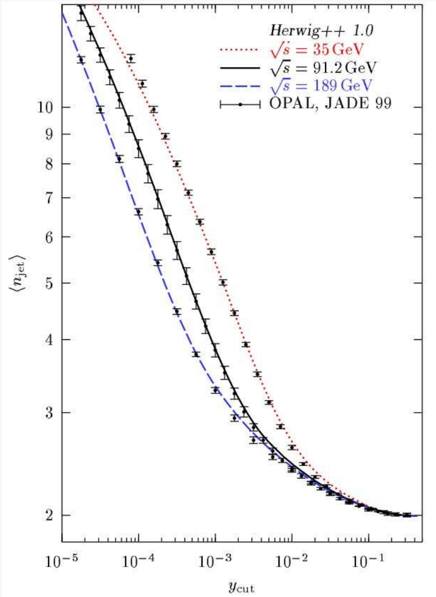
$$y_{ij} = min(E_i^2, E_j^2)(1 - \cos\theta_{ij})$$

'k_t' measure: closeness $\Leftrightarrow$ structure of QCD divergences

2. If all $y_{ij} > y_{\text{cut}}$ *stop.*
3. Otherwise, select the i, j, with the smallest y_{ij} and *cluster* them to make a 'pseudojet'.
4. Go back to step 1.

Number of jets depends on the resolution you choose

Number of jets v. resolution (e^+e^-)



Interim QCD summary

- Gluons carry charge and couple to each other ⇒
  - asymptotic freedom (large Q)
  - confinement (low Q): quarks, gluon $\neq$ physical d.o.f.
- High-energy QCD processes involve whole range of scales ($Q \to \Lambda$)
  - spanned (logarithmically) by soft and collinear gluons
  - amenable to *simulation* by Monte Carlo event generators
  - 'hadronisation' (modelled) connects parton-level ↔ hadron-level

  Crucial for understanding experimental setups
- Choose *Infrared-Collinear Safe* observables for comparison to perturbation theory, *e.g.*
  - total cross sections, jet cross sections
  - weakly sensitive to soft-collinear gluons, hadronisation
  - predictions have residual dependence on *renormalisation scale*
- Jet $\neq$ parton, but rather cluster of partons
  - Must adopt a *conventional* procedure for defining jets
  - Jet-definition ambiguity mirrored in a *jet-resolution* parameter — number of jets depends on resolution.

Next lecture

Processes with incoming protons