

HARD EXCLUSIVE SCATTERING

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Outline:

- **Lecture 1: QCD, factorization, PDFs and all that**
- **Lecture 2: Generalized Parton Distributions (GPDs)**
- **Lecture 3: Applications: Deeply virtual and wide-angle exclusive scattering**

Lecture 1: QCD, factorization, PDFs and all that

Outline:

- QCD, asymptotic freedom, factorization
- Evolution
- Deep inelastic scattering (DIS)
- Parton distribution functions (PDFs)
- Results, interpretation and use of PDFs

Quarks

Multiplet structure of hadron spectrum lead to introduction of constituents

QUARKS u, d, s spin-1/2 Gell-Mann (64); Zweig (64)

today: three more quarks c, b, t (heavy)

light quarks form the fundamental triplets of the group $SU(3)_F$

light hadrons are members of the irreducible representations of the $SU(3)_F$

mesons: $q\bar{q}$ $3 \times \bar{3} = 1 + 8$

baryons: qqq $3 \times 3 \times 3 = 1 + 8 + 8 + 10$

Spectrum in agreement with this structure, $SU(3)_F$ broken by quark masses

Gell-Mann Nobel prize 1969

but - free quarks have not been observed

- baryon wf. symmetric unter quark interchange

e.g. $\Delta^{++}(s_3 = 3/2) = f(r_1, r_2, r_3) |\uparrow\uparrow\uparrow\rangle |uuu\rangle$

symmetric **space** ($L = 0$) **spin** **flavor** wfs

by virtue of $SU(3)_F$ product wf of baryons symmetric - symmetric quark model

in conflict with Spin-Statistik theorem

Color

Greenberg (64), Han-Nambu(65) to cure this problem a new quantum number

color q_c $c = 1, 2, 3$ $SU(3)_C$ triplet

confinement hypothesis: only color singlet states appear as free, asymp. states
excludes existence of free quarks

color structure of baryons: $3 \times 3 \times 3 = 1 + 8 + 8 + 10$

singlet state is total antisymmetric $B = \sum_{i,j,k=1,2,3} \epsilon_{ijk} q_i q_j q_k$

\implies product of space, spin and flavor wfs must be symmetric

other evidence for color:

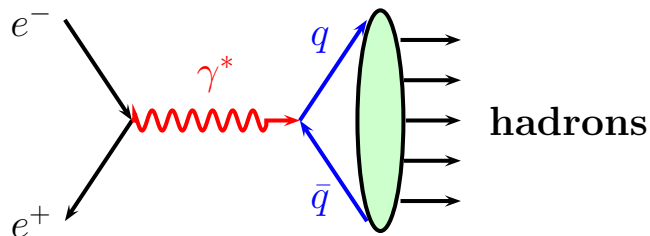
e^+e^- annihilation into hadrons

reaction mech.: $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}$

and soft $q\bar{q} \rightarrow$ hadrons transition

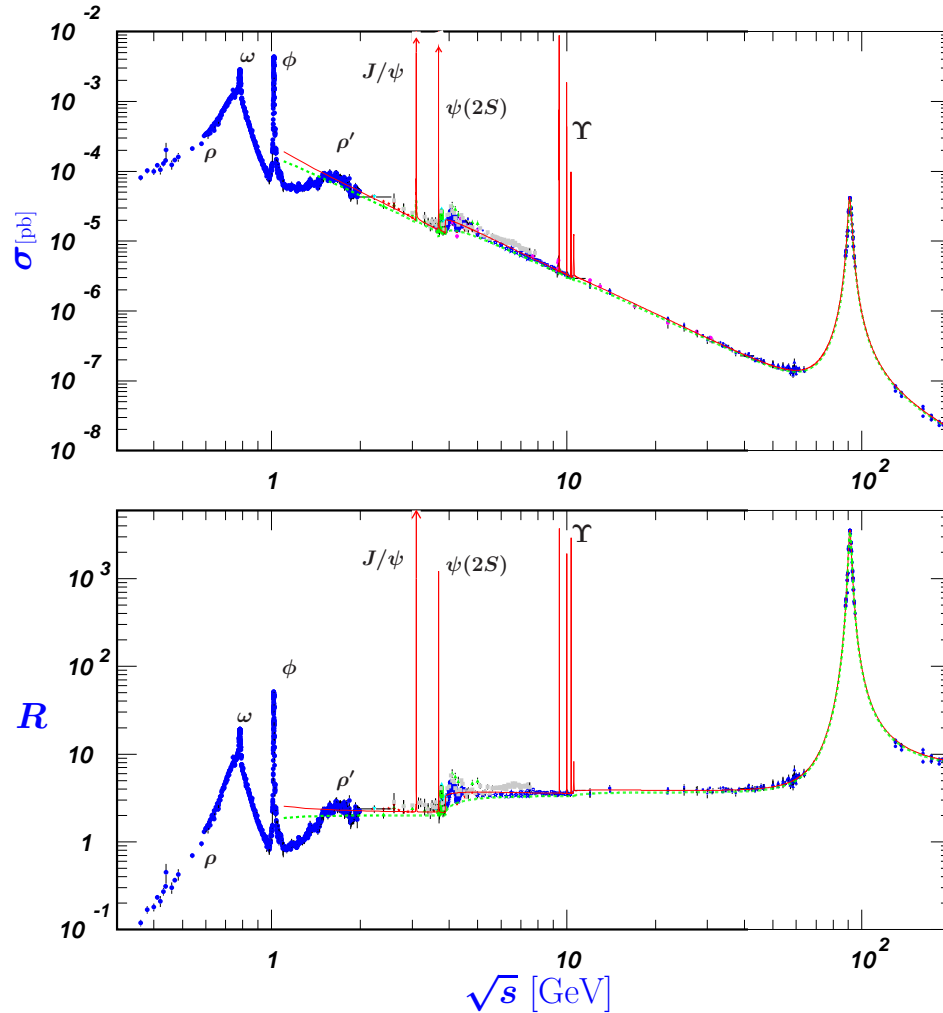
with probability 1 (confinement)

hence, up to charges, same cross section as for $e^+e^- \rightarrow \mu^+\mu^-$



$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3 \sum_a e_a^2$$

$e^+e^- \rightarrow \text{hadrons}$



dashed: quark approach

steps: 2 10/3 11/3

solid: 3-loop pQCD
(NNNLO)

plot taken from PDG

QCD

Weyl (29), Yang-Mills (54): dynamical principle

require a locally phase invariant field with respect to a certain group

QED: U(1)

Fritzsch, Gell-Mann, Leutwyler (73): take $SU(3)_C$ (exact) for interaction among quarks

$$\Psi(x) \rightarrow \Psi'(x) = \exp \left[ig_s \sum_{a=1}^8 T_a \alpha_a(x) \right] \Psi(x)$$

$T_a = \lambda_a/2$ generators of $SU(3)_C$, λ_a traceless, hermitic 3×3 matrices (8), $\alpha_a(x)$ arbitrary functions

free Lagrangian: $\mathcal{L}_0 = i\bar{\Psi} \not{\partial} \Psi - m\bar{\Psi}\Psi$

replace ∂_μ by covariant derivative: $D_\mu = \partial_\mu + ig_s \sum T_a G_{a\mu}$

$$\mathcal{L}_{\text{QCD}} = \bar{\Psi}(i\not{\partial} - m)\Psi - g_s \sum_a \bar{\Psi} \gamma^\mu T_a \Psi G_a^\mu - 1/4 \sum_a G_{\mu\nu}^a G_a^{\mu\nu}$$

field strength tensor of gauge field $G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s \sum_{b,c} f_{abc} G_\mu^b G_\nu^c$

G_μ^a potential of gauge field, 8 gauge fields

quant of gauge field - gluon $J^P = 1^-$, $a = 1..8$

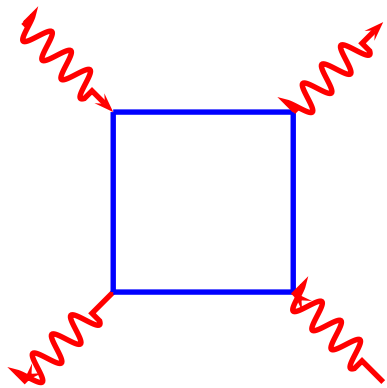
gauge transform $G_{a\mu} \rightarrow G'_{a\mu} = G_{a\mu} - \partial_\mu \alpha_a - g_s \sum_{bc} f_{abc} \alpha_b G_{c\mu}$

f_{abc} structure constants of $SU(3)_C$ ($[\lambda_a, \lambda_b] = f_{abc} \lambda_c$)

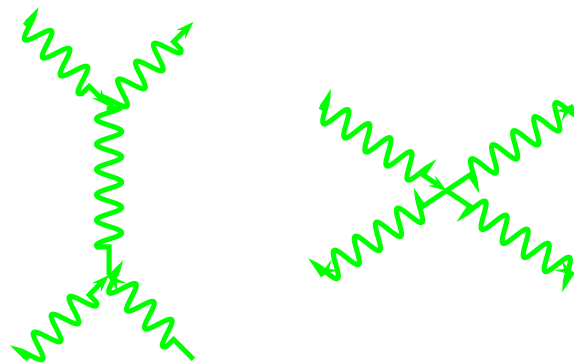
similar to QED but gluon is colored: - self interaction, non-linear theory

$\gamma\gamma \rightarrow \gamma\gamma$ $\sigma \sim \mathcal{O}(e^8)$

$gg \rightarrow gg$ $\sigma \sim \mathcal{O}(g_s^4)$



$e^+e^- \rightarrow \mu^+\mu^-$ of $\mathcal{O}(e^4)$



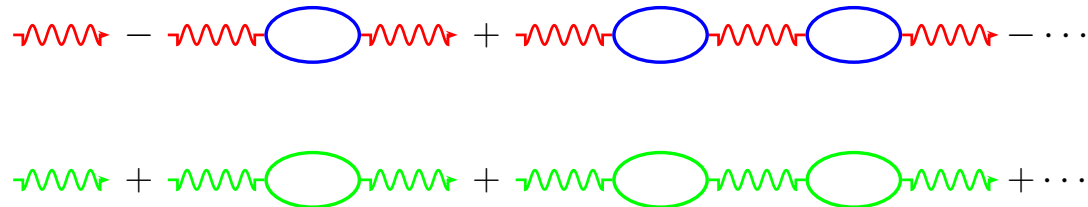
$u\bar{u} \rightarrow d\bar{d}$ of $\mathcal{O}(g_s^4)$

Gliding coupling

fundamental feature of relativistic QFT

- pair creation and annihilation
- i.e. strong fluctuations in particle number (in contrast to non-rel QM)
- complicated vacuum, can be polarized (screening/antiscreening)
- strength of coupling depends on scale/distance

formally - pert. corrections to gauge boson propagator ($Q^2 = -q^2 > 0$)

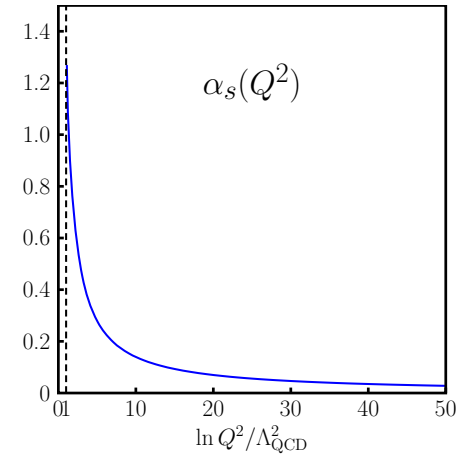
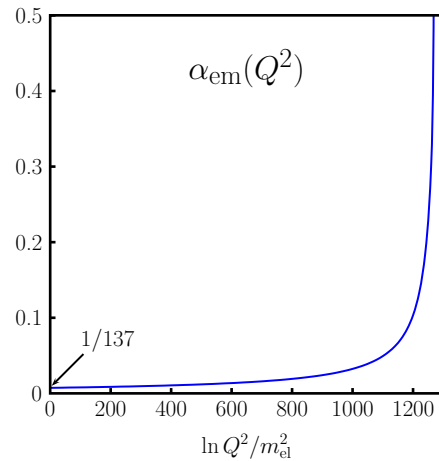


$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \beta_0 \alpha(\mu^2) \ln(Q^2/\mu^2)}$$

QED: $\beta_0 = 1/(3\pi)$

QCD: $\beta_0 = (2/3n_f - 11)/(4\pi) < 0$

breakdown of pert. theory if coupling $\alpha = g^2/4\pi \not\ll 1$



choose $\mu = m_e$ static limit
 $\alpha_{\text{em}}(0) = 1/137.03599976$

singular at $Q^2/m_e^2 \simeq 10^{556}$

$\alpha_{\text{em}}(Q^2 = M_Z^2) \simeq 1/128$

distance $r = 1/Q$, $\hbar = c = 1$, natural units
(not anthropomorphic **Planck**)

with regard to position of singularity

$$\alpha_s(Q^2) = \frac{4\pi}{(11 - 2/3n_f) \ln Q^2/\Lambda_{\text{QCD}}^2}$$

$\Lambda_{\text{QCD}} \simeq 240 \text{ MeV}$

asymptotic freedom - infrared slavery

Gross-Wilczek; Politzer (73); Nobel prize 04

singularity is severe problem - in any event

there are soft regions with large α_s

(where hadrons are formed)

is QCD of any use?

Factorization

IT IS: processes with a hard scale (Q^2) often factorize into

- hard partonic-level subprocesses
- and soft hadronic matrix elements

to all orders of α_s and in all logs of μ_F^2 , partons treated as on-shell part.
(μ_F factorization scale of $\mathcal{O}(Q^2)$, partons=quarks,gluons)

QFT: in higher orders

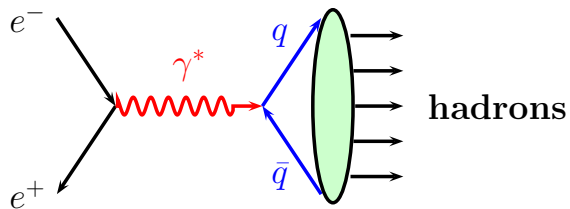
- **ultraviolet** singularities (renormalization of coupling and mass μ_R)
- **infrared** singularities (redefinition of observables)

(e.g. from emission of (almost) collinear or soft gluons)

μ_F - defines separation of short-distance from long-distance effects
i.e. any propagator that is off-shell by a scale $> \mu_F^2$ is considered
as short-range effect, scales $\leq \mu_F^2$ as long range

hard subprocess: calculable in pert. theory, infrared safe,
independent of the hadrons, depends on μ_F
soft hadr. matrix elem.: infrared singular, is specific to the hadrons,
is **universal**, i.e. does not depend on particular hard process
depends on μ_F (**evolution**)

example: $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q} \rightarrow \text{hadrons}$ at large s



$e^+e^- \rightarrow q\bar{q}$	hard (QED)
$q\bar{q} \rightarrow \text{hadrons}$	soft

for a number of processes factorization has been rigorously shown to hold

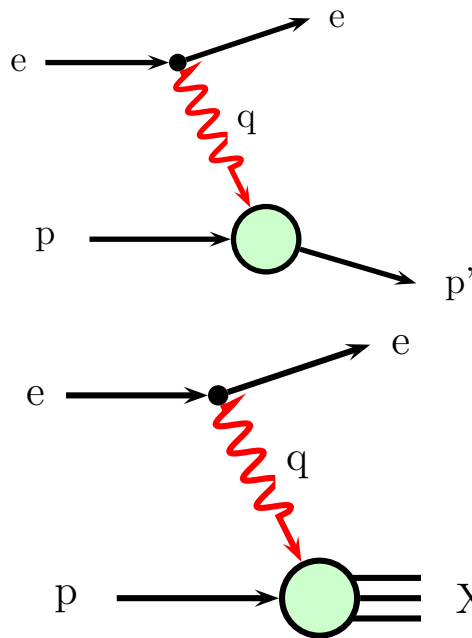
$e^+e^- \rightarrow \text{hadrons}$, $ep \rightarrow e'X$, $AB \rightarrow e^+e^-X$, F_π , $F_{\pi\gamma}$, DVCS, ...

for others still a hypothesis (often in very good agreement with experiment)

e.g. $AB \rightarrow HX$, $AB \rightarrow \text{jets}$, F_p, \dots

Inelastic ep scattering

elastic - form factors $[\tau = -q^2/(4m^2)]$



$$\frac{d\sigma_L}{d\Omega} = \frac{d\sigma_{\text{Mott}}}{d\Omega} \left\{ \frac{G_E^2 + \tau G_M^2}{1 + \tau} - 2\tau G_M^2 \tan^2 \theta/2 \right\}$$

Hoftader et al (56), Nobel prize 1961

$$F_1 = \frac{G_E + \tau G_M}{1 + \tau} \quad F_2 = \frac{G_M - G_E}{1 + \tau}$$

inelastic - structure functions

$$q = k' - k; \quad Q^2 = -q^2 > 0$$

$$\nu = E_L - E'_L = p \cdot q/m$$

$$x_{Bj} = Q^2/(2p \cdot q); \quad 0 \leq x_{Bj} \leq 1$$

$$\frac{d^2\sigma_L}{dQ^2 d\nu} = \frac{4\pi\alpha_{\text{em}}^2 E'}{mEQ^4} \left[2 \sin^2 \theta/2 W_1 + \cos^2 \theta/2 W_2 \right]$$

Bjorken limit

Bjorken limit: $Q^2, \nu \rightarrow \infty$, x_{Bj} fixed

$$\lim_{Bj} mW_1(\nu, Q^2) = F_1(x_{Bj}) \quad \lim_{Bj} \nu W_2(\nu, Q^2) = F_2(x_{Bj})$$

scaling with finite F_i

Friedman-Kendall-Taylor *et al* (72), Nobel prize 1990

scaling already at low Q^2 early scaling parton model

more precise measurements

Chan et al (75)

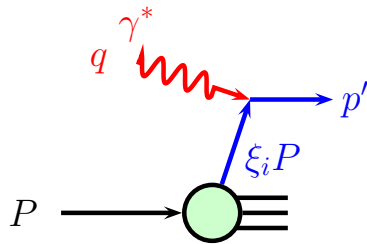
scaling violations $F_i(x_{Bj}, Q^2)$

QCD

Interpretation of scaling

scaling with finite limits is evidence for elastic electron scattering off point-like constituents (partons)

- nucleon at rest - partons possess isotropic momentum distribution (rotational symmetry) $\sum \mathbf{p}_i = 0$ $\Delta p = 1/\Delta x \simeq 200\text{MeV}$
- boost to frame with large momentum P of proton (IMF)
 $\mathbf{p}_{\perp i}/P \ll 1$ neglect **collinear appr.**
 neglect binding energy too, treat partons as quasi free
- $p_i = \xi_i P$, $0 < \xi_i < 1$ momentum fraction (IMF: $\xi_i = p_i^+ / P^+$)



mass-shell condition

$$\begin{aligned} \frac{d^3 p'}{2E'} \delta(p' - q - \xi_i P) &= \delta(Q^2 - \xi_i 2P \cdot q) \\ &= \frac{x_{Bj}}{Q^2} \delta(\xi_i - x_{Bj}) \end{aligned}$$

light-cone coordinates $a = (a_0, a_1, a_2, a_3) \Rightarrow [(a_0 + a_3)/\sqrt{2}, (a_0 - a_3)/\sqrt{2}, a_{\perp}]$

different types of partons may exist

$q_i(\xi) (\geq 0)$ number of partons of type i (with charge $e_i e_0$) with momentum fractions between ξ and $\xi + d\xi$ Feynman (69)

Consequences

- contribution to structure functions

$$2mW_1 = \sum_i e_i^2 \int_0^1 d\xi q_i(\xi) \delta(\xi - x_{Bj}) = \sum_i e_i^2 q_i(x_{Bj}), \quad \nu W_2 = \sum_i e_i^2 x_{Bj} q_i(x_{Bj})$$

only partons with $\xi = x_{Bj}$ contribute at fixed Q^2, ν

- structure fcts. reveal scaling $F_1(x) = 1/2 \sum e_i^2 q_i(x)$
 $F_2(x) = 2xF_1(x)$ Callan-Gross relation characteristic of spin 1/2

- partons as quarks (isospin symmetry $u_p = d_n, d_p = u_n, s_p = s_n$)

$$1/x F_2^{ep}(x) = 4/9[u(x) + \bar{u}(x)] + 1/9[d(x) + \bar{d}(x)] + 1/9[s(x) + \bar{s}(x)] + \dots$$

$$1/x F_2^{en}(x) = 4/9[d(x) + \bar{d}(x)] + 1/9[u(x) + \bar{u}(x)] + 1/9[s(x) + \bar{s}(x)] + \dots$$

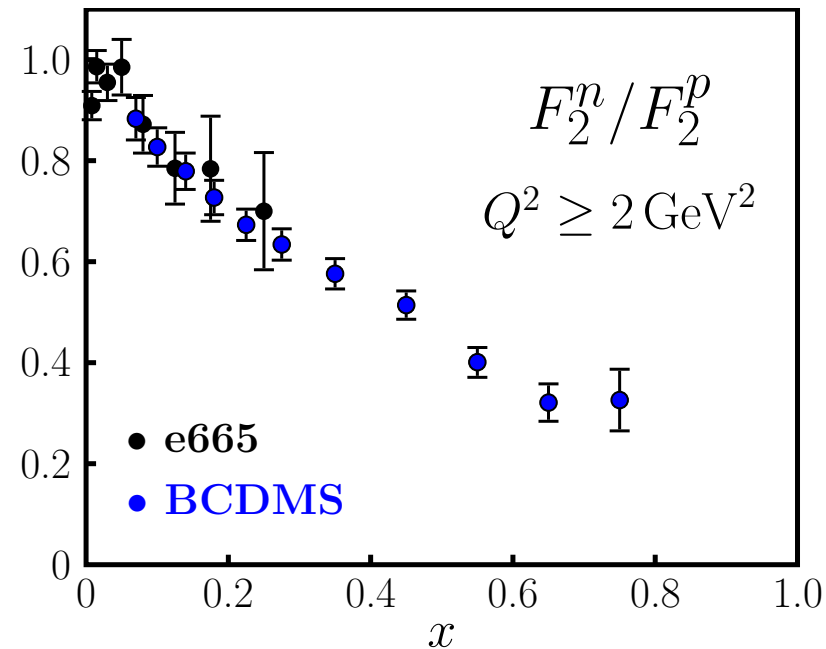
Nachtmann inequality

$$\frac{1}{4} \leq \frac{F_2^{en}(x)}{F_2^{ep}(x)} \leq 4$$

dominance of

$$u + \bar{u}$$

$$d + \bar{d}$$



exp: - dominance of u (d) in proton (neutron) at large x

- $x \rightarrow 0$ many types of partons contribute

- $SU(3)_F$ symmetric PDFs $F_2^{en} / F_2^{ep} = 1$

- how many quarks are in the proton?

$$\int_{x_0}^1 F_2^{ep} dx/x = \sum e_i^2 \int q_i(x) dx = \sum e_i^2 n_i$$

integral seems to diverge

HERA: $x_0 = 10^{-4} \implies n = 12$

- does sea contribute equally to proton and neutron?

Gottfried sum rule $u_v = u - \bar{u}$ $d_v = d - \bar{d}$ (definition)

$$1/x(F_2^{ep} - F_2^{en}) = 1/3[u - d + \bar{u} - \bar{d}] = 1/3[u_v - d_v] + 2/3[\bar{u} - \bar{d}]$$

$$\int_0^1 dx/x(F_2^{ep} - F_2^{en}) = 1/3 + 2/3(n_{\bar{u}} - n_{\bar{d}})$$

integral seems to exist

exp: $0.24 \pm 0.016 \implies n_{\bar{u}} - n_{\bar{d}} \simeq -0.14$

- momentum sum rule $xq_i(x)$ momentum distr. of type i partons

$$F = \int_0^1 dx x [u + \bar{u} + d + \bar{d} + s + \bar{s}]$$

total momentum carried by charged partons (i.e. quarks) exp: $F \simeq 0.5$ at 10 GeV^2

\implies there are uncharged partons in the proton as well:

gluons

Evolution

QFT: due to splitting processes - a particle is always surrounded by a cloud of other particles (with strong fluctuations)

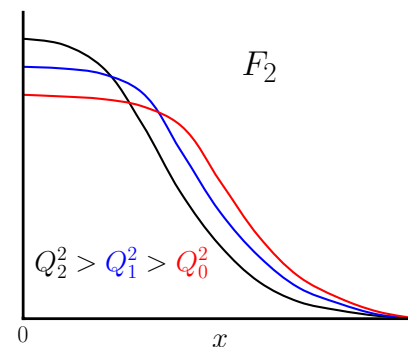
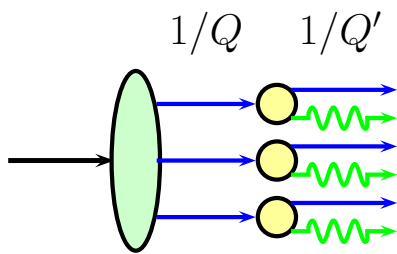


What one sees in an experiment depends on resolution

DIS: resolution set by wave length of virtual photon $1/Q$

with increasing Q one sees more (effective) constituents - with different PDFs

$$F_2(x, Q^2) = x \sum_i e_i^2 q_i(x, Q^2) \quad \text{scaling violations}$$



$$Q < Q'$$

evolution is calculable within pert. theory (in contrast to PDFs at Q_0)

DGLAP equation

- change of PDFs $\propto \alpha_s$
- change of $q(x)$ only depends on $q(y)$ with $x \leq y$ (mom. distr. over larger number)
- change of q can only depend on ratios of momenta (quark masses neglected)

$$\frac{\partial q_i(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} [q_i(y, Q^2) P_{qq}(x/y) + g(y, Q^2) P_{qg}(x/y)]$$

$$\frac{\partial g(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[\sum_j q_j(y, Q^2) P_{gq}(x/y) + g(y, Q^2) P_{gg}(x/y) \right]$$

DGLAP equation continued

splitting function P_{ij} represent leading $\ln Q^2$ of parton cross sections ($\mu_F = Q$)
 (DIS: $\gamma^* q \rightarrow qg$, $\gamma^* g \rightarrow q\bar{q}$, $\gamma^* g \rightarrow q\bar{q}g$)

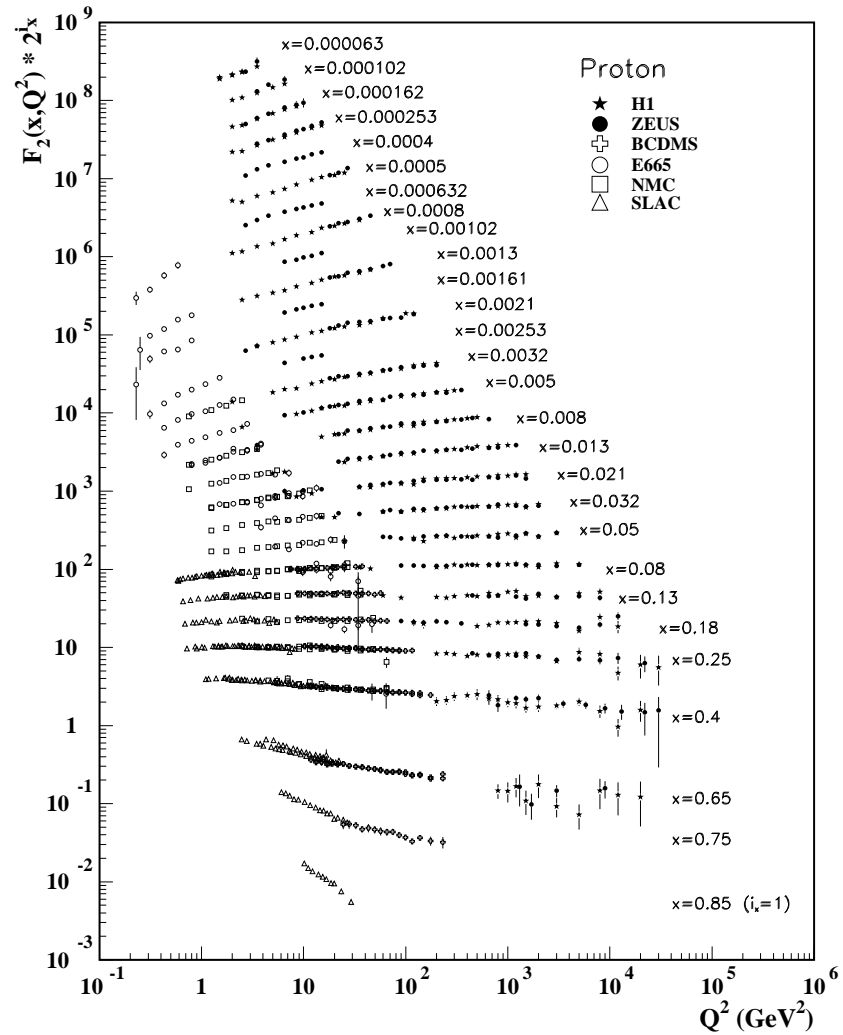
$$P_{qq}(x) = \frac{4}{3} \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right] \quad P_{gq}(x) = \frac{4}{3} \frac{1+(1-x)^2}{x}$$

$$P_{gg}(x) = 6 \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) + \left(\frac{11}{12} - \frac{n_f}{18} \right) \delta(1-x) \right]$$

$$P_{qg}(x) = \frac{1}{2} \left[x^2 + (1-x)^2 \right] \quad \int dx f(x) \frac{1}{(1-x)_+} = \int_0^1 dx \frac{f(x) - f(1)}{1-x}$$

$SU(3)_F$: $q_i^{NS} = q_i - \bar{q}_i$ no mixture with gluons $q^S = \sum_i (q_i + \bar{q}_i)$ mixes
 with gluons

Scaling violation in F_2



taken from PDG

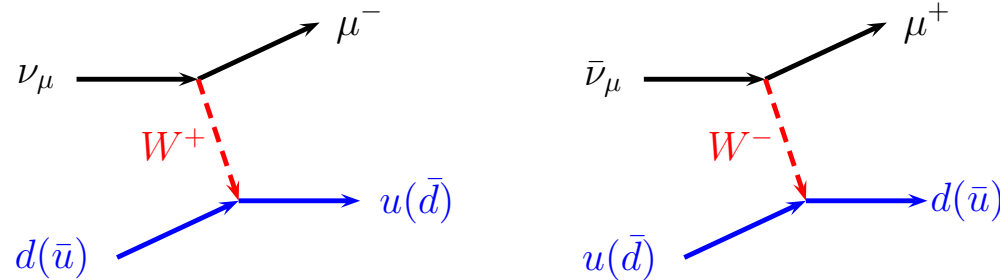
DIS with neutrinos

weak interaction

$$\nu_\mu N \rightarrow \mu^- X \text{ and } \bar{\nu}_\mu N \rightarrow \mu^+ X$$

$$W_1^{\nu, \bar{\nu}}, W_2^{\nu, \bar{\nu}} \text{ and } \pm W_3^{\nu, \bar{\nu}}$$

$$\lim_{Bj} \nu W_3(\nu, Q^2) = -F_3(x_{Bj})$$



+(s, c) and flavor mixing

below charm threshold

$$1/x F_2^\nu(x) \sim 2d(x) + 2\bar{u}(x)$$

$$1/x F_2^{\bar{\nu}}(x) \sim 2u(x) + 2\bar{d}(x)$$

$$1/x F_3^\nu(x) \sim 2d(x) - 2\bar{u}(x)$$

$$1/x F_3^{\bar{\nu}}(x) \sim 2u(x) - 2\bar{d}(x)$$

Parton Distributions

extraction of quark PDFs from DIS eN and νN scattering (and some other inclusive reactions in order to improve the flavor separation)

determination of $g(x)$ with the help of DGLAP (evolution with Q^2)

Polarized DIS: $\Delta q, \Delta g$ $(q(\Delta q) = q(\rightarrow) \pm q(\leftarrow), \quad |\Delta q| \leq q)$

PDFs describe longitudinal momentum distr. of partons within proton (IMF)

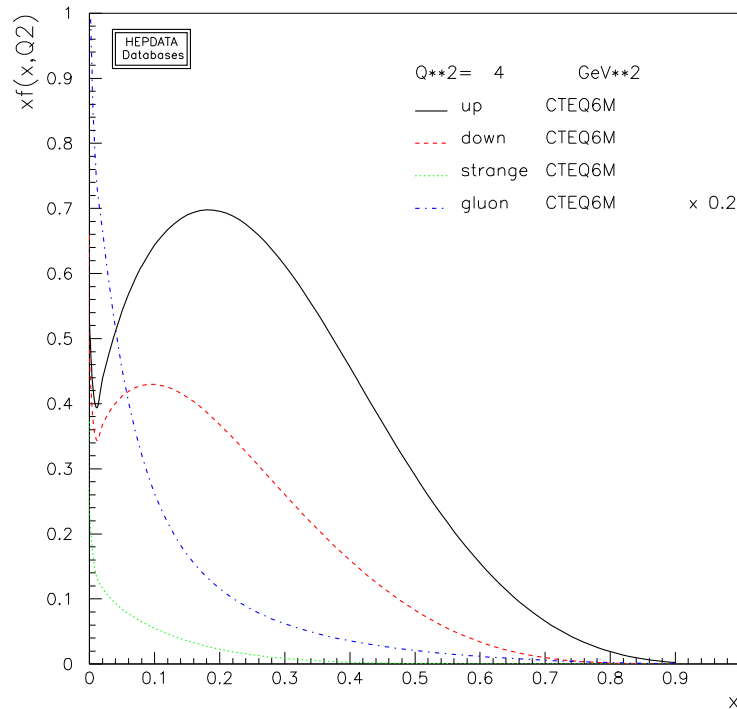
30 years of experimental (SLAC, CERN, HERA, FNAL, JLab)

and theoretical (Barger-Phillips (74), GRV, CTEQ, MRST, ...)

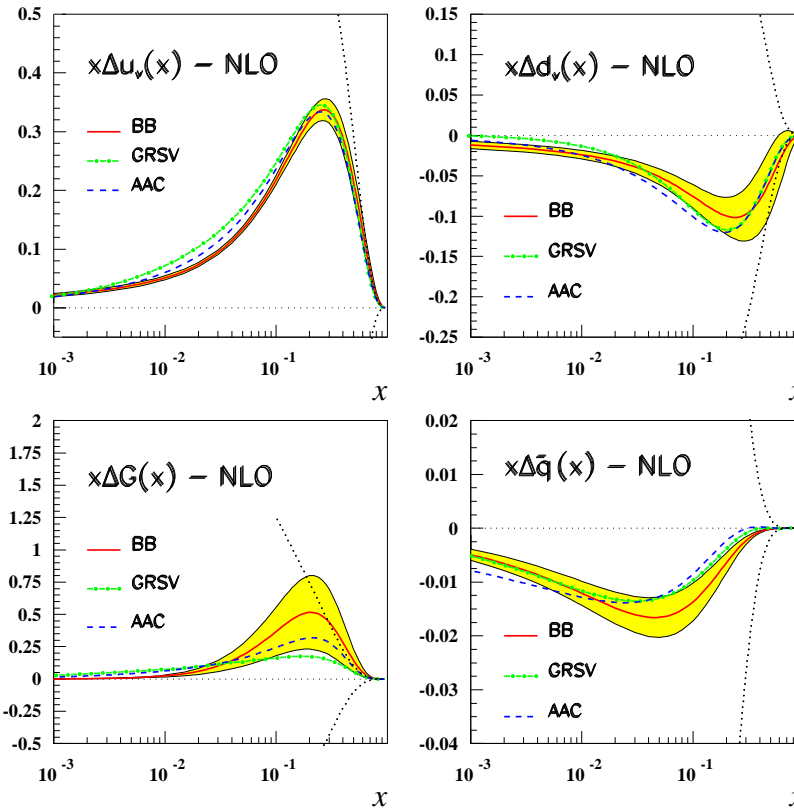
effort has lead to a fair knowledge of the PDFs

(although not perfect, e.g. $x \rightarrow 0, 1, g, \Delta g$)

quark PDFs



$$Q^2 = 4 \text{ GeV}^2$$

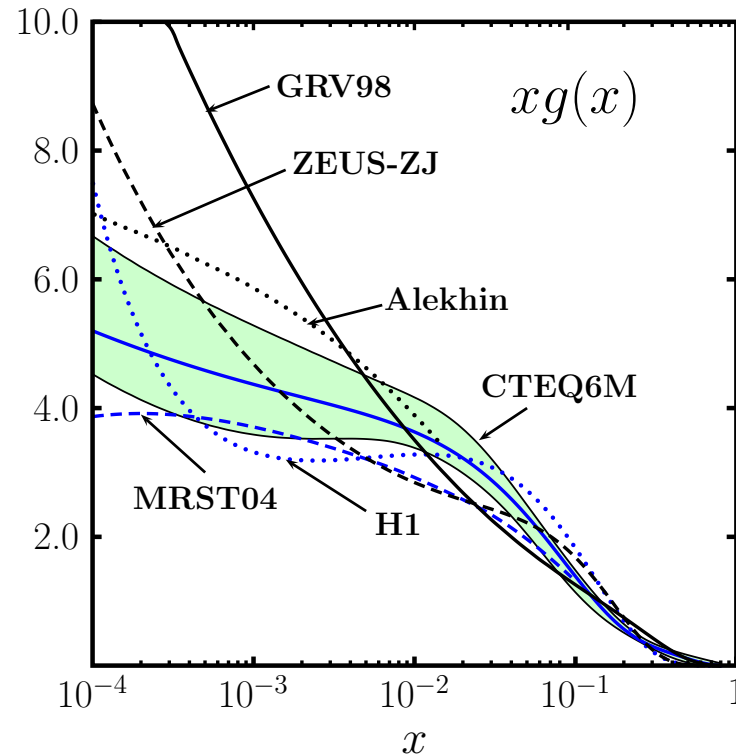


from [Bluemlein-Boettcher \(02\)](#)

dotted line: bound $|\Delta q| \leq 2q$

$SU(3)_F$ symmetric sea assumed

What do we know about the gluon PDF?



not too well constrained in region $\simeq 10^{-2}$

$$Q^2 = 4 \text{ GeV}^2$$

$$s(x) \neq \bar{s}(x)?$$

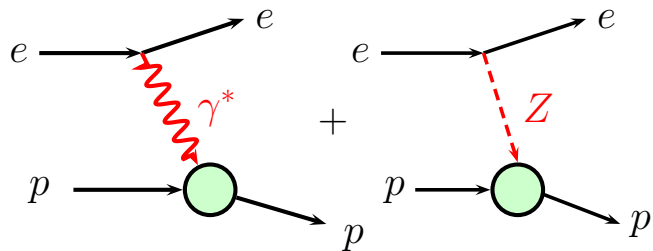
there is no net strangeness in the proton $\implies \int_0^1 dx [s(x) - \bar{s}(x)] = n_s - n_{\bar{s}} = 0$
 CTEQ PDFs: weak indication for $s(x) \neq \bar{s}(x)$

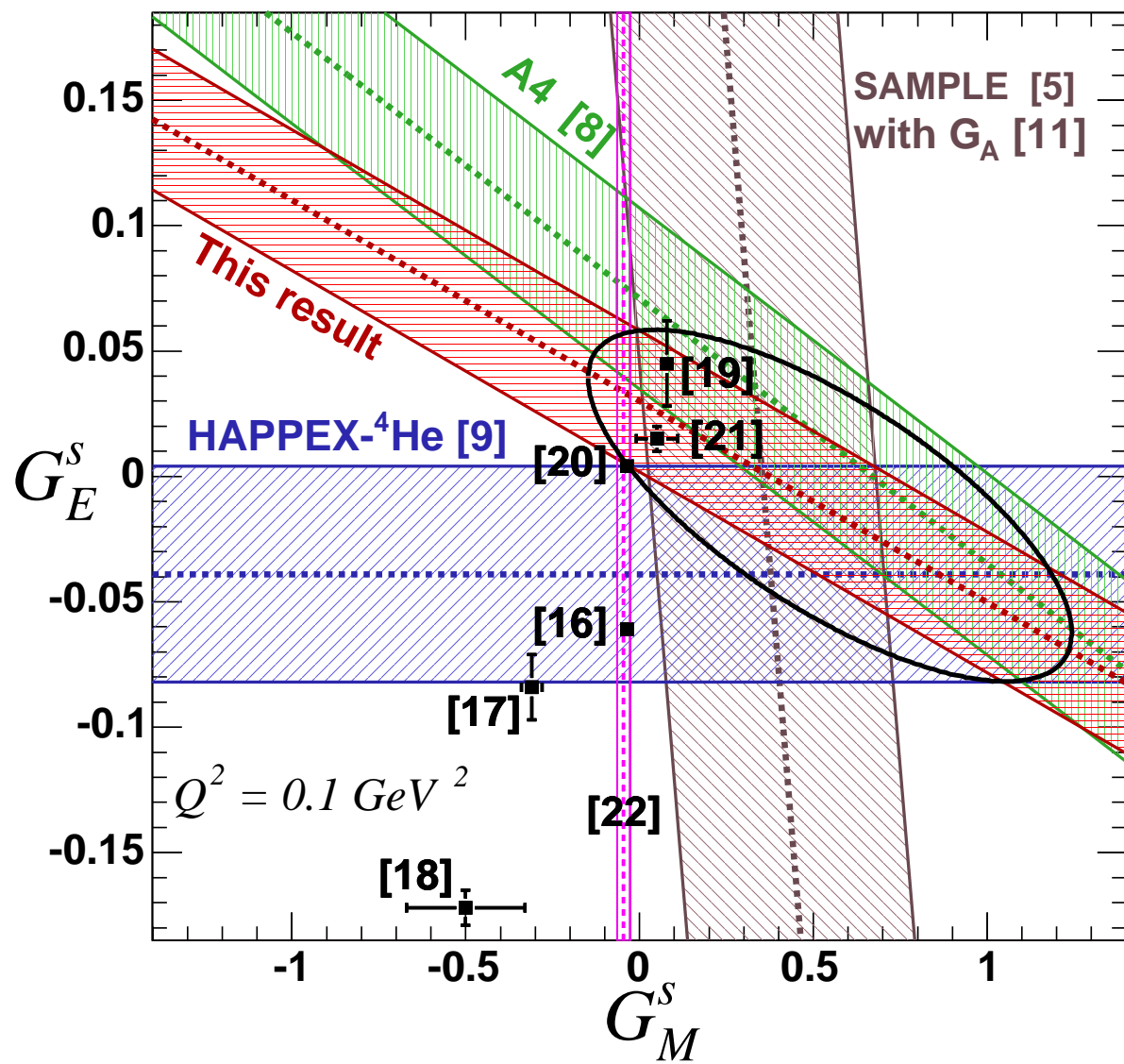
strangeness form factor (g_0 , HAPPEX, A4, SAMPLE)

$$G_{E,M}^a \propto q^a - \bar{q}^a \text{ (see lecture 2)}$$

ep scattering with long. pol. electrons $A^{\text{PV}} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$ **parity violation**

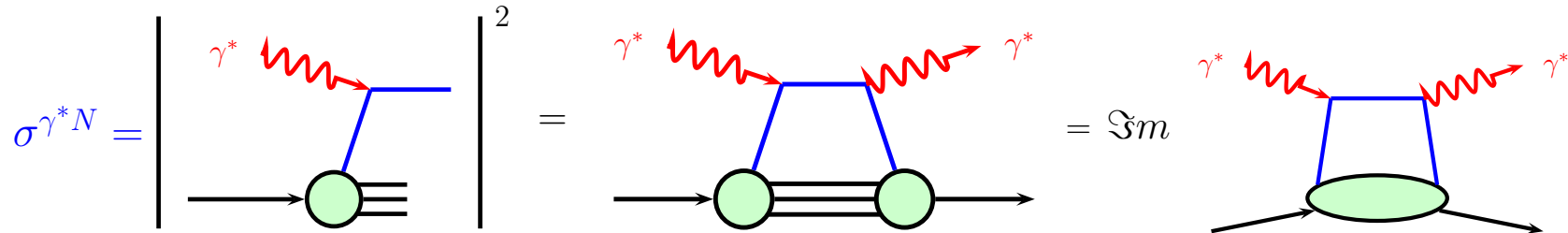
measures interference between elm. FF and strange ones
 data not yet conclusive but probably $s - \bar{s}$ small





taken from HAPPEX 05

Total cross section for absorption of virtual photons by unpolarized protons



optical theorem

$$\sum_n |\langle 1 \dots n | T | \gamma^* \rangle|^2 = 2\Im m \langle \gamma^* p | T(\theta = 0) | \gamma^* p \rangle = \sigma^{\gamma^* N}$$

cross section for fixed helicity of photon

$$\sigma_T = 1/2(\sigma_+ + \sigma_-) = 2mW_1\sigma_0 \quad \sigma_L = 2m[-W_1 + W_2(1 + Q^2/(4m^2x_{Bj}^2))]\sigma_0$$

$$r = \sigma_L/\sigma_T = W_2/W_1(1 + Q^2/(4m^2x_{Bj}^2)) - 1$$

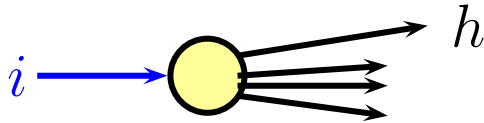
$$\text{parton approach } r = 4m^2x_{Bj}^2/Q^2 \rightarrow 0 \text{ for } Q^2 \rightarrow \infty$$

in agreement with experiment

Fragmentation functions

in hard inclusive processes: sharply collimated jets of particles in final state
single particle spectra described by

fragmentation fct $z = p_h^+ / k^+$



$D_i^h(z)$ probability that a hadron h
with mom. fraction between z
and $z + dz$ is emitted by parton i

satisfy same evolution eq. as the PDFs $\implies D_i^h(z, Q^2)$

momentum conservation: $\sum_h \int_0^1 dz z D_i^h(z) = 1$

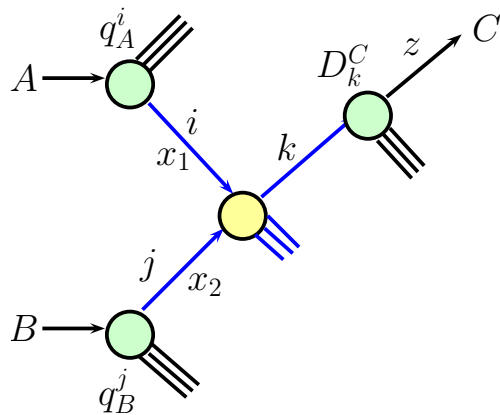
e.g. in $e^+e^- \rightarrow q\bar{q}$: $\sum_h \int_0^1 dz [D_i^h(z) + D_{\bar{i}}^h(z)] = 1$

Applications

PDFs (and frag. fcts) are input to calculations of other hard inclusive processes

QCD gets predictive power

master formula (to leading-twist order or collinear appr.)



$$\sigma(AB \rightarrow CX) \text{ and } \hat{\sigma}(q_i q_j \rightarrow q_k X)$$

$$E_C \frac{d\sigma}{d^3 p_c} = \sum_{i,j,k} \int dx_1 dx_2 \frac{dz}{z^2} q_A^i(x_1) q_B^j(x_2) E_k \frac{d\hat{\sigma}}{d^3 k} D_k^C(z)$$

holds also if A, B or C is elementary

e.g. $q_A^i = \delta_{iA} \delta(1 - x_1)$

if proton PDFs known ($\bar{p}: q \leftrightarrow \bar{q}$)

- $pp(\bar{p}) \rightarrow \pi X$

- $pp(\bar{p}) \rightarrow \gamma X$

prompt photon prod.

- $pp(\bar{p}) \rightarrow e^+ e^- X$

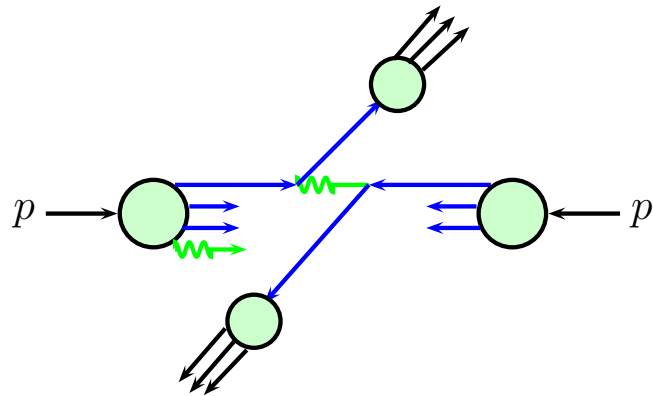
Drell-Yan process

- $pp(\bar{p}) \rightarrow \text{jets}$

“QCD background” understanding important for

discovery of new physics at LHC

Jet cross section at the TEVATRON



reconstruction of jet, i.e. the parton subprocesses to lowest order

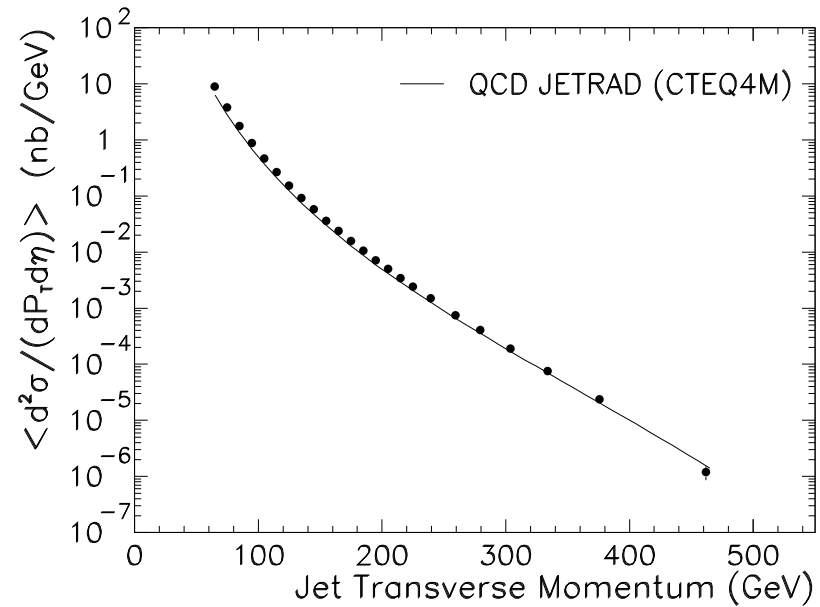
$$qq \rightarrow qq, \quad qg \rightarrow qg, \quad gg \rightarrow gg,$$

$$q\bar{q} \rightarrow gg, \dots$$

4-jet structure

today: NLO (5 jets)

D0 results

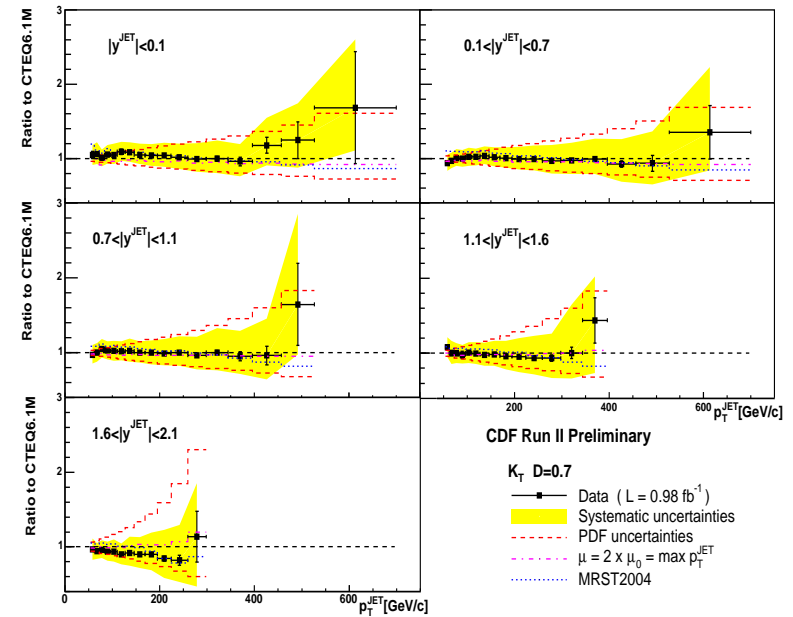
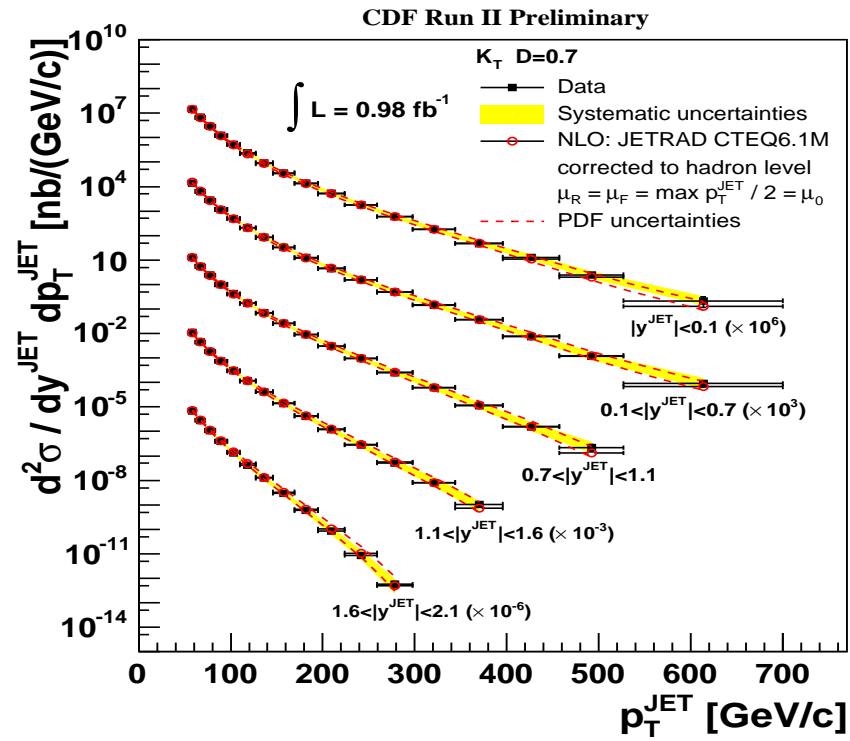


Tevatron $\sqrt{s} = 1.8$ TeV

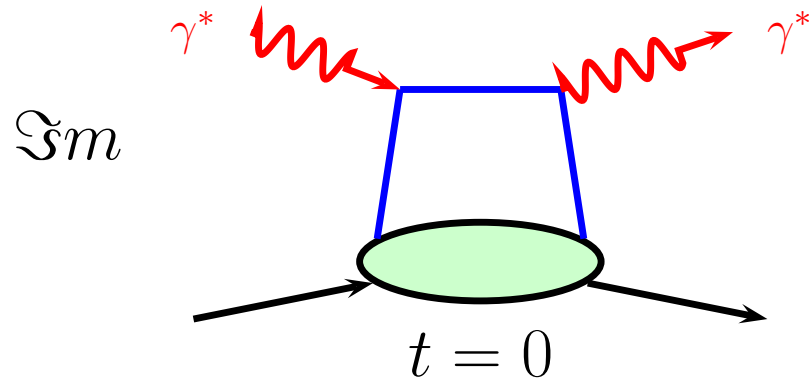
pseudorapidity $|\eta| = |\ln[\tan \theta/2]| < 0.5$

NLO QCD prediction using CTEQ4M

CDF results



Generalization



Only forward imaginary part of graph realized in nature?

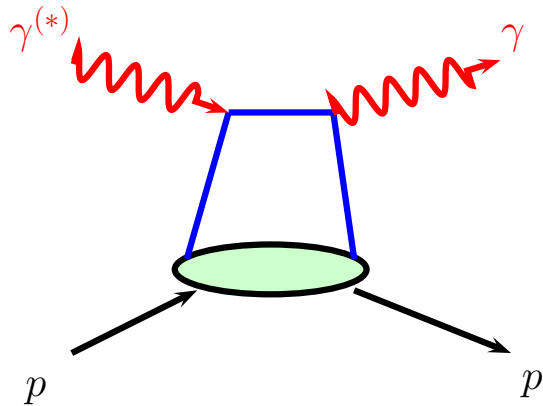
would be a bad feature of the theory

what about

- real part at $t = 0$?
- non-forward directions ($t \neq 0$)?
- unequal virtualities of the photons?

\implies hard exclusive reactions and generalized parton distributions (GPDs)

Handbag factorization in excl. reactions



only one active parton
(others are spectators)

hard process: $\gamma^{(*)}q \rightarrow \gamma q$

soft physics: GPDs

two classes of hard exclusive reactions:

DEEP VIRTUAL Q^2 large $-t/Q^2 \ll 1$

WIDE-ANGLE $-t(-u)$ large $Q^2/(-t) \leq 1$