

LECTURE 23: CP Violation (Part II)

Overview:

- Meson mixing (cont.)
- K meson system
- B meson system

(I used mostly Burgess, and Cheng Li as references, and notes from Frank Wuerthwein)

Meson Mixing (continued from lecture 22)

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We define the states $|Q\rangle$ and $|\bar{Q}\rangle$ (destroyed by the fields $Q(x)$ and $\bar{Q}(x)$ and define $|Z_{\pm}\rangle$ (destroyed by $Z_{\pm}(x)$ and $\bar{Z}_{\pm}(x)$). The normalized states are:

$$|Z_{+}\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} [p|Q\rangle + q|\bar{Q}\rangle]$$

$$|Z_{-}\rangle = \frac{i}{\sqrt{|p|^2 + |q|^2}} [p|Q\rangle - q|\bar{Q}\rangle]$$

$$\frac{p}{q} = z^2 = \sqrt{\frac{C}{B}}$$

If CP is a symmetry: $C = B \Rightarrow z = 1$

$$\Rightarrow Q^{\pm} = \frac{Q + \bar{Q}}{\sqrt{2}}, \quad Q^{-} = i \frac{(\bar{Q} - Q)}{\sqrt{2}}$$

$$M^2_{\pm} \approx M^2_{\pm} - im\Gamma_{\pm} = A \pm B$$

Meson Mixing (cont.)

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eigenstates: $|2_+\rangle = \frac{1}{\sqrt{1+|\xi|^2}} [|q_+\rangle - i\xi |q_-\rangle]$

$$|2_-\rangle = \frac{1}{\sqrt{1+|\xi|^2}} [|q_-\rangle + i\xi |q_+\rangle]$$

with $\xi = \frac{(2z-1)}{1+z^2} = \frac{p-q}{p+q}$

If we start with state $|q\rangle$:

$$\langle q | q \rangle_T = \frac{1}{2} [e^{-iEt} + e^{-iE^*t}], \quad \langle \bar{q} | q \rangle = \frac{q}{2p} [e^{-iEt} - e^{-iE^*t}]$$

$$P_T [q(k) \rightarrow q(k)] = \frac{1}{4} \left[e^{-i\Gamma_+(k)T} + e^{-i\Gamma_-(k)T} + 2 e^{-\frac{[\Gamma_+(k) + \Gamma_-(k)]T}{2}} \cos \Omega_k T \right]$$

$$P_T [q(k) \rightarrow \bar{q}(k)] = \frac{1}{4} \left[\frac{q}{p} \right]^2 \left[e^{-i\Gamma_+(k)T} + e^{-i\Gamma_-(k)T} - 2 e^{-\frac{[\Gamma_+(k) + \Gamma_-(k)]T}{2}} \cos \Omega_k T \right]$$

$$\Omega_k = \frac{\sqrt{k^2 + m_+^2} - \sqrt{k^2 + m_-^2}}{2k} \approx \begin{matrix} m_+ - m_- & \text{if } k \ll m \\ \frac{m_+^2 - m_-^2}{2k} & \text{if } k \gg m \end{matrix}$$

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$$\rightarrow K \left[\sqrt{1 + \frac{M_+^2}{K^2}} - \sqrt{1 + \frac{M_-^2}{K^2}} \right]$$

Taylor $(1+x)^{1/2} = 1 + \frac{x}{2}$

$$\Rightarrow K \left[1 + \frac{M_+^2}{2K^2} - 1 - \frac{M_-^2}{2K^2} \right] = \frac{M_+^2 - M_-^2}{2K}$$

$K^0 - \bar{K}$ mixing (revisited)

With K_{eons} we will have $|K_S\rangle = |2_+\rangle$

$$|K_L\rangle = |2_-\rangle$$

As we saw, To a good approx. (since $|\hat{\xi}| \ll 1$),

$$\Rightarrow |K_L\rangle \approx |K_-\rangle = |0_-\rangle$$

$K^0 - \bar{K}^0$ mixing (cont.)

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For K mesons, we have:

$$m_K = 498 \text{ MeV} \quad (3m_\pi = 420 \text{ MeV})$$

$$\Gamma_S^{-1} = 9 \times 10^{-11} \text{ sec}, \quad \Gamma_L^{-1} = 5 \times 10^{-8} \Rightarrow \Gamma_S = 560 \Gamma_L$$

With $\Gamma_+ \gg \Gamma_-$ and assuming slow-moving K mesons:

$$P_+ [K(K) \rightarrow K(K)] \approx \frac{1}{4} e^{-\Gamma_L(K)T} [1 + 2e^{-\Gamma_S(K) - \Gamma_L(K)T/2} \cos(\Delta m t)]$$

$$P_+ [K(K) \rightarrow \bar{K}(\bar{K})] \approx \frac{1}{4} e^{-\Gamma_L(K)T} |g|^2 [1 - 2e^{-\Gamma_S(K) - \Gamma_L(K)T/2} \cos(\Delta m t)]$$

$$\Delta m = 3.5 \times 10^{-6} \text{ eV} \quad (m_{K_L} - m_{K_S}) > 0$$

Frequency comparable to K_S lifetime (how would you measure K^0 vs \bar{K}^0 ?)

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CP violation in K_{non} decays

- CP violation can occur through mixing:

$$|K_{-}\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}} [|K_{-}\rangle + i\epsilon |K_{+}\rangle]$$

With \mathcal{L}_W denoting weak interaction Lagrangian
($\Delta S = \pm 1$) we write $\langle \pi\pi | \mathcal{L}_W | K_{\pm} \rangle$

- CP violation can also occur at the decay:

$\langle \pi\pi | \mathcal{L}_W | K_{-} \rangle$ i.e. \mathcal{L}_W itself breaks CP (K_{non}
as "direct" CP violation.

To determine relative size of these contributions
we can use the observables:

$$\eta_{00} = \frac{\langle \pi^0\pi^0 | \mathcal{L}_W | K_L \rangle}{\langle \pi^0\pi^0 | \mathcal{L}_W | K_S \rangle} \quad \eta_{+-} = \frac{\langle \pi^+\pi^- | \mathcal{L}_W | K_L \rangle}{\langle \pi^+\pi^- | \mathcal{L}_W | K_S \rangle}$$

CP violation in K_{con} decays ⑦

If mixing is sole source of CP violation: $\eta_{00} = \eta_{+-}$

Note that the K_{con} decaying to two pions have either isospin 0 or 2. Amplitudes are:

$$\langle \pi^+ \pi^- | \mathcal{H}_W | K_{\text{con}}^0 \rangle = A_0 e^{i\zeta_0} + A_2 e^{i\zeta_2}$$

$$\langle \pi^+ \pi^- | \mathcal{H}_W | \bar{K}_{\text{con}}^0 \rangle = A_0 e^{-i\zeta_0} + A_2 e^{-i\zeta_2}$$

A_0, A_2 are CP-conserving strong interaction matrix elements for $\pi\pi$ isospin channel, ζ_0, ζ_2 are the CP-violating phases due to \mathcal{H}_W (assuming CP-violating \mathcal{H}_W)

→ Physical decay rates prop. to $|A_0 e^{i\zeta_0} + A_2 e^{i\zeta_2}|^2$

⇒ relative phase is relevant

CP violation in K_{long} decays (cont.)

with $\epsilon = -\bar{\epsilon} + \zeta_0$

$$\epsilon' = \left(\frac{A_2}{A_0 + A_2} \right) \boxed{(\epsilon_2 - \epsilon_0)}$$

we get: $\eta_{+-} = \epsilon + \epsilon'$, $\eta_{00} = \epsilon - 2\epsilon'$

Experiments (KTeV, NA48) have confirmed that

$$\frac{\epsilon'}{\epsilon} \neq 0, \quad \text{Re} \left| \frac{\epsilon'}{\epsilon} \right| = 1.7 \times 10^{-3}$$

→ ϵ' small relative to ϵ

Measurements with B mesons also have observed direct CP violation

$B-\bar{B}$ Mixing

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Similar to $K-\bar{K}$ mixing but b quark mass $\gg s$ quark mass and well above QCD scale:

- theoretical uncertainties are reduced.
- much larger phase space eliminates (essentially) the lifetime difference. Simplifies expressions ...
- B decays are more CKM suppressed

With $\Gamma_s \approx \Gamma_b$, we'll denote the two states by " H " for heavy, and " L " for light. Previous

oscillation probabilities become (non-relativistic B 's):

$$P_T [B^0 \rightarrow B^0] = e^{-\Gamma_T} \cos^2 \left(\frac{\Delta m T}{2} \right)$$

$$P_T [B^0 \rightarrow \bar{B}^0] = \left| \frac{q}{p} \right|^2 e^{-\Gamma_T} \sin^2 \left(\frac{\Delta m T}{2} \right)$$

$B-\bar{B}$ Mixing (cont.)

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In our example (non-rel, e.g. CLEO), it is difficult to measure time t elapsed since the B was in a pure B^0 eigenstate. We start from:



→ relative angular momentum $l=1$

$$CP|B\bar{B}\rangle = -|B\bar{B}\rangle$$

$$\text{initial state } |B\bar{B}\rangle = \frac{1}{\sqrt{2}} [|B(K)\bar{B}(-K)\rangle - |B(-K)\bar{B}(K)\rangle]$$

We can then "tag" the flavour of the B using semi-leptonic decay of one B (there are other ways to tag).

By reconstructing decay vertex, we can determine prob. of observing (for instance) same-sign leptons vs. distance i.e. versus Time → asymmetric B factories make this easier

B- \bar{B} Mixing (in more detail)

$$\begin{aligned}
 |\langle \bar{F} | H | B^0 \rangle|^2 &= |\langle \bar{F} | H | B^0(t) \rangle|^2 \\
 &= \frac{1}{4|p|^2} |\langle \bar{F} | B_L(t) \rangle + \langle \bar{F} | B_H(t) \rangle|^2 \\
 &= \frac{1}{4|p|^2} |pA e^{(-im_L - \Gamma_L/2)t} + pA e^{(-im_H - \Gamma_H/2)t}|^2 \\
 &= \frac{1}{4} |A|^2 (e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2e^{-(\Gamma_H + \Gamma_L)t/2} \cos \Delta m t)
 \end{aligned}$$

$$\begin{aligned}
 |\langle \bar{F} | H | B^0 \rangle|^2 &= |\langle \bar{F} | B^0(t) \rangle|^2 \\
 &\approx \frac{1}{4|p|^2} |\langle \bar{F} | B_L(t) \rangle + \langle \bar{F} | B_H(t) \rangle|^2 \\
 &= \frac{1}{4|p|^2} |q\bar{A} e^{(-im_L - \Gamma_L/2)t} - q\bar{A} e^{(-im_H - \Gamma_H/2)t}|^2 \\
 &= \frac{1}{4} |q|^2 |\bar{A}|^2 (e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_H + \Gamma_L)t/2} \cos \Delta m t)
 \end{aligned}$$

$B-\bar{B}$ Mixing (in more detail, cont.)

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Remember we have:

$$|B_H\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$$

$$|B_L\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$$

$$\rightarrow \langle F|B^0\rangle = A, \quad \langle \bar{F}|B^0\rangle = 0$$

$$\langle \bar{F}|\bar{B}^0\rangle = \bar{A}, \quad \langle F|\bar{B}^0\rangle = 0$$

$$B_H \rightarrow F \quad \text{provides } p A$$

$$B_L \rightarrow \bar{F} \quad \text{provides } -q \bar{A}$$

\rightarrow if $A \neq \bar{A}$, CP violation in decay

\rightarrow if $|\frac{q}{p}| \neq 1$, CP violation in mixing

Without CP violation we would have

$$|\langle F|H|B^0\rangle|^2 + |\langle \bar{F}|H|\bar{B}^0\rangle|^2 = \frac{1}{2}|A|^2 (e^{-\Gamma_L t} + e^{-\Gamma_H t})$$

CKM Matrix

In SM, Flavour changing and CP-violating physics is due to 4 parameters in unitary CKM matrix.

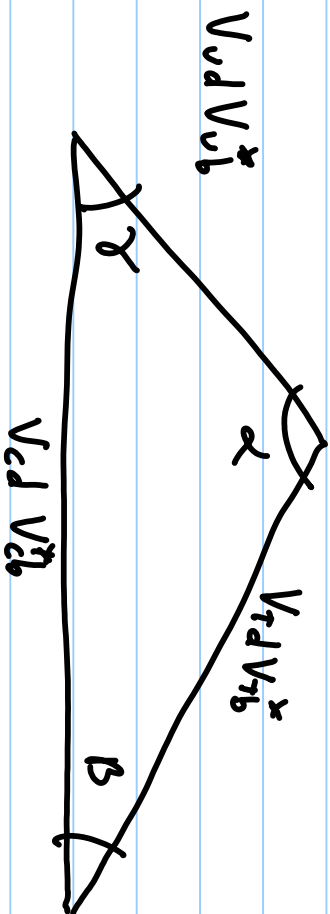
Physics beyond SM could provide new contributions. B physics provides many opportunities of testing SM.

Unitarity conditions $\rightarrow \sum_i V_{in} V_{im}^* = \delta_{nm}$

With Bd mesons, we have $m=b$, $n=d$, which implies:

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

\rightarrow sum of 3 complex numbers vanish.
 \rightarrow can be expressed 3 vectors in complex plane

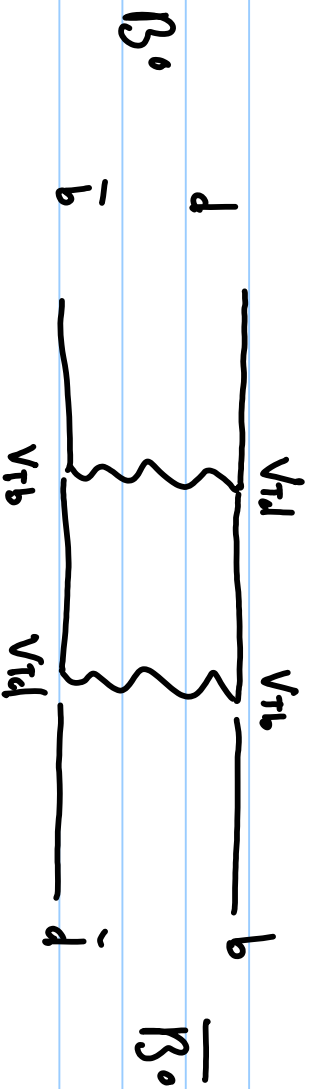


$$\alpha = \arg \left(-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right)$$

$$\beta = \arg \left(\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right)$$

CKM Matrix and B physics

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$$\beta = \arg \left(\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right)$$

$B^0 \rightarrow 2K_S : V_{td} V_{tb}$ from mixing

V_{cb} from $b \rightarrow c$

V_{cd} from K^0 mixing

In asymmetric B factories:

- reconstruct one B in CP eigenstate e.g. $2K_S$
- reconstruct decay of other B \rightarrow determine Flavor
- Measure distance between mesons and convert to proper time

-FIT decay time difference T_0

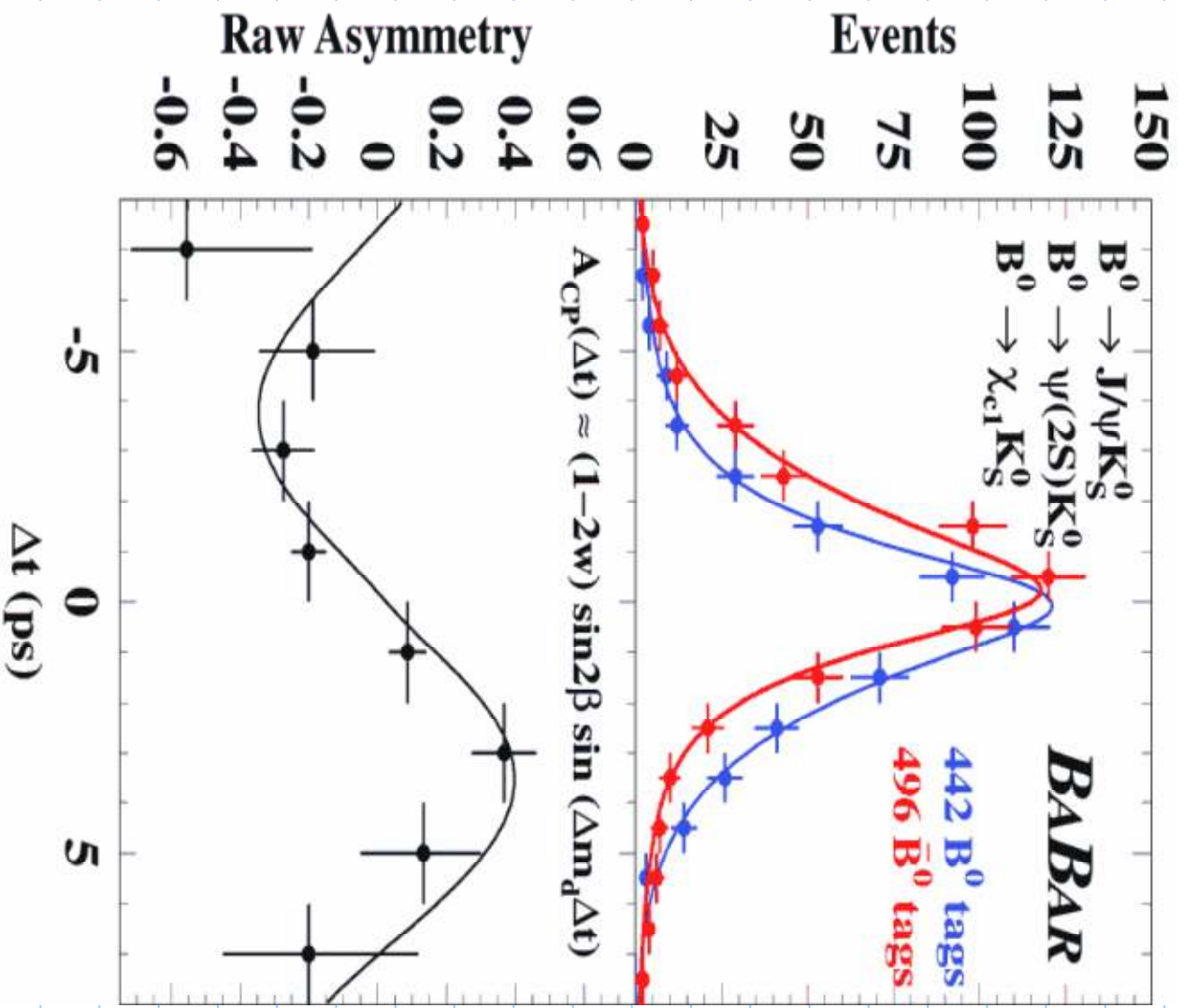
$$\frac{dN}{dt} \propto e^{-T(t)} [1 \pm F_{sc} (\sin 2\theta) \sin(\Delta n t)]$$

↳ CP of final state

- Determine θ

Note that:
$$A = \frac{N(B_1 \rightarrow F_{CP}, \bar{B}_2 \rightarrow \bar{F}) - N(B_1 \rightarrow F_{CP}, B_2 \rightarrow F)}{\text{sum}} = 0$$

$$B_{UT} A(T_1, T_2) \propto \sin 2\theta_{CP}$$



Other sources of CP violation

CP violation due to CKM matrix is too small to explain observed matter anti-matter asymmetry in the Universe.

Are there other potential sources of CP violation in SM?

Yes: — lepton sector (next lecture)

— Strong CP violation (very small if θ exists)

Physics beyond SM could also contribute

Strong CP Problem

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Most general renorm. Lagrangian involving SM fields includes:

$$\mathcal{L}_S = -\frac{1}{4} G_{\mu\nu}^2 G^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^2 W^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$- \frac{g_3^2 \Theta_3}{64\pi^2} \epsilon^{\mu\nu\lambda\rho} G^{\mu\nu} G^{\lambda\rho} - \text{similar terms for } W, B$$

→ This term is effectively a total derivative and would be expected to have no physical implications

$$\propto \int d^4x K$$

Term determined by change in charge $\int d^3x K^0 \equiv N_{CS}$

Change in charge need not be zero in ϵ vacuum
To vacuum process → QCD vacuum Topologically non-trivial

Axions are a potential solution (need to add scalar field)

