LECTURE 13: Z Boson Physics and Neutral Currents (Part 2)

Overview:

-A closer look at the structure of the neutral current

-Neutral currents and neutrino scattering

-Forward-backward asymmetries

I used Quigg and Halzen-Martin as references

See also chapters 15 and 16 of Thomson

NEUTRAL CURRENT

We wrote the NC as:

$$-\frac{g}{2\cos\theta_W}\sum_{\psi_i=\nu,\ell}\bar{\psi}_i\gamma^\mu\big(\underbrace{g_V^i}_{T_3^i-2Q_i\sin^2\theta_W}-\underbrace{g_A^i}_{T_3^i}\gamma^5\big)\psi_i Z_\mu$$

Electrons : $T_3=-1/2$, $Q=-1$

neutrinos : $T_3=1/2$, $Q=0$

$$\Rightarrow \text{electrons}: \quad \frac{-g}{2\cos\theta_W}\,\bar{e}\gamma^\mu\left[-1/2+2\sin^2\theta_W+\tfrac{1}{2}\gamma^5\right]e Z_\mu \qquad (1)$$

we could write: $(1)=\dfrac{+g}{2\cos\theta_W}\cdot\dfrac{1}{2}\cdot\bar{e}\gamma^\mu\left[1-4\sin^2\theta_W-\gamma^5\right]e Z_\mu$

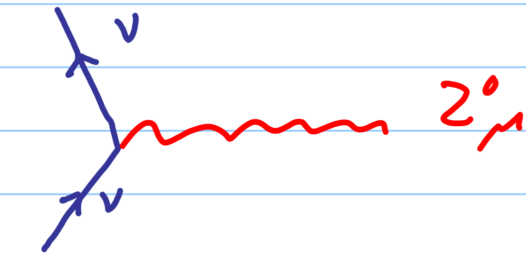
or $\quad = \dfrac{-g}{2\cos\theta_W}\,\dfrac{1}{2}\,\bar{e}\gamma^\mu\left[-1+4\sin^2\theta_W+\gamma^5\right]e Z_\mu$

$$=\frac{-g}{2\cos\theta_W}\cdot\frac{1}{2}\cdot\bar{e}\gamma^\mu\Big[\underbrace{2\sin^2\theta_W}_{R_e}(1+\gamma^5)+\underbrace{(2\sin^2\theta_W-1)}_{L_e}(1-\gamma^5)\Big]e Z_\mu$$

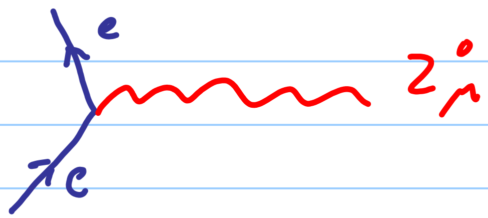
$$\Rightarrow \text{neutrinos}: \quad \frac{-g}{2\cos\theta_W}\,\bar{\nu}\gamma^\mu\tfrac{1}{2}(1-\gamma^5)\,\nu Z_\mu$$

NEUTRAL CURRENT (cont.)

FROM WHICH WE GET THE VERTEX FACTORS:



$$\frac{-i}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{1/2} \bar{\nu}\gamma_\mu(1-\gamma_5)\nu$$



$$\frac{-i}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{1/2} \bar{e}\gamma_\mu\left[2s_w^2(1+\gamma_5) + (2s_w^2-1)(1-\gamma_5)\right]e$$

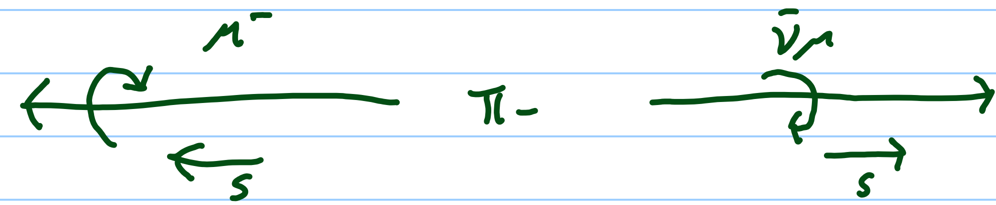
$s_w = \sin\theta_w$

Pion decay interlude:

Pion decay width: $\Gamma = \frac{f_\pi^2}{\pi m_\pi^3}\left(\frac{g_w}{4 m_w}\right)^4 m_\ell^2 (m_\pi^2 - m_\ell^2)^2$

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = 1.3 \times 10^{-4} \qquad \text{why?}$$

Hint: pion has spin 0, $\bar{\nu}$ is right-handed $\Rightarrow$ electron must be right-handed... !?



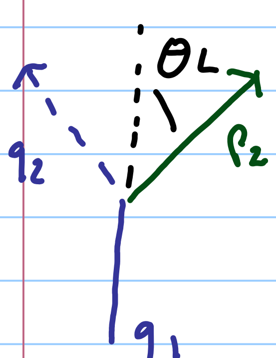
→ have to take some care when we talk about "handedness" and helicity. Helicity is not a Lorentz invariant. Also note that the required addition of ν_R in the SM has almost no impact on the results we've obtained so far since m_ν is so small.

Consider the reactions:

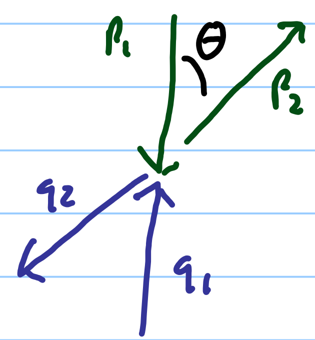
$\bar{\nu}_e e \rightarrow \bar{\nu}_e e$ } FROM REACTORS

$\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e$

$\nu_\mu e \rightarrow \nu_\mu e$ } FROM ACCELERATORS



LAB



CM

NEUTRAL CURRENTS (CONT.)

4 vectors:

in LAB	in CM
$q_1^\mu = (E, 0, 0, E)$	$q_1^\mu = (p^*, 0, 0, p^*)$
$p_1^\mu = (m, 0, 0, 0)$	$p_1^\mu = (\omega^*, 0, 0, -p^*)$
$q_2^\mu = (E+m-E', -p\sin\theta_L, 0, -p\cos\theta_L)$	$q_2^\mu = (p^*, -p^*\sin\theta, 0, -p^*\cos\theta)$
$p_2^\mu = (E', p\sin\theta_L, 0, p\cos\theta_L)$	$p_2^\mu = (\omega^*, p^*\sin\theta, 0, p^*\cos\theta)$

$$E' = \sqrt{p'^2 + m^2} \equiv yE$$

Invariants:

in LAB	in CM
$(p_1 \cdot q_1) = (p_2 \cdot q_2) = mE$	$= p^*(\omega^* + p^*) \approx 2p^{*2}$
$(p_1 \cdot q_2) = (p_2 \cdot q_1) \approx mE(1-y)$	$= p^*(\omega^* - p^*\cos\theta) \approx p^{*2}(1-\cos\theta)$
$(p_1 \cdot p_2) = myE$	$\simeq p^{*2}(1+\cos\theta)$
$(q_1 \cdot q_2) = m(E'-m) \simeq mEy$	$\simeq p^{*2}(1+\cos\theta)$

NEUTRAL CURRENTS (CONT.)

NOTE FROM THE ABOVE THAT

$$mE \approx 2p^{2*}$$

$$p^{*2}(1-\cos\theta) \simeq mE(1-y) \Rightarrow \boxed{1-\cos\theta = 2(1-y)}$$

In Quigg's book, a general matrix element for charged-current interaction is studied: $\sum M_i = \sum C_i \, \bar{\nu} O_i e \, \bar{e} O_i (1-\gamma_5)\nu$

O_i can be vector, scalar, axial vector, tensor, pseudoscalar. We use some of the results in the following.

V-A interaction for $\nu_e e \to \nu_e e$ (we'll deal with constants and propagators and other details later)

$$M = \bar{u}_\nu \gamma_\mu (1-\gamma_5) u_e \; \bar{u}_e \gamma_\mu (1-\gamma_5) u_\nu$$

$$|M|^2 = \mathrm{Tr}\left[\gamma_\mu (1-\gamma_5)(\not{p}_1 + m)\gamma_\nu (1-\gamma_5)\not{q}_2\right]$$

$$\times \mathrm{Tr}\left[\gamma^\mu (1-\gamma^5)\not{q}_1 \gamma^\nu (1-\gamma^5)(\not{p}_2 + m)\right]$$

NEUTRAL CURRENTS (cont.)

TRACE 1: $2\,\mathrm{Tr}\left[(1+\gamma_5)\gamma_\nu \not{q}_2 \gamma_\mu (\not{p}_1 + m)\right]$

$$= 8\left(q_{2\nu} p_{1\mu} - g_{\mu\nu}(q_2\cdot p_1) + q_{2\mu} p_{1\nu}\right) - 8\,\varepsilon_{\mu\nu\rho\sigma}\, q_2^\rho p_1^\sigma$$

TRACE 2: $2\,\mathrm{Tr}\left[(1+\gamma^5)\gamma^\mu \not{q}_1 \gamma^\nu (\not{p}_2 + m)\right]$

$$= 8\left(q_1^\mu p_2^\nu - g^{\mu\nu}(q_1\cdot p_2) + q_1^\nu p_2^\mu\right) + 8\,\varepsilon^{\mu\nu\kappa\lambda}\, q_1^\kappa p_2^\lambda$$

$$|M|^2 = 128\left(q_1\cdot q_2\, p_1\cdot p_2 + q_1\cdot p_1\, q_2\cdot p_2\right)$$

$$\left.\begin{aligned} &- 64\, i\,\varepsilon_{\mu\nu\rho\sigma}\, q_2^\rho p_1^\sigma \left(q_1^\nu p_2^\mu + q_1^\mu p_2^\nu\right) \\ &+ 64\, i\,\varepsilon^{\mu\nu\kappa\lambda}\, q_{1\kappa} p_{2\lambda}\left(q_{2\nu} p_{1\mu} + q_{2\mu} p_{1\nu}\right)\end{aligned}\right\} = 0$$

3 indep. momenta

$$\left[+ 64\,\varepsilon_{\mu\nu\rho\sigma}\,\varepsilon^{\mu\nu\kappa\lambda}\, q_2^\rho p_1^\sigma q_{1\kappa} p_{2\lambda}\right.$$

$$= 128\left(q_1\cdot p_1\, q_2\cdot p_2 - q_1\cdot q_2\, p_1\cdot p_2\right)$$

$$\Rightarrow |M|^2 = 256\, q_1\cdot p_1\, q_2\cdot p_2 = 256\,(mE)^2$$

NEUTRAL CURRENTS

$$\frac{d\sigma}{d\Omega_{cm}} = \frac{|M|^2}{64\pi^2 s} \quad , \quad s = (p_1+q_1)^2 \simeq 2(p_1 \cdot q_1)$$

$$= 4p^{*2} = 2mE$$

note factor of $\frac{1}{2}$ for average spin of e

$$\sigma = \frac{4mE}{\pi} \qquad \text{note that} \quad \frac{d\sigma}{dy} = \frac{4mE}{\pi}$$

$$\frac{d\sigma}{dy} = 4\pi \frac{d\sigma}{d\Omega}$$

V-A interaction for $\bar{\nu}_e e \to \bar{\nu}_e e$

we need to change $q_1 \longleftrightarrow q_2$

this give $|M|^2 = 256 \; q_2 \cdot p_1 \; q_1 \cdot p_2$

$$= 256 (mE)^2 (1-y)^2$$

NEUTRAL CURRENTS (CONT.)

$$\frac{d\sigma}{dy} = 256\,(mE)^2\,(1-y)^2\,\frac{\cdot 4\pi}{64\pi^2\cdot 2mE}\cdot\frac{1}{2}$$

$$= \frac{4mE}{\pi}(1-y)^2 \quad , \quad \sigma = \int_0^1 \frac{4mE}{\pi}(1-y)^2\,dy$$

$$= \frac{4mE}{3\pi}$$

V+A for $\nu_\mu e \to \nu_\mu e$

$$M = \bar{u}_\nu \gamma_\mu (1+\gamma_5) \nu_e \;\; \bar{u}_e \gamma_\mu (1-\gamma_5) \nu_e$$

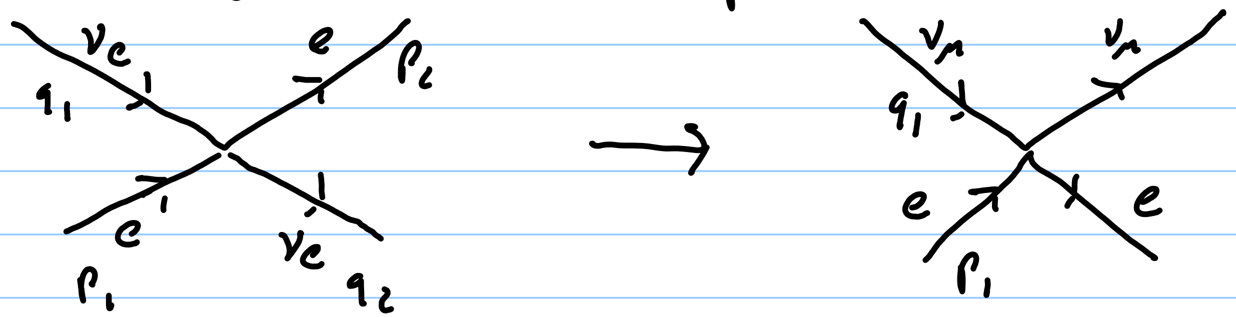
This changes the sign of the $\varepsilon_{\mu\nu\kappa\sigma}$ term

which gives $|M|^2 = 256\, q_1\cdot q_2\; p_1\cdot p_2 = 256\,(mE)^2 y^2$

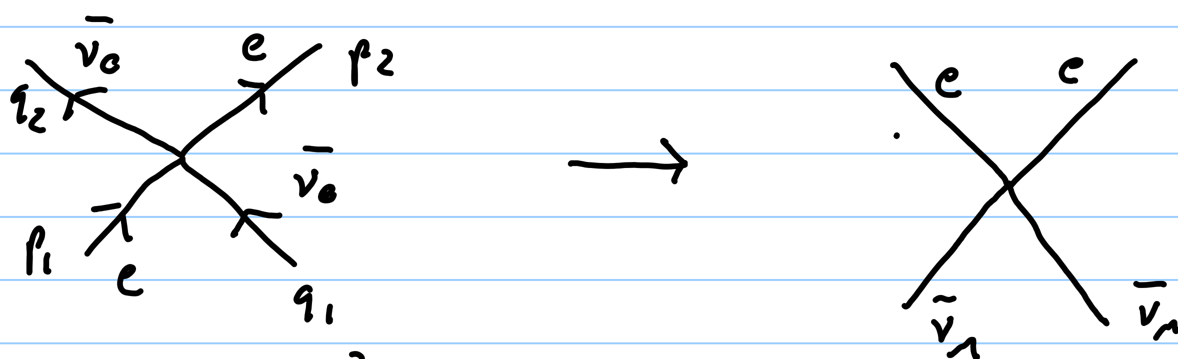
$$\frac{d\sigma}{dy} = \frac{4mE}{\pi}y^2 \quad\rightarrow\quad \sigma = \frac{4mE}{3\pi}$$

NEUTRAL CURRENTS (cont.)

Now, to get actual SM processes with $s \ll M_Z^2$:



$p_2 \leftrightarrow q_2$



$p_1 \leftrightarrow q_2$

with $s \ll M_Z^2$

We get:

	$\frac{\pi}{mE}\frac{d\sigma}{dy}(\nu_\mu e \to \nu_\mu e)$	$\frac{\pi}{mE}\frac{d\sigma}{dy}(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)$
$\lvert V-A\rvert^2$	4	$4(1-y)^2$
$\lvert V+A\rvert^2$	$4(1-y)^2$	4

Neutral currents (cont.)

Using $\frac{-i}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{1/2} \bar{e}\gamma_\mu \left[R_e(1+\gamma_5) + L_e(1-\gamma_5)\right] e$

The processes $e\nu_\mu \to e\nu_\mu$, $e\bar{\nu}_\mu \to e\bar{\nu}_\mu$ give:

$$\frac{d\sigma}{dy}(\nu_\mu e) = \frac{4mE}{\pi}\frac{G_F^2}{8}\left[L_e^2 + R_e^2(1-y)^2\right]$$

(cross terms cancel $\to$ $(1-\gamma_5)(1+\gamma_5)=0$)

$$= \frac{G_F^2 mE}{2\pi}\left[(2\sin^2\theta_W - 1)^2 + 4\sin^4\theta_W(1-y)^2\right]$$

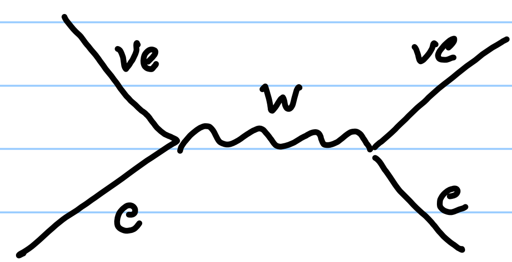
$$\frac{d\sigma}{dy}(\bar{\nu}_\mu e) = \frac{G_F^2 mE}{2\pi}\left[L_e^2(1-y)^2 + R_e^2\right]$$

$$\sigma(\nu_\mu e) = \frac{G_F^2 mE}{2\pi}\left(L_e^2 + \frac{R_e^2}{3}\right)$$

$$\sigma(\bar{\nu}_\mu e) = \frac{G_F^2 mE}{2\pi}\left(\frac{L_e^2}{3} + R_e^2\right)$$

NEUTRAL CURRENTS AND NEUTRINO SCATTERING

$\nu_e\, e \to \nu_e\, e$: Two contributions



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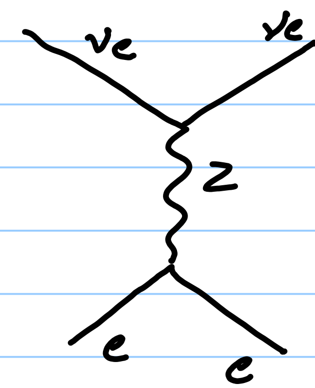


Fierz reordering Theorem:

$$-\left[\bar{e}\gamma^\mu(1-\gamma^5)\nu_e\right]\left[\bar{\nu}_e\gamma_\mu(1-\gamma_5)e\right] = \left[\bar{\nu}_e\gamma^\mu(1-\gamma^5)\nu_e\right]\left[\bar{e}\gamma_\mu(1-\gamma_5)e\right]$$

note sign

$\nu_e e \to \nu_e e$ is obtained from $\nu_\mu e \to \nu_\mu e$ with: $L_e \to L_e + 2$

and $\bar{\nu}_e e$ is obtained from $\bar{\nu}_\mu e \to \bar{\nu}_\mu e$ with

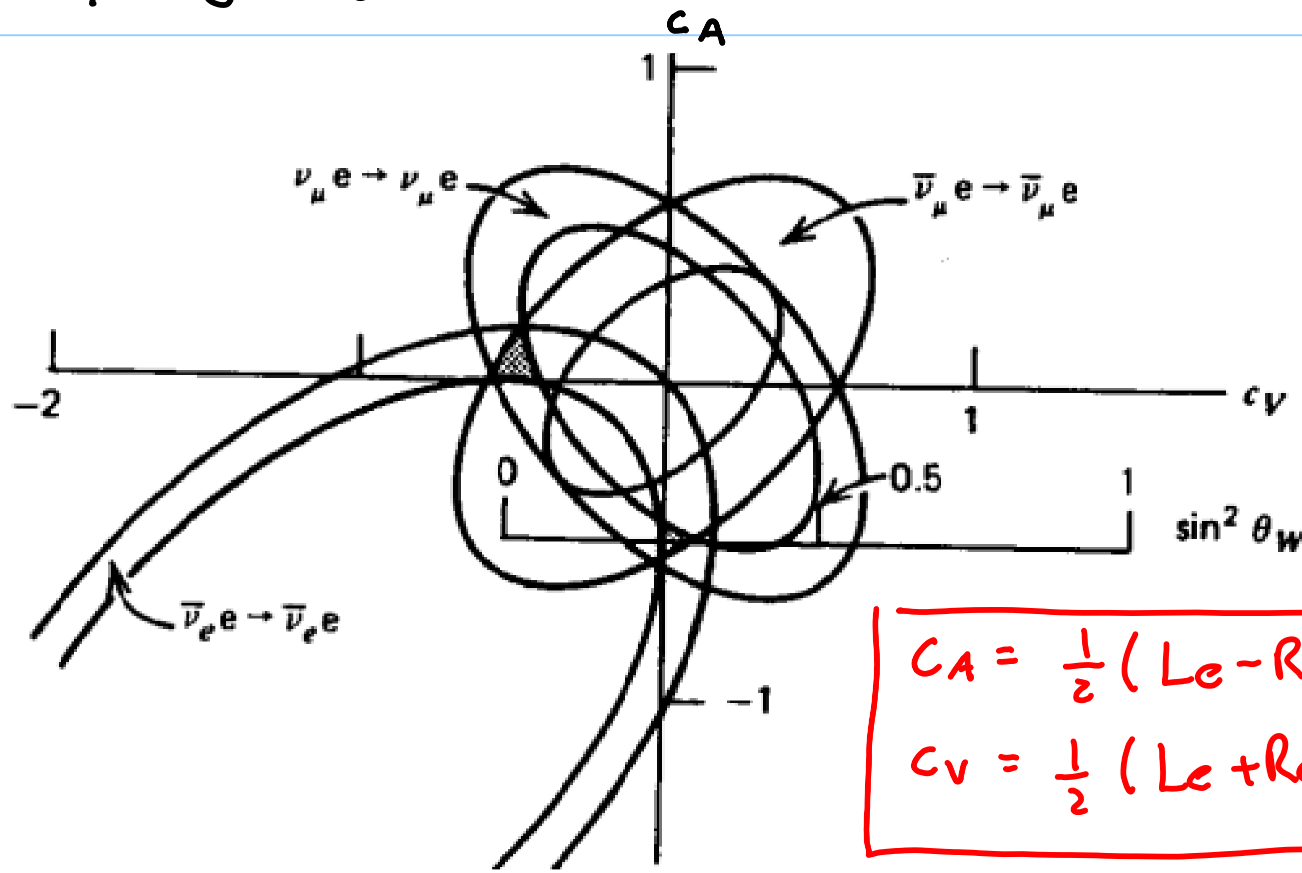
we have: $\sigma(\bar{\nu}_e e \to \bar{\nu}_e e) = \dfrac{G_F^2 m E}{2\pi}\left[\dfrac{(L_e+2)^2}{3} + R_e^2\right]$

$g_a \to g_a + 1$, $g_v \to g_v + 1$

$g_a = \frac{1}{2}(L_e - R_e)$, $g_v = \frac{1}{2}(L_e + R_e)$

Neutral currents (cont.)

Putting everything together with experimental results we get:

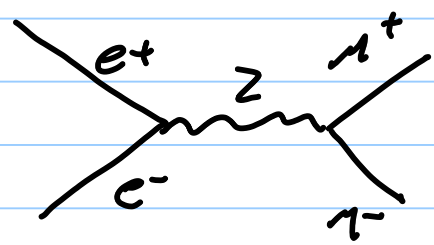


$$\boxed{\begin{aligned} c_A &= \tfrac{1}{2}(L_e - R_e) \\ c_V &= \tfrac{1}{2}(L_e + R_e) \end{aligned}}$$

Note that there are two solutions.
How do we determine which one is correct?

NEUTRAL CURRENTS (CONT.)

We can study $e^+e^- \to \mu^+\mu^-$ for example. Two diagrams contribute:



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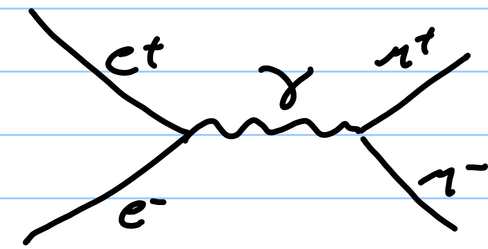
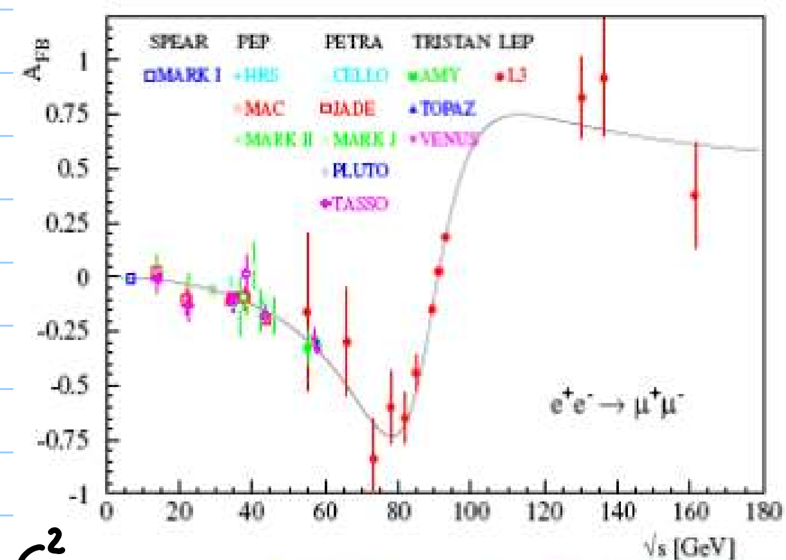


This is done in Thomson. Here we need the γ and γZ terms too.

We then measure the forward-backward asymmetry defining $z = \cos\theta_{cm}$

$$A_{FB} \equiv \frac{\int_0^1 dz \frac{d\sigma}{dz} - \int_{-1}^0 dz \frac{d\sigma}{dz}}{\int_{-1}^1 dz \frac{d\sigma}{dz}}$$



J. Mnich *Phys. Rep.* **271**, 181-266 (1996)

$$\lim_{s/M_Z^2 \to 0} A_{FB} \propto (R_e - L_e)(R_\mu - L_\mu) \propto C_A^2$$

Resolves ambiguity, measures $\sin^2\theta_W$