

## LECTURE 14: Z Boson Physics and Neutral Currents (Part 3)

### Overview:

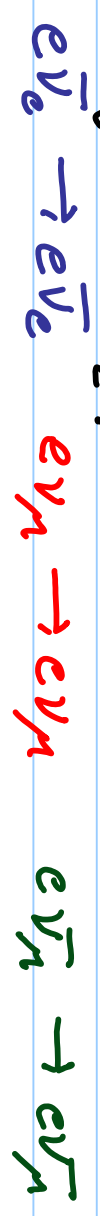
- Production asymmetries
- Neutral current and quarks

(I used Quigg and Halzen-Martin, K. Graham's thesis as references)

# NEUTRAL CURRENT AND ASYMMETRIES

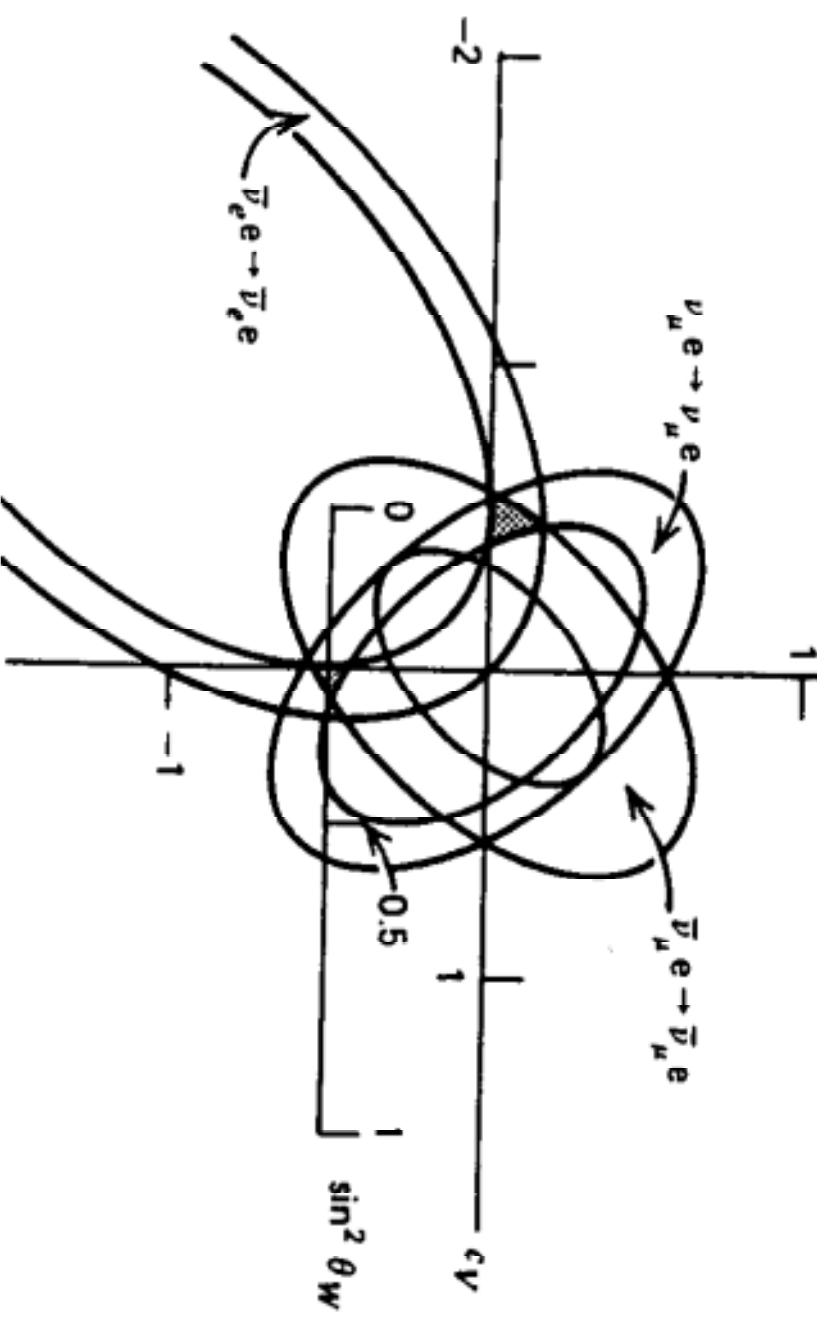
(2)

Last lecture we calculated the cross sections for the processes:  $(s \ll M_Z^2)$



We saw that the measurements gave two values for  $C_V$  and  $C_A$ .

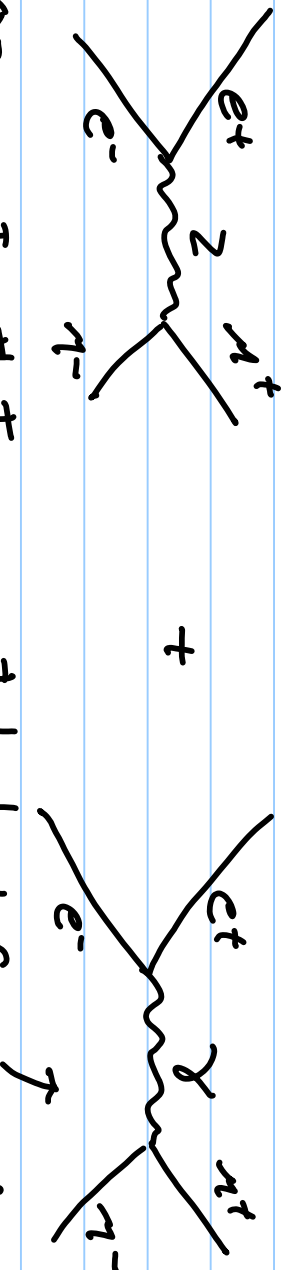
$$\frac{g}{2 \cos \theta_W} \sum_{i=1}^3 \bar{\psi}_i \gamma^\mu (c_V^i - c_A^i \gamma^5) \psi_i \quad Z_\mu$$



## NEUTRAL CURRENT AND ASYMMETRIES

We saw that this ambiguity could be removed with an asymmetry measurement. Let's look at this in more detail.

Two diagrams contribute to  $e^+e^- \rightarrow \mu^+\mu^-$ :



The QED part that we studied before gives us:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} (1 + \cos^2\theta), \quad \sigma = \frac{4\pi\alpha^2}{3s} \quad (\text{Full calculation in lecture 4})$$

You are calculating the pure Z contribution.

The full calculation involves an interference term.

## NEUTRAL CURRENTS AND ASYMMETRIES

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$$M_Y = \frac{-e^2}{k^2} [\bar{u} \gamma^\nu v] [\bar{e} \gamma_\nu e]$$

$$M_Z = \frac{-\sqrt{e} G M_Z^2}{s - M_Z^2} [R_\mu (\bar{\mu}_R \gamma^\nu \mu_L) + L_\mu (\bar{\mu}_L \gamma^\nu \mu_L)] [R_e (\bar{e}_R \gamma_\nu e_R) + L_e (\bar{e}_L \gamma_\nu e_L)]$$

$$L_C \equiv C_V + C_A = g_V + g_A, \quad R_C = C_V - C_A = g_V - g_A$$

$$\text{i.e. } C_V - C_A \gamma^5 = (C_V - C_A) \frac{1}{2} (1 + \gamma^5) + (C_V + C_A) \frac{1}{2} (1 - \gamma^5)$$

Doing the full calculation yields:

$$\frac{d\sigma}{d\Omega} (\bar{e} \bar{e} e^+ \rightarrow \mu^+ \mu^- e^+) = \frac{\alpha^2}{4s} (1 + \cos)^2 |1 + r L_C L_\mu|^2$$

$$\frac{d\sigma}{d\Omega} (\bar{e} \bar{e} e^+ \rightarrow \mu^+ \mu^- e^+) = \frac{\alpha^2}{4s} (1 - \cos)^2 |1 + r R_C R_\mu|^2$$

+ 2 other helicity contributions

$$r = \frac{\sqrt{2} G M_Z^2}{s - M_Z^2 + i M_Z \Gamma_Z} \left( \frac{s}{e^2} \right)$$

note that  $R_C = R_\mu, L_C = L_\mu$

## NEUTRAL CURRENT AND ASYMMETRIES

Results above averaged over all helicities give:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \left[ A_0 (1 + \cos^2\theta) + A_1 \cos\theta \right]$$

↪ symmetric wrt  $\cos\theta$ 
↪ asymmetric wrt  $\cos\theta$

$$A_0 = 1 + \frac{1}{2} \text{Re}(r) (L_e + R_e)^2 + \frac{1}{4} |r|^2 (L_e^2 + R_e^2)^2$$

$$= 1 + 2 \text{Re}(r) C_V^2 + |r|^2 (C_V^2 + C_A^2)$$

$$A_1 = \text{Re}(r) (L_e - R_e)^2 + \frac{1}{2} |r|^2 (L_e^2 - R_e^2)^2$$

$$= 4 \text{Re}(r) C_A^2 + 8 |r|^2 C_V C_A^2$$

Note that with QED only we get:  $A_0 = 1$ ,  $A_1 = 0$   
giving a symmetric distribution.

The weak interaction introduces an asymmetry in the  $\cos\theta$  distribution  $\rightarrow$  a "forward-backward" asymmetry

# NEUTRAL CURRENT AND ASYMMETRIES

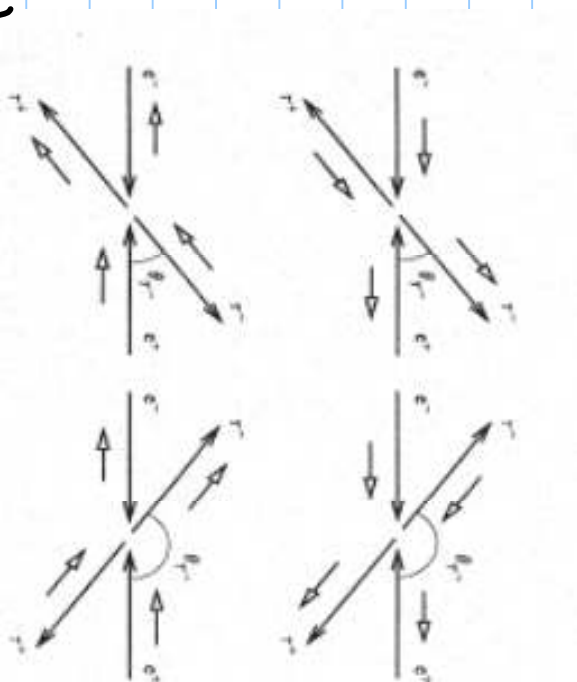
⑥

$$A_{FB} \equiv \frac{\int_0^1 dz \frac{d\sigma}{dz} - \int_1^0 dz \frac{d\sigma}{dz}}{\int_{-1}^1 dz \frac{d\sigma}{dz}}$$

For  $s \ll M_Z^2 \rightarrow |r| \ll 1$

$$A_{FB} = \frac{A_1}{8A_0/3} \approx \frac{3}{2} \text{Re} C_A^2 \approx -\frac{3}{\sqrt{2}} C_A^2 \left( \frac{G_S}{e^2} \right)$$

$e^+e^- \rightarrow \gamma\gamma\gamma^-$



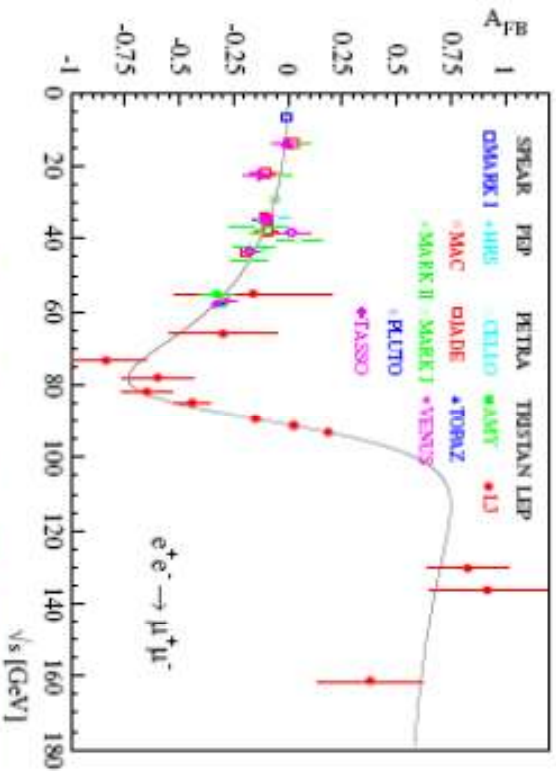
$\rightarrow$  does not offer accurate determination

of  $\sin^2 \theta_w$  (since  $C_V \approx 0$ ). We integrate  $d\sigma$  and get  $\sigma = \sigma_0 A_0$

where  $\sigma_0 \equiv$  QED cross section

$$R_\mu \equiv \frac{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}{\sigma_0} =$$

$$1 + 2 \text{Re} |r| C_V^2 + |r|^2 (C_V^2 + C_A^2)^2$$



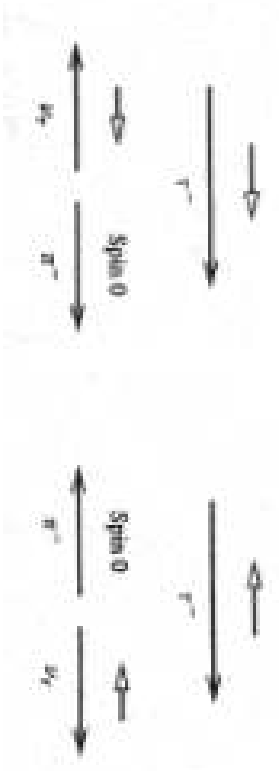
# NEUTRAL CURRENT AND ASYMMETRIES

AT the Z pole, we get very high values of  $R_n$  (~150) since  $r$  hits its maximum.

$$A_{FB}^{S\bar{S}} = 3 \frac{C_V C_A^2}{(C_V^2 - C_A^2)^2}$$

This can get more interesting if we can measure the polarization of the final state leptons. This can be done with  $e^+e^- \rightarrow \gamma^* \gamma^-$

→ The  $\tau$  decays very quickly vis the charged current interaction →  $\nu_\tau$  will be left-handed.  $P_{ion}$  momentum depends on whether we started with  $\tau_L$  or  $\tau_R$



$$P_\tau \equiv \frac{(\sigma_R - \sigma_L)}{(\sigma_R + \sigma_L)}$$

$\sigma_R$  ( $\sigma_L$ ) is  $\sigma$  for producing right (left)-handed  $\tau^-$   
→  $P_\tau = 0$  if parity is conserved.

⑧

# NEUTRAL CURRENT AND ASYMMETRIES

$$\frac{1}{\sigma_{\text{Tot}}} \frac{d\sigma_R}{d\cos\theta_{\nu^-}} = \frac{3}{16} [(1 + \langle P_{\nu^-} \rangle)(1 + \cos^2\theta_{\nu^-}) + \frac{8}{3} (A_{\text{FB}} + A_{\text{pol}}^{\text{FB}}) \cos\theta_{\nu^-}]$$

$$\frac{1}{\sigma_{\text{Tot}}} \frac{d\sigma_L}{d\cos\theta_{\nu^-}} = \frac{3}{16} [(1 - \langle P_{\nu^-} \rangle)(1 + \cos^2\theta_{\nu^-}) + \frac{8}{3} (A_{\text{FB}} - A_{\text{pol}}^{\text{FB}}) \cos\theta_{\nu^-}]$$

$$\langle P_{\nu^-} \rangle = \frac{\sigma_R - \sigma_L}{\sigma_{\text{Tot}}}, \quad A_{\text{FB}}^{\text{pol}} = \frac{(\sigma_R - \sigma_L) \cos\theta_{\nu^0} - (\sigma_L - \sigma_R) \cos\theta_{\nu^0}}{\sigma_{\text{Tot}}}, \quad A_{\text{FB}} \text{ as prev. defined}$$

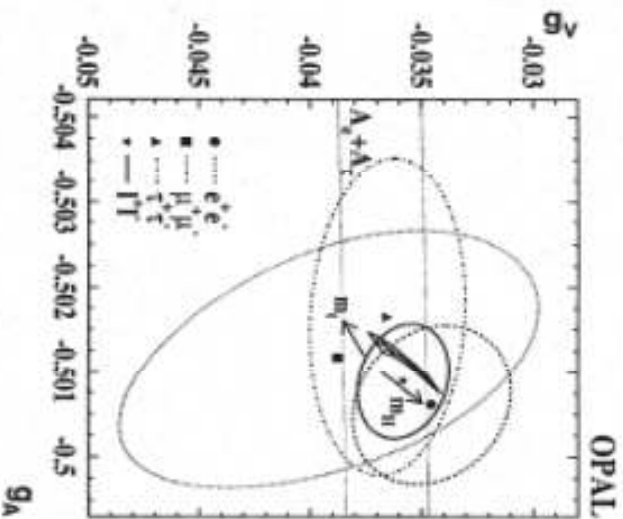
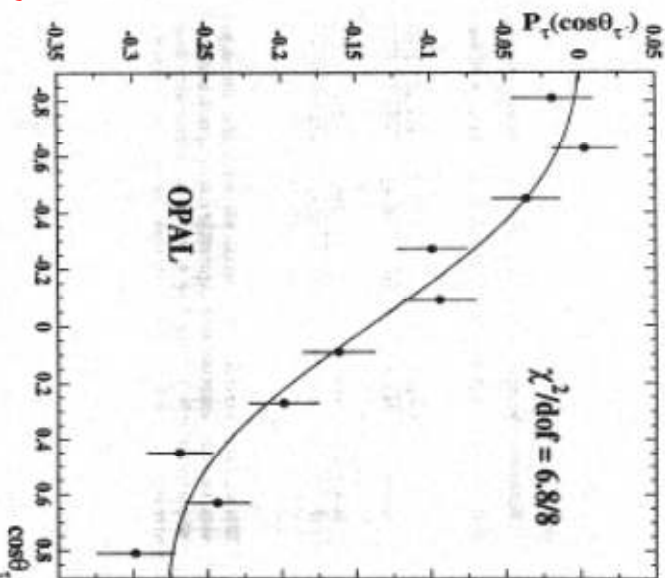
$$\langle P_{\nu^-} \rangle = -A_{\nu^-}, \quad A_{\text{pol}}^{\text{FB}} = -\frac{3}{4} A_e, \quad A_{\text{FB}} = \frac{3}{4} A_e A_{\nu^-}$$

$$A_R = \frac{2g_V/g_A}{(1 + 5v/g_A)^2}$$

note that  $g_V$  is very small. If  $g_V = 0$  ( $\sin^2\theta_w = 0.25$ )

$\Rightarrow A_R = 0$ , no parity violation.

$$\frac{g_V}{g_A} = 1 - 4\sin^2\theta_w$$





# NEUTRAL CURRENT FOR QUARKS

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For charged leptons, we had the following vertex factor:

$$-\frac{i}{\sqrt{2}} \left( \frac{G_F M_Z^2}{\sqrt{2}} \right)^{1/2} \bar{e} \gamma_\lambda [R_e (1 + \gamma_5) + L_e (1 - \gamma_5)] e$$

$$R_e = 2 \sin^2 \theta_w, \quad L_e = 2 \sin^2 \theta_w - 1$$

For quarks we have:

$$-\frac{i}{\sqrt{2}} \left( \frac{G_F M_Z^2}{\sqrt{2}} \right)^{1/2} \bar{q} \gamma_\lambda [R_q (1 + \gamma_5) + L_q (1 - \gamma_5)] q$$

$$R_q = -2 Q_q \sin^2 \theta_w$$

$$L_q = 1 - 2 Q_q \sin^2 \theta_w$$

Recall that  $\Gamma_{e^+e^-} = (R_e^2 + L_e^2) \Gamma_{\nu\bar{\nu}}$        $R_e = \frac{1}{2} (C_V - C_A)$

Now we set:  $\Gamma_{\bar{q}q} = 3 (R_q^2 + L_q^2) \Gamma_{\nu\bar{\nu}}$        $L_e = \frac{1}{2} (C_V + C_A)$

For  $\nu$  quark:  $Q_q = \frac{2}{3} \rightarrow R_q = -\frac{4}{3} \sin^2 \theta_w, L_q = 1 - \frac{4}{3} \sin^2 \theta_w$

For  $d$  quark:  $Q_q = -\frac{1}{3} \rightarrow R_q = \frac{2}{3} \sin^2 \theta_w, L_q = -1 + \frac{2}{3} \sin^2 \theta_w$

# NEUTRAL CURRENT FOR QUARKS

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$$R_U^2 = 4 \cdot \left(\frac{2}{3}\right)^2 \sin^2 \theta_w = \frac{16}{9} \sin^2 \theta_w$$

$$L_U^2 = \left(1 - \frac{4}{3} \sin^2 \theta_w\right) = 1 + \frac{16}{9} \sin^4 \theta_w - \frac{8}{3} \sin^2 \theta_w$$

$$R_D^2 = \frac{4}{9} \sin^2 \theta_w$$

$$L_D^2 = 1 + \frac{2}{3} \sin^2 \theta_w = 1 + \frac{4}{9} \sin^4 \theta_w - \frac{4}{3} \sin^2 \theta_w$$

$$\Gamma_{\bar{u}u} = 3 \cdot \left(1 - \frac{8}{3} x_w + \frac{32}{9} x_w^2\right) \Gamma(2 \rightarrow \bar{u}u) \approx 3 \cdot 0.58 \cdot \Gamma(2 \rightarrow \bar{u}u)$$

$$\Gamma_{\bar{d}d} = 3 \cdot \left(1 - \frac{4}{3} x_w + \frac{8}{9} x_w^2\right) \Gamma(2 \rightarrow \bar{d}d) \approx 3 \cdot 0.75 \cdot \Gamma(2 \rightarrow \bar{d}d)$$

So what is approx.  $\Gamma(2 \rightarrow q\bar{q}) / \Gamma(2 \rightarrow \nu\bar{\nu})$  ?   
 $\rightarrow \sin^2 \theta_w = x_w$

Also  $R_{\text{PEAK}} \equiv \sigma(e^+e^- \rightarrow F\bar{F}) / \sigma_{\text{QED}}(e^+e^- \rightarrow \mu^+\mu^-)$    
 $\hookrightarrow$  2 peak  $\sim 0.5$

$$R_{\text{PEAK}}(e^+e^- \rightarrow \mu^+\mu^-) = \left(1 - 4x_w + 8x_w^2\right) R_{\text{PEAK}}(e^+e^- \rightarrow \nu\bar{\nu})$$

$$R_{\text{PEAK}}(e^+e^- \rightarrow u\bar{u}) = 3 \cdot \left(1 - \frac{8}{3} x_w + \frac{32}{9} x_w^2\right) R_{\text{PEAK}}(e^+e^- \rightarrow \nu\bar{\nu})$$

$$R_{\text{PEAK}}(e^+e^- \rightarrow d\bar{d}) = 3 \cdot \left(1 - \frac{4}{3} x_w + \frac{8}{9} x_w^2\right) R_{\text{PEAK}}(e^+e^- \rightarrow \nu\bar{\nu})$$

# NEUTRAL CURRENT FOR QUARKS

FOR ASYMMETRIES WE HAVE:

$$A(\bar{q}\bar{q}) = \frac{3G_F s}{16\pi\alpha Q_f \sqrt{2}} (R_e - L_e)(R_f - L_f) \quad \text{FOR } s \ll M_Z^2$$

$$R_e - L_e = R_d - L_d = -(R_u - L_u) = 1$$

$$\Rightarrow A(\nu\nu) = \frac{3}{2} A(\mu^+\mu^-)$$

$$A(\bar{d}\bar{d}) = 3A(\mu^+\mu^-)$$

$$A_{\text{PEAK}}(\bar{p}\bar{p}) = \frac{3(L_e^2 - R_e^2)(L_f^2 - R_f^2)}{4(L_e^2 + R_e^2)(L_f^2 + R_f^2)} = \frac{3c_A^e c_V^e c_A^f c_V^f}{(c_A^e + c_V^e)(c_A^f + c_V^f)}$$

	$\nu$	$e$	$\nu$	$d$
$C_V$	0.5	-0.05	0.2	-0.35

# Early asymmetry measurement (DESY $\sqrt{s} = 30$ GeV)

