LECTURE 17: Hadron Structure (Part 3) and QCD (Part 1)

Overview:

-Deep Inelastic Scattering (charged current)

-Deep Inelastic Scattering (neutral current)

(I used Quigg, Thomson, and Giffiths as references)

Deep Inelastic scattering (charged current)

In your first problem (in problem set #3), you calculate the cross section for the reaction:

$$\nu + N \rightarrow \mu + X$$

→ lepton Tensor now has (V-A) structure

The result is:

$$\frac{d^2\sigma^\nu}{dQ^2 d\nu} = \frac{G_F^2}{2\pi}\frac{E'}{E}\left[2W_1^\nu \sin^2\left(\frac{\theta}{2}\right) + W_2^\nu \cos^2\left(\frac{\theta}{2}\right) + W_3^\nu \frac{(E+E')}{M}\sin^2\left(\frac{\theta}{2}\right)\right]$$

In Terms of x and y:

$$\frac{d^2\sigma^\nu}{dx\,dy} = \frac{G_F^2}{\pi} ME\left[F_1(x)\, xy^2 + F_2(x)(1-y) \pm F_3\, xy\,(1-y/2)\right]$$

$$F_3 \equiv \nu W_3^\nu(x)$$

Parton model prediction: $\boxed{\sigma_{Total}(\nu N \rightarrow \ell + X) \propto E}$

Deep Inelastic scattering (charged current)

$$\frac{\sigma}{E}(\nu N \to \ell) \simeq 6 \times 10^{-39} \ \mathrm{cm}^{-2} / \mathrm{GeV}$$

$$\frac{\sigma}{E}(\bar{\nu} N \to \ell) \simeq 3 \times 10^{-39} \ \mathrm{cm}^{-2} / \mathrm{GeV}$$

Also:

$$\frac{x F_3(x)}{F_2(x)} = \begin{array}{ll} +1 & \text{Fermion Target} \\ -1 & \text{anti-F Target} \end{array}$$

Combine with $2x F_1(x) = F_2(x)$ for spin ½ partons

gives:

$$\frac{d^2\sigma}{dx\,dy}(\nu q) = \frac{G_F^2 ME}{\pi} F_2(x)$$

$$\frac{d^2\sigma}{dx\,dy}(\bar{\nu} q) = \frac{G_F^2 ME}{\pi} (1-y)^2 F_2(x)$$

why?

→ reproduces the form of previous xsections with charged current interaction.

Deep Inelastic scattering (charged current)

with the approx.: $\cos\theta_c = 1$

$$F_2^{\nu}(x) = 2x\,(d(x) + \bar{u}(x))$$

$$F_3^{\nu}(x) = 2\,(d(x) - \bar{u}(x))$$

and

$$F_2^{\bar{\nu}}(x) = 2x\,(u(x) + \bar{d}(x))$$

$$F_3^{\bar{\nu}}(x) = 2\,(u(x) - \bar{d}(x))$$

Note That:

$$F_2^{\nu p} + F_2^{\nu n} = 2x\,(u(x) + \bar{u}(x) + d(x) + \bar{d}(x))$$

is proportional To: $\dfrac{F_2^{ep} + F_2^{en}}{F_2^{\nu p} + F_2^{\nu n}} = \dfrac{5}{18}$ (neg. s quarks)

→ ~ gluons carry ~ 50% of proton momentum which does not interact through weak and em Forces

Deep Inelastic scattering (charged current)

→ also note that F_3^ν measures the difference between quark and anti-quark contributions and F_2 measures the sum.

$$F_3^{\nu p} + F_3^{\nu n} = 2(u - \bar{u} + d - \bar{d})$$

→ baryon number sum rule : $\int_0^1 dx \, (F_3^{\nu p}(x) + F_3^{\nu n}(x)) = 6$

Consider a nucleon with $N \equiv \frac{1}{2}(p+n)$

the charged xs are:

$$\frac{d^2\sigma}{dx\,dy}(\nu N \to \mu^- X) = \frac{G_F^2 ME}{\pi} \times \left[u(x) + d(x) + (\bar{u}(x) + \bar{d}(x))(1-y)^2 \right]$$

$$\frac{d^2\sigma}{dx\,dy}(\bar{\nu} N \to \mu^+ X) = \frac{G_F^2 ME}{\pi} \times \left[\bar{u}(x) + \bar{d}(x) + (u(x) + d(x))(1-y)^2 \right]$$

Deep Inelastic scattering (neutral current)

For neutral currents:

$$\frac{d\sigma^2}{dx\,dy}(\nu N \to \nu X) = \frac{G_F^2 ME}{4\pi} \times \left[(L_u^2 + L_d^2)\left[(u(x)+d(x)) + (\bar{u}(x)+\bar{d}(x))(1-y)^2\right] + (R_u^2 + R_d^2)\left[(u(x)+d(x))(1-y)^2 + (\bar{u}(x)+\bar{d}(x))\right]\right]$$

$$\frac{d\sigma^2}{dx\,dy}(\bar{\nu} N \to \bar{\nu} X) = \ldots$$ except swap $(1-y)^2$ Terms

if we neglect anti-quark Terms we obtain the ratios:

$$R_\nu \equiv \frac{\sigma(\nu N \to \nu X)}{\sigma(\nu N \to \mu^- X)} = \frac{1}{2} - x_w + \frac{20}{27} x_w^2$$

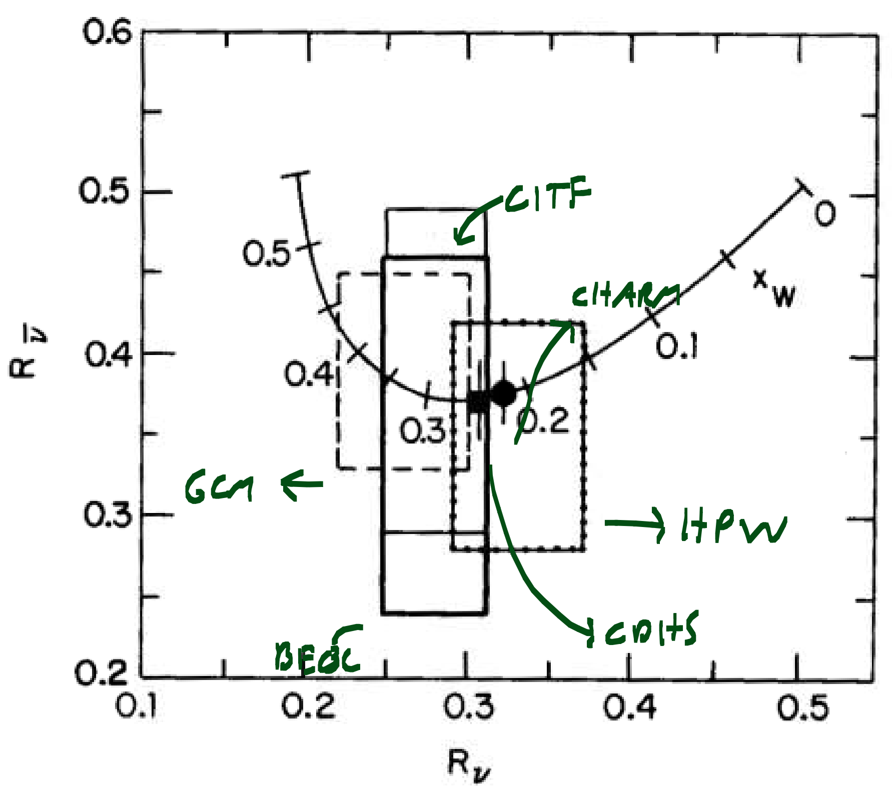
$$R_{\bar{\nu}} \equiv \frac{1}{2} - x_w + \frac{20}{9} x_w^2$$

$x_w \simeq 0.23 \quad \to \quad R_\nu = 0.31$

$R_{\bar{\nu}} = 0.39$

Deep Inelastic scattering (charged current)

Ratios of R_ν and $R_{\bar\nu}$ from various experiments:



We can also observe γ-Z interference effects in deep inelastic scattering.

$$\frac{d^2\sigma}{dx\,dy}(e\,p \to e\,X) = \text{em Term} + \text{weak Term} + \text{interference Term}$$

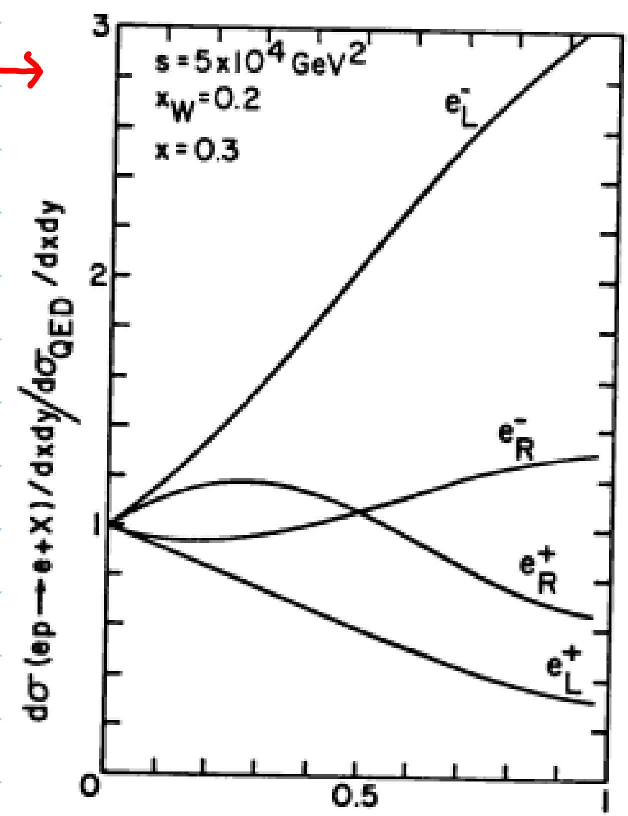
Deep Inelastic scattering (neutral currents)

Asymmetry for $Q^2 \ll M_Z^2$:

$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \simeq \frac{-G_F Q^2}{4\pi\alpha\sqrt{2}} \cdot \frac{9}{5}\left(1 - \frac{20}{9} x_w\right)$$

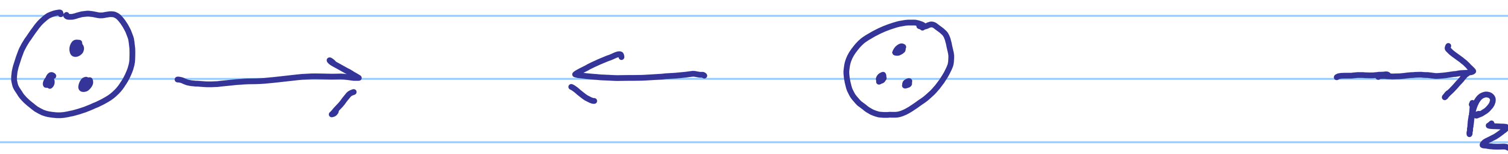
$$\frac{A}{Q^2} \approx -7 \times 10^{-5}$$ → small, observed by SLAC-YALE collaboration

AT higher energies effect becomes of order 1



Polarized high energy beams were used at DESY.

HADRON-HADRON COLLISIONS



Some kinematic variables:

$$p_T = \sqrt{p_x^2 + p_y^2}$$

$$\text{Rapidity} = y = \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right)$$

$\rightarrow$ rapidity differences invariant under boosts

If we neglect the mass of outgoing particles

$$y \approx \frac{1}{2} \ln \left(\frac{1 + \cos\theta}{1 - \cos\theta} \right) = \frac{1}{2} \ln \left(\cot^2 \theta/2 \right)$$

$$\rightarrow \eta \equiv -\ln \left(\tan \theta/2 \right)$$

Hadron-Hadron Interactions

$$\sigma(a+b \to c+X) = \sum_{i,j} F_i^{(a)}(x_a)\, F_j^{(b)}(x_b)\, \hat{\sigma}(i+j \to c+X')$$

- Invariant mass of i,j system $M = \sqrt{s\tau}$

$\hookrightarrow$ dimensionless parameter

i,j system longitudinal momentum in hadron-hadron in c.m.:

$$p = x\sqrt{s}/2$$

- $x_{a,b} = \frac{1}{2}\left[(x^2 + 4\tau)^{1/2} \pm x\right]$

Drell-Yan Production

$a+b \to \ell^+\ell^- + X$

$q+\bar{q} \to \gamma \to \ell^+\ell^-$

invariant mass M

HADRON-HADRON INTERACTIONS

diff. cross section given by:

$$\frac{d\sigma}{dM^2 dx} = \underbrace{\left(\frac{4\pi\alpha^2}{3M^4}\right)}_{\text{as in } e^+e^- \to \mu^+\mu^-} F(\tau, x)$$

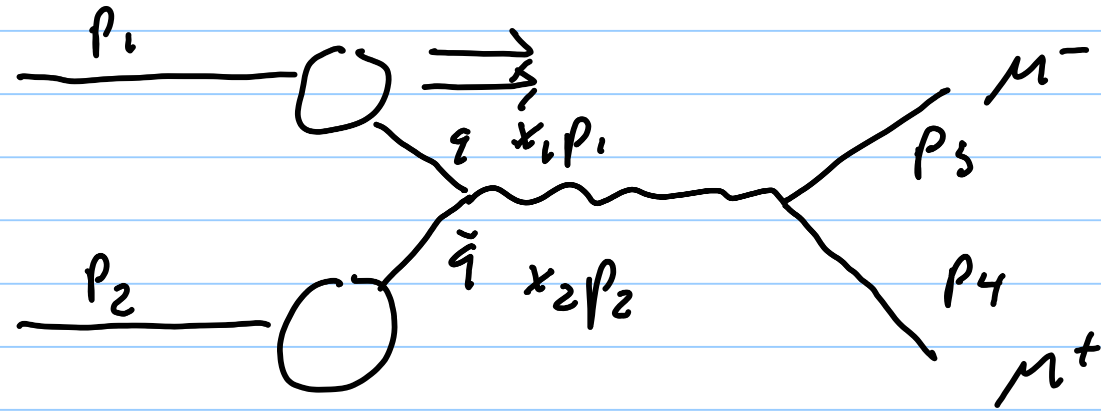
$$F(\tau, x) = \frac{x_a x_b}{(x^2 + 4\tau)^{1/2}} \; g(x_a, x_b)$$

$$\to g(x_a, x_b) = \frac{1}{3} \sum_i e_i^2 \left[q_i^a(x_a)\, \bar{q}_i^b(x_b) + \bar{q}_i^a(x_a) + q_i^b(x_b) \right]$$

THE Drell-Yan process

- consider a $p\bar{p}$ collider i.e. TEVATRON
- LET'S FOCUS ON QED FIRST

HADRON-HADRON COLLISIONS



USING QED RESULTS:

$$\sigma(q\bar{q} \to \mu^+\mu^-) = \frac{1}{N_c} Q_q^2 \frac{4\pi\alpha^2}{3\hat{s}}$$

N_c → 3 colours

We then have

$$d^2\sigma = Q_u^2 \frac{4\pi\alpha^2}{9\hat{s}} u^p(x_1)\,dx_1\, \bar{u}^{\bar{p}}(x_2)\,dx_2$$

$$\to 4/9 \cdot \frac{4\pi\alpha^2}{9\hat{s}} u(x_1)u(x_2)\,dx_1\,dx_2$$

HADRON-HADRON COLLISIONS

we have:

$$\hat{s} = (x_1 p_1 + x_2 p_2)^2 = x_1^2 p_1^2 + x_2^2 p_2^2 + 2 x_1 x_2 p_1 p_2$$

if we neglect proton mass: $\hat{s} \simeq x_1 x_2 (2 p_1 \cdot p_2) = x_1 x_2 s$

$$\rightarrow d^2\sigma = \frac{4}{9} \frac{4\pi\alpha^2}{9 x_1 x_2 s} u(x_1) u(x_2) \, dx_1 \, dx_2$$

accounting for sea quarks

$$\rightarrow \frac{4\pi\alpha^2}{9 x_1 x_2 s} \left[\frac{4}{9} \left(u(x_1) u(x_2) + \bar{u}(x_1) \bar{u}(x_2) \right) + \frac{1}{9} \left(d(x_1) d(x_2) + \bar{d}(x_1) \bar{d}(x_2) \right) dx_1 \, dx_2 \right.$$

Inv. mass of $\mu^+\mu^-$ system: $M^2 = x_1 x_2 s$

Rapidity of the system:

$$y = \frac{1}{2} \ln \left(\frac{E_3 + E_4 + p_{3z} + p_{4z}}{E_3 + E_4 - p_{3z} - p_{4z}} \right) = \frac{1}{2} \ln \left(\frac{E_q + E_{\bar{q}} + p_{qz} + p_{\bar{q}z}}{E_q + E_{\bar{q}} - p_{qz} - p_{\bar{q}z}} \right)$$

Hadron - Hadron Collisions

$$p_q = \frac{\sqrt{s}}{2}(x_1, 0, 0, x_1) \quad , \quad p_{\bar{q}} = \frac{\sqrt{s}}{2}(x_2, 0, 0, -x_2)$$

$$\Rightarrow y = \frac{1}{2}\ln\frac{x_1}{x_2}$$

We can also write: $x_1 = \frac{M}{\sqrt{s}}e^{y}$, $x_2 = \frac{M}{\sqrt{s}}e^{-y}$

$$dy\, dM = \frac{s}{2M}\, dx_1\, dx_2$$

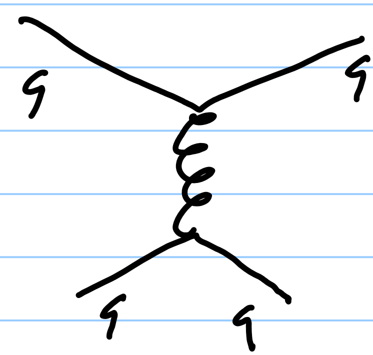
$$\Rightarrow d^2\sigma = \frac{4\pi\alpha^2}{9M^2} F(x_1, x_2)\frac{2M}{s}\, dy\, dM$$

with $F(x_1, x_2)$ given by expression on last page.

$$\frac{d^2\sigma}{dy\, dM} = \frac{8\pi\alpha^2}{9Ms} F(x_1, x_2)$$

MORE JETS

FROM



$qq \to qq$

Dijets will be back-to-back in (x,y) plane but not (in general) along beam axis

$\to (x_1 - x_2)\sqrt{s}/2$

XS: $$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha_s^2}{9Q^4}\left[1 + \left(1 - \frac{Q^2}{\hat{s}}\right)^2\right]$$

with $Q^2 = -q^2$, $\hat{s} = x_1 x_2 s$

$$\to \frac{d\sigma}{dQ^2} = \frac{4\pi\alpha_s^2}{9Q^4}\left[1 + \left(1 - \frac{Q^2}{s x_1 x_2}\right)^2\right] g(x_1, x_2)\, dx_1\, dx_2$$

MORE JETS

$$g(x_1, x_2) = \left[u(x_1)\, u(x_2) + u(x_1)\, d(x_2) + d(x_1)\, u(x_2) + d(x_1)\, d(x_2) \right]$$

$$\frac{d^3\sigma}{dQ^2\, dx_1\, dx_2} = \frac{4\pi\alpha_s^2}{9Q^4} \left[1 + \left(1 - \frac{Q^2}{s\, x_1 x_2} \right)^2 \right] g(x_1, x_2)$$

→ problem 10.6 in Thomson