



LECTURE 23: CP Violation (Part II)

Overview:

-Meson mixing (cont.)

-K meson system

-B meson system

(I used mostly Burgess, and Cheng Li as references, and notes from Frank Wuerthwein)

Meson Mixing (continued from lecture 22)

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We define the states $|q\rangle$ and $|\bar{q}\rangle$ (destroyed by the fields $q(x)$ and $\bar{q}(x)$) and define $|q_{\pm}\rangle$ (destroyed by $\psi_{\pm}(x)$ and $\bar{\psi}_{\pm}(x)$). The normalized states are:

$$|q_{+}\rangle = \frac{1}{\sqrt{|p|^2 + |q|^2}} [p|q\rangle + q|\bar{q}\rangle]$$

$$|q_{-}\rangle = \frac{i}{\sqrt{|p|^2 + |q|^2}} [p|q\rangle - q|\bar{q}\rangle]$$

$$\frac{p}{q} = z^2 = \sqrt{\frac{C}{B}}$$

If CP is a symmetry : $C = B \Rightarrow z = 1$

$$\Rightarrow q^{+} = \frac{q + \bar{q}}{\sqrt{2}}, \quad q^{-} = i \frac{(\bar{q} - q)}{\sqrt{2}}$$

$$m_{\pm}^2 \approx m_{\frac{1}{2}}^2 - im \Gamma_{\pm} = A \pm B$$

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Meson Mixing (cont.)

eigenstates: $|\psi_+\rangle = \frac{1}{\sqrt{1+|\hat{\xi}|^2}} [|\varphi_+\rangle - i\hat{\xi}|\varphi_-\rangle]$

$$|\psi_-\rangle = \frac{1}{\sqrt{1+|\hat{\xi}|^2}} [|\varphi_-\rangle + i\hat{\xi}|\varphi_+\rangle]$$

with $\hat{\xi} = \frac{(z^2-1)}{1+z^2} = \frac{p-q}{p+q}$

If we start with state $|\varphi\rangle$:

$$\langle \varphi | \varphi \rangle_T = \frac{1}{2} [e^{-iE_+T} + e^{-iE_-T}], \quad \langle \bar{\varphi} | \varphi \rangle = \frac{q}{2p} [e^{-iE_-T} - e^{-iE_+T}]$$

$$P_T [\varphi(k) \rightarrow \varphi(k)] = \frac{1}{4} \left[e^{-\Gamma_+(k)T} + e^{-\Gamma_-(k)T} + 2e^{-\frac{[\Gamma_+(k) + \Gamma_-(k)]T}{2}} \cos \Omega_k T \right]$$

$$P_T [\varphi(k) \rightarrow \bar{\varphi}(k)] = \frac{1}{4} \left| \frac{q}{p} \right|^2 \left[e^{-\Gamma_+(k)T} + e^{-\Gamma_-(k)T} - 2e^{-\frac{[\Gamma_+(k) + \Gamma_-(k)]T}{2}} \cos \Omega_k T \right]$$

$$\Omega_k = \sqrt{k^2 + m_+^2} - \sqrt{k^2 + m_-^2} \approx \begin{array}{ll} m_+ - m_- & \text{if } k \ll m \\ \frac{m_+^2 - m_-^2}{2k} & \text{if } k \gg m \end{array}$$

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$$\rightarrow K \left[\sqrt{1 + \frac{M_+^2}{K^2}} - \sqrt{1 + \frac{M_-^2}{K^2}} \right]$$

$$\text{Taylor } (1+x)^{1/2} = 1 + \frac{x}{2}$$

$$\Rightarrow K \left[1 + \frac{M_+^2}{2K^2} - 1 - \frac{M_-^2}{2K^2} \right] = \frac{M_+^2 - M_-^2}{2K}$$

$K^0 - \bar{K}$ mixing (revisited)

With K_{aons} we will have $|K_S\rangle = |\varphi_+\rangle$

$$|K_L\rangle = |\varphi_-\rangle$$

As we saw, To a good approx. (since $|\hat{\epsilon}| \ll 1$),

$$\Rightarrow |K_L\rangle \approx |K_-\rangle = |\varphi_-\rangle$$

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$K^0 - \bar{K}^0$ mixing (cont.)

For kaons, we have:

$$m_K = 498 \text{ MeV} \quad (3m_\pi = 420 \text{ MeV})$$

$$\Gamma_S^{-1} = 9 \times 10^{-11} \text{ sec}, \quad \Gamma_L^{-1} = 5 \times 10^{-8} \Rightarrow \Gamma_S = 580 \Gamma_L$$

With $\Gamma_+ \gg \Gamma_-$ and assuming slow-moving kaons:

$$P_+ [K(K) \rightarrow K(K)] \approx \frac{1}{4} e^{-\Gamma_L(K)T} \left[1 + 2 e^{-[\Gamma_S(K) - \Gamma_L(K)]T/2} \cos(\Delta m t) \right]$$

$$P_+ [K(K) \rightarrow \bar{K}(K)] \approx \frac{1}{4} e^{-\Gamma_L(K)T} \left| \frac{q}{p} \right|^2 \left[1 - 2 e^{-[\Gamma_S(K) - \Gamma_L(K)]T/2} \cos(\Delta m t) \right]$$

$$\Delta m = 3.5 \times 10^{-6} \text{ eV} \quad (m_{K_L} - m_{K_S}) > 0$$

Frequency comparable to K_S lifetime (how would you measure K^0 vs \bar{K}^0 ?)

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CP violation in Kaon decays

- CP violation can occur through mixing:

$$|K_{-}\rangle = \frac{1}{\sqrt{1+|\xi|^2}} [|K_{-}^0\rangle + i\xi |K_{+}^0\rangle]$$

With \mathcal{L}_w denoting weak interaction Lagrangian ($\Delta S = \pm 1$) we write $\langle \pi\pi | \mathcal{L}_w | K_{\pm} \rangle$

- CP violation can also occur at the decay:

$\langle \pi\pi | \mathcal{L}_w | K_{-} \rangle$ i.e. \mathcal{L}_w itself breaks CP (known as "direct" CP violation).

To determine relative size of these contributions we can use the observables:

$$\eta_{00} = \frac{\langle \pi^0 \pi^0 | \mathcal{L}_w | K_L \rangle}{\langle \pi^0 \pi^0 | \mathcal{L}_w | K_S \rangle}$$

$$\eta_{+-} = \frac{\langle \pi^+ \pi^- | \mathcal{L}_w | K_L \rangle}{\langle \pi^+ \pi^- | \mathcal{L}_w | K_S \rangle}$$

CP violation in Kaon decays

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If mixing is sole source of CP violation: $\gamma_{00} = \gamma_{+-}$

Note that the Kaons decaying to two pions have either isospin 0 or 2. Amplitudes are:

$$\langle \pi^+ \pi^- | \mathcal{L}_w | K^0 \rangle = A_0 e^{i\xi_0} + A_2 e^{i\xi_2}$$

$$\langle \pi^+ \pi^- | \mathcal{L}_w | \bar{K}^0 \rangle = A_0 e^{-i\xi_0} + A_2 e^{-i\frac{\xi_2}{2}}$$

A_0, A_2 are CP-conserving strong interaction matrix elements for pion isospin channel, ξ_0, ξ_2 are the CP-violating phases due to \mathcal{L}_w (assuming CP-violating \mathcal{L}_w)

→ Physical decay rates prop. to $|A_0 e^{i\xi_0} + A_2 e^{i\xi_2}|^2$

⇒ relative phase is relevant

CP violation in K_{long} decays (cont.)

with $\epsilon = -\hat{\epsilon} + \xi_0$

$$\epsilon' = \left(\frac{A_2}{A_0 + A_2} \right) (\epsilon_2 - \epsilon_0)$$

we get : $\eta_{+-} = \epsilon + \epsilon'$, $\eta_{00} = \epsilon - 2\epsilon'$

Experiments (KTeV, NA48) have confirmed that

$$\frac{\epsilon'}{\epsilon} \neq 0 \quad , \quad \text{Re} \left| \frac{\epsilon'}{\epsilon} \right| = 1.7 \times 10^{-3}$$

$\rightarrow \epsilon'$ small relative to ϵ

Measurements with B mesons also have observed direct CP violation

⑨

B- \bar{B} Mixing

Similar To $K-\bar{K}$ mixing but b quark mass $\gg s$ quark mass and well above QCD scale:

- theoretical uncertainties are reduced.
- much larger phase space eliminates (essentially) the lifetime difference. Simplifies expressions ...
- B decays are more CKM suppressed

With $\Gamma_- \approx \Gamma_+$, we'll denote the two states

by "H" for heavy, and "L" for light. Previous

oscillation probabilities become (non-relativistic B's):

$$P_T [B^0 \rightarrow B^0] = e^{-\Gamma_T} \cos^2 \left(\frac{\Delta m T}{2} \right)$$

$$P_T [B^0 \rightarrow \bar{B}^0] = \left| \frac{q}{p} \right|^2 e^{-\Gamma_T} \sin^2 \left(\frac{\Delta m T}{2} \right)$$

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B- \bar{B} Mixing (cont.)

In our example (non-rel, e.g. CLEO), it is difficult to measure time t elapsed since the B was in a pure B^0 eigenstate. We start from:

$$e^+e^- \rightarrow \gamma^* \rightarrow B\bar{B}$$

→ relative angular momentum $l=1$

$$CP|B\bar{B}\rangle = -|B\bar{B}\rangle$$

$$\text{initial state } |B\bar{B}\rangle = \frac{1}{\sqrt{2}} [|B(k)\bar{B}(-k)\rangle - |B(-k)\bar{B}(k)\rangle]$$

We can then "tag" the flavour of the B using semi-leptonic decay of one B (there are other ways to tag).

By reconstructing decay vertex, we can determine prob. of observing (for instance) same-sign leptons vs distance i.e. versus Time
 → asymmetric B factories make this easier

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B- \bar{B} Mixing (in more detail)

$$\begin{aligned}
 |\langle F | H | B^0 \rangle|^2 &= |\langle F | H | B^0(t) \rangle|^2 \\
 &= \frac{1}{4|p|^2} |\langle F | B_L(t) \rangle + \langle F | B_H(t) \rangle|^2 \\
 &= \frac{1}{4|p|^2} |pA e^{(-i\mu_L - \Gamma_L/2)t} + pA e^{(-i\mu_H - \Gamma_H/2)t}|^2 \\
 &= \frac{1}{4} |A|^2 (e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2 e^{-(\Gamma_H + \Gamma_L)t/2} \cos \Delta m t)
 \end{aligned}$$

$$\begin{aligned}
 |\langle \bar{F} | H | B^0 \rangle|^2 &= |\langle \bar{F} | B^0(t) \rangle|^2 \\
 &= \frac{1}{4|p|^2} |\langle \bar{F} | B_L(t) \rangle + \langle \bar{F} | B_H(t) \rangle|^2 \\
 &= \frac{1}{4|p|^2} |q\bar{A} e^{(-i\mu_L - \Gamma_L/2)t} - q\bar{A} e^{(-i\mu_H - \Gamma_H/2)t}|^2 \\
 &= \frac{1}{4} \left| \frac{p}{q} \right|^2 |\bar{A}|^2 (e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2 e^{-(\Gamma_H + \Gamma_L)t/2} \cos \Delta m t)
 \end{aligned}$$

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B- \bar{B} Mixing (in more detail, cont.)

Remember we have:

$$|B_H\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$$

$$|B_L\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$$

$$\rightarrow \langle F|B^0\rangle = A, \quad \langle \bar{F}|B^0\rangle = 0$$

$$\langle \bar{F}|\bar{B}^0\rangle = \bar{A}, \quad \langle F|\bar{B}^0\rangle = 0$$

$$B_H \rightarrow F \quad \text{provides} \quad pA$$

$$B_L \rightarrow \bar{F} \quad \text{provides} \quad -q\bar{A}$$

\rightarrow if $A \neq \bar{A}$, CP violation in decay

\rightarrow if $|\frac{q}{p}| \neq 1$, CP violation in mixing

Without CP violation we would have

$$|\langle F|H|B^0\rangle|^2 + |\langle \bar{F}|H|B^0\rangle|^2 = \frac{1}{2}|A|^2 (e^{-\Gamma_L t} + e^{-\Gamma_H t})$$

CKM Matrix

In SM, Flavour changing and CP-violating physics is due to 4 parameters in unitary CKM matrix.

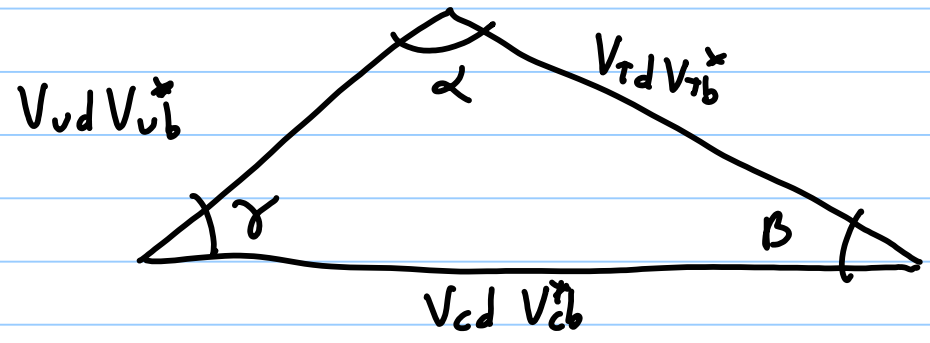
Physics beyond SM could provide new contributions. B physics provides many opportunities of testing SM.

Unitarity conditions $\rightarrow \sum_i V_{in} V_{im}^* = \delta_{mn}$

With B_d mesons, we have $m=b, n=d$, which implies:

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

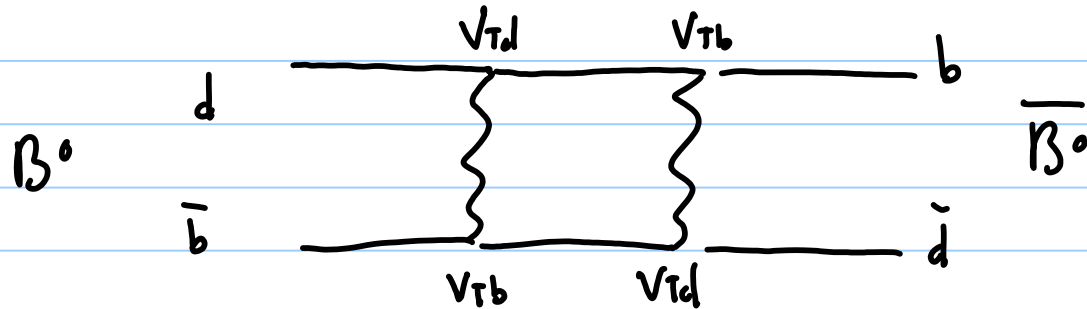
- \rightarrow sum of 3 complex numbers vanish.
- \rightarrow can be expressed 3 vectors in complex plane



$$\alpha = \arg \left(- \frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right)$$

$$\beta = \arg \left(\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right)$$

CKM Matrix and B physics



$$\beta = \arg \left(\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} \right)$$

$B^0 \rightarrow 2K_S : V_{td} V_{tb}$ From mixing

V_{cb} from $b \rightarrow c$

V_{cd} from K^0 mixing

In asymmetric B factories:

- reconstruct one B in CP eigenstate e.g. $2K_S$
- reconstruct decay of other B \rightarrow determine Flavour
- Measure distance between mesons and convert to proper time

$$\epsilon = \left(\frac{M_{12}^* - i/2 \Gamma_{12}^*}{M_{12} - i/2 \Gamma_{12}} \right)^{1/2} \approx \frac{M_{12}^*}{|M_{12}|}$$

$$\rightarrow M_{12}^* \propto (V_{Td} V_{Tb}^*)^2, \quad \epsilon = \frac{M_{12}^*}{|M_{12}|} = \frac{(V_{Td} V_{Tb}^*)^2}{|(V_{Td} V_{Tb}^*)^2|} = \frac{V_{Td}^2}{|V_{Td}^2|}$$

$$V_{Td} = |V_{Td}| e^{-i\beta}$$

$$|B_L\rangle = \frac{1}{\sqrt{2}} (|B^0\rangle + e^{-i2\beta} |\bar{B}^0\rangle), \quad |B_H\rangle = \dots - \dots$$

$$\Delta m_d = 2|M_{12}| \propto |(V_{Td} V_{Tb}^*)^2| \rightarrow \Delta m_d \propto |V_{Td}|^2$$

$$\text{Now: } P(\bar{B}_{T=0}^0 \rightarrow \bar{B}^0) = e^{-\Gamma T} \cos^2\left(\frac{\Delta m_d T}{2}\right)$$

$$P(B_{T=0}^0 \rightarrow B^0) = \left|\frac{1}{\epsilon}\right|^2 e^{-\Gamma T} \sin^2\left(\frac{\Delta m_d T}{2}\right)$$

For this system, very few decays are common, interference between decays is small

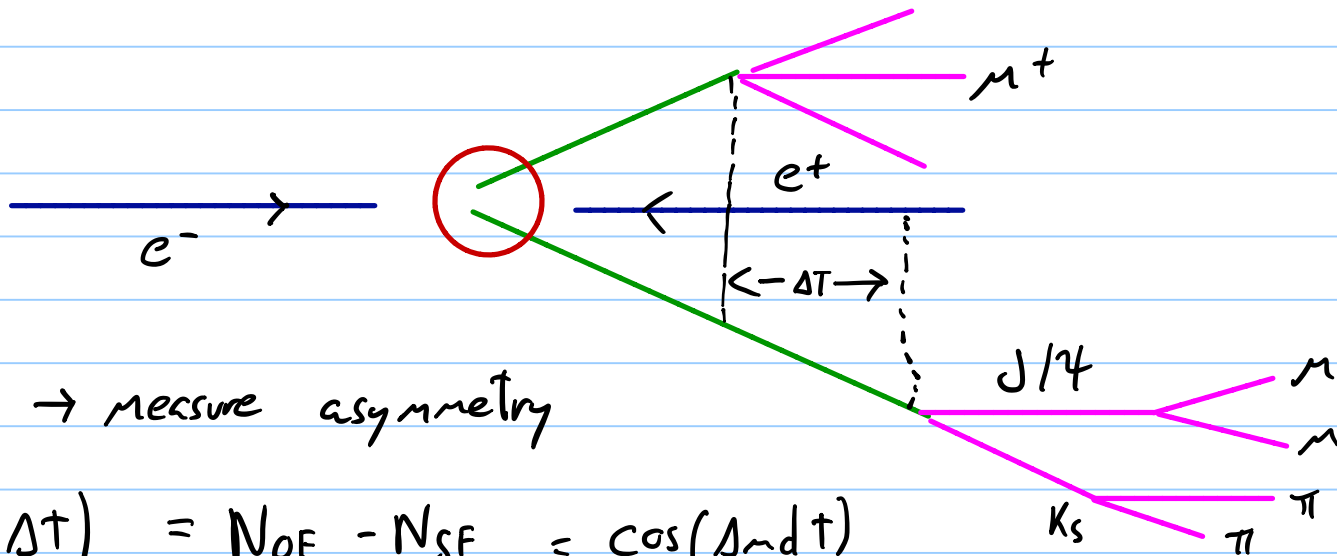
→ Hard To observe CP violation in mixing

What to do?

CP violation can be observed:

- directly: $\Gamma(A \rightarrow X) \neq \Gamma(\bar{A} \rightarrow \bar{X})$
- through mixing, as we saw for Kaon system
- CP violation in interference between decays

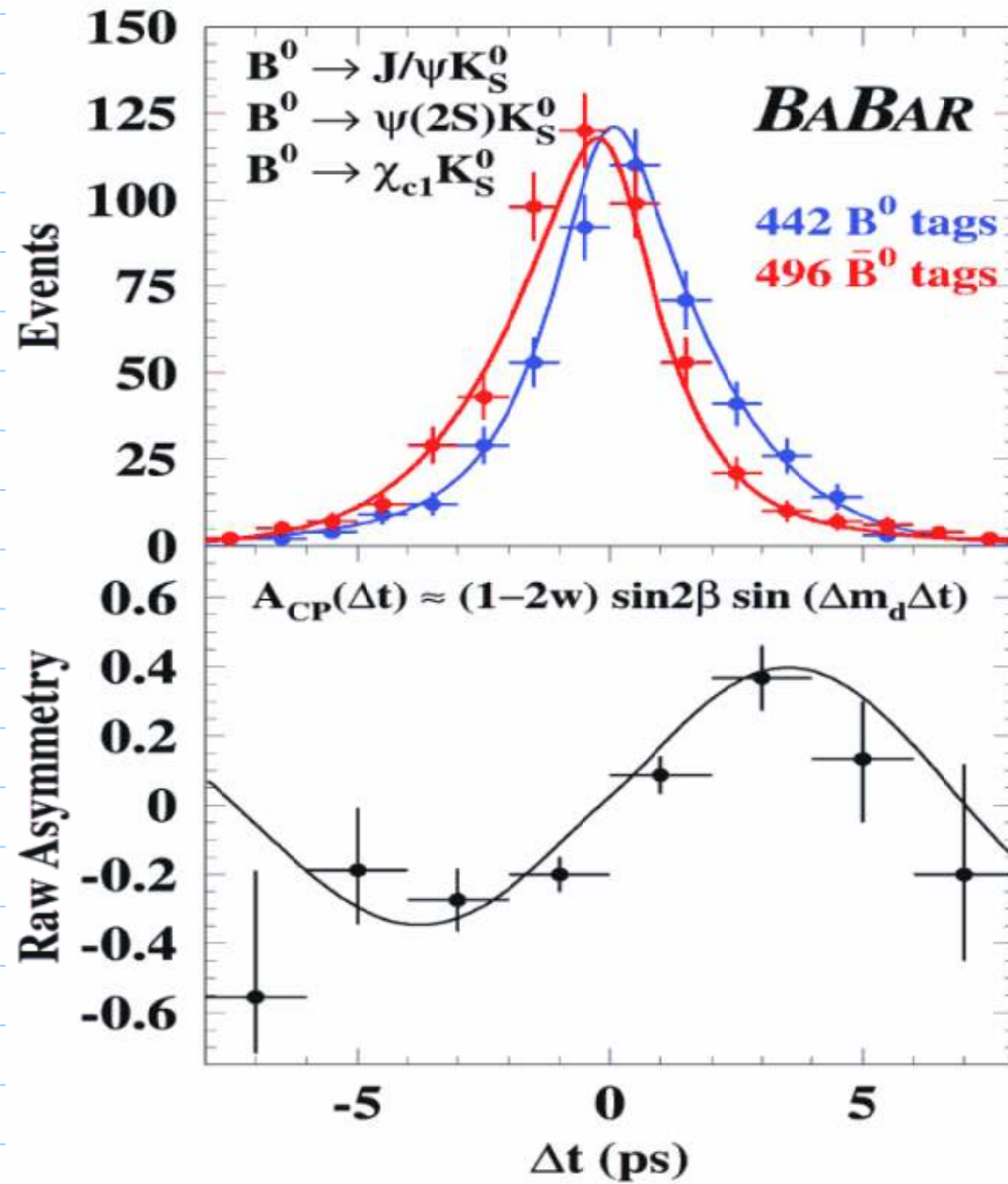
$$B^0 \rightarrow F, \quad B^0 \rightarrow \bar{B}^0 \rightarrow F$$



$\Delta m_d \rightarrow$ measure asymmetry

$$A(\Delta T) = \frac{N_{OF} - N_{SF}}{N_{OF} + N_{SF}} = \cos(\Delta m_d t)$$

$$A_{CP}^{K_S} = \frac{\Gamma(\bar{B}_{T=0}^0 \rightarrow J/\psi K_S) - \Gamma(B_{T=0}^0 \rightarrow J/\psi K_S)}{\Gamma(\bar{B}_{T=0}^0 \rightarrow J/\psi K_S) + \Gamma(B_{T=0}^0 \rightarrow J/\psi K_S)} = \sin 2\beta \sin \Delta m_d T$$



Other sources of CP violation

CP violation due to CKM matrix is too small to explain observed matter anti-matter asymmetry in the Universe.

Are there other potential sources of CP violation in SM?

- Yes:
- lepton sector (next lecture)
 - Strong CP violation (very small if it exists)

Physics beyond SM could also contribute

Strong CP Problem

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Most general renorm. Lagrangian involving SM fields includes:

$$\mathcal{L}_{FS} = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} - \frac{1}{4} W_{\mu\nu}^i W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$- \frac{g_s^2 \Theta_3}{64\pi^2} \epsilon_{\mu\nu\lambda\rho} G^{a\mu\nu} G^{a\lambda\rho} - \text{similar terms for } W, B$$

→ this term is effectively a total derivative and would be expected to have no physical implications

$$\propto \int d^4x K_\mu$$

Term determined by change in charge $\int d^3x K^0 \equiv N_{CS}$

Change in charge need not be zero in a vacuum
To vacuum process → QCD vacuum topologically non-trivial

Axions are a potential solution (need to add scalar field)

