

## LECTURE 6: Spontaneous Symmetry Breaking (Part II)

### Overview:

- Recap of Abelian case
- Ginzburg-Laudau
- Higgs Mechanism (non-Abelian case)

(This lecture mostly follows Quigg Chapters 4-5)

## Higgs Mechanism (Abelian case recap)

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We saw that spontaneous breaking of a continuous symmetry leads to massless bosons (Goldstone bosons). We expect one massless boson per broken generator.

We saw however that in the case of a local gauge theory, the massless

gauge boson and the massless Goldstone boson conspire to give us a massive gauge boson without the massless Goldstone boson.

In the case we studied, we had before symmetry breaking:

2 scalars: 2 degrees of freedom

1 massless vector boson: 2 degrees of freedom

Total = 4

After breaking we had (explicit in unitary gauge):

1 massive vector boson: 3 degrees of freedom

1 massive Higgs scalar: 1 degree of freedom      Total = 4

# Ginzburg Landau Superconductivity (3)

4: macroscopic wave function describing condensate  
Free energy of superconductor can be written as:

$$G_{\text{super}}[\psi] = G_{\text{normal}}[\psi] + \alpha |\psi|^2 + \beta |\psi|^4$$

$$G_{\text{super}}(\mathbf{B}) = G_{\text{super}}(0) + \frac{\mathbf{B}^2}{2} + \frac{1}{2\pi^*} \nabla^2 (-i\nabla \cdot \mathbf{e}^* A)^2$$

in weak field approx. , field equations  
derived using  $G_{\text{super}}(\mathbf{B})$  lead to massive photon

Meissner Effect:

- Cooper pairs form BEC condensate below  $T_c \sim 10^0 - 10^2$  K. Condensate disturbed by EM field
- Short range force, attenuation length  $\sim 10^{-6}$  cm
- equivalent to photon acquiring a mass

Electroweak symmetry breaking:

- Higgs condenses below  $T_c \sim 10^{15}$  K. Condensate disturbed by gauge bosons
- Short range force, attenuation length  $\sim 10^{-18}$  cm
- W/Z bosons acquire mass



# Higgs Mechanism (non-Abelian case) (4)

We will study an  $SU(2)$  doublet of complex scalar fields:

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

The Lagrangian is:  $(\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$   
The covariant derivative:  $D_\mu = \partial_\mu + ig \frac{\tau_a}{2} B_\mu^a$

Under infinitesimal Transformation:  $\phi(x)' = (1 + i \frac{\alpha(x)}{2} \cdot \gamma) \phi$

$$B_\mu' = B_\mu - \frac{1}{g} \partial_\mu \alpha - \alpha \times B_\mu$$

We obtain

$$\mathcal{L} = (\partial_\mu \phi + ig \frac{\tau}{2} \cdot B_\mu \phi)^\dagger (\partial^\mu \phi + ig \frac{\tau}{2} \cdot B_\mu \phi) - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$V(\phi) = \mu^2 (\phi^\dagger \phi) + \lambda (\phi^\dagger \phi)^2$$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu - g B_\mu \times B_\nu$$

# Higgs Mechanism (non-Abelian case) (5)

minimum of potential at  $\Phi^\dagger \Phi = \frac{1}{2} (\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = -\frac{\mu^2}{2\lambda}$

we chose minimum around which To do our expansion:  $\phi_3^2 = -\frac{\mu^2}{\lambda} \equiv v^2$   $\phi_1 = \phi_2 = \phi_4 = 0$

We parametrize fluctuations from the vacuum  $\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$  in terms of 4 real scalar fields  $\xi_1, \xi_2, \xi_3, \eta$

$$Q(x) = e^{i\gamma \cdot \xi(x)/v} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \eta \end{pmatrix} \approx \frac{1}{\sqrt{2}} \begin{pmatrix} 1 + i\xi_3/v & i(\xi_1 - \xi_2)/v \\ i(\xi_1 + \xi_2)/v & 1 - i\xi_3/v \end{pmatrix} \begin{pmatrix} 0 \\ v + \eta \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} \xi_2 + i\xi_1 \\ v + \eta - i\xi_3 \end{pmatrix}, \quad \text{so } \Phi^\dagger \Phi =$$

$$\frac{1}{2} (\xi_1^2 - i\xi_1, v + \eta + i\xi_3) \frac{1}{\sqrt{2}} \begin{pmatrix} \xi_2 + i\xi_1 \\ v + \eta - i\xi_3 \end{pmatrix} = \xi_1^2 + \xi_2^2 + \xi_3^2 + v^2 + \eta^2 + 2v\eta$$

We know all terms from  $v$  will cancel save  $\mu^2/\eta$   
 $\rightarrow$  for small oscillations massive scalar

# Higgs Mechanism (non-Abelian case) (6)

Let's move to the unitary gauge right away:

$$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \eta \end{pmatrix}$$

$$D_\mu \varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 2m \begin{pmatrix} 0 \\ \eta \end{pmatrix} + \frac{ig}{2} \begin{pmatrix} b_\mu^3 & \sqrt{2} b_\mu^- \\ \sqrt{2} b_\mu^+ & -b_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ v + \eta \end{pmatrix} \end{pmatrix}$$

where  $b_\mu^\pm = \frac{1}{\sqrt{2}} (b_\mu^1 \pm i b_\mu^2)$

$$|D_\mu \varphi|^2 = \frac{1}{2} \lambda m \eta^2 \lambda^2 \eta + \frac{1}{4} g^2 v^2 (b_\mu^+ b_\mu^- + \frac{1}{2} b_\mu^3 b_\mu^3) + \frac{1}{4} g^2 \eta^2 (b_\mu^+ b_\mu^- + \frac{1}{2} b_\mu^3 b_\mu^3) + \frac{1}{2} g^2 v \eta (b_\mu^+ b_\mu^- + \frac{1}{2} b_\mu^3 b_\mu^3)$$

→ 3 bosons with mass  $\frac{gv}{2}$

# Higgs Mechanism (non-Abelian case) (7)

Summary:

we started with: 4 scalars : 4 dof  
3 massless bosons:  $3 \times 2 = 6$  dof

$$\boxed{\text{Total} = 10 \text{ dof}}$$

we end up with: 1 scalar (massive) : 1 dof  
3 massive bosons:  $3 \times 3 = 9$  dof

$$\boxed{\text{Total} = 10 \text{ dof}}$$

In the Standard Model we have 3 massive vector bosons (2 charged, one neutral) and one massless boson (neutral).

→ see problem set #1

## Problem set 1 (8)

### Problem 1

Due Date: Monday 20th of October  
at 13:00

Analyse the spontaneous breaking of a global SU(2) symmetry for the case of 3 real scalar fields in an SU(2) Triplet:

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi) \cdot (\partial^\mu \phi) - V(\phi)$$

$$V(\phi) = \frac{1}{2} \mu^2 \phi \cdot \phi + \frac{1}{4} \lambda (\phi \cdot \phi)^2$$

### Problem 2

Analyse the spontaneous breaking of a local SU(2) symmetry for the case of 3 real scalar fields in an SU(2) Triplet:

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

→ use  $\mathcal{L}$  and  $V$  from page 4



## Problem set # 1 (cont)

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Problem 3:

We now turn to the following SU(2) x U(1) Lagrangian:

$$\mathcal{L} = \left| (i D_\mu - g \frac{\tau}{2} \cdot W_\mu - \frac{g'}{2} B_\mu) \phi \right|^2 - V(\phi)$$

We use the following doublet of complex scalar fields

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \phi^\dagger = (\phi_1 + i\phi_2)/\sqrt{2}$$

$$\phi^0 = (\phi_3 + i\phi_4)/\sqrt{2}$$

Obtain the mass of the vector bosons using the relevant Term:  $\left| \left( -i g \frac{\tau}{2} \cdot W_\mu - i g' \frac{B_\mu}{2} \right) \phi \right|^2$

you will get an off-diagonal Term for  $W_\mu^3$  and  $B_\mu$

Express your result in Terms of physical fields that diagonalize the mass matrix.

you can use  $\frac{g'}{g} = \tan \theta_w$

