LECTURE 8: Weak Interactions (Part Deux)

Overview:

-Neutrino scattering (cont.)

-W decay

(This lecture mostly follows Griffiths Chapter 10
and Quigg Chapter 6)

Neutrino scattering (cont.)

We saw that the cross section for inverse muon decay was given by:

$$\sigma_{\nu e \to \mu\nu} = \frac{G_F^2 s}{\pi}$$

This process will violate unitarity at ~ 300 GeV

→ note that this is in the context of Fermi's theory with 4-particle coupling

→ we used $q \ll M_W$

What do we get if we use the full propagator?

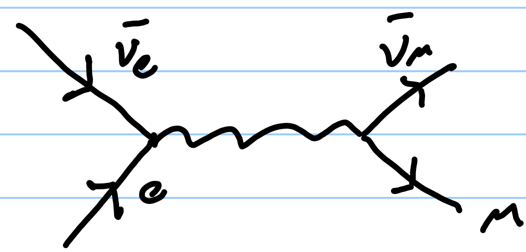
$$\sigma(\nu_\mu e \to \mu \nu_e) \simeq \frac{2G_F^2}{\pi} m_e E \frac{[1-(m_\mu^2 - m_e^2)/2mE]^2}{(1+2mE/m_W^2)}$$

at high energies: $\sigma \sim \frac{G_F^2 M_W^2}{\pi}$

(Still violates unitarity but at very, very high energies)

Neutrino scattering (cont.)

For the s-channel process $\bar{\nu}_e e \to \bar{\nu}_\mu \mu$:



$$\sigma = \frac{2mE\, G_F^2 \left[1 - (m_\mu^2 - m_e^2)/2mE\right]^2}{3\pi\,(1 - 2mE/M_W^2)^2}$$

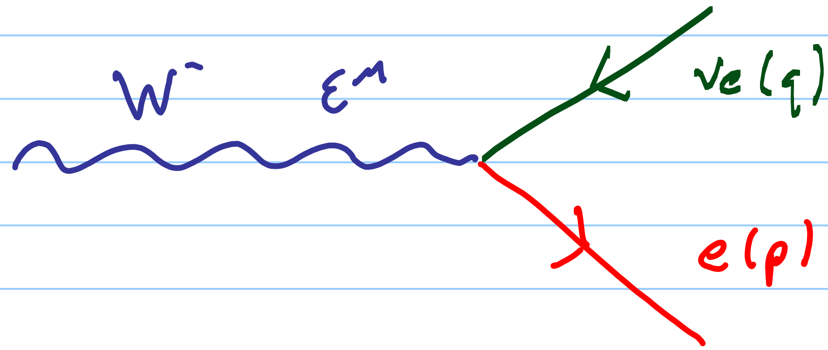
For very high energies $\sigma \approx \dfrac{G_F^2 M_W^4}{5\pi m E}$

→ Tends towards 0 as $E \to \infty$

→ adding the boson propagator fixed unitarity problems*

→ $(1 - 2mE/M_W^2)$ will make $\sigma \to \infty$ if $2mE = M_W^2$
we need to include the W width

W decay



→ $\bar{\nu}_e$ going forward, ν_e going backward

$$\mathcal{M} = -i\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{1/2} \bar{u}(p)\,\gamma_\mu\,(1-\gamma_5)\,v(q)\,\varepsilon^\mu$$

$\varepsilon^\mu \equiv (0, \hat{\varepsilon})$ is the polarization vector of the W

→ we neglect the electron mass

$$|\mathcal{M}|^2 = \frac{G_F M_W^2}{\sqrt{2}} \,\mathrm{Tr}\left[\not{\varepsilon}\,(1-\gamma_5)\,\not{q}\,(1+\gamma_5)\,\not{\varepsilon}^*\,\not{p}\right]$$

$$= \frac{G_F M_W^2}{\sqrt{2}} \; 2\,\mathrm{Tr}\left[(1+\gamma_5)\,\not{\varepsilon}\,\not{q}\,\not{\varepsilon}^*\,\not{p}\right]$$

W decay

$$|M|^2 = \frac{8 G_F M_W^2}{\sqrt{2}} \Big((\varepsilon \cdot q)(\varepsilon^* \cdot p) - (\varepsilon \cdot \varepsilon^*)(p \cdot q) + (\varepsilon \cdot p)(\varepsilon^* \cdot q)$$
$$+ i \varepsilon_{\mu\nu\rho\sigma} \varepsilon^\mu q^\nu \varepsilon^{*\rho} p^\sigma \Big]$$

let's pick the longitudinal polarization for the W : $\varepsilon^\mu = (0; 0, 0, 1) = \varepsilon^{\mu *}$ (helicity 0)

$\rightarrow$ the $\varepsilon_{\mu\nu\rho\sigma}$ term vanishes

$$p = \frac{M_W}{2}(1, \sin\theta, 0, \cos\theta)$$
$$q = \frac{M_W}{2}(1, -\sin\theta, 0, -\cos\theta)$$

$$|M|^2 = \frac{8 G_F M_W^2}{\sqrt{2}} \cdot \frac{M_W^2}{4} \left(-\cos^2\theta - -1 \cdot [1 + \sin^2\theta + \cos^2\theta] - \cos^2\theta \right.$$
$$= \frac{4 G_F M_W^4}{\sqrt{2}} \sin^2\theta$$

W Decay

$$\frac{d\Gamma}{d\Omega} = \frac{|M|^2}{64\pi^2 M_W} = \frac{G_F M_W^3}{16\pi^2\sqrt{2}} \sin^2\theta$$

$$d\Gamma = \cdot \frac{G_F M_W^3}{16\pi^2\sqrt{2}} \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta \, \sin^2\theta \, d\theta$$

$$\text{int} = -\cos\theta + \frac{\cos^3\theta}{3} \Big|_0^{\pi} = 1 - \frac{1}{3} - (-1 + 1/3)$$

$$= \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$$

$$\boxed{\Gamma = \frac{G_F M_W^3}{16\pi^2\sqrt{2}} \cdot 2\pi \cdot \frac{4}{3} = \frac{G_F M_W^3}{6\pi\sqrt{2}}}$$

$= 227$ MeV (for $M_W = 80.4$)

$G_F = 1.166 \times 10^{-5}$ GeV^{-2}

Total width: 2.06 GeV

W decay

You can check that you get the same result for helicity +1:

$$\varepsilon_\mu = (0, -1, -i, 0)/\sqrt{2}$$

→ note that $\frac{d\Gamma}{d\Omega} = \frac{G_F M_W^3}{32\pi^2\sqrt{2}}(1-\cos\theta)^2$

helicity −1 will give $\frac{G_F M_W^3}{32\pi^2\sqrt{2}}(1+\cos\theta)^2$