Some of the following slides are taken from the website of the textbook Modern Particle Physics by Mark Thomson (the main suggested textbook for this course)

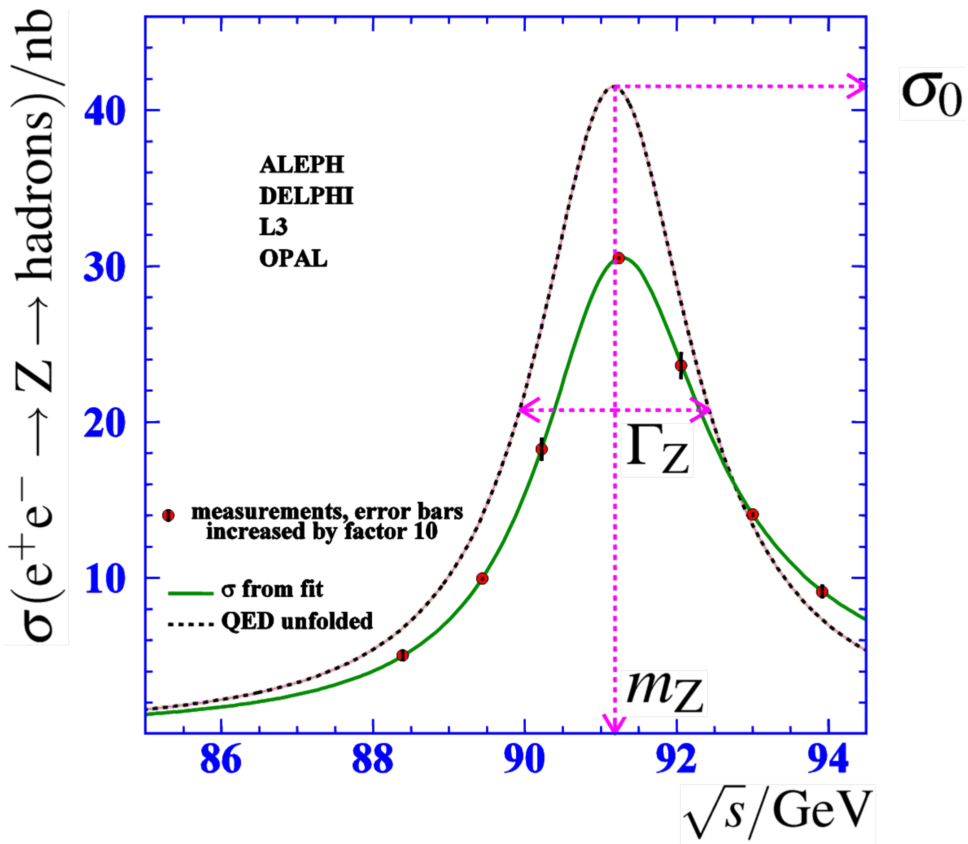
The full set of slides is available online here:

http://www.hep.phy.cam.ac.uk/~thomson/MPP/ModernParticlePhysics.html

The book (if you don't already have it) and slides are a good reference

Particle Physics

Michaelmas Term 2010
Prof Mark Thomson

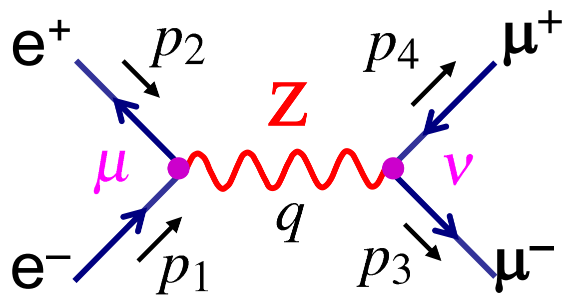


Handout 14 : Precision Tests of the Standard Model

The Z Resonance

★ **Want to calculate the cross-section for** $e^+e^- \to Z \to \mu^+\mu^-$

- **Feynman rules for the diagram below give:**



e^+e^- vertex: $\overline{v}(p_2)\cdot -ig_Z\gamma^\mu \frac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)$

Z propagator: $\dfrac{-ig_{\mu\nu}}{q^2 - m_Z^2}$

$\mu^+\mu^-$ vertex: $\overline{u}(p_3)\cdot -ig_Z\gamma^\nu \frac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)$

➡ $$-iM_{fi} = [\overline{v}(p_2)\cdot -ig_Z\gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)].\frac{-ig_{\mu\nu}}{q^2 - m_Z^2}.[\overline{u}(p_3)\cdot -ig_Z\gamma^\nu \tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)]$$

➡ $$M_{fi} = -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}[\overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(c_V^e - c_A^e\gamma^5)\cdot u(p_1)].[\overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(c_V^\mu - c_A^\mu\gamma^5)\cdot v(p_4)]$$

★ **Convenient to work in terms of helicity states by explicitly using the Z coupling to LH and RH chiral states (ultra-relativistic limit so helicity = chirality)**

$$\tfrac{1}{2}(c_V - c_A\gamma^5) = c_L\tfrac{1}{2}(1-\gamma^5) + c_R\tfrac{1}{2}(1+\gamma^5)$$

LH and RH projections operators

hence $c_V = (c_L + c_R), \;\; c_A = (c_L - c_R)$

and $$\tfrac{1}{2}(c_V - c_A\gamma^5) = \tfrac{1}{2}(c_L + c_R - (c_L - c_R)\gamma^5) = c_L\tfrac{1}{2}(1-\gamma^5) + c_R\tfrac{1}{2}(1+\gamma^5)$$

with $c_L = \frac{1}{2}(c_V + c_A), \;\; c_R = \frac{1}{2}(c_V - c_A)$

★ **Rewriting the matrix element in terms of LH and RH couplings:**

$$M_{fi} = -\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}[c_L^e \overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(1-\gamma^5)u(p_1) + c_R^e \overline{v}(p_2)\gamma^\mu \tfrac{1}{2}(1+\gamma^5)u(p_1)]$$
$$\times[c_L^\mu \overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(1-\gamma^5)v(p_4) + c_R^\mu \overline{u}(p_3)\gamma^\nu \tfrac{1}{2}(1+\gamma^5)v(p_4)]$$

★ **Apply projection operators remembering that in the ultra-relativistic limit**

$$\tfrac{1}{2}(1-\gamma^5)u = u_\downarrow; \quad \tfrac{1}{2}(1+\gamma^5)u = u_\uparrow, \quad \tfrac{1}{2}(1-\gamma^5)v = v_\uparrow, \quad \tfrac{1}{2}(1+\gamma^5)v = v_\downarrow$$

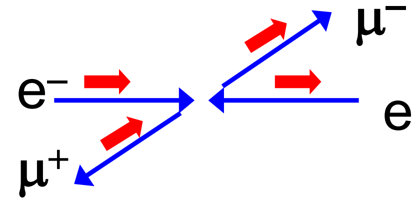
⇨ $$M_{fi} = -\frac{g_Z}{q^2 - m_Z^2} g_{\mu\nu}[c_L^e \overline{v}(p_2)\gamma^\mu u_\downarrow(p_1) + c_R^e \overline{v}(p_2)\gamma^\mu u_\uparrow(p_1)]$$
$$\times[c_L^\mu \overline{u}(p_3)\gamma^\nu v_\uparrow(p_4) + c_R^\mu \overline{u}(p_3)\gamma^\nu v_\downarrow(p_4)]$$

★ **For a combination of V and A currents,** $\overline{u}_\uparrow \gamma^\mu v_\uparrow = 0$ **etc, gives four orthogonal contributions**

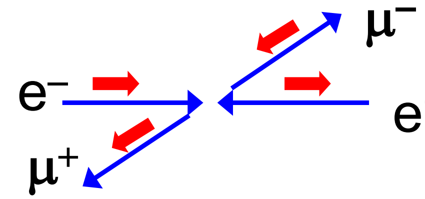
⇨ $$-\frac{g_Z^2}{q^2 - m_Z^2} g_{\mu\nu}[c_L^e \overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1) + c_R^e \overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)]$$
$$\times[c_L^\mu \overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4) + c_R^\mu \overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$

★ **Sum of 4 terms**

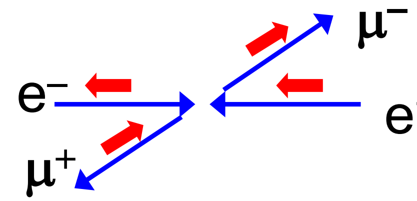
$$M_{RR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_R^\mu g_{\mu\nu} [\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



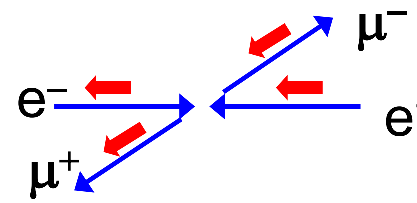
$$M_{RL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_R^e c_L^\mu g_{\mu\nu} [\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



$$M_{LR} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_R^\mu g_{\mu\nu} [\overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]$$



$$M_{LL} = -\frac{g_Z^2}{q^2 - m_Z^2} c_L^e c_L^\mu g_{\mu\nu} [\overline{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1)][\overline{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]$$



Remember: the L/R refer to the helicities of the initial/final state particles

★ **Fortunately we have calculated these terms before when considering**

$e^+e^- \to \gamma \to \mu^+\mu^-$ **giving:** **(pages 137-138)**

$$[\overline{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1)][\overline{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)] = s(1+\cos\theta)$$ **etc.**

★ **Applying the QED results to the Z exchange with**

$$\frac{e^2}{q^2} \rightarrow \frac{g_Z^2}{q^2 - m_Z^2} c^e c^\mu$$

gives:

$$|M_{RR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_R^\mu)^2 (1 + \cos\theta)^2$$

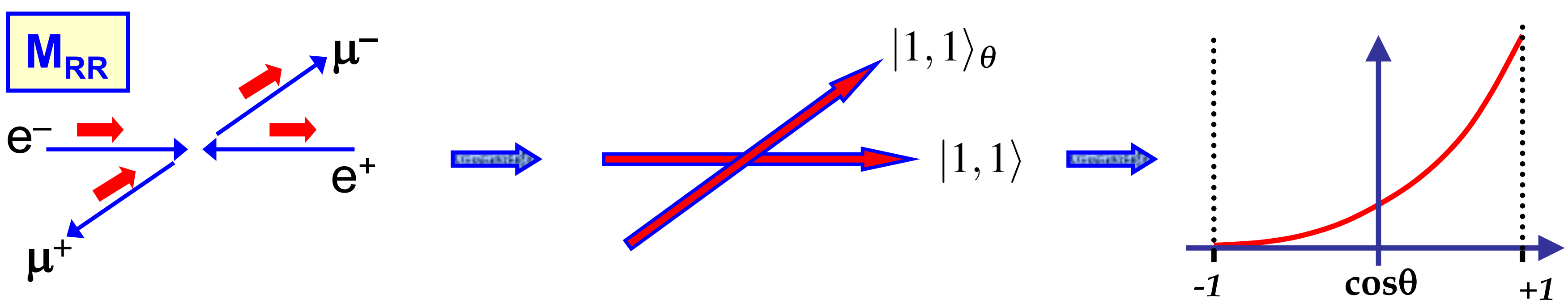
$$|M_{RL}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_R^e)^2 (c_L^\mu)^2 (1 - \cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_R^\mu)^2 (1 - \cos\theta)^2$$

$$|M_{LR}|^2 = s^2 \left| \frac{g_Z^2}{s - m_Z^2} \right|^2 (c_L^e)^2 (c_L^\mu)^2 (1 + \cos\theta)^2$$

where $q^2 = s = 4E_e^2$

★ **As before, the angular dependence of the matrix elements can be understood in terms of the spins of the incoming and outgoing particles e.g.**



The Breit-Wigner Resonance

★ Need to consider carefully the propagator term $1/(s-m_Z^2)$ which diverges when the C.o.M. energy is equal to the rest mass of the Z boson

★ To do this need to account for the fact that the Z boson is an unstable particle

- For a stable particle at rest the time development of the wave-function is:

$$\psi \sim e^{-imt}$$

- For an unstable particle this must be modified to

$$\psi \sim e^{-imt} e^{-\Gamma t/2}$$

so that the particle probability decays away exponentially

$$\psi^* \psi \sim e^{-\Gamma t} = e^{-t/\tau} \qquad \text{with} \qquad \tau = \frac{1}{\Gamma_Z}$$

- Equivalent to making the replacement

$$m \rightarrow m - i\Gamma/2$$

★ In the Z boson propagator make the substitution:

$$m_Z \rightarrow m_Z - i\Gamma_Z/2$$

★ Which gives:

$$(s-m_Z^2) \longrightarrow [s-(m_Z - i\Gamma_Z/2)] = s - m_Z^2 + im_Z\Gamma_Z + \tfrac{1}{4}\Gamma_Z^2 \approx s - m_Z^2 + im_Z\Gamma_Z$$

where it has been assumed that $\Gamma_Z \ll m_Z$

★ Which gives

$$\left|\frac{1}{s-m_Z^2}\right|^2 \rightarrow \left|\frac{1}{s-m_Z^2+im_Z\Gamma_Z}\right|^2 = \frac{1}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}$$

★ **And the Matrix elements become**

$$|M_{RR}|^2 = \frac{g_Z^4 s^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2$$ **etc.**

★ **In the limit where initial and final state particle mass can be neglected:**

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|M_{fi}|^2$$

(page 31)

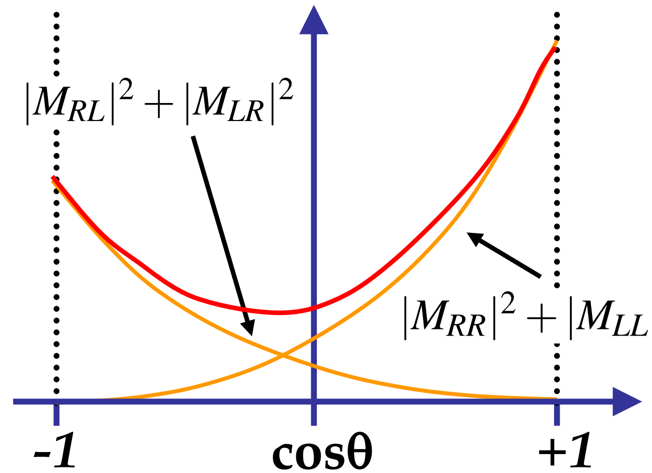
★ **Giving:**

$$\frac{d\sigma_{RR}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_R^e)^2(c_R^\mu)^2(1+\cos\theta)^2$$

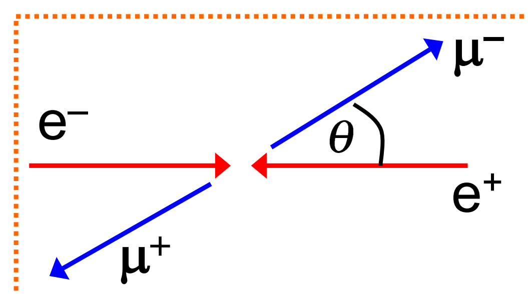
$$\frac{d\sigma_{LL}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_L^e)^2(c_L^\mu)^2(1+\cos\theta)^2$$

$$\frac{d\sigma_{LR}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$

$$\frac{d\sigma_{RL}}{d\Omega} = \frac{1}{64\pi^2}\frac{g_Z^4 s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}(c_L^e)^2(c_R^\mu)^2(1-\cos\theta)^2$$



★ **Because** $|M_{LL}|^2 + |M_{RR}|^2 \neq |M_{LR}|^2 + |M_{RL}|^2$ **, the differential cross section is asymmetric, i.e. parity violation (although not maximal as was the case for the W boson).**



Cross section with unpolarized beams

★ To calculate the total cross section need to sum over all matrix elements and **average** over the **initial spin states**. Here, assuming unpolarized beams (i.e. both e^+ and both e^- spin states equally likely) there a four combinations of initial electron/positron spins, so

$$\langle |M_{fi}|^2 \rangle = \frac{1}{2}\cdot\frac{1}{2}\cdot(|M_{RR}|^2 + |M_{LL}|^2 + |M_{LR}|^2 + |M_{RL}|^2)$$

$$= \tfrac{1}{2}\cdot\tfrac{1}{2}\frac{g_Z^4 s^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} \times \{[(c_R^e)^2(c_R^\mu)^2 + (c_L^e)^2(c_L^2)^2](1+\cos\theta)^2 + [(c_L^e)^2(c_R^\mu)^2 + (c_R^e)^2(c_L^2)^2](1-\cos\theta)^2\}$$

★ The part of the expression {…} can be rearranged:

$$\{\ldots\} = [(c_R^e)^2 + (c_L^e)^2][(c_R^\mu)^2 + (c_L^\mu)^2](1+\cos^2\theta) + 2[(c_R^e)^2 - (c_L^e)^2][(c_R^\mu)^2 - (c_L^\mu)^2]\cos\theta \qquad (1)$$

and using $c_V^2 + c_A^2 = 2(c_L^2 + c_R^2)$ and $c_V c_A = c_L^2 + c_R^2$

$$\{\ldots\} = \frac{1}{4}[(c_V^e)^2 + (c_A^e)^2][(c_V^\mu)^2 + (c_A^\mu)^2](1+\cos^2\theta) + 2c_V^e c_A^e c_V^\mu c_A^\mu \cos\theta$$

★ **Hence the complete expression for the unpolarized differential cross section is:**

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \frac{1}{64\pi^2 s}\langle|M_{fi}|^2\rangle \\ &= \frac{1}{64\pi^2}\cdot\frac{1}{4}\cdot\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\times \\ &\quad \left\{\tfrac{1}{4}[(c_V^e)^2+(c_A^e)^2][(c_V^\mu)^2+(c_A^\mu)^2](1+\cos^2\theta)+2c_V^e c_A^e c_V^\mu c_A^\mu\cos\theta\right\}\end{aligned}$$

★ **Integrating over solid angle** $d\Omega = d\phi\, d(\cos\theta) = 2\pi\, d(\cos\theta)$

$$\int_{-1}^{+1}(1+\cos^2\theta)d(\cos\theta) = \int_{-1}^{+1}(1+x^2)dx = \tfrac{8}{3} \quad \textbf{and} \quad \int_{-1}^{+1}\cos\theta\, d(\cos\theta) = 0$$

$$\boxed{\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\left[(c_V^e)^2+(c_A^e)^2\right]\left[(c_V^\mu)^2+(c_A^\mu)^2\right]}$$

★ **Note: the total cross section is proportional to the sums of the squares of the vector- and axial-vector couplings of the initial and final state fermions**

$$(c_V^f)^2+(c_A^f)^2$$

Connection to the Breit-Wigner Formula

★ **Can write the total cross section**

$$\sigma_{e^+e^-\to Z\to\mu^+\mu^-} = \frac{1}{192\pi}\frac{g_Z^4 s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\left[(c_V^e)^2+(c_A^e)^2\right]\left[(c_V^\mu)^2+(c_A^\mu)^2\right]$$

in terms of the Z boson decay rates (partial widths) from page 473 (question 26)

$$\Gamma(Z\to e^+e^-) = \frac{g_Z^2 m_Z}{48\pi}\left[(c_V^e)^2+(c_A^e)^2\right] \quad \textbf{and} \quad \Gamma(Z\to\mu^+\mu^-) = \frac{g_Z^2 m_Z}{48\pi}\left[(c_V^\mu)^2+(c_A^\mu)^2\right]$$

$$\Rightarrow \quad \sigma = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma(Z\to e^+e^-)\Gamma(Z\to\mu^+\mu^-)$$

★ **Writing the partial widths as** $\Gamma_{ee}=\Gamma(Z\to e^+e^-)$ **etc., the total cross section can be written**

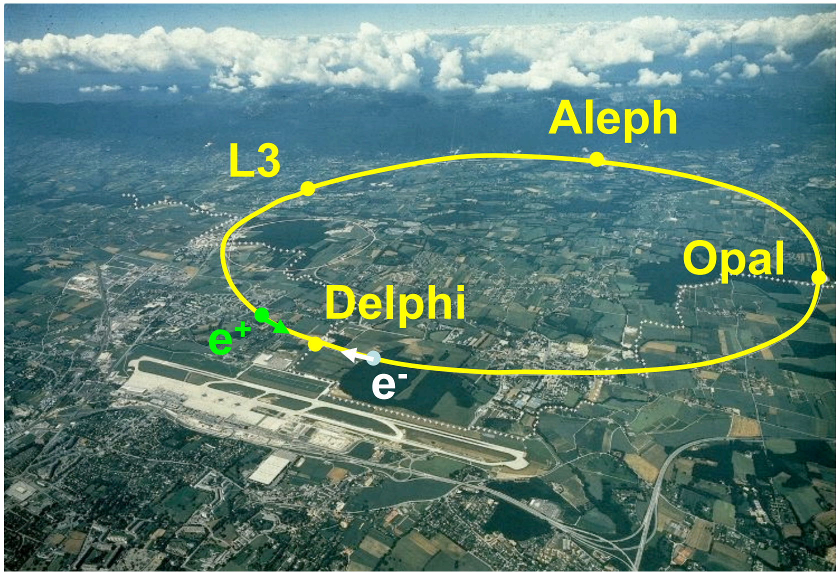
$$\sigma(e^+e^-\to Z\to f\overline{f}) = \frac{12\pi}{m_Z^2}\frac{s}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\Gamma_{ee}\Gamma_{ff} \qquad \textbf{(2)}$$

where f is the final state fermion flavour:

(The relation to the non-relativistic form of the part II course is given in the appendix)

Electroweak Measurements at LEP

★ The Large Electron Positron (LEP) Collider at CERN (1989-2000) was designed to make precise measurements of the properties of the Z and W bosons.



- 26 km circumference accelerator straddling French/Swiss boarder
- Electrons and positrons collided at 4 interaction points
- 4 large detector collaborations (each with 300-400 physicists):
  - ALEPH,
  - DELPHI,
  - L3,
  - OPAL

Basically a large Z and W factory:

★ **1989-1995:** Electron-Positron collisions at $\sqrt{s}$ = 91.2 GeV

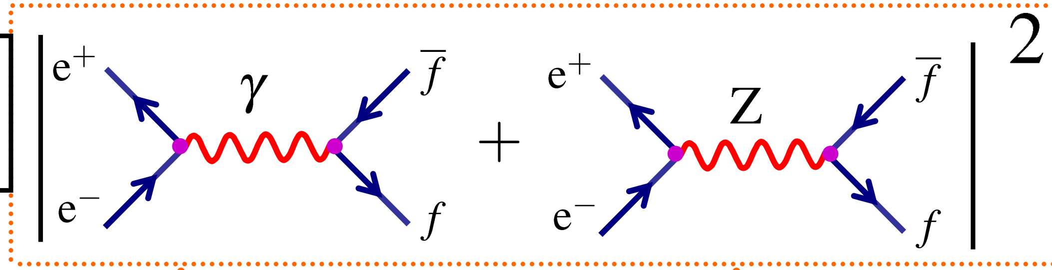
- 17 Million Z bosons detected

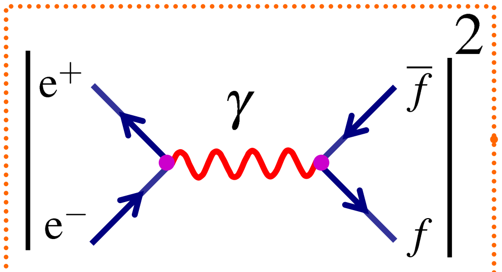
★ **1996-2000:** Electron-Positron collisions at $\sqrt{s}$ = 161-208 GeV

- 30000 W^+W^- events detected

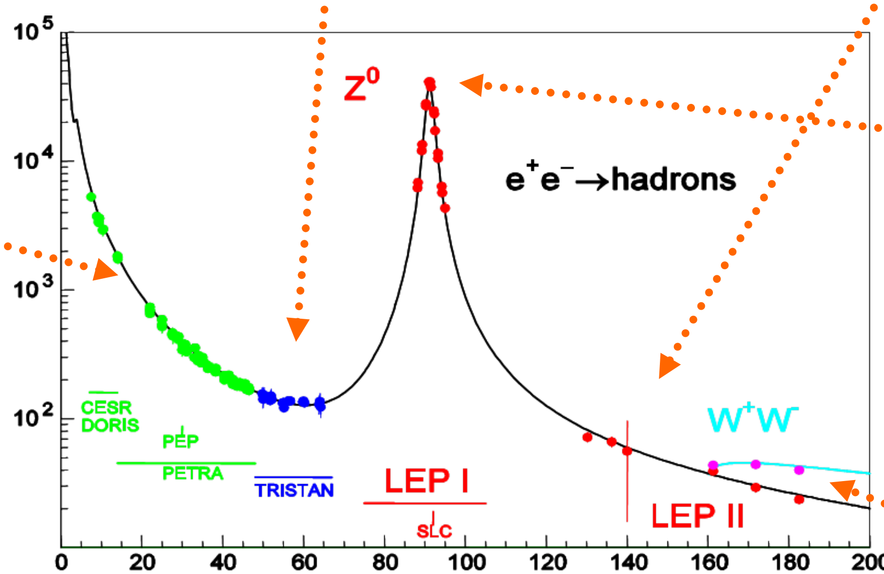
e^+e^- Annihilation in Feynman Diagrams

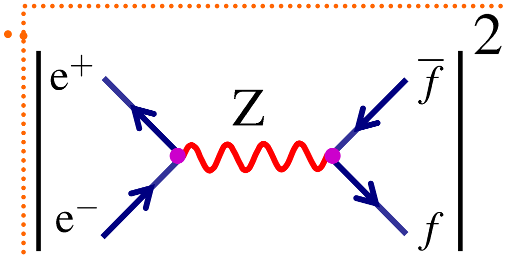
In general e^+e^- annihilation involves both photon and Z exchange : + interference





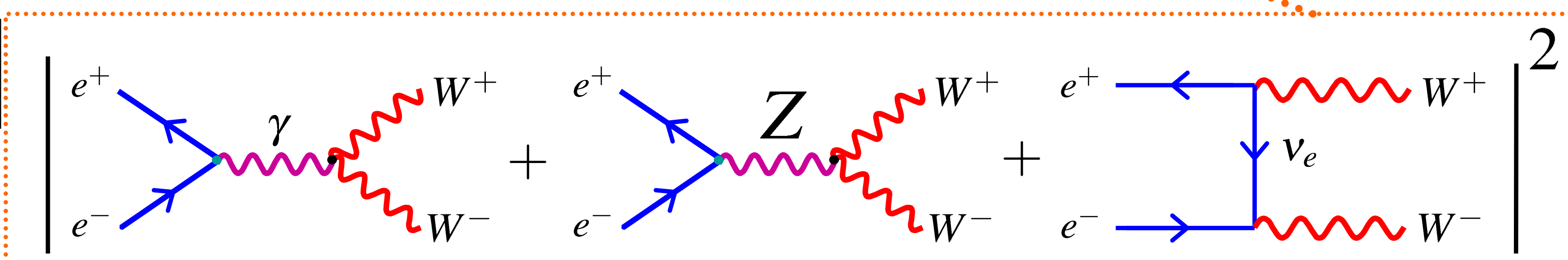
Well below Z: photon exchange dominant





At Z resonance: Z exchange dominant

High energies: WW production



Cross Section Measurements

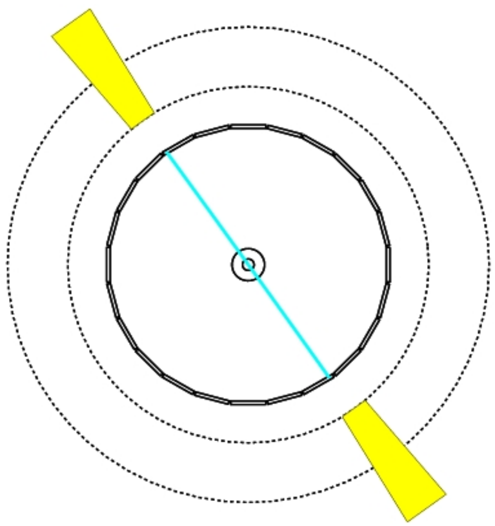
- At Z resonance mainly observe four types of event:

$$e^+e^- \to Z \to e^+e^- \qquad e^+e^- \to Z \to \mu^+\mu^- \qquad e^+e^- \to Z \to \tau^+\tau^-$$

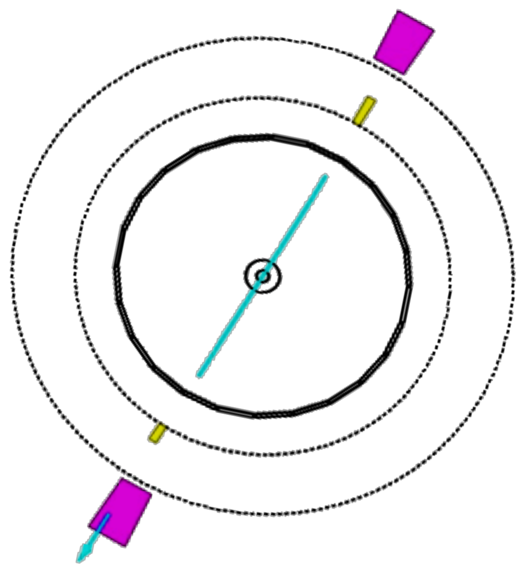
$$e^+e^- \to Z \to q\bar{q} \to \text{hadrons}$$

- Each has a distinct topology in the detectors, e.g.

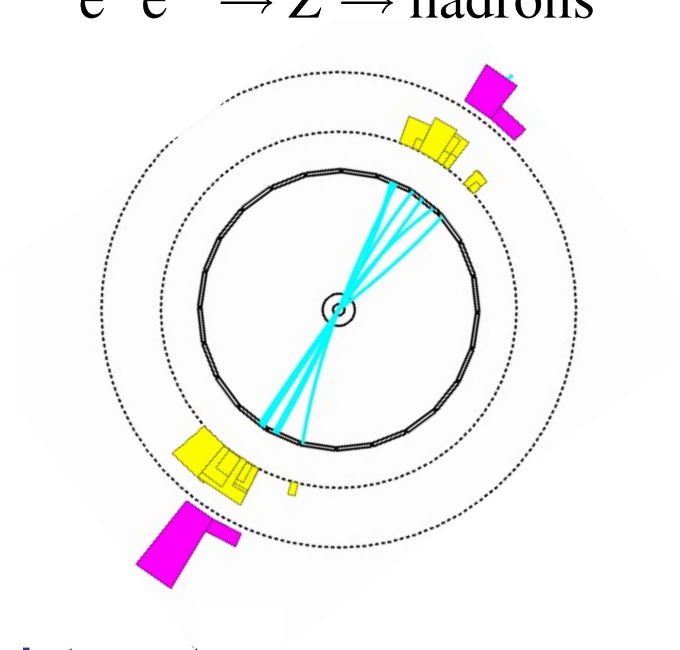
$$e^+e^- \to Z \to e^+e^-$$



$$e^+e^- \to Z \to \mu^+\mu^-$$



$$e^+e^- \to Z \to \text{hadrons}$$



- To work out cross sections, first count events of each type
- Then need to know "integrated luminosity" of colliding beams, i.e. the relation between cross-section and expected number of interactions

$$N_{\text{events}} = \mathscr{L}\sigma$$

Forward-Backward Asymmetry

★ On page 495 we obtained the expression for the differential cross section:

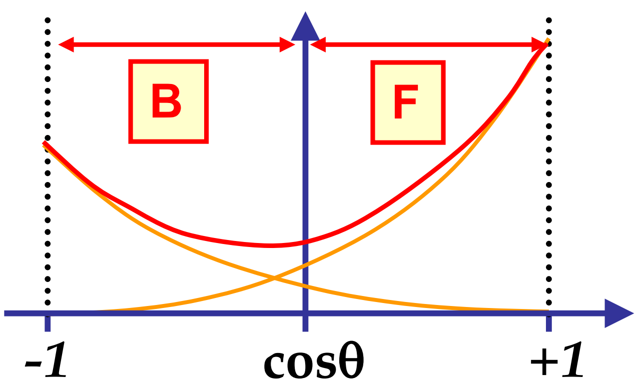
$$\langle |M_{fi}|\rangle^2 \propto [(c_L^e)^2+(c_R^e)^2][(c_L^\mu)^2+(c_R^\mu)^2](1+\cos^2\theta)+[(c_L^e)^2-(c_R^e)^2][(c_L^\mu)^2-(c_R^\mu)^2]\cos\theta$$

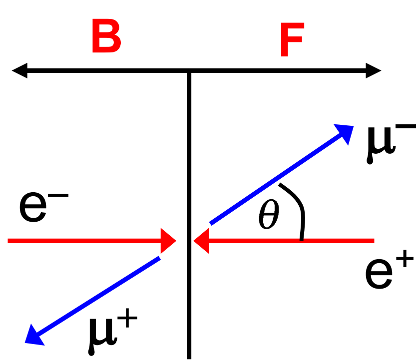
★ The differential cross sections is therefore of the form:

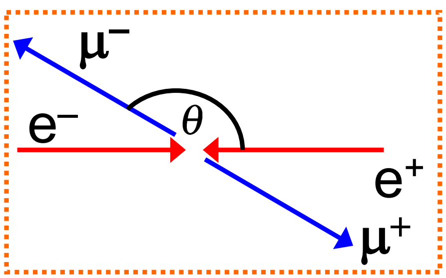
$$\frac{d\sigma}{d\Omega} = \kappa \times [A(1+\cos^2\theta)+B\cos\theta] \qquad \begin{cases} A = [(c_L^e)^2+(c_R^e)^2][(c_L^\mu)^2+(c_R^\mu)^2] \\ B = [(c_L^e)^2-(c_R^e)^2][(c_L^\mu)^2-(c_R^\mu)^2] \end{cases}$$

★ Define the **FORWARD** and **BACKWARD** cross sections in terms of angle incoming electron and out-going particle

$$\sigma_F \equiv \int_0^1 \frac{d\sigma}{d\cos\theta}\,d\cos\theta \qquad \sigma_B \equiv \int_{-1}^0 \frac{d\sigma}{d\cos\theta}\,d\cos\theta$$

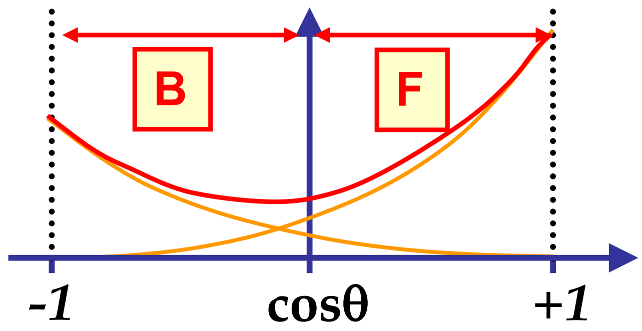






★ The level of asymmetry about **cosθ=0** is expressed in terms of the Forward-Backward Asymmetry

$$A_{\rm FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$



• Integrating equation **(1)**:

$$\sigma_F = \kappa \int_0^1 [A(1+\cos^2\theta) + B\cos\theta]\,\mathrm{d}\cos\theta = \kappa \int_0^1 [A(1+x^2) + Bx]\,\mathrm{d}x = \kappa\left(\frac{4}{3}A + \frac{1}{2}B\right)$$

$$\sigma_B = \kappa \int_{-1}^0 [A(1+\cos^2\theta) + B\cos\theta]\,\mathrm{d}\cos\theta = \kappa \int_{-1}^0 [A(1+x^2) + Bx]\,\mathrm{d}x = \kappa\left(\frac{4}{3}A - \frac{1}{2}B\right)$$

★ Which gives:

$$A_{\rm FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{B}{(8/3)A} = \frac{3}{4}\left[\frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2}\right] \cdot \left[\frac{(c_L^\mu)^2 - (c_R^\mu)^2}{(c_L^\mu)^2 + (c_R^\mu)^2}\right]$$

★ This can be written as

$$A_{\rm FB} = \frac{3}{4}A_e A_\mu$$

with

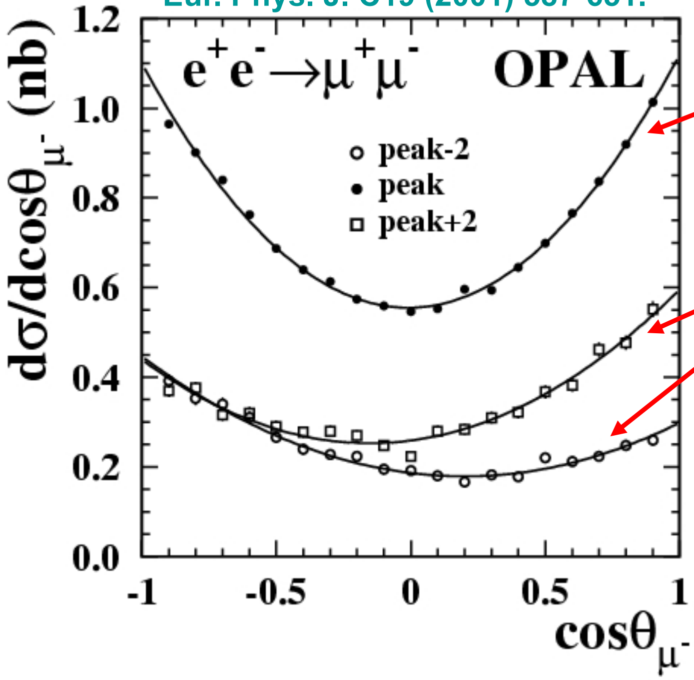
$$A_f \equiv \frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2} = \frac{2c_V^f c_A^f}{(c_V^f)^2 + (c_A^f)^2} \qquad \textbf{(4)}$$

★ Observe a non-zero asymmetry because the couplings of the Z to LH and RH particles are different. Contrast with QED where the couplings to LH and RH particles are the same (parity is conserved) and the interaction is FB symmetric

Measured Forward-Backward Asymmetries

★ Forward-backward asymmetries can only be measured for final states where the charge of the fermion can be determined, e.g. $e^+e^- \to Z \to \mu^+\mu^-$

OPAL Collaboration,
Eur. Phys. J. C19 (2001) 587-651.



Because $\sin^2\theta_W \approx 0.25$, the value of A_{FB} for leptons is almost zero

For data above and below the peak of the Z resonance interference with $e^+e^- \to \gamma \to \mu^+\mu^-$ leads to a larger asymmetry

★LEP data combined:

$$A_{FB}^{0,e} = 0.0145 \pm 0.0025$$
$$A_{FB}^{0,\mu} = 0.0169 \pm 0.0013$$
$$A_{FB}^{0,\tau} = 0.0188 \pm 0.0017$$

★ To relate these measurements to the couplings uses $A_{FB} = \frac{3}{4} A_e A_\mu$

★ In all cases asymmetries depend on A_e

★ To obtain A_e could use $A_{FB}^{0,e} = \frac{3}{4} A_e^2$ (also see Appendix II for A_{LR})

Determination of the Weak Mixing Angle

★ **From LEP :** $A_{FB}^{0,f} = \frac{3}{4} A_e A_f$

★ **From SLC :** $A_{LR} = A_e$

$\Big\}\ A_e, A_\mu, A_\tau, \ldots$

Putting everything together ⇨

$$A_e = 0.1514 \pm 0.0019$$
$$A_\mu = 0.1456 \pm 0.0091$$
$$A_\tau = 0.1449 \pm 0.0040$$

includes results from other measurements

with $$A_f \equiv \frac{2 c_V^f c_A^f}{(c_V^f)^2 + (c_A^f)^2} = 2 \frac{c_V / c_A}{1 + (c_V/c_A)^2}$$

★ **Measured asymmetries give ratio of vector to axial-vector Z coupings.**

★ **In SM these are related to the weak mixing angle**

$$\frac{c_V}{c_A} = \frac{I_W^3 - 2Q \sin^2\theta_W}{I_W^3} = 1 - \frac{2Q}{I_3}\sin^2\theta_W = 1 - 4|Q|\sin^2\theta_W$$

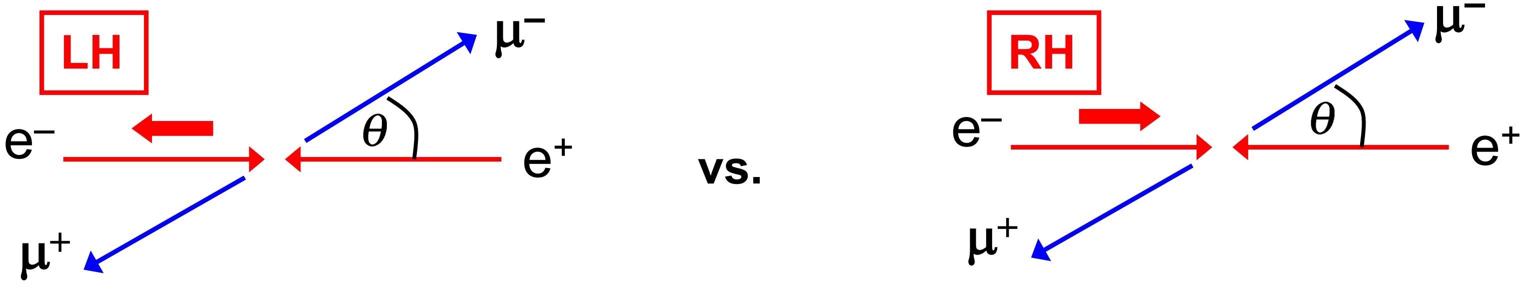
★ **Asymmetry measurements give precise determination of** $\sin^2\theta_W$

$$\sin^2\theta_W = 0.23154 \pm 0.00016$$

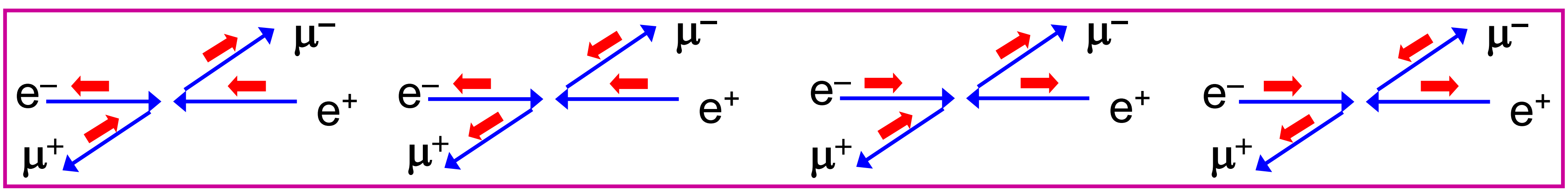
Appendix II: Left-Right Asymmetry, A_{LR}

★ At an e^+e^- linear collider it is possible to produce polarized electron beams e.g. SLC linear collider at SLAC (California), **1989-2000**

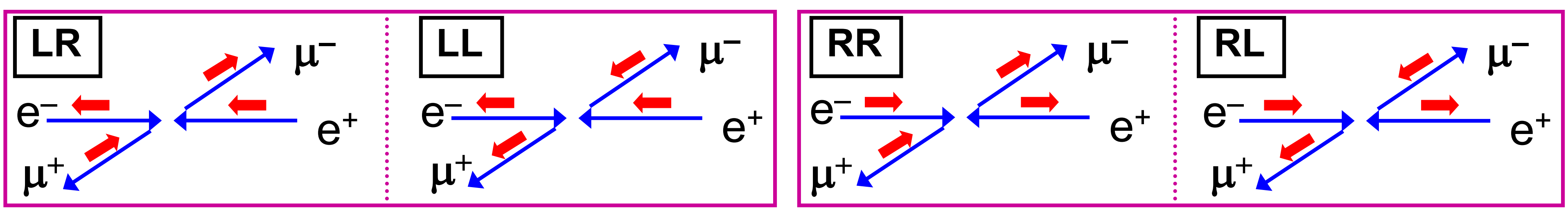
★ Measure cross section for any process for LH and RH electrons separately



- At LEP measure total cross section: sum of 4 helicity combinations:



- At SLC, by choosing the polarization of the electron beam are able to measure cross sections **separately** for LH / RH electrons



★ Averaging over the two possible polarization states of the positron for a given electron polarization:

$$\langle|M_L|\rangle^2 = \tfrac{1}{2}(|M_{LL}|^2 + |M_{LR}|^2) \qquad \langle|M_R|\rangle^2 = \tfrac{1}{2}(|M_{RL}|^2 + |M_{RR}|^2)$$

⇨ $$\sigma_L = \tfrac{1}{2}(\sigma_{LR} + \sigma_{LL}) \qquad \sigma_R = \tfrac{1}{2}(\sigma_{RR} + \sigma_{RL})$$

★ Define cross section asymmetry:

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

★ Integrating the expressions on page 494 gives:

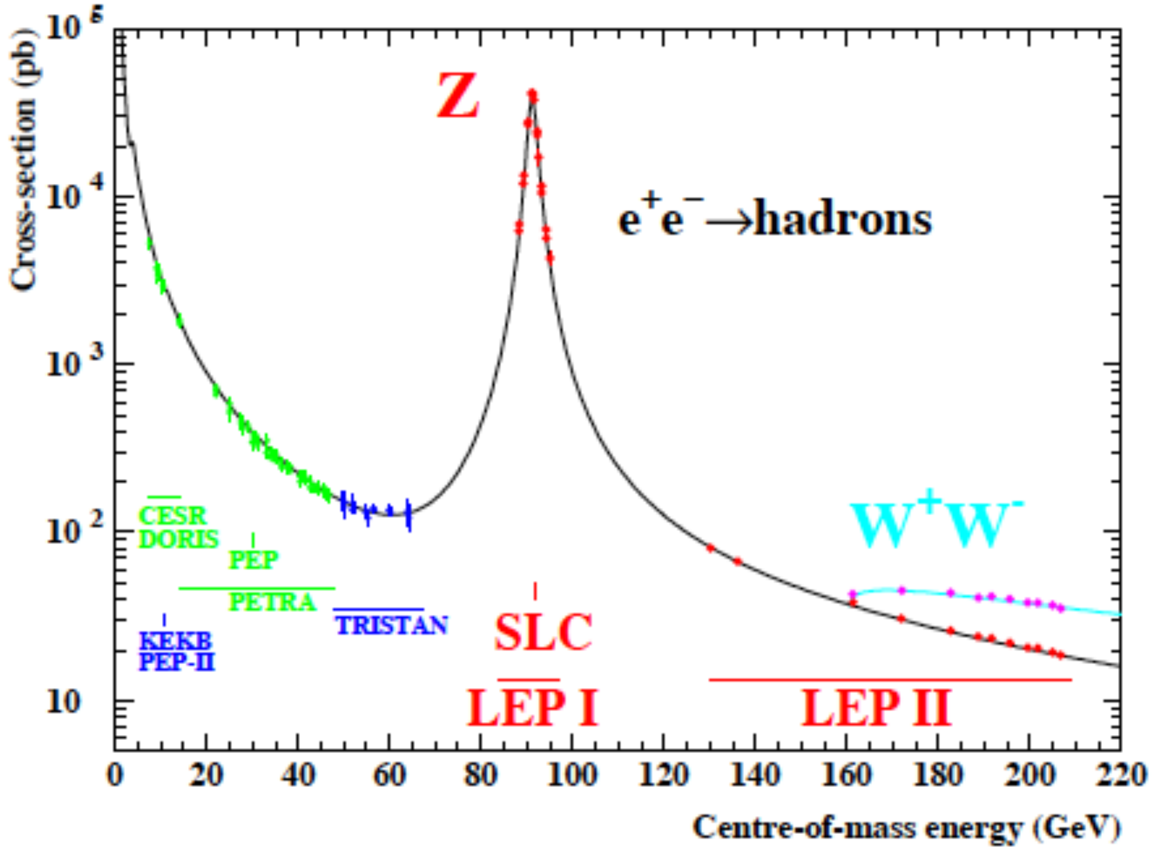
$$\sigma_{LL} \propto (c_L^e)^2(c_L^\mu)^2 \quad \sigma_{LR} \propto (c_L^e)^2(c_R^\mu)^2 \quad \sigma_{RL} \propto (c_R^e)^2(c_L^\mu)^2 \quad \sigma_{RR} \propto (c_R^e)^2(c_R^\mu)^2$$

⇨ $$\sigma_L \propto (c_L^e)^2[(c_L^\mu)^2 + (c_R^\mu)^2] \qquad \sigma_R \propto (c_R^e)^2[(c_L^\mu)^2 + (c_R^\mu)^2]$$

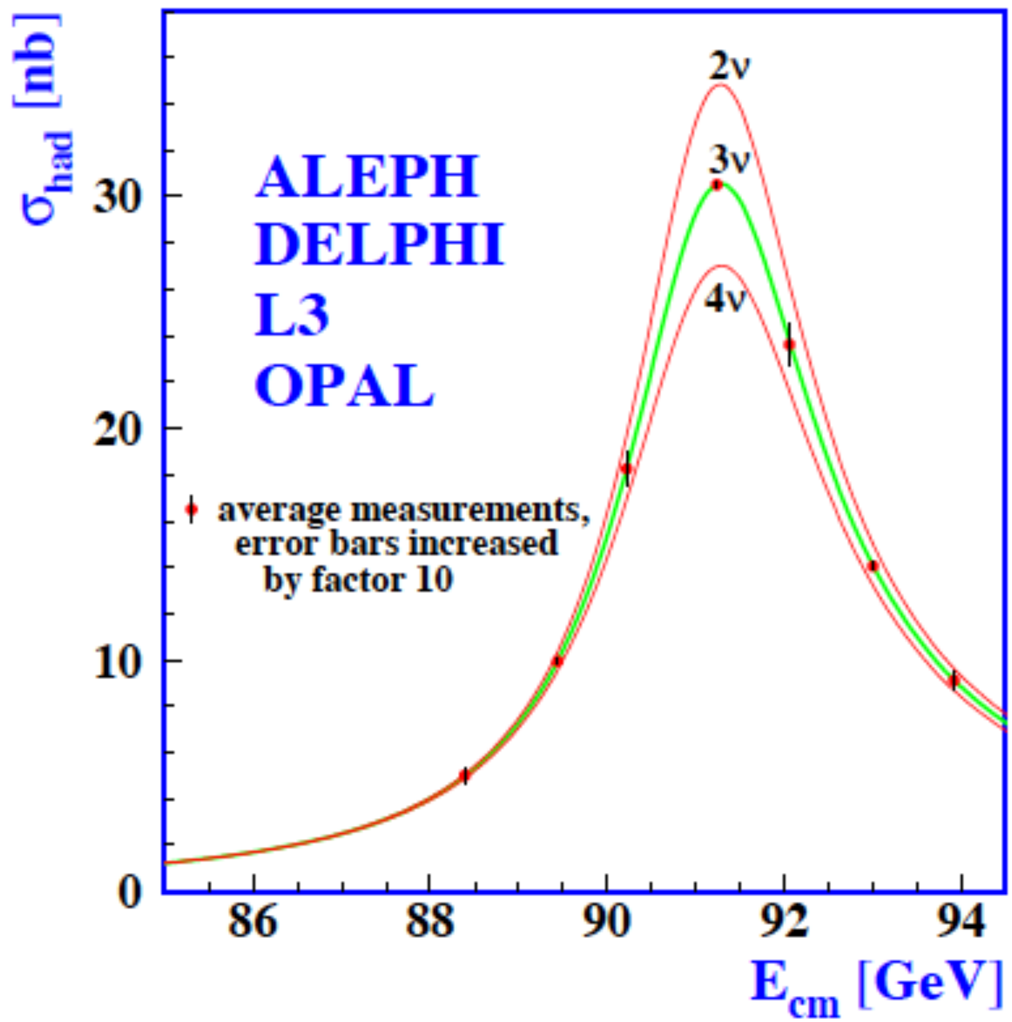
$$A_{LR} = \frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2} = A_e$$

★ Hence the Left-Right asymmetry for any cross section depends only on the couplings of the electron

Z/γ* lineshape



Neutrinos from Lineshape



$$N_\nu = 2.9840 \pm 0.0082$$

$$\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{hadrons}} + \Gamma_{\nu_1\nu_1} + \Gamma_{\nu_2\nu_2} + \Gamma_{\nu_3\nu_3} + ?$$

Z Partial Widths

Parameter $\Gamma_{f\bar{f}}$	Average [MeV]	Correlations						
Without Lepton Universality								
		Γ_{had}	Γ_{ee}	$\Gamma_{\mu\mu}$	$\Gamma_{\tau\tau}$	$\Gamma_{b\bar{b}}$	$\Gamma_{c\bar{c}}$	Γ_{inv}
Γ_{had}	1745.8 ± 2.7	1.00						
Γ_{ee}	83.92 ± 0.12	−0.29	1.00					
$\Gamma_{\mu\mu}$	83.99 ± 0.18	0.66	−0.20	1.00				
$\Gamma_{\tau\tau}$	84.08 ± 0.22	0.54	−0.17	0.39	1.00			
$\Gamma_{b\bar{b}}$	377.6 ± 1.3	0.45	−0.13	0.29	0.24	1.00		
$\Gamma_{c\bar{c}}$	300.5 ± 5.3	0.09	−0.02	0.06	0.05	−0.12	1.00	
Γ_{inv}	497.4 ± 2.5	−0.67	0.78	−0.45	−0.40	−0.30	−0.06	1.00
With Lepton Universality								
		Γ_{had}	$\Gamma_{\ell\ell}$	$\Gamma_{b\bar{b}}$	$\Gamma_{c\bar{c}}$	Γ_{inv}		
Γ_{had}	1744.4 ± 2.0	1.00						
$\Gamma_{\ell\ell}$	83.985 ± 0.086	0.39	1.00					
$\Gamma_{b\bar{b}}$	377.3 ± 1.2	0.35	0.13	1.00				
$\Gamma_{c\bar{c}}$	300.2 ± 5.2	0.06	0.03	−0.15	1.00			
Γ_{inv}	499.0 ± 1.5	−0.29	0.49	−0.10	−0.02	1.00		

Left-Right Asymmetries

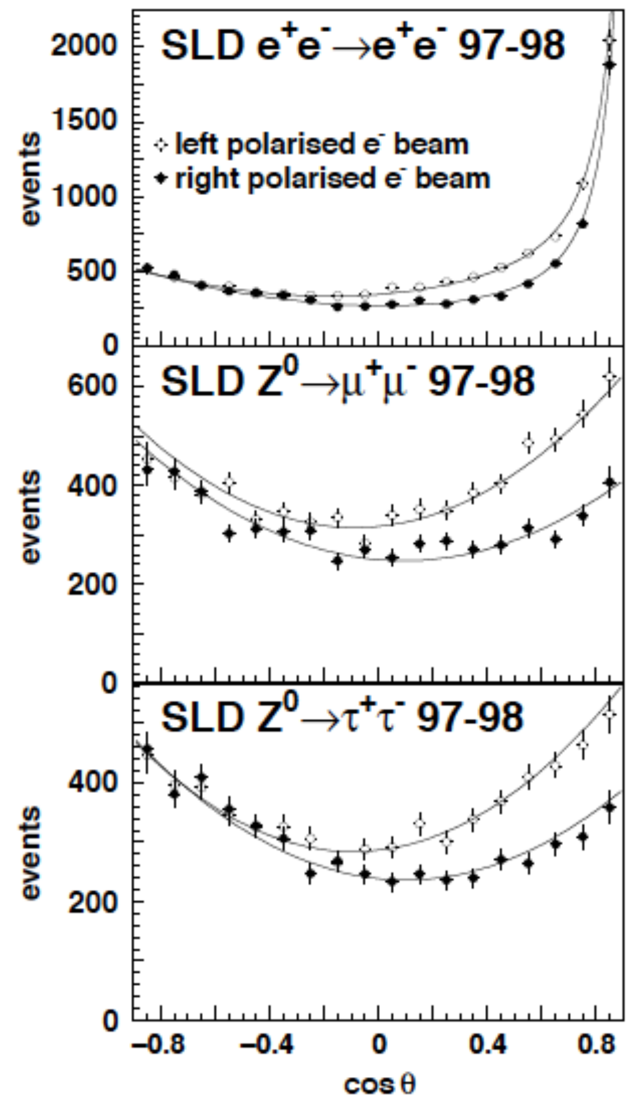
Stanford Linear Collider could produced polarized e beams

$$\sigma_L = \frac{1}{2}(\sigma_{LR} + \sigma_{LL})$$

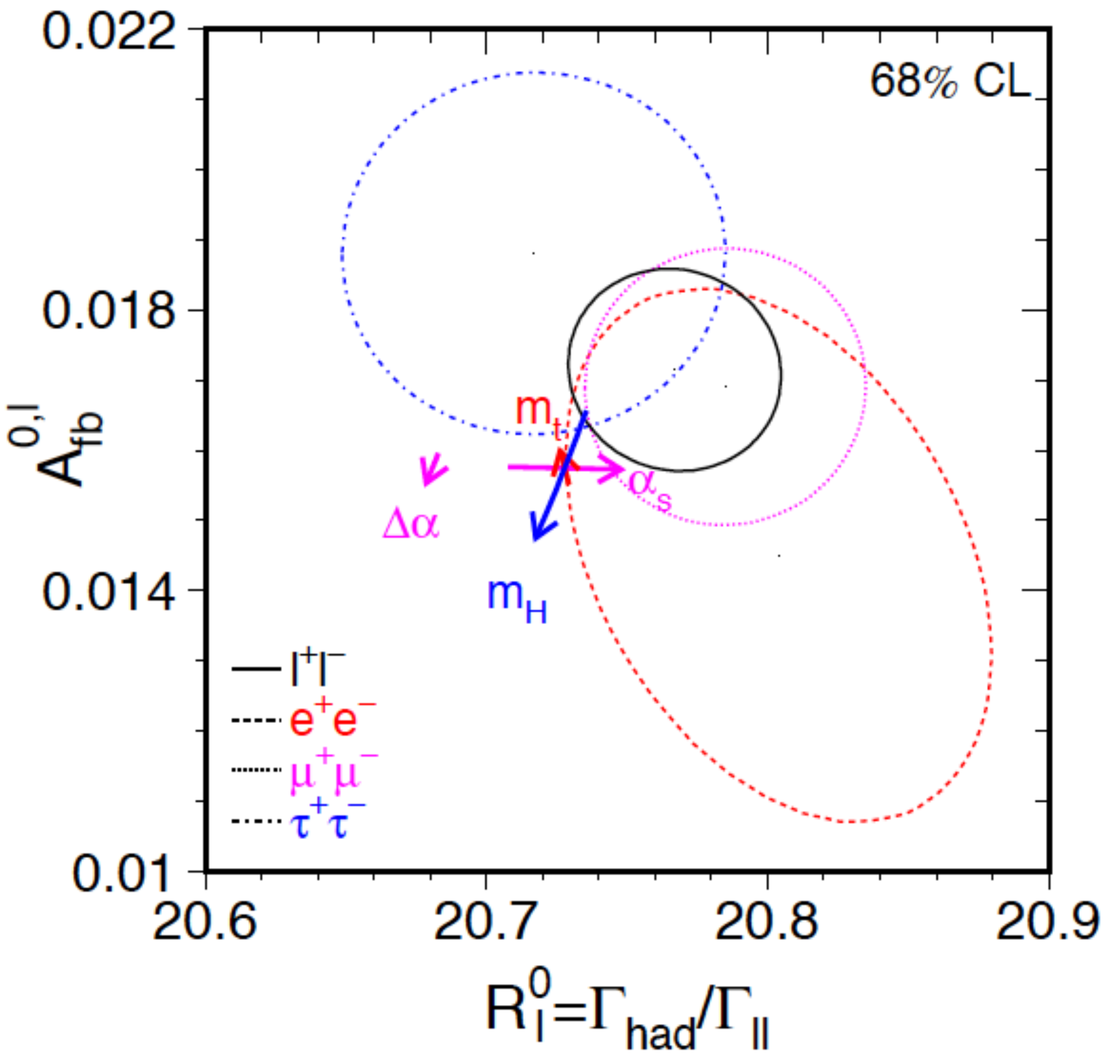
$$\sigma_R = \frac{1}{2}(\sigma_{RR} + \sigma_{RL})$$

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

$$A_{LR} = \frac{(c_L^e)^2 - (c_R^e)^2}{(c_L^e)^2 + (c_R^e)^2} = A_e$$

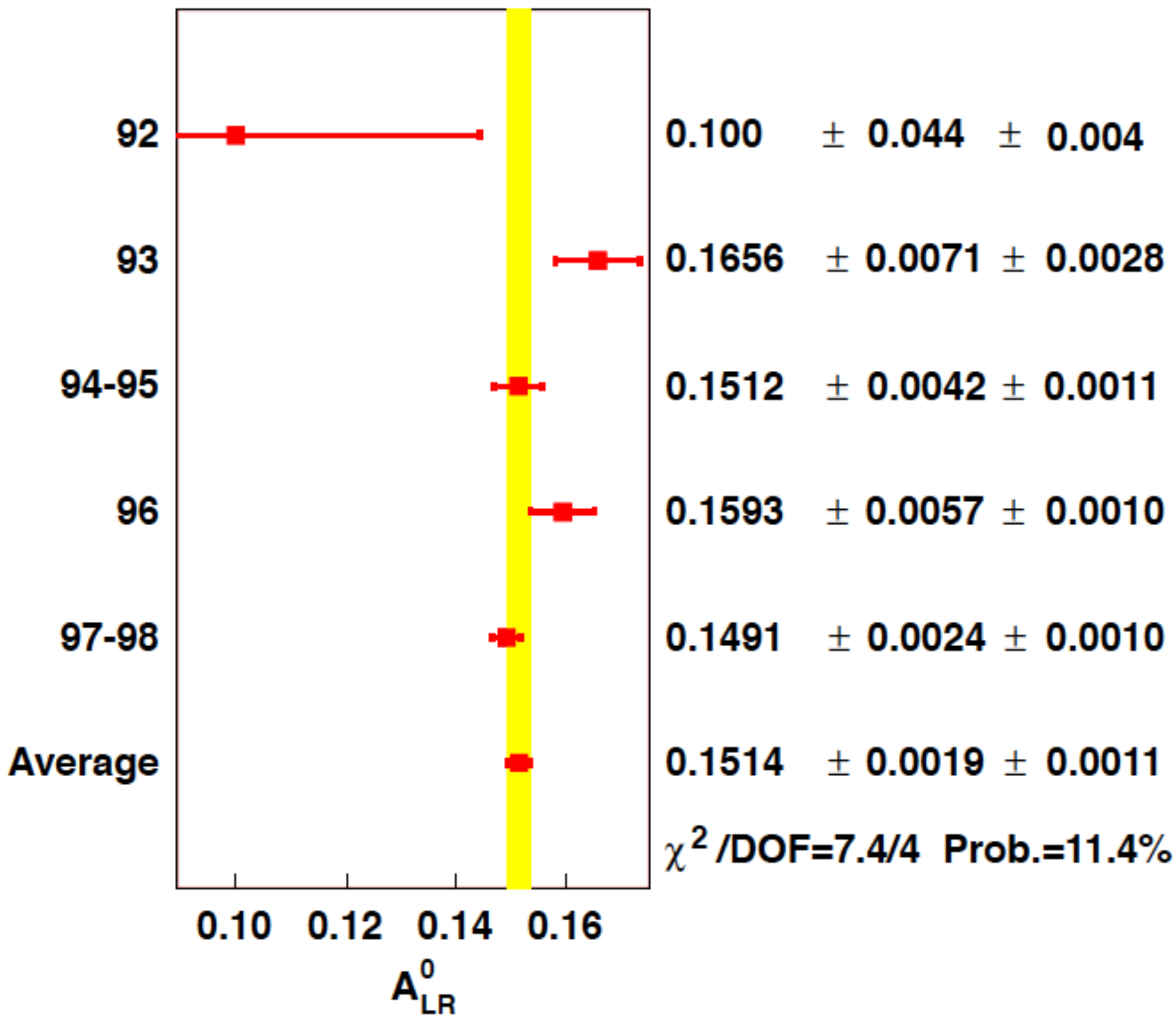


LEP Results



0.022
0.018
0.014
0.01
20.6
20.7
20.8
20.9
68% CL
$A_{fb}^{0,l}$
$R_l^0=\Gamma_{had}/\Gamma_{ll}$
m_t
m_H
$\Delta\alpha$
α_s
l^+l^-
e^+e^-
$\mu^+\mu^-$
$\tau^+\tau^-$

Left-Right Asymmetries



Z Measurements

	Measurement with Total Error	Systematic Error	Standard Model High-Q^2 Fit	Pull
$\Delta\alpha_{had}^{(5)}(m_Z^2)$ [59]	0.02758 ± 0.00035	0.00034	0.02767 ± 0.00035	0.3
m_Z [GeV]	91.1875 ± 0.0021	[a]0.0017	91.1874 ± 0.0021	0.1
Γ_Z [GeV]	2.4952 ± 0.0023	[a]0.0012	2.4965 ± 0.0015	0.6
σ_{had}^0 [nb]	41.540 ± 0.037	[a]0.028	41.481 ± 0.014	1.6
R_ℓ^0	20.767 ± 0.025	[a]0.007	20.739 ± 0.018	1.1
$A_{FB}^{0,\ell}$	0.0171 ± 0.0010	[a]0.0003	0.01642 ± 0.00024	0.8
+ correlation matrix Table 2.13				
$\mathcal{A}_\ell$ (P_τ)	0.1465 ± 0.0033	0.0015	0.1480 ± 0.0011	0.5
$\mathcal{A}_\ell$ (SLD)	0.1513 ± 0.0021	0.0011	0.1480 ± 0.0011	1.6
R_b^0	0.21629 ± 0.00066	0.00050	0.21562 ± 0.00013	1.0
R_c^0	0.1721 ± 0.0030	0.0019	0.1723 ± 0.0001	0.1
$A_{FB}^{0,b}$	0.0992 ± 0.0016	0.0007	0.1037 ± 0.0008	2.8
$A_{FB}^{0,c}$	0.0707 ± 0.0035	0.0017	0.0742 ± 0.0006	1.0
$\mathcal{A}_b$	0.923 ± 0.020	0.013	0.9346 ± 0.0001	0.6
$\mathcal{A}_c$	0.670 ± 0.027	0.015	0.6683 ± 0.0005	0.1
+ correlation matrix Table 5.11				
$\sin^2\theta_{eff}^{lept}$ (Q_{FB}^{had})	0.2324 ± 0.0012	0.0010	0.23140 ± 0.00014	0.8

Compatibility with SM

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4965
σ_{had}^{0} [nb]	41.540 ± 0.037	41.481
R_l	20.767 ± 0.025	20.739
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01642
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21562
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1037
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.425 ± 0.034	80.389
Γ_W [GeV]	2.133 ± 0.069	2.093
m_t [GeV]	178.0 ± 4.3	178.5