LECTURE 10: Standard Model (Part 2)

Overview:

-Construction of the Standard Model

-SM Higgs Mechanism

(I used Quigg and Novaes as references)

Standard Model Lagrangian

Last lecture, we ended with:

$$\mathcal{L}_{lep.} = \mathcal{L}_{\ell ep} + \underbrace{\bar{L}\, i\gamma^\mu \left(i\frac{g}{2}\tau^i W_\mu^i + i\frac{g'}{2} Y B_\mu \right) L}$$
$$+ \bar{R}\, i\gamma^\mu \left(i\frac{g'}{2} B_\mu \right) R$$

expanding:

$$-g\bar{L}\gamma^\mu \left(\frac{\tau^1}{2} W_\mu^1 + \frac{\tau^2}{2} W_\mu^2 \right) L \qquad (1)$$

$$-g\bar{L}\gamma^\mu \frac{\tau^3}{2} L W_\mu^3 - \frac{g'}{2} Y \bar{L}\gamma^\mu L B_\mu \qquad (2)$$

We saw that Term (1) involves charged current. We can write it as:

$$-\frac{g}{2}\bar{L}\gamma^\mu \begin{pmatrix} 0 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & 0 \end{pmatrix} L$$

We can define the charged bosons as: $W_\mu^{\pm} = \frac{1}{\sqrt{2}}(W_\mu^1 \mp W_\mu^2)$

Standard Model Lagrangian (cont.)

Term (1) becomes:

$$-\frac{g}{2\sqrt{2}}\left[\bar{\nu}\gamma^\mu(1-\gamma^5)\ell\, W_\mu^+ + \bar{\ell}\gamma^\mu(1-\gamma^5)\nu\, W_\mu^-\right]$$

note that $\frac{g}{2\sqrt{2}} = \left(\frac{M_W^2 G_F}{\sqrt{2}}\right)^{1/2}$

We now concentrate on the neutral current terms which involves both L and R components:

$$-g\,\bar{L}\,\gamma^\mu \frac{\tau^3}{2} L\, W_\mu^3 - \frac{g'}{2}\left(\bar{L}\gamma^\mu Y L + \bar{R}\gamma^\mu Y R\right)B_\mu$$

$$= -g\, J_3^\mu W_\mu^3 - \frac{g'}{2} J_Y^\mu B_\mu$$

$\longrightarrow$ from last lecture: $J_3^\mu = \frac{1}{2}\left(\bar{\nu}_L\gamma^\mu\nu_L - \bar{\ell}\gamma^\mu\ell\right)$

$J_Y^\mu = -\left(\bar{\nu}_L\gamma^\mu\nu_L + \bar{\ell}_L\gamma^\mu\ell_L + 2\,\bar{\ell}_R\gamma^\mu\ell_R\right)$

Standard Model Lagrangian (cont.)

Remember that $J_{em} = J_3 + \frac{1}{2} J_Y$

We want the right combination of fields that couple to J_{em}.

We can do this by rotating the fields:

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}$$

$$W_\mu^3 = \sin\theta_W A_\mu + \cos\theta_W Z_\mu$$

$$B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu$$

with $\sin\theta_W = \dfrac{g'}{\sqrt{g^2+g'^2}}$ $\qquad$ $\cos\theta_W = \dfrac{g}{\sqrt{g^2+g'^2}}$

$\dfrac{g'}{g} = \tan\theta_W$, $\qquad g\sin\theta_W = g'\cos\theta_W$

Standard Model Lagrangian (cont.)

With the rotated fields, we get:

$$\left[-\left(g\sin\theta_W J_3^\mu + \tfrac{1}{2}g'\cos\theta_W J_Y^\mu\right)A_\mu\right] \text{ correct form for em interaction}$$

$$+\left[\left(-g\cos\theta_W J_3^\mu + \tfrac{1}{2}g'\sin\theta_W J_Y^\mu\right)Z_\mu\right] \text{ something new ...}$$

First Term

$$= -g\sin\theta_W\,(\bar{\ell}\gamma^\mu\ell)A_\mu \qquad e = g\sin\theta_W$$

Second Term: $\dfrac{-g}{2\cos\theta_W}\left(2\cos^2\theta_W J_3^\mu - \dfrac{g'}{g}\cos\theta_W\sin\theta_W J_Y^\mu\right)Z_\mu$

$$= \frac{-g}{2\cos\theta_W}\left(2\cos^2\theta_W J_3^\mu - \frac{g}{g}\sin^2\theta_W J_Y^\mu\right)Z_\mu$$

$$= \frac{-g}{2\cos\theta_W}\left(2(1-\sin^2\theta_W)J_3^\mu - \sin^2\theta_W J_Y^\mu\right)Z_\mu$$

$J_Y^\mu = -(\bar{\nu}_L\gamma_\mu\nu_L + \bar{\ell}_L\gamma_\mu\ell_L + 2\bar{\ell}_R\gamma_\mu\ell_R)$, $J_3^\mu = \frac{1}{2}(\bar{\nu}_L\gamma_\mu\nu_L - \bar{\ell}_L\gamma_\mu\ell_L)$

note that $\sin^2\theta_W$ terms cancel for neutrinos

Standard Model Lagrangian (cont.)

For electrons we have for $\sin^2\theta_W$ terms:

$$+\bar{\ell}_L\gamma^\mu\ell_L + \bar{\ell}_L\gamma^\mu\ell_L + 2\bar{\ell}_R\gamma^\mu\ell_R = 2(\bar{\ell}_L\gamma^\mu\ell_L + \bar{\ell}_R\gamma^\mu\ell_R)$$

other terms for neutrinos: $\bar{\nu}_L\gamma^\mu\nu_L$

" " " electrons: $-\bar{\ell}_L\gamma^\mu\ell_L$

Which can be summarized as:

$$-\frac{g}{2\cos\theta_W}\sum_{\psi_i=\nu,\ell}\bar{\psi}_i\gamma^\mu(\underbrace{g_V^i}_{T_3^i-2Q_i\sin^2\theta_W} - \underbrace{g_A^i}_{T_3^i}\gamma^5)\psi_i Z_\mu$$

→ note that SM predicted neutral weak interactions which were observed ~5 years after model was proposed
more on that later...

Fermions and bosons are massless

Standard Model Lagrangian (cont.)

Back to the Higgs mechanism. We introduce the scalar doublet:

$$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \qquad \text{with } Y = 1$$
$$T = 1/2 \,, \quad T^3 = -1/2$$

$$\mathcal{L}_{scalar}: \partial_\mu \Phi^\dagger \partial^\mu \Phi - V(\Phi^\dagger \Phi)$$

$$V(\Phi^\dagger \Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

We introduce the gauge-covariant derivative:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + i g \frac{\tau^i}{2} W_\mu^i + i g' Y B_\mu$$

chose VEV: $\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$, $v = \sqrt{\frac{-\mu^2}{\lambda}}$

$\rightarrow$ we need to keep $U(1)$ a symmetry of the vacuum (keep photon massless)

$$Q \langle \Phi \rangle_0 = (T_3 + \tfrac{1}{2} Y) \langle \Phi \rangle_0 = \frac{1}{2} \left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0$$

Standard Model Lagrangian (cont.)

As you've done in problem set 1, we parametrize the Higgs doublet

$$\Phi \equiv \exp\left(\frac{i\tau^i \chi_i}{2v}\right) \begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix}.$$

we make a gauge trans. $\Phi \to \Phi' = \exp\left(-i\frac{\tau^i}{2v}\chi_i\right)\Phi$

$$= \frac{(v+\eta)}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\mathcal{L}_{scalar}: \left| \left(\partial_\mu + ig\frac{\tau^i}{2}W_\mu^i + i\frac{g'}{2}Y B_\mu\right) \frac{(v+\eta)}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2$$

$$-\frac{\mu^2}{2}(v+\eta)^2 - \frac{\lambda}{4}(v+\eta)^4$$

$$\left| \begin{pmatrix} 0 \\ \partial_\mu \eta/\sqrt{2} \end{pmatrix} + \frac{ig}{2}(v+\eta) \begin{pmatrix} W_\mu^+ \\ \frac{-1}{\sqrt{2}\cos\theta_W} Z_\mu \end{pmatrix} \right|^2$$

Standard Model Lagrangian (cont.)

note:

$$-gW_\mu^3 + g'B_\mu$$

$$= -g\sin\theta_W A_\mu - g\cos\theta_W Z_\mu + g'\cos\theta_W A_\mu - g'\sin\theta_W Z_\mu$$

$$= -g\sin\cancel{\theta_W} A_\mu - g\cos\theta_W Z_\mu + g\sin\cancel{\theta_W} A_\mu - g'\sin\theta_W Z_\mu$$

$$= \frac{-g}{\cos\theta_W}\left(\cos\theta_W Z_\mu + \frac{g'}{g}\sin\theta_W \cos\theta_W Z_\mu\right)$$

$$= \frac{-g}{\cos\theta_W}\left(\cos^2\theta_W Z_\mu + g\sin\theta_W \frac{1}{g}\sin\theta_W Z_\mu\right)$$

$$= \frac{-g}{\cos\theta_W} Z_\mu$$

so:

$$\left|\begin{pmatrix} 0 \\ \partial_\mu\eta/\sqrt{2}\end{pmatrix} + \frac{ig}{2}(v+\eta)\begin{pmatrix} W_\mu^+ \\ \frac{-1}{\sqrt{2}\cos\theta_W} Z_\mu\end{pmatrix}\right|^2 \quad \text{becomes:}$$

$$\partial^\mu\eta\,\partial_\mu\eta + \frac{g^2}{4}(v+\eta)^2\left(W_\mu^+ W^{-\mu} + \frac{1}{2\cos^2\theta_W} Z_\mu Z^\mu\right)$$

Standard Model Lagrangian (cont.)

The quadratic terms give:

$$\frac{g^2 v^2}{4} W_\mu^+ W^{-\mu} + \frac{g^2 v^2}{4} \cdot \frac{1}{2\cos^2\theta_W} Z_\mu Z^\mu$$

with $W_\mu^\pm = \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp i W_\mu^2 \right)$

note that boson mass terms in Lagrangians have the form

$\frac{1}{2} m^2 B_\mu B^\mu$. So $\Rightarrow$ $\boxed{M_W = \frac{gv}{2} \quad , \quad M_Z = \frac{gv}{2\cos\theta_W}}$

$M_W = M_Z \cos\theta_W$

with $v = \left(\sqrt{2} G_F\right)^{1/2}$ measured to be $\simeq 246$ GeV

and $\sin^2\theta_W \sim 0.22$, we get

$$M_W^2 = \frac{e^2 v^2}{4\sin^2\theta_W} = \frac{\pi\alpha}{\sin^2\theta_W} v^2 \simeq \left(\frac{37.2 \text{ GeV}}{\sin\theta_W} \right)^2 \simeq (80 \text{ GeV})^2$$

$$M_Z^2 = \left(\frac{37.2 \text{ GeV}}{\sin\theta_W \cos\theta_W} \right)^2 \simeq (90 \text{ GeV})^2$$

Standard Model Lagrangian (cont.)

The terms involving the scalar field:

$$\underbrace{-\frac{1}{2}(-2\mu^2)\eta^2}\ +\ \frac{1}{4}\mu^2 v^2\left(\frac{4}{v^3}H^3 + \frac{1}{v^4}H^4 - 1\right)$$

$\rightarrow m_H = \sqrt{-2\mu^2}$

$\longrightarrow$ no prediction for Higgs mass

Another way to get the mass terms for Z^μ and A^μ:

$$\mathcal{L}_{scalar}:\ \frac{v^2}{2}\left|\left(g\frac{\tau^3}{2}W^3_\mu + \frac{g'}{2}YB_\mu\right)\begin{pmatrix}0\\1\end{pmatrix}\right|^2$$

$$\frac{v^2}{8}\left[(B_\mu\ W^3_\mu)\begin{pmatrix}g'^2 & -gg'\\ -gg' & g^2\end{pmatrix}\begin{pmatrix}B^\mu\\ W_3^\mu\end{pmatrix}\right]$$

Two eigenvalues for the mass matrix: 0, $\frac{1}{2}\frac{(g^2+g'^2)v^2}{4}$

$\hookrightarrow = m_A \qquad\qquad = \frac{1}{2}m_Z^2$

Standard Model Lagrangian (cont.)

The ρ parameter : $$\rho = \frac{M_W^2}{\cos^2\theta_W M_Z}$$

represents the relative strengths of the neutral and charged effective currents:

$$J^{0\mu} J^0_\mu \,/\, J^{+\mu} J^-_\mu$$

$\rho = 1$ in the SM (at tree level)

In model with arbitrary number of Higgs multiplets φ_i with isospin T_i and 3rd component T^3_i and VEV v_i,

ρ is given by : $$\frac{\sum_i \left[T_i (T_i + 1) - (T_{3i})^2 \right] v_i^2}{2 \sum_i (T_{3i})^2 v_i^2}$$

will give $\rho = 1$ for doublets

$\rightarrow$ Tests isospin structure of Higgs sector

Standard Model Lagrangian (cont.)

Lepton masses:

We saw that adding explicit mass terms in the Lagrangian breaks gauge invariance. We therefore add another term:

$$\mathcal{L}_{Yukawa} = -G_\ell \left[\bar{R} (\varphi^\dagger L) + (\bar{L} \varphi) R \right]$$

$$= -G_\ell \frac{(v+\eta)}{\sqrt{2}} \left[\bar{\ell}_R (0 \;\; 1) \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} + (\bar{\nu}_L \;\; \bar{\ell}_L) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \ell_R \right]$$

$$= -\frac{G_\ell v}{\sqrt{2}} \bar{\ell}\ell - \frac{G_\ell}{\sqrt{2}} \bar{\ell}\ell\eta$$

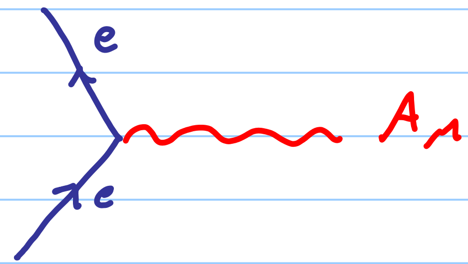
we get $m_\ell = \dfrac{G_\ell v}{\sqrt{2}}$ $\rightarrow$ no prediction for G_ℓ

$\rightarrow G_{Top} = 1$ (within 1%)

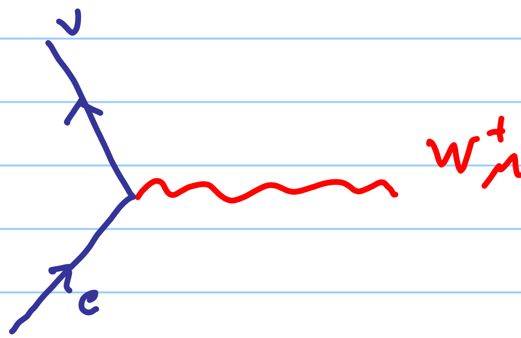
coupling strength $\bar{\ell}\ell H = \dfrac{m_\ell}{v}$ $\rightarrow$ $\boxed{\text{a Testable prediction}}$

The Standard Model (cont.)

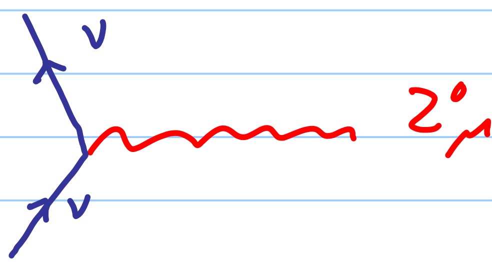
From the previous pieces of the SM Lagrangian we can deduce the following Feynman rules:



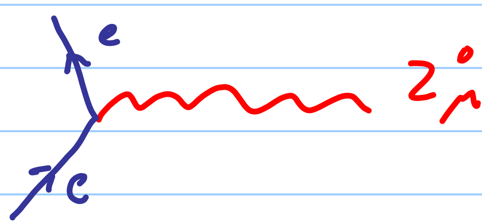
$$= ie\,\bar{e}\gamma_\mu e$$



$$-i\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{1/2} \bar{\nu}\gamma_\mu(1-\gamma_5)e$$



$$\frac{-i}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{1/2} \bar{\nu}\gamma_\mu(1-\gamma_5)\nu$$



$$\frac{-i}{\sqrt{2}}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)^{1/2} \bar{e}\gamma_\mu\left[2s_W^2(1+\gamma_5) + (2s_W^2-1)(1-\gamma_5)\right]e$$

$$s_W = \sin\theta_W$$

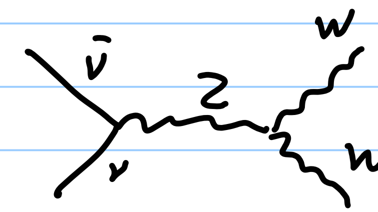
The Standard Model (cont.)

$$ie \cot\theta_W \left[g_{\alpha\beta}(k_+ - k_-)_\mu - g_{\alpha\mu}(p + k_+)_\beta + g_{\beta\mu}(p + k_-)_\alpha \right]$$

In lecture 9 we saw that the $\nu\bar{\nu} \to WW$ cross section was not well behaved at high energies:

$$\sigma \sim \frac{G_F^2 s}{3\pi}$$

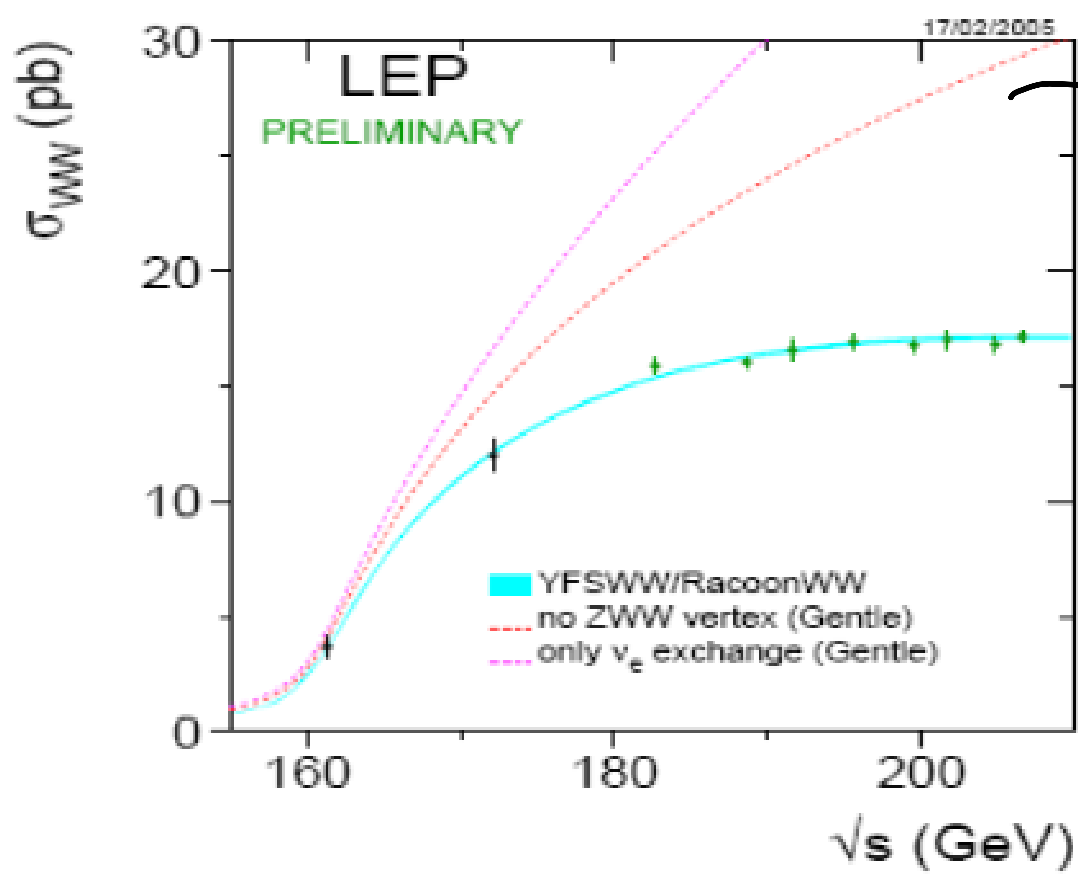
We know we were missing one diagram:



$$M_Z = \frac{-ig^2}{4(s - M_Z^2)} \bar{v}\gamma_\mu (1-\gamma_5) u \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{M_Z^2} \right) \times \varepsilon_+^{*\alpha} \varepsilon_-^{*\beta}$$
$$\left[g_{\alpha\beta}(k_- - k_+)_\nu + g_{\alpha\nu}(k_+ + p)_\beta - g_{\beta\nu}(k_- + p)_\alpha \right]$$

PROBLEM SET 2, PROBLEM #1:
show that extra diagram cures our problem.

$e^+e^- \to WW$ cross section from LEP II :



adding photon