

LECTURE 13: Z Boson Physics and Neutral Currents (Part 2)

Overview:

- A closer look at the structure of the neutral current
- Neutral currents and neutrino scattering
- Forward-backward asymmetries

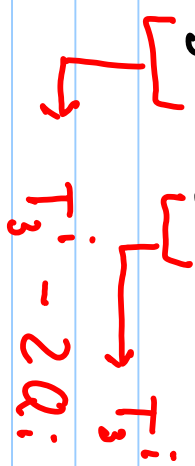
(I used Quigg and Halzen-Martin as references)

NEUTRAL CURRENT

②

We wrote the NC as:

$$-\frac{g}{2 \cos \theta_w} \sum_i \bar{\psi}_i \gamma^\mu (g_V^i - g_A^i \gamma^5) \psi_i Z_\mu$$



Electrons: $T_3 = -1/2$, $Q = -1$
 neutrinos: $T_3 = 1/2$, $Q = 0$

$$\Rightarrow \text{electrons: } -\frac{g}{2 \cos \theta_w} \bar{e} \gamma^\mu \left[-1/2 + 2 \sin^2 \theta_w + \frac{1}{2} \gamma^5 \right] e Z_\mu \quad \text{①}$$

we could write: ① = $\frac{+g}{2 \cos \theta_w} \cdot \frac{1}{2} \cdot \bar{e} \gamma^\mu [1 - 4 \sin^2 \theta_w - \gamma^5] e Z_\mu$

$$\text{or } = \frac{-g}{2 \cos \theta_w} \frac{1}{2} \bar{e} \gamma^\mu [-1 + 4 \sin^2 \theta_w + \gamma^5] e Z_\mu$$

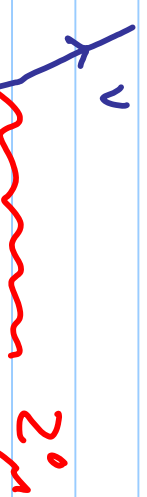
$$= \frac{-g}{2 \cos \theta_w} \cdot \frac{1}{2} \cdot \bar{e} \gamma^\mu \left[\underbrace{2 \sin^2 \theta_w (1 + \gamma^i)}_{Re} + \underbrace{(2 \sin^2 \theta_w - 1) (1 - \gamma^i)}_{Le} \right] e Z_\mu$$

$$\Rightarrow \text{neutrinos: } \frac{-g}{2 \cos \theta_w} \bar{\nu} \gamma^\mu \frac{1}{2} (1 - \gamma^i) \nu Z_\mu$$

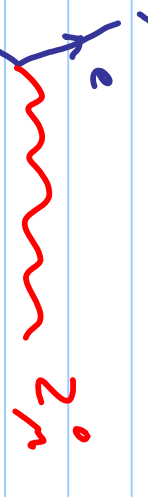
NEUTRAL CURRENT (cont.)

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From which we get the vertex factors:



$$-\frac{i}{\sqrt{2}} \left(\frac{GF M_Z^2}{\sqrt{2}} \right)^{1/2} \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu$$



$$\frac{-i}{\sqrt{2}} \left(\frac{GF M_Z^2}{\sqrt{2}} \right)^{1/2} \bar{e} \gamma_\mu [2s_w^2 (1 + \gamma_5) + (2s_w^2 - 1) (1 - \gamma_5)] e$$

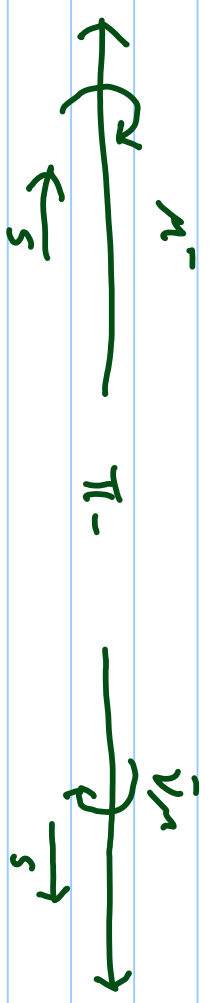
$$s_w = \sin \theta_w$$

Pion decay interlude:

Pion decay width: $\Gamma = \frac{F_\pi^2}{\pi M_\pi^3} \left(\frac{g_w}{4M_W} \right)^4 M_\pi^2 (M_\pi^2 - M_\pi^2)^2$

$$\frac{\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} = 1.3 \times 10^{-4} \quad \text{why?}$$

Hint: pion has spin 0, $\bar{\nu}$ is right-handed \Rightarrow electron must be right-handed... !?



→ have to take some care when we talk about "handedness" and helicity. Helicity is not a Lorentz invariant. Also note that the required addition of v_R in the SM has almost no impact on the results we've obtained so far since m_ν is so small.

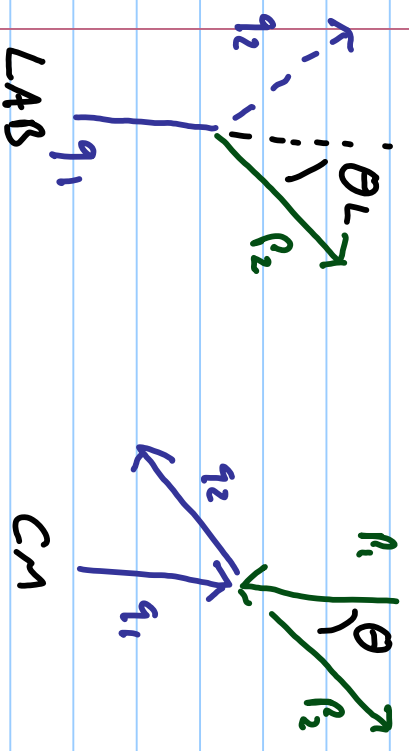
Consider the reactions:

$$\bar{\nu}_e \rightarrow \bar{\nu}_e \quad] \text{ FROM REACTORS}$$

$$\bar{\nu}_e \rightarrow \bar{\nu}_e$$

$$\nu_e \rightarrow \nu_e$$

} FROM ACCELERATORS



NEUTRAL CURRENTS (cont.)

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4 vectors: in LAB

$$q_1^m = (E, 0, 0, E)$$

$$p_1^m = (m, 0, 0, 0)$$

$$q_2^m = (E + m - E', -p \sin \theta_L, 0, -p \cos \theta_L)$$

$$p_2^m = (E', p \sin \theta_L, 0, p \cos \theta_L)$$

$$E' = \sqrt{p'^2 + m^2} \equiv \gamma E$$

Invariants: in LAB

$$(p_1 \cdot q_1) = (p_2 \cdot q_2) = mE$$

$$(p_1 \cdot q_2) = (p_2 \cdot q_1) \approx mE(1 - \gamma)$$

$$(p_1 \cdot p_2) = m\gamma E$$

$$(q_1 \cdot q_2) = m(E' - m) \approx mE\gamma$$

in CM

$$q_1^m = (p^*, 0, 0, p^*)$$

$$p_1^m = (w^*, 0, 0, -p^*)$$

$$q_2^m = (p^*, -p^* \sin \theta, 0, -p^* \cos \theta)$$

$$p_2^m = (w^*, p^* \sin \theta, 0, p^* \cos \theta)$$

in CM

$$= p^* (w^* + p^*) \approx 2p^{*2}$$

$$= p^* (w^* - p^* \cos \theta) \approx p^{*2} (1 - \cos \theta)$$

$$\approx p^{*2} (1 + \cos \theta)$$

$$= p^{*2} (1 + \cos \theta)$$

NEUTRAL CURRENTS (CONT.)

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NOTE FROM THE ABOVE THAT

$$ME \approx 2p^{2*}$$

$$p^{*2} (1 - \cos \Theta) \approx ME (1 - \gamma) \Rightarrow \boxed{1 - \cos \Theta \approx 2(1 - \gamma)}$$

In Quigg's book, a general matrix element for charged-current interaction is studied: $\sum M_i = \sum C_i \bar{\psi} O_i \psi$
 O_i can be vector, scalar, axial vector, Tensor, pseudoscalar. We use some of the results in the following.

V-A interaction for $\nu_e e \rightarrow \nu_e e$ (we'll deal with constants

$$M = \bar{\nu}_\nu \gamma_\mu (1 - \gamma_5) \nu_e \quad \bar{\nu}_e \gamma_\mu (1 - \gamma_5) \nu_\nu$$

and propagators and other details later)

$$|M|^2 = \text{Tr} [\gamma_\mu (1 - \gamma_5) (\not{p}_1 + m) \gamma_\nu (1 - \gamma_5) \not{p}_2]$$

$$\times \text{Tr} [\gamma^\mu (1 - \gamma_5) \not{q}_1 \gamma^\nu (1 - \gamma_5) (\not{q}_2 + m)]$$

NEUTRAL CURRENTS (cont.)

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$$\text{TRACE 1: } 2\text{Tr} [(0 + \gamma_5) \gamma_\nu q_2 \gamma_\mu (p_1 + m)]$$

$$= 8 (q_{2\nu} p_{1\mu} - g_{\mu\nu} (q_2 \cdot p_1) + g_{2\mu} p_{1\nu}) - 8 \epsilon_{\mu\nu\rho\sigma} q_2^\rho p_1^\sigma$$

$$\text{TRACE 2: } 2\text{Tr} [(1 + \gamma_5) \gamma^\mu q_1 \gamma^\nu (p_2 + m)]$$

$$= 8 (q_1^\mu p_2^\nu - g^{\mu\nu} (q_1 \cdot p_2) + q_1^\nu p_2^\mu) + 8 \epsilon^{\mu\nu\kappa\lambda} q_{1\kappa} p_{2\lambda}$$

$$|M|^2 = 128 (q_{1\cdot} q_{2\cdot} p_{1\cdot} p_{2\cdot} + q_{1\cdot} p_{1\cdot} q_{2\cdot} p_{2\cdot})$$

$$- 64 i \epsilon_{\mu\nu\rho\sigma} q_2^\rho p_1^\sigma (q_1^\nu p_2^\mu + q_1^\mu p_2^\nu)$$

$$+ 64 i \epsilon^{\mu\nu\kappa\lambda} q_{1\kappa} p_{2\lambda} (q_{2\nu} p_{1\mu} + q_{2\mu} p_{1\nu})$$

$$\left[+ 64 \epsilon_{\mu\nu\rho\sigma} \epsilon^{\mu\nu\kappa\lambda} q_2^\rho p_1^\sigma q_{1\kappa} p_{2\lambda} \right]$$

$$= 128 (q_{1\cdot} p_{1\cdot} q_{2\cdot} p_{2\cdot} - q_{1\cdot} q_2 \cdot p_{1\cdot} p_{2\cdot})$$

$$\Rightarrow |M|^2 = 256 (q_{1\cdot} p_{1\cdot} q_{2\cdot} p_{2\cdot} = 256 (mE)^2)$$

= 0
3 indep.
moments

NEURAL CURRENTS

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$$\frac{d\sigma}{d\Omega_{cm}} = \frac{|M|^2}{64\pi^2 s}, \quad s = (p_1 + q_1)^2 \approx 2(p_1 \cdot q_1)$$

note Factor of $\frac{1}{2}$ for average spin of e
 $\approx 4 p_1^2 = 2mE$

$$\sigma = \frac{4mE}{\pi} \quad \text{note that} \quad \frac{d\sigma}{dy} = \frac{4mE}{\pi}$$

$$\frac{d\sigma}{dy} = 4\pi \frac{d\sigma}{d\Omega}$$

V-A interaction for $\bar{\nu}_0 e \rightarrow \bar{\nu}_e e$

we need to change $q_1 \leftrightarrow q_2$

$$\begin{aligned} \text{this give } |M|^2 &= 256 q_2 \cdot p_1 q_1 \cdot p_2 \\ &= 256 (mE)^2 (1-y)^2 \end{aligned}$$

NEUTRAL CURRENTS (cont.)

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$$\begin{aligned} \frac{d\sigma}{dy} &= 256 (nE)^2 (1-y)^2 \cdot \frac{4\pi}{64\pi^2} \cdot 2nE \cdot \frac{1}{2} \\ &= \frac{4nE}{11} (1-y)^2, \quad \sigma = \int_0^1 \frac{4nE}{11} (1-y)^2 dy \\ &= \frac{4nE}{3\pi} \end{aligned}$$

V+A for $\nu_{ne} \rightarrow \nu_{ne}$

$$M = \bar{\nu}_\nu \gamma_\mu (1+\gamma_5) \nu_e \bar{\nu}_e \gamma_\mu (1-\gamma_5) \nu_e$$

This changes the sign of the $\epsilon_{\mu\nu\kappa\sigma}$ Term

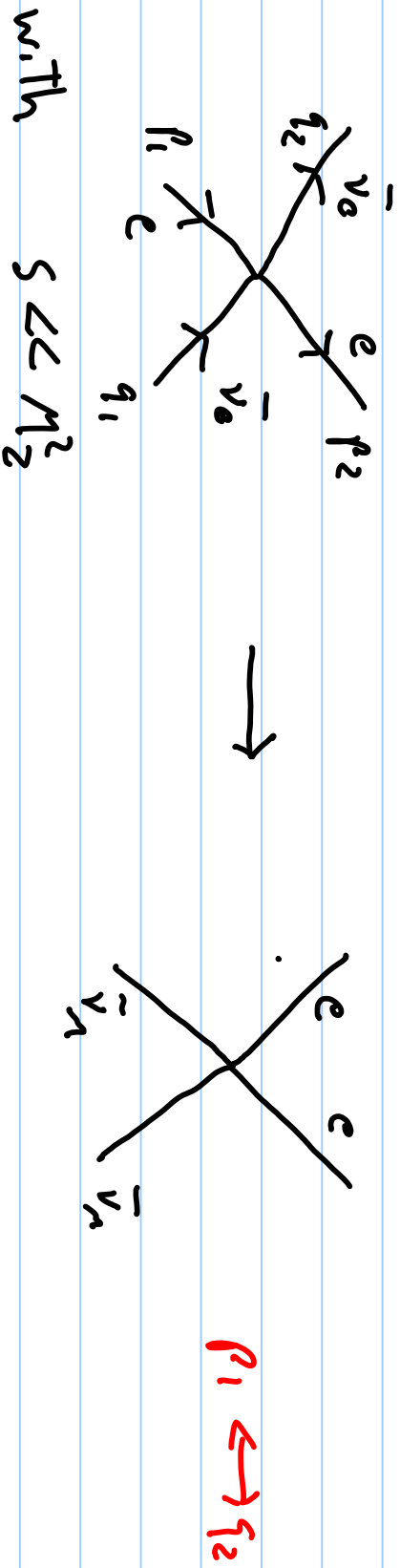
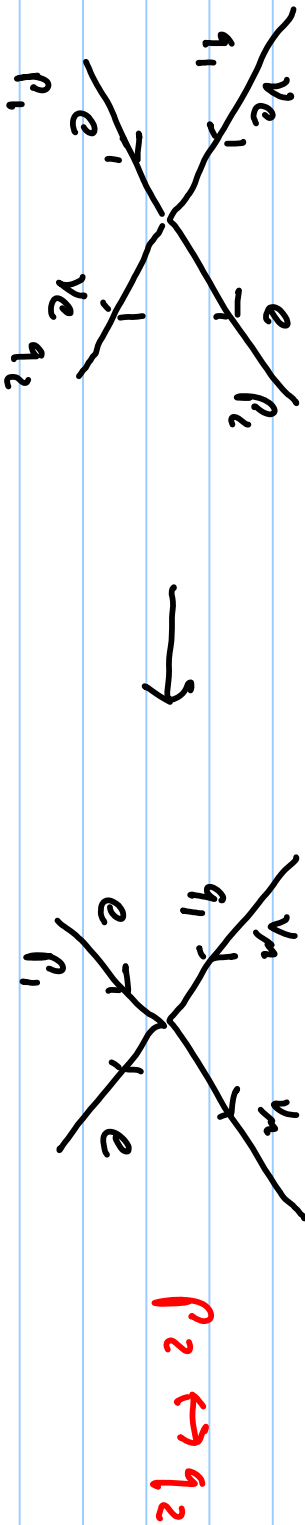
which gives $|M|^2 = 256 q_1 \cdot q_2 p_1 \cdot p_2 = 256 (nE)^2 y^2$

$$\frac{d\sigma}{dy} = \frac{4nE}{11} y^2 \rightarrow \sigma = \frac{4nE}{3\pi}$$

NEUTRAL CURRENTS (cont.)

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Now, To get actual SM processes with $s \ll M_Z^2$:



with $s \ll M_Z^2$

We get:

$$\frac{\mathbb{I}}{ME} \frac{d\sigma}{dy} (v_1 e \rightarrow v_2 e) \quad \frac{\mathbb{I}}{ME} \frac{d\sigma}{dy} (\bar{v}_1 e \rightarrow \bar{v}_2 e)$$

$ V-A ^2$	$ V+A ^2$
4	4
$4(1-y)^2$	$4(1-y)^2$
4	4

Neutral currents (cont.)

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$$\text{Using } \frac{-i}{\sqrt{2}} \left(\frac{G_F M_Z^2}{\sqrt{2}} \right)^{1/2} \bar{e} \gamma_\mu [R_e (1+\gamma_5) + L_e (1-\gamma_5)] e$$

The processes $e \nu_\mu \rightarrow e \nu_\mu$, $e \bar{\nu}_\mu \rightarrow e \bar{\nu}_\mu$ give:

$$\frac{d\sigma}{dy} (\nu_\mu e) = \frac{4 M^2 E}{11} \frac{G_F^2}{8} [L_e^2 + R_e^2 (1-y)^2] \quad (\text{cross terms cancel} \rightarrow (1-\gamma_5)(1+\gamma_5) = 0)$$

$$= \frac{G_F^2 M^2 E}{2\pi} [12 \sin^2 \theta_W - 1]^2 + 4 \sin^4 \theta_W (1-y)^2]$$

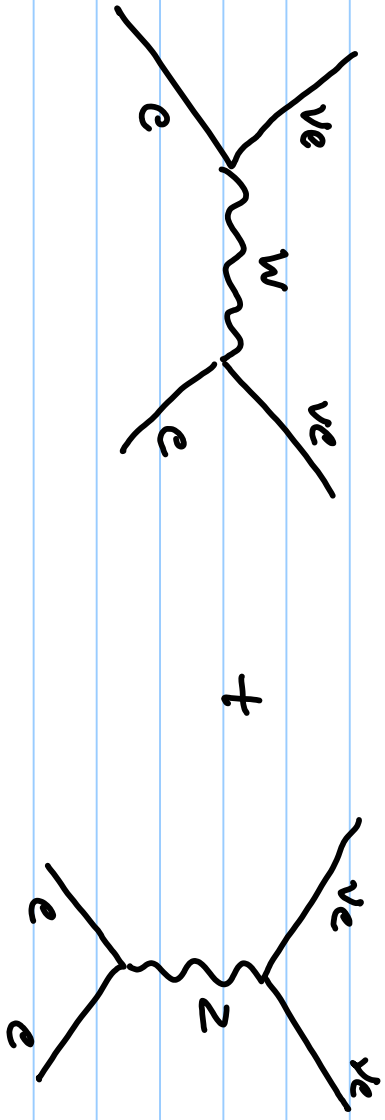
$$\frac{d\sigma}{dy} (\bar{\nu}_\mu e) = \frac{G_F^2 M^2 E}{2\pi} [L_e^2 (1-y)^2 + R_e^2]$$

$$\sigma(\nu_\mu e) = \frac{G_F^2 M^2 E}{2\pi} \left(L_e^2 + \frac{R_e^2}{3} \right)$$

$$\sigma(\bar{\nu}_\mu e) = \frac{G_F^2 M^2 E}{2\pi} \left(\frac{L_e^2}{3} + R_e^2 \right)$$

NEURAL CURRENTS AND NEUTRINO SCATTERING

$\nu_e e \rightarrow \nu_e e$: Two contributions



Fierz reordering Theorem:

$$- [\bar{e} \gamma^\mu (1 - \gamma^5) \nu_e] [\bar{\nu}_e \gamma_\mu (1 - \gamma^5) e] = [\bar{\nu}_e \gamma^\mu (1 - \gamma^5) \nu_e] [\bar{e} \gamma_\mu (1 - \gamma^5) e]$$

note sign

$\nu_e e \rightarrow \nu_e e$ is obtained from $\nu_n e \rightarrow \nu_n e$ with: $L_e \rightarrow L_e + 2$

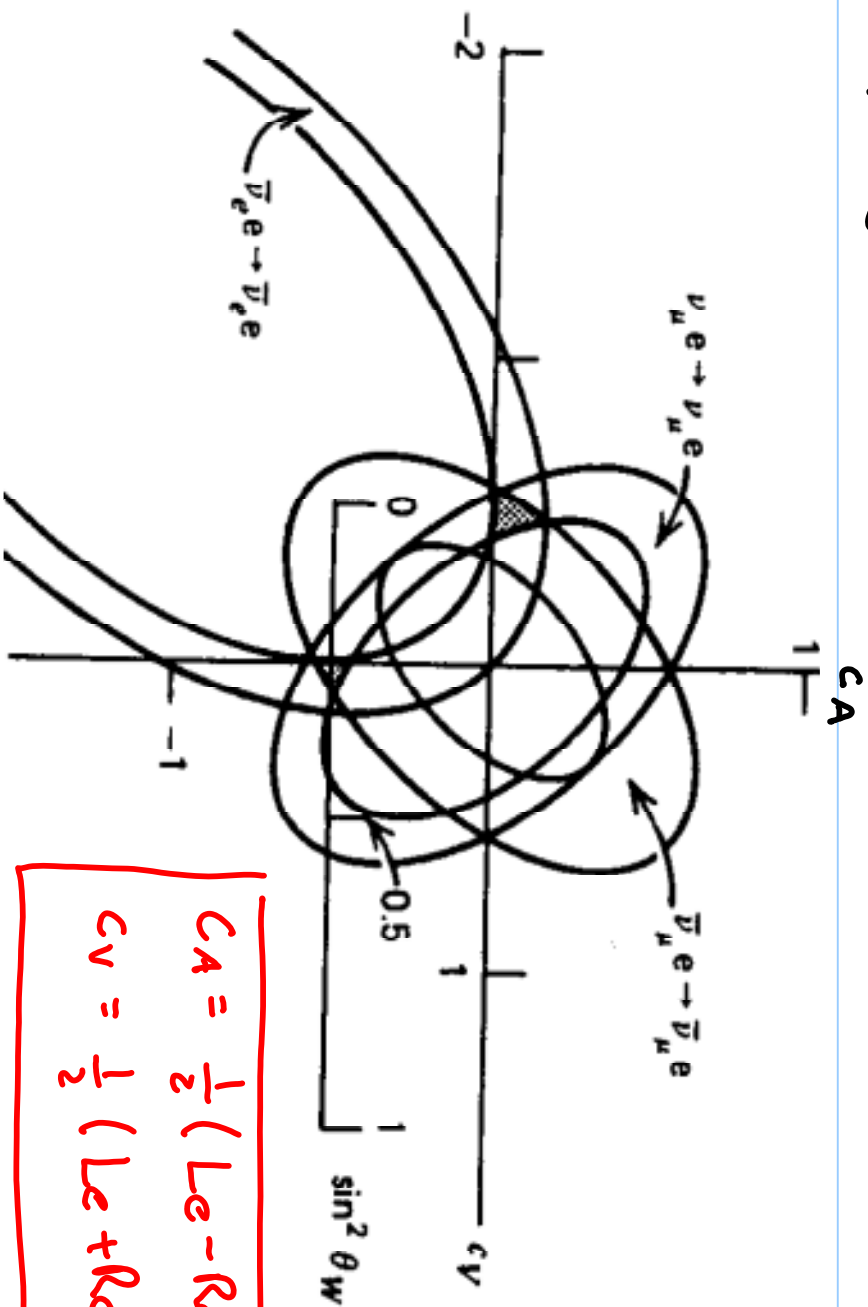
and $\bar{\nu}_e e$ is obtained from $\bar{\nu}_n e \rightarrow \bar{\nu}_n e$ with \rightarrow

we have : $\sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e) = \frac{G_F^2 m^2 E}{2\pi} \left[\frac{(L_e + 2)^2 + R_e^2}{3} \right]$

$g_A \rightarrow g_A + 1$
 $g_V \rightarrow g_V + 1$
 $g_A = \frac{1}{2} (L_e - R_e)$, $g_V = \frac{1}{2} (L_e + R_e)$

Neutral currents (cont.)

Putting everything together with experimental results we get:



$$c_A = \frac{1}{2} (L_e - R_e)$$

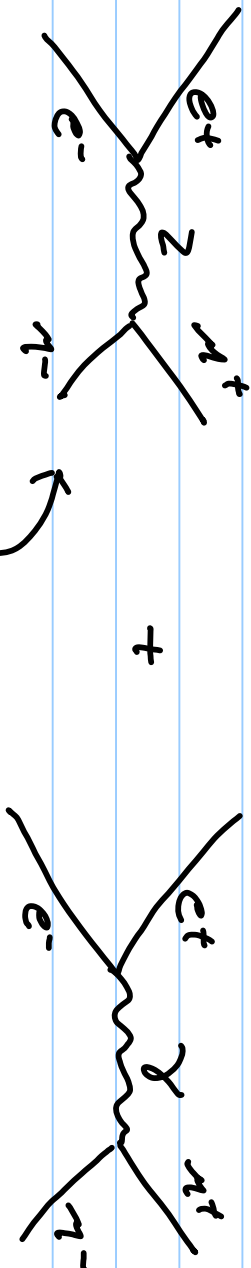
$$c_V = \frac{1}{2} (L_e + R_e)$$

Note that there are two solutions.
How do we determine which one is correct?

NEUTRAL CURRENTS (cont.)

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We can study $e^+e^- \rightarrow \mu^+\mu^-$ for example. Two diagrams contribute:



You are calculating this in problem set. Here we need the γ and γ^2 terms in T_{00} .

We then measure the forward-backward asymmetry defining $z = \cos \theta_{cm}$

$$A_{FB} \equiv \frac{\int_0^1 dz \frac{d\sigma^0}{dz} - \int_{-1}^0 dz \frac{d\sigma^0}{dz}}{\int_{-1}^1 dz \frac{d\sigma^0}{dz}}$$

$$\lim_{s/M_Z^2 \rightarrow 0} A_{FB} \propto (\text{Re} - \text{Im}) (\text{Re} - \text{Im}) \propto c_A^2$$

Resolves ambiguity, measures $\sin^2 \theta_w$

