

LECTURE 14: Z Boson Physics and Neutral Currents (Part 3)

Overview:

- Production asymmetries
- Neutral current and quarks

(I used Quigg and Halzen-Martin, K. Graham's thesis as references)

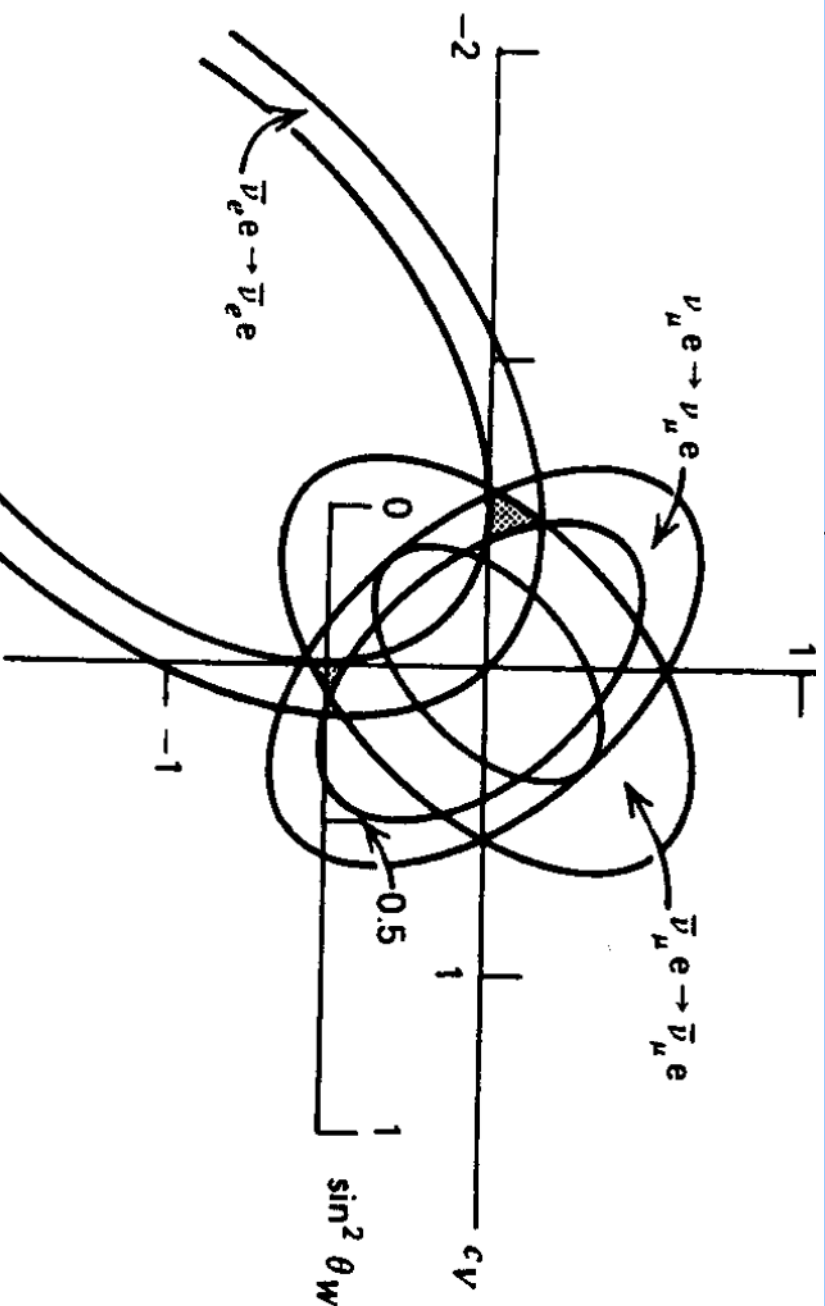
NEUTRAL CURRENT AND ASYMMETRIES

Last lecture we calculated the cross sections for the processes: $(s \ll M_Z^2)$



We saw that the measurements gave two values for C_V and C_A .

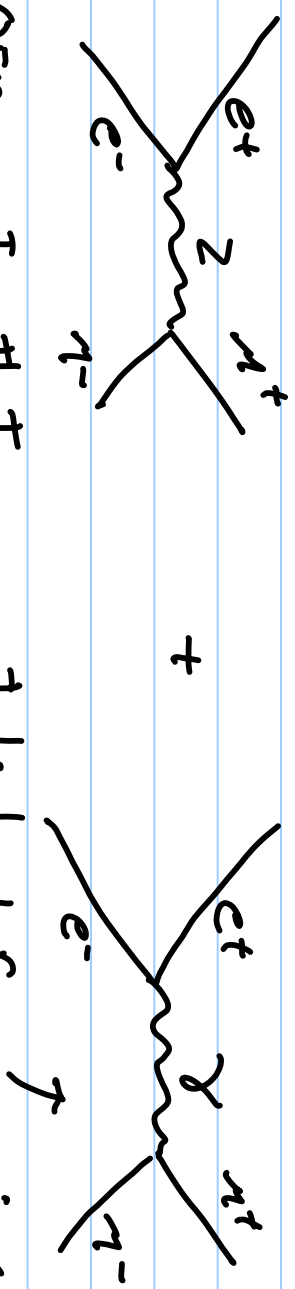
$$\frac{g}{2 \cos \theta_w} \sum_i \bar{\psi}_i \gamma^\mu (c_V^i - c_A^i \gamma_5) \psi_i \quad Z_\mu$$



NEUTRAL CURRENT AND ASYMMETRIES

We saw that this ambiguity could be removed with an asymmetry measurement. Let's look at this in more detail.

Two diagrams contribute to $e^+e^- \rightarrow \mu^+\mu^-$:



The QED part that we studied before gives us:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} (1 + \cos^2\theta) , \quad \sigma = \frac{4\pi\alpha^2}{3s} \quad (\text{Full calculation in lecture 4})$$

You are calculating the pure Z contribution.

The full calculation involves an interference term.

NEUTRAL CURRENT AND ASYMMETRIES

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$$M_Y = \frac{-e^2}{k^2} [\bar{u} \gamma^\nu v] [\bar{e} \gamma_\nu e]$$

$$M_Z = \frac{-\sqrt{2} G M_Z^2}{s - M_Z^2} [R_\mu (\bar{\mu}_R \gamma^\nu \mu_L) + L_\mu (\bar{\mu}_L \gamma^\nu \mu_L)] [R_e (\bar{e}_R \gamma_\nu e_L) + L_e (\bar{e}_L \gamma_\nu e_L)]$$

$$L_C \equiv C_V + C_A = g_V + g_A, \quad R_C \equiv C_V - C_A = g_V - g_A$$

$$\text{i.e. } C_V - C_A \gamma^5 = (C_V - C_A) \frac{1}{2} (1 + \gamma^5) + (C_V + C_A) \frac{1}{2} (1 - \gamma^5)$$

Doing the full calculation yields:

$$\frac{d\sigma}{d\Omega} (\bar{e} \bar{e} e^+ \rightarrow \mu \bar{\mu} \mu e^+) = \frac{\alpha^2}{4s} (1 + \cos)^2 |1 + r L e L_\mu|^2$$

$$\frac{d\sigma}{d\Omega} (\bar{e} \bar{e} e^+ \rightarrow \mu \bar{\mu} \mu e^+) = \frac{\alpha^2}{4s} (1 - \cos)^2 |1 + r R e R_\mu|^2$$

+ 2 other helicity contributions

$$r = \frac{\sqrt{2} G M_Z^2}{s - M_Z^2 + i M_Z \Gamma_Z} \left(\frac{s}{e^2} \right) \quad \text{note that } R_e = R_\mu, L_e = L_\mu$$

NEUTRAL CURRENT AND ASYMMETRIES

Results above averaged over all helicities give:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \left[A_0 (1 + \cos^2 \theta) + A_1 \cos \theta \right]$$

↪ symmetric wrt $\cos \theta$
↪ asymmetric wrt $\cos \theta$

$$A_0 = 1 + \frac{1}{2} \operatorname{Re}(r) (L_e + R_e)^2 + \frac{1}{4} |r|^2 (L_e^2 + R_e^2)^2$$

$$= 1 + 2 \operatorname{Re}(r) C_V^2 + |r|^2 (C_V^2 + C_A^2)$$

$$A_1 = \operatorname{Re}(r) (L_e - R_e)^2 + \frac{1}{2} |r|^2 (L_e^2 - R_e^2)^2$$

$$= 4 \operatorname{Re}(r) C_A^2 + 8 |r|^2 C_V C_A^2$$

Note that with QED only we get: $A_0 = 1$, $A_1 = 0$
giving a symmetric distribution.

The weak interaction introduces an asymmetry in the $\cos \theta$ distribution \rightarrow a "forward-backward" asymmetry

NEUTRAL CURRENT AND ASYMMETRIES

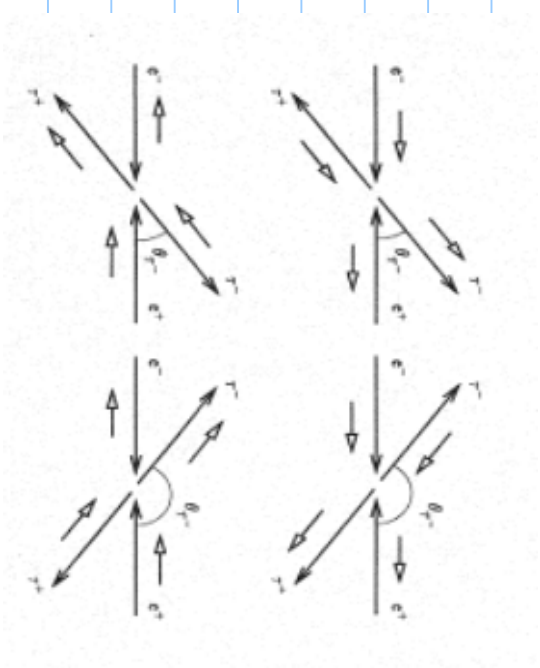
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$$A_{FB} \equiv \frac{\int_0^1 dz \frac{d\sigma}{dz} - \int_1^0 dz \frac{d\sigma}{dz}}{\int_{-1}^1 dz \frac{d\sigma}{dz}}$$

For $s \ll M_Z^2 \rightarrow |r| \ll 1$

$$A_{FB} = \frac{A_1}{8A_0/3} \approx \frac{3}{2} \text{Re}(\lambda^2) \approx -\frac{3}{\sqrt{2}} C_A^2 (G_S)$$

$e^+e^- \rightarrow \gamma\gamma$



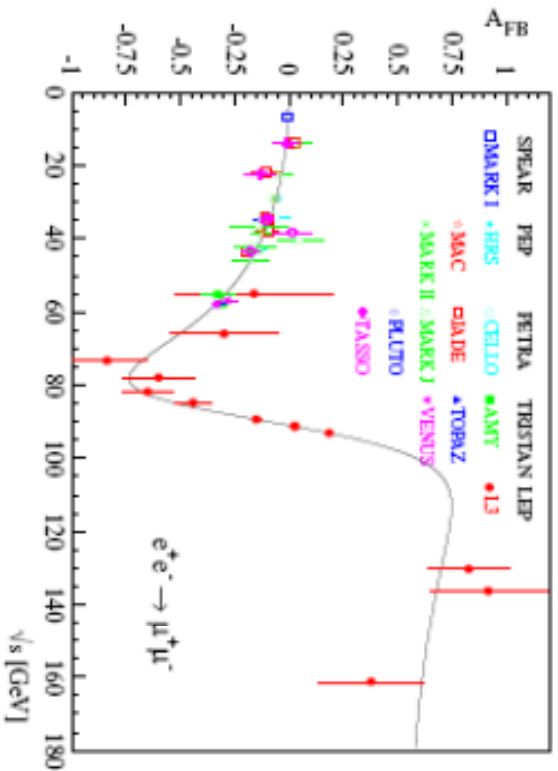
\rightarrow does not offer accurate determination

of $\sin^2 \theta_w$ (since $C_V \approx 0$). We integrate $d\sigma$ and get $\sigma = \sigma_0 A_0$

where $\sigma_0 \equiv$ QED cross section

$$R_\mu \equiv \frac{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}{\sigma_0} =$$

$$1 + 2 \text{Re}(|r| C_V^2 + |r|^2 (C_V^2 + C_A^2)^2)$$



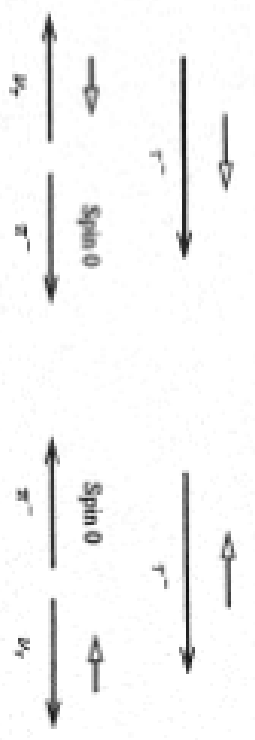
NEUTRAL CURRENT AND ASYMMETRIES

AT the Z pole, we get very high values of R_n (~150) since r hits its maximum.

$$A_{FB}^{s\nu\nu^2} = 3 \frac{c_V c_A^2}{(c_V^2 + c_A^2)^2}$$

This can get more interesting if we can measure the polarization of the final state leptons. This can be done with $e^+e^- \rightarrow \gamma^*\gamma^*$

→ The τ decays very quickly via the charged current interaction → ν_τ will be left-handed. P_{ion} momentum depends on whether we started with γ_L or γ_R



$$P_\gamma \equiv \frac{(\sigma_R - \sigma_L)}{(\sigma_R + \sigma_L)}$$

σ_R (σ_L) is σ for producing right (left)-handed γ^-
 → $P_\gamma = 0$ if parity is conserved.

NEUTRAL CURRENT AND ASYMMETRIES

$$\frac{1}{\sigma_{\text{Tot}}} \frac{d\sigma_R}{d\cos\theta_{\gamma^-}} = \frac{3}{16} [(1 + \langle P_T \rangle)(1 + \cos^2\theta_{\gamma^-}) + \frac{8}{3} (A_{\text{FB}} + A_{\text{pol}}^{\text{FB}}) \cos\theta_{\gamma^-}]$$

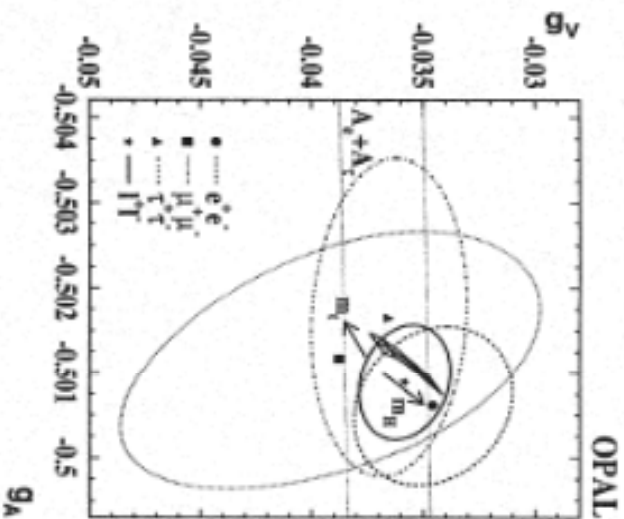
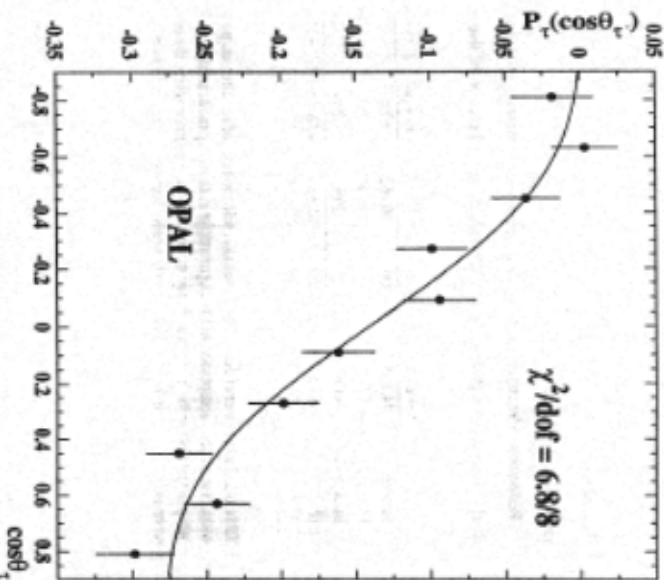
$$\frac{1}{\sigma_{\text{Tot}}} \frac{d\sigma_L}{d\cos\theta_{\gamma^-}} = \frac{3}{16} [(1 - \langle P_T \rangle)(1 + \cos^2\theta_{\gamma^-}) + \frac{8}{3} (A_{\text{FB}} - A_{\text{pol}}^{\text{FB}}) \cos\theta_{\gamma^-}]$$

$$\langle P_T \rangle = \frac{\sigma_R - \sigma_L}{\sigma_{\text{Tot}}}, \quad A_{\text{FB}}^{\text{pol}} = \frac{(\sigma_R - \sigma_L) \cos\theta_{\gamma^0} - (\sigma_R - \sigma_L) \cos\theta_{\gamma^0}}{\sigma_{\text{Tot}}}, \quad A_{\text{FB}} \text{ as prev. defined}$$

$$\langle P_T \rangle = -A_T, \quad A_{\text{pol}}^{\text{FB}} = -\frac{3}{4} A_e, \quad A_{\text{FB}} = \frac{3}{4} A_e A_T$$

$$A_R = \frac{2g_V/g_A}{(1 + 5v/g_A)^2}$$

note that g_V is very small. If $g_V = 0$ ($\sin^2\theta_w = 0.25$) $\Rightarrow A_R = 0$, no parity violation. Why?



NEUTRAL CURRENT FOR QUARKS

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For charged leptons, we had the following vertex factor:

$$-\frac{i}{\sqrt{2}} \left(\frac{G_F M_Z^2}{\sqrt{2}} \right)^{1/2} \bar{e} \gamma_\lambda [R_e (1 + \gamma_5) + L_e (1 - \gamma_5)] e$$

$$R_e = 2 \sin^2 \theta_w, \quad L_e = 2 \sin^2 \theta_w - 1$$

For quarks we have:

$$-\frac{i}{\sqrt{2}} \left(\frac{G_F M_Z^2}{\sqrt{2}} \right)^{1/2} \bar{q} \gamma_\lambda [R_q (1 + \gamma_5) + L_q (1 - \gamma_5)] q$$

$$R_q = -2 Q_q \sin^2 \theta_w$$

$$L_q = 1 - 2 Q_q \sin^2 \theta_w$$

Recall that $\Gamma_{e^+e^-} = (R_e^2 + L_e^2) \Gamma_{\nu\bar{\nu}}$ $R_e = \frac{1}{2} (C_V - C_A)$

Now we set: $\Gamma_{q\bar{q}} = 3 (R_q^2 + L_q^2) \Gamma_{\nu\bar{\nu}}$ $L_e = \frac{1}{2} (C_V + C_A)$

For ν quark: $Q_q = \frac{2}{3} \rightarrow R_q = -\frac{4}{3} \sin^2 \theta_w, L_q = 1 - \frac{4}{3} \sin^2 \theta_w$

d quark: $Q_q = -\frac{1}{3} \rightarrow R_q = \frac{2}{3} \sin^2 \theta_w, L_q = -1 + \frac{2}{3} \sin^2 \theta_w$

NEUTRAL CURRENT FOR QUARKS

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$$R_U^2 = 4 \cdot \left(\frac{2}{3}\right)^2 \sin^2 \theta_w = \frac{16}{9} \sin^2 \theta_w$$

$$L_U^2 = \left(1 - \frac{4}{3} \sin^2 \theta_w\right) = 1 + \frac{16}{9} \sin^4 \theta_w - \frac{8}{3} \sin^2 \theta_w$$

$$R_D^2 = \frac{4}{9} \sin^2 \theta_w$$

$$L_D^2 = -1 + \frac{2}{3} \sin^2 \theta_w = 1 + \frac{4}{9} \sin^4 \theta_w - \frac{4}{3} \sin^2 \theta_w$$

$$\Gamma_{\nu\nu} = 3 \cdot \left(1 - \frac{8}{3} x_w + \frac{32}{9} x_w^2\right) \Gamma(Z \rightarrow \nu\bar{\nu}) \approx 3 \cdot 0.58 \cdot \Gamma(Z \rightarrow \nu\bar{\nu})$$

$$\Gamma_{dd} = 3 \cdot \left(1 - \frac{4}{3} x_w + \frac{8}{9} x_w^2\right) \Gamma(Z \rightarrow \nu\bar{\nu}) \approx 3 \cdot 0.75 \cdot \Gamma(Z \rightarrow \nu\bar{\nu})$$

$$\rightarrow \sin^2 \theta_w = x_w$$

So what is approx. $\Gamma(Z \rightarrow q\bar{q}) / \Gamma(Z \rightarrow \nu\bar{\nu})$?

Also $R_{\text{PEAK}} \equiv \sigma(e^+e^- \rightarrow F\bar{F}) / \sigma_{\text{QED}}(e^+e^- \rightarrow \mu^+\mu^-)$
 $\hookrightarrow 2 \text{ peak} \sim 0.5$

$$R_{\text{PEAK}}(e^+e^- \rightarrow \mu^+\mu^-) = \left(1 - 4x_w + 8x_w^2\right) R_{\text{PEAK}}(e^+e^- \rightarrow \nu\bar{\nu})$$

$$R_{\text{PEAK}}(e^+e^- \rightarrow \nu\bar{\nu}) = 3 \cdot \left(1 - \frac{8}{3} x_w + \frac{32}{9} x_w^2\right) R_{\text{PEAK}}(e^+e^- \rightarrow \nu\bar{\nu})$$

$$R_{\text{PEAK}}(e^+e^- \rightarrow d\bar{d}) = 3 \cdot \left(1 - \frac{4}{3} x_w + \frac{8}{9} x_w^2\right) R_{\text{PEAK}}(e^+e^- \rightarrow \nu\bar{\nu})$$

NEUTRAL CURRENT FOR QUARKS

FOR ASYMMETRIES WE HAVE:

$$A(\bar{q}\bar{q}) = \frac{3G_F s}{16\pi\alpha} Q_f \sqrt{2} (R_e - L_e) (R_f - L_f) \quad \text{FOR } s \ll M_Z^2$$

$$R_e - L_e = R_d - L_d = - (R_u - L_u) = 1$$

$$\Rightarrow A(\bar{u}\bar{u}) = \frac{3}{2} A(\mu^+\mu^-)$$

$$A(\bar{d}\bar{d}) = 3 A(\mu^+\mu^-)$$

$$A_{\text{PEAK}}(\bar{P}\bar{P}) = \frac{3(L_e^2 - R_e^2)(L_f^2 - R_f^2)}{4(L_e^2 + R_e^2)(L_f^2 + R_f^2)} = \frac{3C_A^e C_V^e C_A^f C_V^f}{(C_A^e + C_V^e)(C_A^f + C_V^f)}$$

	ν	e	ν	d
C_V	0.5	-0.05	0.2	-0.35

Early asymmetry measurement (DESY $\sqrt{s} = 30$ GeV)

