

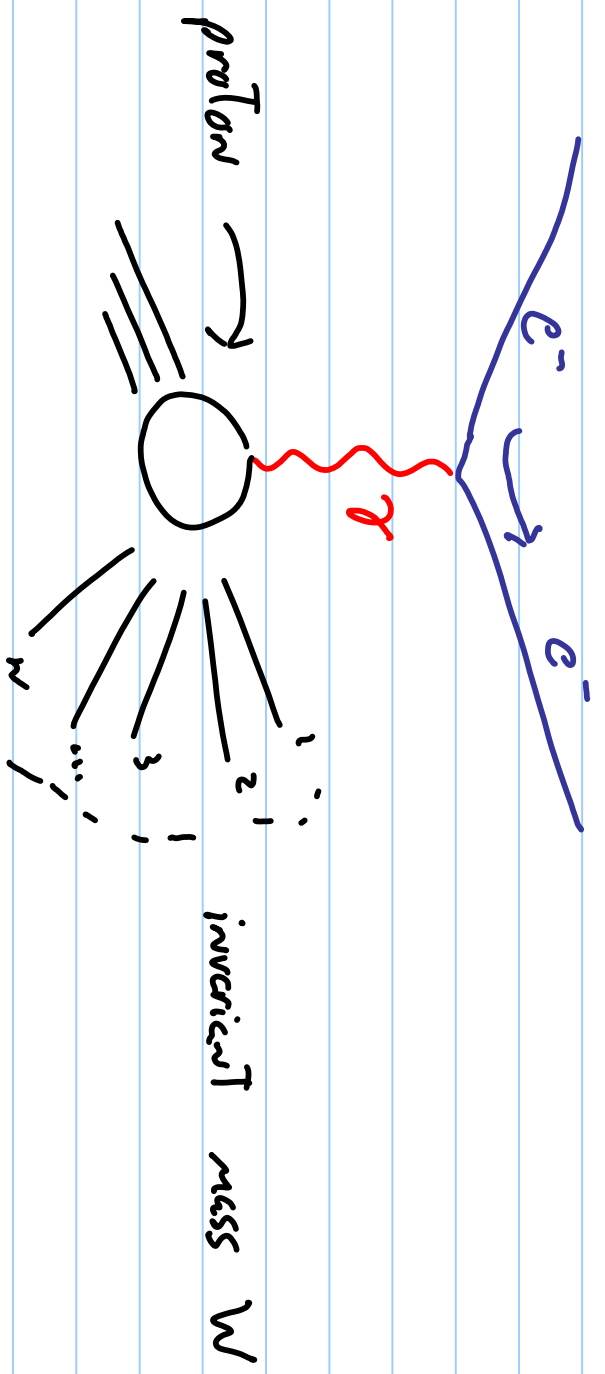
LECTURE 16: Hadron Structure (Part 2)

Overview:

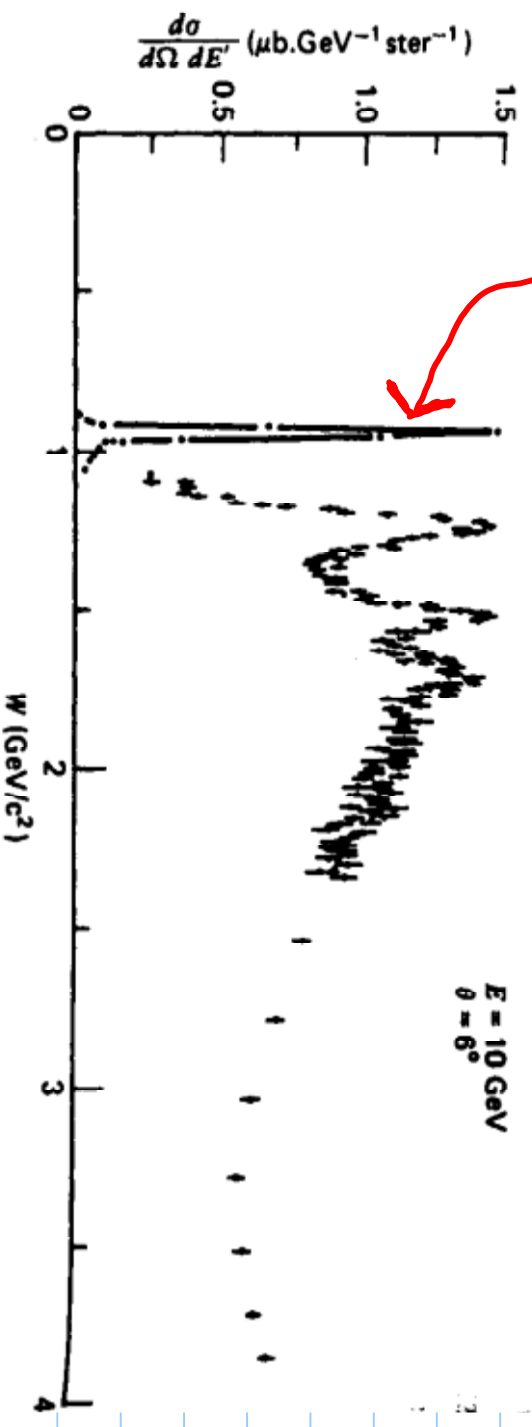
- Inelastic Scattering of protons and electrons
- Contents of the proton and neutron
- Parton Distribution Functions

(I used Quigg and mostly Halzen-Martin as references)

Inelastic Electron-Proton Scattering



elastic contribution



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INELASTIC ELECTRON-PROTON SCATTERING

We had before for e_{μ} scattering: $d\sigma \sim L_{\mu\nu}^{(e)} L^{\mu\nu(p)}$

We'll try: $L_{\mu\nu}^{(e)} W^{\mu\nu}$

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^\mu p^\nu + \frac{W_4}{M^2} q^\mu q^\nu + \frac{W_5}{M^2} (p^\mu q^\nu + p^\nu q^\mu)$$

$L_{\mu\nu}$ is symmetric \rightarrow antisymmetric terms will vanish

W_3 will be parity violating term

We use $q^\mu L_{\mu\nu}^{(e)} = q^\nu L_{\mu\nu}^{(e)} = 0$ (check this)

We can also show that $q_\mu W^{\mu\nu} = 0$ which

follows from $d_\mu j^\mu = 0$

$$\Rightarrow W_5 = -\frac{p \cdot q}{q^2} W_2, \quad W_4 = \left(\frac{p \cdot q}{q^2}\right)^2 W_2 + \frac{M^2}{q^2} W_1$$

\Rightarrow only two structure functions are indep.

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INELASTIC ELECTRON-PROTON SCATTERING

$$W_{\mu\nu} = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + W_2 \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) \cdot \frac{1}{M^2}$$

We can choose two indep. variables: q^2 , $\frac{p \cdot q}{M^2} \equiv \nu$

or dimensionless: $X = \frac{-q^2}{2p \cdot q} = \frac{-q^2}{2M\nu}$, $Y = \frac{p \cdot q}{p \cdot k}$

Invariant mass: $W^2 = (p+q)^2 = M^2 + 2M\nu + q^2$

In rest frame of proton: $\nu = E - E'$

$$Y = \frac{E - E'}{E}$$

We now have:

$$L^e / \nu^\nu W_{\mu\nu} = 4W_1 (k \cdot k') + \frac{2W_2}{M^2} \left[2(p \cdot k)(p \cdot k') - M^2 k \cdot k' \right]$$

INELASTIC ELECTRON-PROTON SCATTERING

1st Lab Frame:

$$L_{\mu\nu} W_{\mu\nu} = 4EE' \left[\cos^2 \frac{\theta}{2} W_2(\nu, q^2) + \sin^2 \frac{\theta}{2} 2W_1(\nu, q^2) \right]$$

$$d\sigma = \frac{1}{4(K \cdot p)^2 - m^2 M^2}^{1/2} \left[\frac{e^4}{q^4} L_{\mu\nu} W_{\mu\nu} Y_{\text{PM}} \right] \frac{d^3 k'}{2E' (2\pi)^3}$$

$\underbrace{\hspace{10em}}_{|M|^2} \rightarrow \text{normalization of } W_{\mu\nu}$

$$\frac{d\sigma}{dE' d\Omega} \Big|_{1st} = \frac{\alpha^2}{4E^2 \sin^2 \frac{\theta}{2}} \left[W_2(\nu, q^2) \cos^2 \frac{\theta}{2} + 2W_1(\nu, q^2) \sin^2 \frac{\theta}{2} \right]$$

→ neglected electron mass.

INELASTIC ELECTRON-PROTON SCATTERING

Summary of recent results:

For all reactions, the diff. cross section can be written as

$$\frac{d\sigma}{d\Omega} = \frac{4\alpha^2 E'^2}{q^4} []$$

point particle (we did more but we'll deal with quarks...)

① $[] = \left(\cos^2 \frac{\theta}{2} - \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2} \right) \delta \left(\nu + \frac{q^2}{2M} \right)$

elastic proton-electron:

② $[] = \left(\frac{6E^2 + \tau G_N^2}{1+\tau} \cos^2 \frac{\theta}{2} + 2\tau G_N^2 \sin^2 \frac{\theta}{2} \right) \delta \left(\nu + \frac{q^2}{2M} \right), \quad \tau = \frac{-q^2}{4M^2}$

inelastic proton-electron:

③ $[] = W_2(\nu, q^2) \cos^2 \frac{\theta}{2} + 2W_1(\nu, q^2) \sin^2 \frac{\theta}{2}$

INELASTIC ELECTRON-PROTON SCATTERING

If point-like spin $1/2$ quarks live inside the proton, we should be able to resolve them with photons that have small enough wavelength.

→ structure functions would become: $Q^2 \equiv -q^2$

$$2W_1 \rightarrow \frac{Q^2}{2m^2} \delta\left(\nu - \frac{Q^2}{2m}\right), \quad W_2 = \delta\left(\nu - \frac{Q^2}{2m}\right)$$

inelastic proton-electron scattering → elastic electron-quark scattering

$$\text{Using } f\left(\frac{x}{z}\right) = c f(x) : \quad 2m W_1(\nu, Q^2) = \frac{Q^2}{2m\nu} \delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

$$\nu W_2(\nu, Q^2) = \delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

→ W_1, W_2 now functions of $\frac{Q^2}{2m\nu}$ and not

Q^2 and ν independently.

INELASTIC ELECTRON-PROTON SCATTERING

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For elastic scattering of ep with $K=0 \Rightarrow G_E = G_M = G$

$$W_1 = \frac{Q^2}{4M^2} G^2(Q^2) \delta\left(\nu - \frac{Q^2}{2m}\right)$$

$$W_2 = G^2(Q^2) \delta\left(\nu - \frac{Q^2}{2m}\right)$$

↳ reflects size of proton

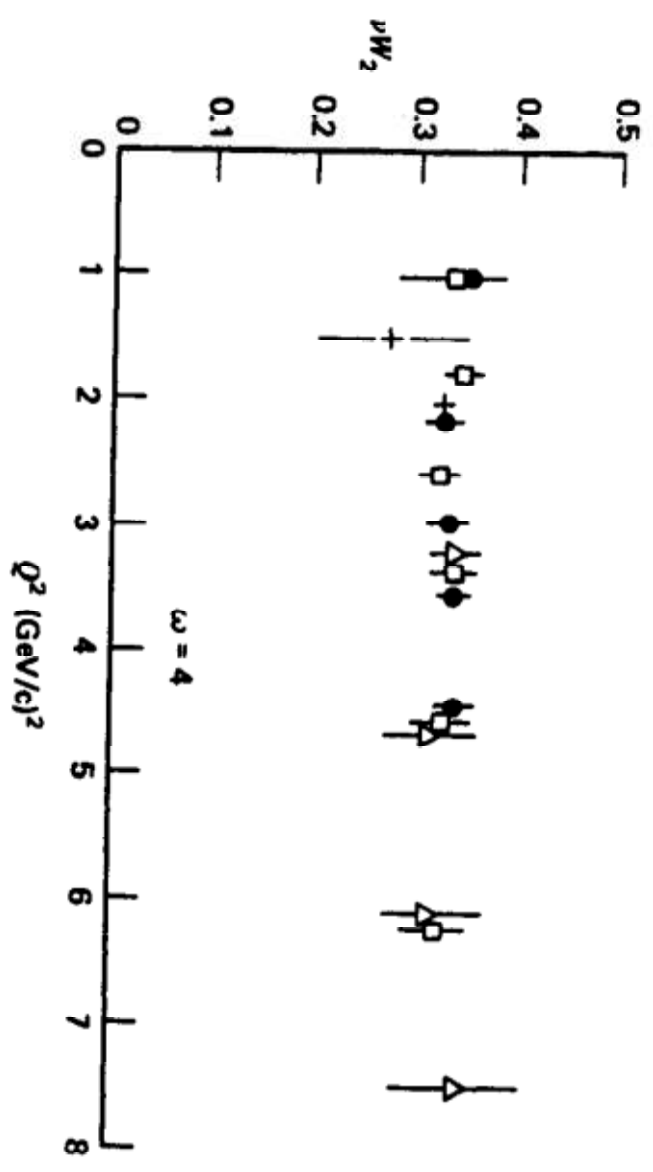
So far point constituents probed by large Q^2 photons:

$$M W_1(\nu, Q^2) \rightarrow F_1(\omega) \quad , \quad \omega = \frac{2q \cdot p}{Q^2} = \frac{2M\nu}{Q^2}$$
$$\nu W_2(\nu, Q^2) \rightarrow F_2(\omega)$$

→ we should see that νW_2 is independent of Q^2
For a given ω

- evidence for "partons"
- Bjorken Scaling

Inelastic Electron-Proton Scattering



"PDFs": parton distribution functions

$F_i(x) = \frac{dP_i}{dx}$ describes probability that a parton will carry a fraction x of the proton's momentum

$$\sum_i \int dx x F_i(x) = 1$$

INELASTIC ELECTRON-PROTON SCATTERING

In terms of x and w , structure functions given by:

$$F_1(w) = \frac{Q^2}{2m\nu x} \delta\left(1 - \frac{Q^2}{2m\nu}\right) = \frac{1}{2x^2 w} \delta\left(1 - \frac{1}{xw}\right)$$

$$F_2(w) = \delta\left(1 - \frac{Q^2}{2m\nu}\right) = \delta\left(1 - \frac{1}{xw}\right)$$

$$\Rightarrow |F_2(w)| = \sum_i \int dx e_i^2 f_i(x) x \delta\left(x - \frac{1}{w}\right)$$

$$F_1(w) = \frac{w}{2} F_2(w)$$

In terms of x :

$$\nu W_2(\nu, Q^2) \rightarrow F_2(x) = \sum_i e_i^2 x f_i(x)$$

$$M W_1(\nu, Q^2) \rightarrow F_1(x) = \frac{1}{2x} F_2(x)$$

satisfies Bjorken
scaling

$$x = \frac{1}{w} = \frac{Q^2}{2m\nu}$$

INELASTIC ELECTRON-PROTON SCATTERING

We can reexpress the results we obtained in terms of x and $y = \frac{v}{E} = \frac{p \cdot q}{p \cdot K}$, $1-y \approx \frac{1}{2}(1+\cos\theta)$

$$dE' d\Omega = \frac{\pi}{EE'} dQ^2 dv = \frac{2ME}{E'} \pi y dx dy$$

$$M_{\nu_{max}} \frac{d\sigma}{dx dy} = \frac{2\pi\alpha^2}{x^2 y^2} \left\{ x y^2 F_1 + \left[(1-y) - \frac{xy}{2\nu_{max}} \right] F_2 \right\}$$

$\nu_{max} = E$ in the lab frame

Rarita model predicts:

$$\frac{d\sigma}{dx dy} = \frac{2\pi\alpha^2}{Q^4} s [1 + (1-y)^2] \sum_i e_i^2 x F_i(x)$$

Proton Quark Content

We had: $F_2(x) = \sum_i e_i^2 x F_i(x)$

$$F_1(x) = \frac{1}{2x} F_2(x)$$

$$\rightarrow \frac{1}{x} F_2^{ep} = \left(\frac{2}{3}\right)^2 [\sum u^p(x) + \bar{u}^p(x)] + \left(\frac{1}{3}\right)^2 [\sum d^p(x) + \bar{d}^p(x)] + \left(\frac{1}{3}\right)^2 [\sum s^p(x) + \bar{s}^p(x)]$$

→ neglect other heavy quarks

We have a similar expression for neutrons.

$$\frac{1}{x} F_2^{en} = \left(\frac{2}{3}\right)^2 [\sum u^p(x) + \bar{u}^p(x)] + \dots$$

$$\begin{aligned} u^p(x) &= d^u(x) \equiv v(x) \\ d^p(x) &= u^u(x) \equiv d(x) \\ s^p(x) &= s^u(x) \equiv s(x) \end{aligned}$$

proton $u v u v d v$ (valence)
 $u_s \bar{u}_s, d_s \bar{d}_s, s_s \bar{s}_s$ (sea)

Proton Quark Content

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$$v_s(x) = \bar{v}_s(x) = d_s(x) = \dots = s(x)$$

$$v(x) = v_u(x) + v_s(x)$$

$$d(x) = d_u(x) + d_s(x)$$

$$\int_0^1 [v(x) - \bar{v}(x)] dx = 2$$

$$\int_0^1 [d(x) - \bar{d}(x)] dx = 1$$

$$\int_0^1 [s(x) - \bar{s}(x)] dx = 0$$

$$\Rightarrow \frac{1}{x} F_2^{ep} = \frac{1}{9} [4v_u + d_u] + \frac{4}{3} s$$

$$\Rightarrow \frac{1}{x} F_2^{en} = \frac{1}{9} [4v_u + 4d_u] + \frac{4}{3} s$$

At low x we expect:

$$\frac{\bar{F}_2^{en}(x)}{\bar{F}_2^{ep}(x)} \xrightarrow{x \rightarrow 0} 1$$

At high x we expect:

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} \rightarrow \frac{v + 4d}{4v_u + d_u}$$

→ proton $v_u \gg d_u$: ratio tends towards 0.25

Proton Quark Content

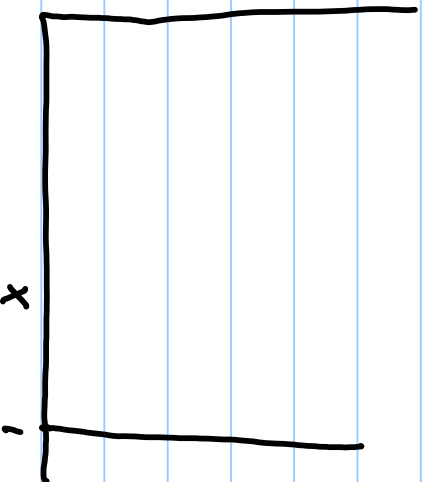
$$\frac{1}{x} F_2^{ep} = \frac{1}{9} [4v_v + d_v] + \frac{4}{3} S$$

$$\frac{1}{x} F_2^{en} = \frac{1}{9} [u_v + 4d_v] + \frac{4}{3} S$$

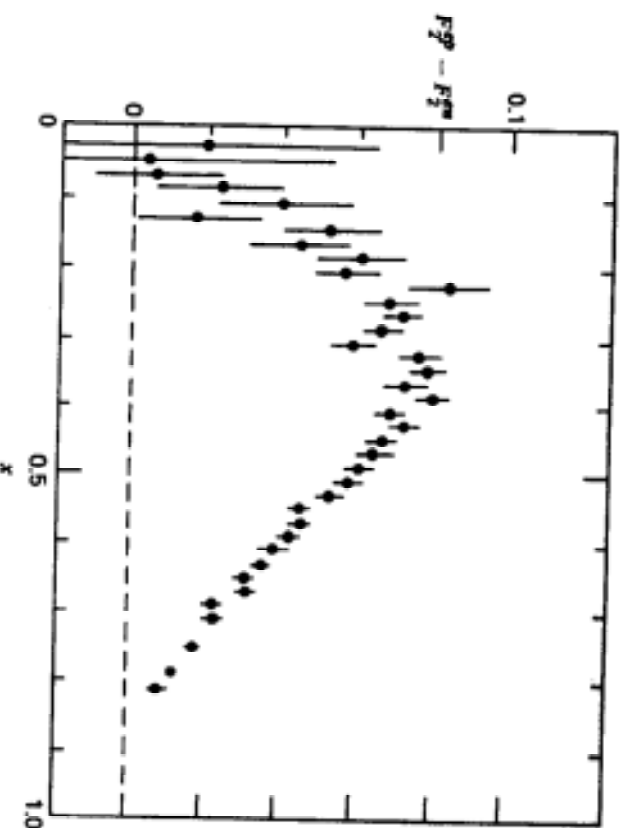
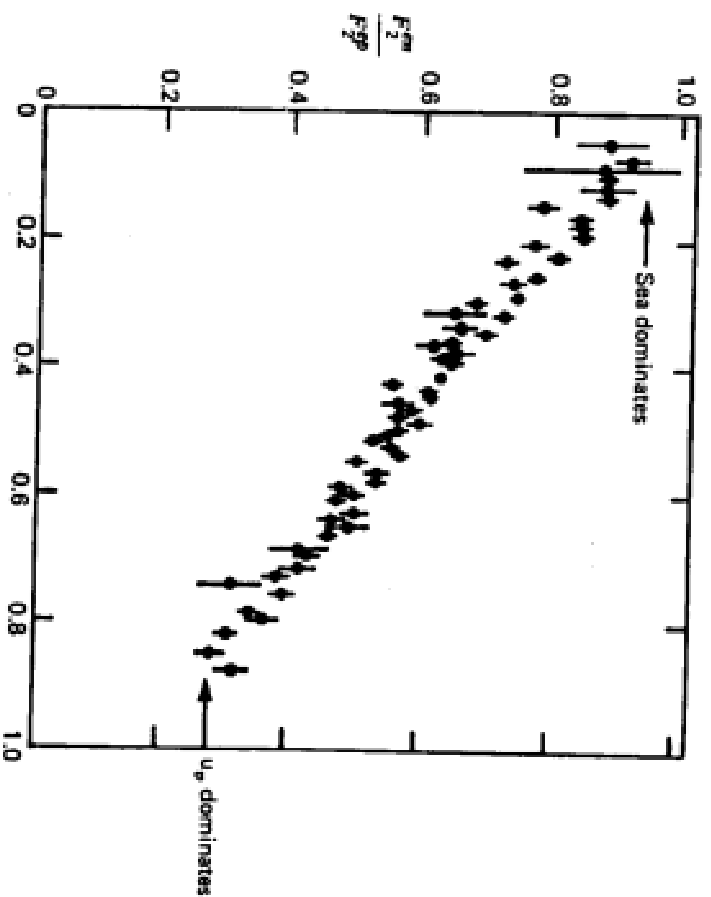
If we subtract the two equations above, we get:

$$\frac{1}{3} [u_v(x) - d_v(x)]$$

For 1 quark, we should get:



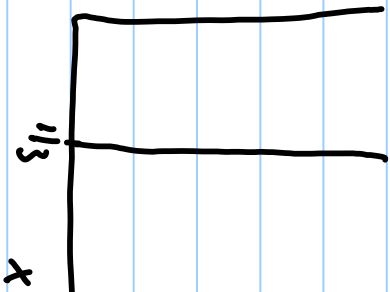
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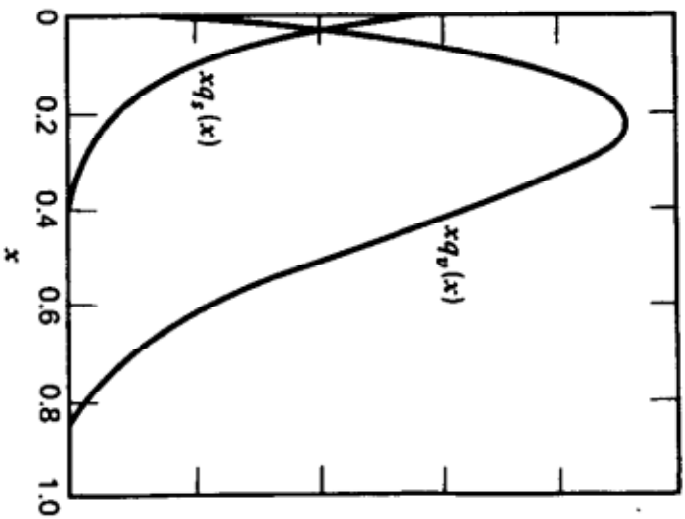
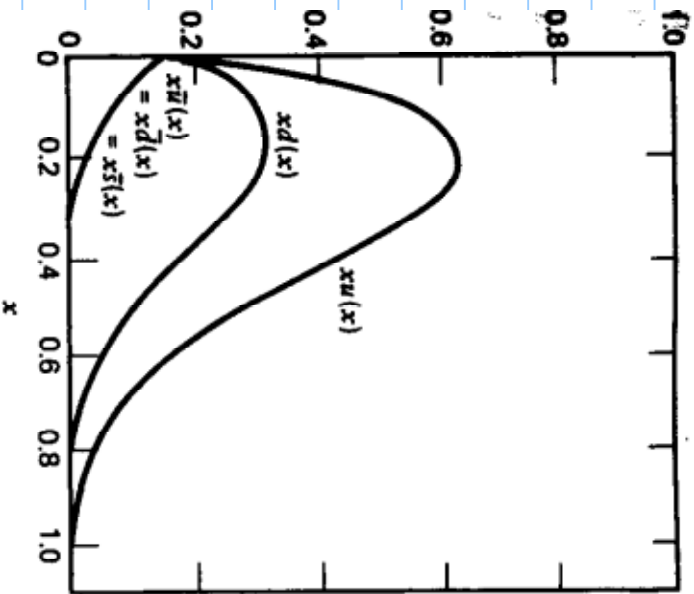
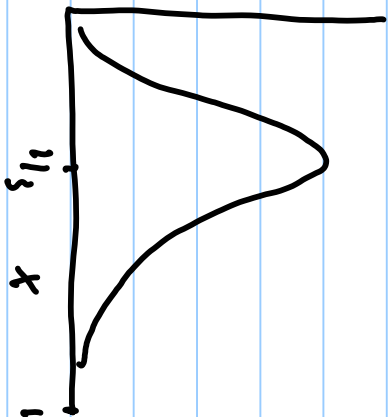
Proton Quark Content

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For 3 quarks:



3 quarks + interactions:



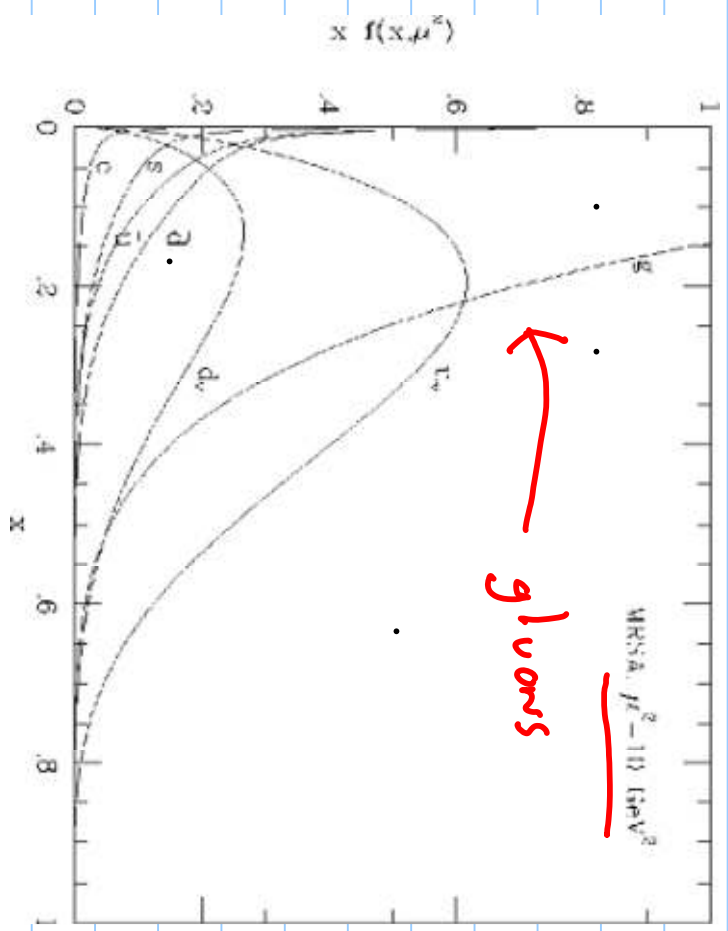
Proton Gluon Content

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If we integrate $\int_0^1 dx x [u + \bar{u} + d + \bar{d} + s + \bar{s}] = 1 - F_{gluon}$

The photon does not interact with the gluon ...

We get $F_{gluon} \sim 0.45!$



Note That these depend on $Q^2 \dots$

