LECTURE 18: QCD (Part 1)

Overview:

- -QCD introduction
- -Feynman rules
- -Calculating colour factors
- -quark anti-quark annihilation

(I used Quigg and mostly Giffiths as references)

QCD (I)

Evidence that quarks come in 3 different "colours" include

→ spin-stat. problem for baryons

→ cross section $e^-e^+ \to$ hadrons

→ Υ branching ratios

→ π^0 lifetime

→ anomaly cancelation

Could we use $U(3)$, $SO(3)$ as candidate gauge groups for the strong interaction?

$SO(3)$ → no asymptotic freedom

→ existence of diquark states, no distinction between quarks and anti-quarks

QCD (II)

$U(3)$ we get a singlet gauge boson $\rightarrow$ long-range interaction.

For the Lagrangian :

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{2}\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu})$$

$$\psi = \begin{pmatrix} q \\ q \\ q \end{pmatrix} \qquad D_\mu = \partial_\mu + igB_\mu$$

$\hookrightarrow$ 3x3 matrix : $B_\mu = \frac{1}{2}\lambda \cdot b_\mu = \frac{1}{2}\lambda^\ell b_\mu^\ell$

b_μ : 8 colour gauge fields

$$G_{\mu\nu} = \frac{1}{2} G_{\mu\nu} \cdot \lambda = \frac{1}{2} G_{\mu\nu}^\ell \lambda^\ell$$

$$= (ig)^{-1}[D_\nu, D_\mu] = \partial_\nu B_\mu - \partial_\mu B_\nu + ig[B_\nu, B_\mu]$$

QCD (III)

Properties of the λ matrices:

$$\mathrm{Tr}(\lambda^\ell) = 0$$
$$\mathrm{Tr}(\lambda^k \lambda^\ell) = 2\delta^{k\ell}$$
$$[\lambda^j, \lambda^k] = 2i f^{jk\ell} \lambda^\ell$$

$f_{123} = 1$, $f_{458} = f_{678} = \sqrt{3}/2$

$f_{147} = f_{246} = f_{345} = f_{516} = f_{637} = 1/2$

$$G^\ell_{\mu\nu} = \partial_\nu b^\ell_\mu - \partial_\mu b^\ell_\nu + g f^{jk\ell} b^j_\mu b^k_\nu$$

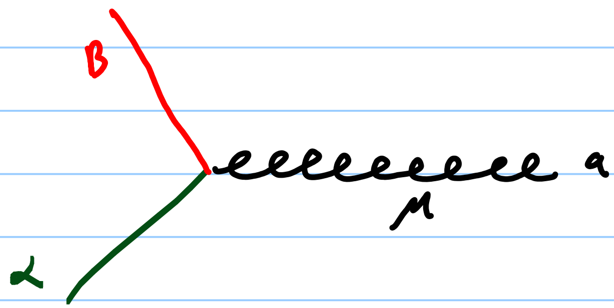
$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

QCD IV

quark - gluon interaction: $\mathcal{L}_{int} = -\frac{g}{2} b^a_\mu \bar{\psi} \gamma^\mu \lambda^a \psi$



$-\frac{ig}{2} \lambda^a_{\alpha\beta} \gamma_\mu$

$\rightarrow$ one gluon exchange force prop. $\frac{g^2}{4} \sum_a \lambda^a_{\alpha\beta} \lambda^a_{\gamma\delta}$

$\alpha + \gamma \rightarrow \beta + \delta$

Gluons are massless particles with spin 1 which we will represent by pol. vector ϵ^μ.

$\rightarrow \quad \epsilon^\mu p_\mu = 0$ (Lorentz Condition)

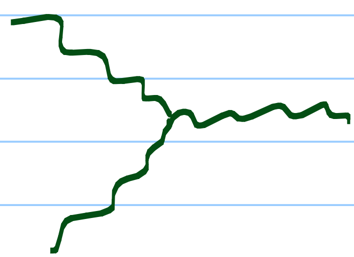
QCD (V)

We will also use Coulomb gauge: $\varepsilon^0 = 0$ $\quad \vec{\varepsilon}\cdot\vec{p} = 0$

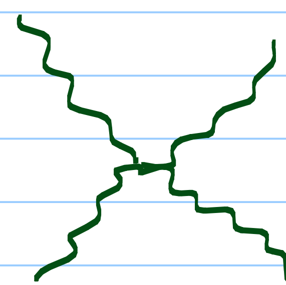
To describe colour state of gluon, we'll use column vector:

$$a = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \equiv |1\rangle \qquad \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \equiv |4\rangle$$

Non-Abelian group $\rightarrow$ gauge mediators interact with themselves:



3-gluon vertex

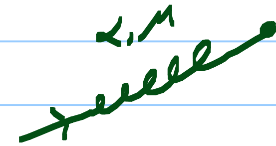


4-gluon vertex

QCD (VI)

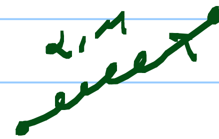
Feynman Rules

incoming gluon :



$\varepsilon_\mu(p)\, a^\alpha$

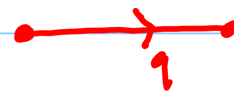
outgoing gluon



$\varepsilon^*_\mu(p)\, a^{\alpha *}$

Propagators

$q\bar{q}$

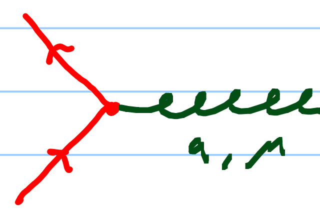


$$\frac{i(\not{q}+m)}{q^2-m^2}$$

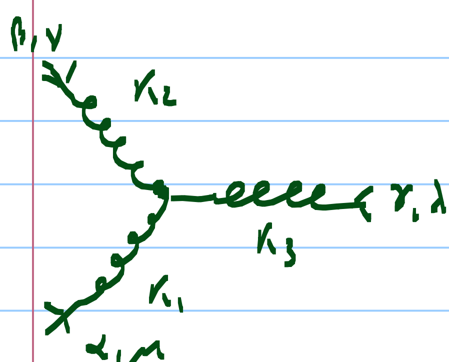
gluon

$$\frac{-i g_{\mu\nu}\,\delta^{\alpha\beta}}{q^2}$$

Vertices



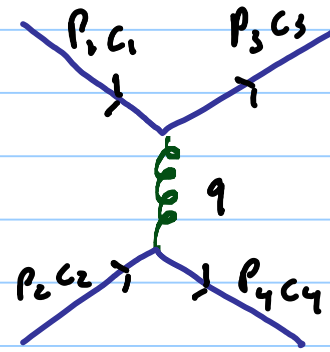
$$-\frac{i g_s}{2}\,\lambda^\alpha \gamma^\mu$$



$$-g_s f^{\alpha\beta\gamma}\left[g_{\mu\nu}(K_1-K_2)_\lambda + g_{\nu\lambda}(K_2-K_3)_\mu + g_{\lambda\mu}(K_3-K_1)_\nu \right]$$

QCD

Consider $u + \bar{d} \to u + \bar{d}$



$$-i\mathcal{M} = [\bar{u}(3)\, c_3^\dagger] \left[-i \frac{g_s}{2} \lambda^\alpha \gamma^\mu\right] [u(1)\, c_1] \left[-i\frac{g_{\mu\nu}\, \delta^{\alpha\beta}}{q^2}\right]$$

$$\times [\bar{v}(2)\, c_2^\dagger] \left[-i \frac{g_s}{2} \lambda^\beta \gamma^\nu\right] [v(4)\, c_4]$$

$$\mathcal{M} = -\frac{g_s^2}{4} \frac{1}{q^2} [\bar{u}(3)\gamma^\mu u(1)] [\bar{v}(2)\gamma_\mu v(4)] \underbrace{(c_3^\dagger \lambda^\alpha c_1)(c_2^\dagger \lambda^\alpha c_4)}_{\text{colour Factor}}$$

$\to$ same as electron-positron scattering with $g_e \to g_s$

but with extra colour Factor

Potential similar To EM case $\quad V \propto \frac{\boxed{-F}\,\alpha_s}{r}$

QCD

- we'll consider the colour octet case e.g. $r\bar{b}$

$$c_1 = c_3 = \begin{pmatrix}1\\0\\0\end{pmatrix}, \quad c_2 = c_4 = \begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$F = \frac{1}{4}\left[(1\,0\,0)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[(0\,1\,0)\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right] = \frac{1}{4}\lambda^\alpha_{11}\lambda^\alpha_{22}$$

$\rightarrow \lambda^3$ and λ^8 are the only relevant matrices

$$f = \frac{1}{4}\left(\lambda^3_{11}\lambda^3_{22} + \lambda^8_{11}\lambda^8_{11}\right) = -1/6$$

- now we consider the singlet state: $\frac{1}{\sqrt{3}}(r\bar{r} + b\bar{b} + g\bar{g})$

$$f = \frac{1}{4}\cdot\frac{1}{\sqrt{3}}\left[\left(c_3^\dagger\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\right)\left[(1\,0\,0)\lambda^\alpha c_4\right] + \left(c_3^\dagger\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}\right)\left[(0\,1\,0)\lambda^\alpha c_4\right]\right.$$

$$\left. + \left(c_3^T\lambda^\alpha\begin{pmatrix}0\\0\\1\end{pmatrix}\right)\left[(0,0,1)\lambda^\alpha c_4\right]\right]$$

QCD

outgoing state also in colour singlet state

$\rightarrow$ we get 9 Terms

$$f = \frac{1}{4} \cdot \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} \left(\lambda_{ij}^{\alpha} \lambda_{ji}^{\alpha} \right) = \frac{1}{12} \mathrm{Tr} \left(\lambda^{\alpha} \lambda^{\alpha} \right)$$

$\rightarrow \mathrm{Tr}(\lambda^{\alpha} \lambda^{\beta}) = 2 \delta^{\alpha\beta}$

$\rightarrow \mathrm{Tr}(\lambda^{\alpha} \lambda^{\alpha}) = 16$

$\Rightarrow f = +\frac{4}{3}$

$$\rightarrow \quad V_{q\bar{q}}(r) = \underbrace{-\frac{4}{3} \frac{\alpha_s}{r}}_{\text{singlet}}, \quad V_{q\bar{q}}(r) = \underbrace{\frac{1}{6} \frac{\alpha_s}{r}}_{\text{octet}}$$

QCD

quark - quark interaction

$$M = -\frac{g_s^2}{4}\frac{1}{q^2}[\bar{u}(3)\gamma^\mu u(1)][\bar{u}(4)\gamma_\mu u(2)](c_3^\dagger \lambda^\alpha c_1)(c_4^\dagger \lambda^\alpha c_2)$$

$$f = \frac{1}{4}(c_3^\dagger \lambda^\alpha c_1)(c_4^\dagger \lambda^\alpha c_2)$$

we get a Triplet:

$$\left.\begin{array}{l}(rb - br)/\sqrt{2}\\(bg - gb)/\sqrt{2}\\(gr - rg)/\sqrt{2}\end{array}\right\}\ \text{anti-sym combinations}$$

and a sextet:

$rr,\ gg,\ bb$

+ symmetric combinations

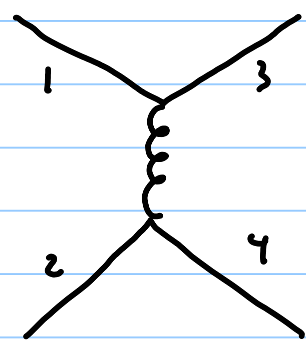
QCD

sextet : $\quad c_1 = c_2 = c_3 = c_4 \quad = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad$ (for rr)

$$f = \frac{1}{4}\left[(1\,0\,0)\,\lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[(1\,0\,0)\,\lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix}\right] = \frac{1}{4}\lambda^\alpha_{11}\lambda^\alpha_{11}$$

$$= \frac{1}{4}\left[\lambda^3_{11}\lambda^3_{11} + \lambda^8_{11}\lambda^8_{11}\right] = \frac{1}{4}\left[1 + \frac{1}{3}\right] = \frac{1}{3}$$

For Triplet: $\quad$ for example $\quad (rb - br)/\sqrt{2}$



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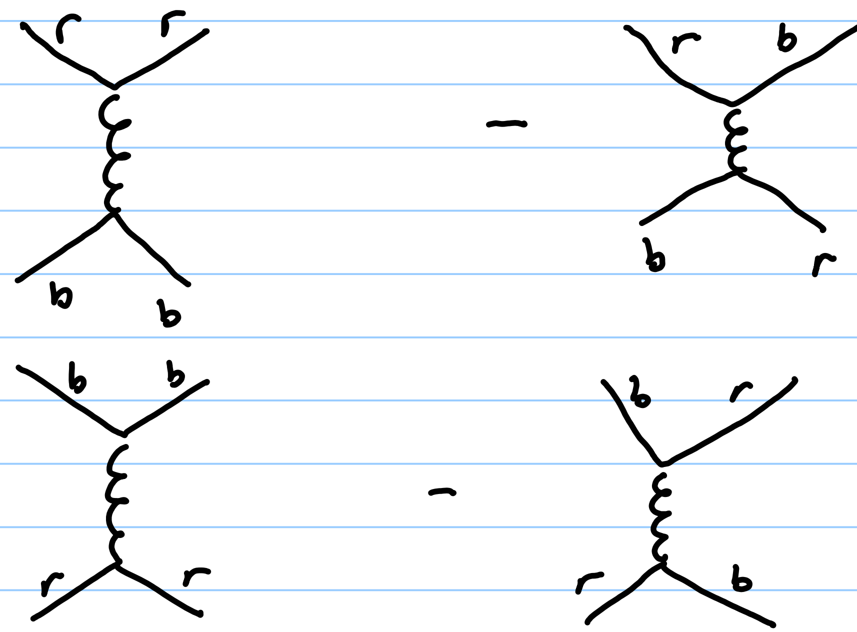


QCD

We get:

$$F = \frac{1}{4}\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\Big[\left[(1\,0\,0)\,\lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[(0\,1\,0)\,\lambda^\alpha \begin{pmatrix}0\\1\\0\end{pmatrix}\right]$$

$$- \left[(0\,1\,0)\,\lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[(1\,0\,0)\,\lambda^\alpha \begin{pmatrix}0\\1\\0\end{pmatrix}\right]$$

$$+ \left[(0\,1\,0)\,\lambda^\alpha \begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[(1\,0\,0)\,\lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix}\right]$$

$$- \left[(1\,0\,0)\,\lambda^\alpha \begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[(0\,1\,0)\,\lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix}\right]\Big]$$

$$= \frac{1}{8}\left[\lambda^\alpha_{11}\lambda^\alpha_{22} - \lambda^\alpha_{21}\lambda^\alpha_{12} + \lambda^\alpha_{22}\lambda^\alpha_{11} - \lambda_{12}\lambda_{21}\right]$$

$$= \frac{1}{4}\left[\lambda^\alpha_{11}\lambda^\alpha_{22} - \lambda^\alpha_{12}\lambda^\alpha_{21}\right]$$

$$= \frac{1}{4}\left[\lambda^3_{11}\lambda^3_{22} + \lambda^8_{11}\lambda^8_{22} - \lambda^1_{12}\lambda^1_{21} - \lambda^2_{12}\lambda^2_{21}\right] = -\frac{2}{3}$$

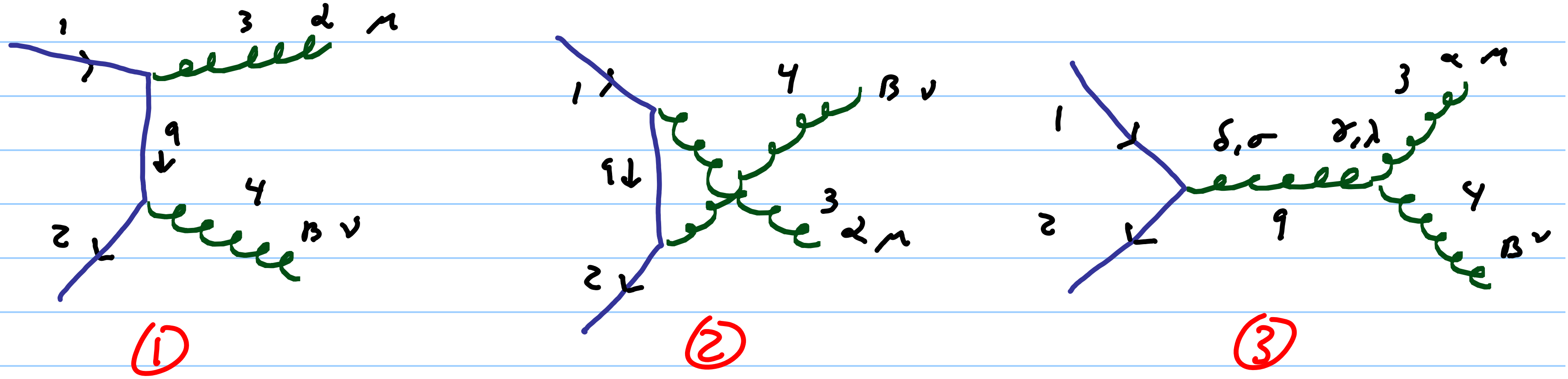
QCD

$$V_{qq}(r) = \frac{-2}{3}\frac{\alpha_s}{r} \quad \text{Triplet}$$

$$V_{qq}(r) \quad \frac{1}{3}\frac{\alpha_s}{r} \quad \text{sextet}$$

not seen...

PAIR ANNIHILATION



①: $-i\mathcal{M}_1 = \bar{v}(2)\,c_2^{\dagger}\left[-i\frac{g_s}{2}\lambda^{B}\gamma^{\nu}\right]\left[\varepsilon^{*}_{4\nu}\,a^{B*}_{4}\right]\left[\frac{i(\not{q}+m)}{q^2-m^2}\right]$

$\times\left[-i\frac{g_s}{2}\lambda^{\alpha}\gamma^{\mu}\right]\left[\varepsilon^{*}_{3}\,a^{\alpha *}_{3}\right]u(1)\,c_1$

QCD

$q = p_1 - p_3 \quad \rightarrow$

$$q^2 - m^2 = p_1^2 - 2\, p_1 \cdot p_3 + p_3^2 - m^2 = -2\, p_1 \cdot p_3$$

$$M_1 = -\frac{g_s^2}{8} \frac{1}{p_1 \cdot p_3} \bar{v}(2) \left[\not{\varepsilon}_4 \left(\not{p}_1 - \not{p}_3 + m \right) \not{\varepsilon}_3 \right] u(1)$$

$$\times\; a_3^\alpha a_4^\beta \left(c_2^\dagger \lambda^\beta \lambda^\alpha c_1 \right)$$

Diagram 2 : switch $\quad 3 \longleftrightarrow 4$

colour factor: $a_3^\alpha a_4^\beta \left(c_2^\dagger \lambda^\alpha \lambda^\beta c_1 \right)$

3 : $-iM_3 = \bar{v}(2)\, c_2^\dagger \left[-i \frac{g_s}{2} \lambda^\delta \gamma_\mu \right] u(1)\, c_1 \left[-i g^{\mu\lambda} \frac{\delta^{\delta\lambda}}{q^2} \right]$

$$\cdot \left[-g_s f^{\alpha\beta\lambda} \left[g_{\mu\nu} (-p_3 + p_4)_\lambda + g_{\nu\lambda} (-p_4 - q)_\mu + g_{\lambda\mu} (q + p_3)_\nu \right] \right]$$

$$\cdot\; \varepsilon_3^\mu a_3^\alpha \; \varepsilon_4^\nu a_4^\beta$$

A

QCD

$q = p_3 + p_4$, $q^2 = 2p_3 \cdot p_4$, $\varepsilon_3 \cdot p_3 = 0 = \varepsilon_4 \cdot p_4$

$$\mathcal{M}_3 = \frac{i g_s^2}{4} \frac{1}{p_3 \cdot p_4} \bar{v}(2)\left[(\varepsilon_3 \cdot \varepsilon_4)(\not{p}_4 - \not{p}_3) + 2(p_3 \cdot \varepsilon_4)\not{\varepsilon}_3 - 2(p_4 \cdot \varepsilon_3)\not{\varepsilon}_4 \right] u(1)$$
$$\times f^{\alpha\beta\lambda} a_3^{\alpha} a_4^{\beta} (c_2^{\dagger} \lambda^{\gamma} c_1) \quad \text{(B)}$$

PROBLEM SET # 2, problem 1 : show how we get (B) from (A) (Griffiths)

To simplify things, we'll assume that the particles are at rest.

$p_1 = p_2 = (m, 0)$, $p_3 = (E, p)$, $p_4 = (E, -p)$

$p_1 \cdot p_3 = p_1 \cdot p_4 = m^2$, $p_3 \cdot p_4 = 2m^2$

$p_3 \cdot \varepsilon_4 = -\vec{p} \cdot \varepsilon_4 = -p_4 \cdot \varepsilon_4 = 0$ (same for $p_4 \cdot \varepsilon_3$)

QCD

We now get a total amplitude:

$$\mathcal{M} = -\frac{g_s}{8m^2}\, a_3^{\alpha} a_4^{\beta}\, \bar{v}(2)\, c_2^{\dagger} \left[\not{\varepsilon}_3 \not{\varepsilon}_4 \not{p}_4 \lambda^{\alpha}\lambda^{\beta} + \not{\varepsilon}_4 \not{\varepsilon}_3 \not{p}_3 \lambda^{\beta}\lambda^{\alpha} - i(\varepsilon_3\cdot\varepsilon_4)(\not{p}_4 - \not{p}_3) f^{\alpha\beta\gamma}\lambda^{\gamma} \right] c_1 v(1)$$

orient coordinates with $\vec{p}$ along z:

$$\not{p}_3 = m(\gamma^0 - \gamma^3), \quad \not{p}_4 = m(\gamma^0 + \gamma^3), \quad (\not{p}_4 - \not{p}_3) = 2m\gamma^3$$

using

$$\not{\varepsilon}_3 \not{\varepsilon}_4 = -(\varepsilon_3\cdot\varepsilon_4) - i(\varepsilon_3 \times \varepsilon_4)\cdot\Sigma \qquad \Sigma = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix}$$

$$\not{\varepsilon}_4 \not{\varepsilon}_3 = -(\varepsilon_3\cdot\varepsilon_4) + i(\varepsilon_3 \times \varepsilon_4)\cdot\Sigma$$

we get

$$\mathcal{M} = \frac{g_s^2}{8m}\, a_3^{\alpha} a_4^{\beta}\, \bar{v}(2)\, c_2^{\dagger} \Big[(\varepsilon_3\cdot\varepsilon_4)\{\lambda^{\alpha},\lambda^{\beta}\}\gamma^0 + i(\varepsilon_3\times\varepsilon_4)\cdot\Sigma\left([\lambda^{\alpha},\lambda^{\beta}]\gamma^0 + \{\lambda^{\alpha},\lambda^{\beta}\}\gamma^3\right)\Big] c_1 v(1)$$

$\rightarrow$ same as QED result with $\lambda = 1$, no colour states

QCD

Now we put the quarks in singlet state (spin 0)

$$M = (M_{\uparrow\downarrow} - M_{\downarrow\uparrow}) / \sqrt{2}$$

For $M_{\uparrow\downarrow}$ we have: $\bar{v}(2)\gamma^0 u(1) = \bar{v}(2)\Sigma\gamma^0 u(1) = 0$

$$\bar{v}(2)\Sigma\gamma^3 u(1) = -2m\hat{z}$$

$$M_{\downarrow\uparrow} = -M_{\uparrow\downarrow}$$

$$M = -i\sqrt{2}\,\frac{g_s^2}{4}\,(\epsilon_3 \times \epsilon_4)_z\, a_3^{\alpha} a_4^{\beta}\,(c_2^{\dagger}\{\lambda^{\alpha},\lambda^{\beta}\}c_1)$$

$$\text{colour Factor} = \frac{1}{8}\, a_3^{\alpha} a_4^{\beta}\,(c_2^{\dagger}\{\lambda^{\alpha},\lambda^{\beta}\}c_1)$$

Problem set 2, problem # 2

if quarks occupy colour singlet state $\frac{1}{\sqrt{3}}(r\bar{r} + b\bar{b} + g\bar{g})$

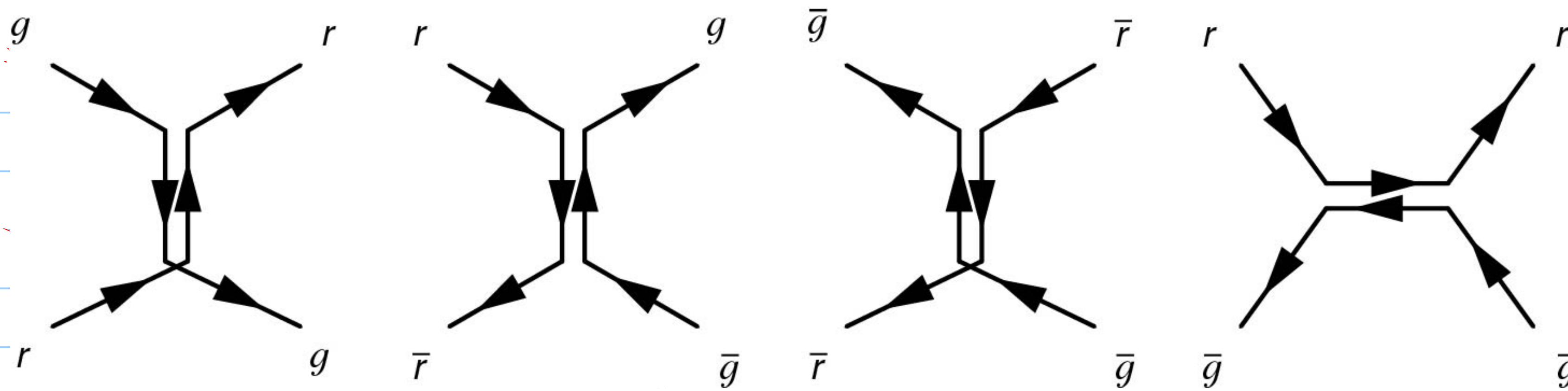
Find the colour Factor.

QCD

$$\mathcal{M} = -4 \cdot \sqrt{2/3}\ g_s^2$$

$$\rightarrow \frac{d\sigma}{d\Omega} = \left[\frac{1}{8\pi(E_1+E_2)}\right]^2 \frac{|p_f|}{|p_i|} |\mathcal{M}|^2$$

$$\Rightarrow \sigma \propto \frac{\alpha_s^2}{s} \quad *$$



Colour Factor Summary

From slide 5: colour factor $C(\alpha\beta \to \delta\gamma) = \frac{g_s^2}{4} \sum_{a=1}^{8} \lambda_{\alpha\beta} \lambda_{\delta\gamma}$

For quark-quark: $C(rr \to rr) = 1/3 = C(gg \to gg) = \text{etc.}$

$C(rb \to rb) = -1/6 = C(rg \to rg) = C(gr \to gr) = \text{etc.}$

$C(rg \to gr) = \frac{1}{2} = C(rb \to br) = \text{etc.}$

note that $C(rb \to bg) = \frac{1}{4} \sum_{a=1}^{8} \lambda^a_{31} \lambda^a_{23} = 0$

why?

Averaged colour factor for quark-quark:

$$\langle |M|^2 \rangle = \frac{1}{9} \sum_{\alpha\beta\delta\gamma=1}^{3} |M(\alpha\beta \to \delta\gamma)|^2$$

$$\langle |C|^2 \rangle = \frac{1}{9} \sum_{\alpha\beta\delta\gamma=1}^{3} |C(\alpha\beta \to \delta\gamma)|^2$$

3 × rr-Type with $C = \frac{1}{3}$

6 × rb → rb-Type with $C = -1/6$

6 × rb → br-Type with $C = 1/2$

$$\Rightarrow \langle |C|^2 \rangle = \frac{1}{9} \cdot \left[3 \cdot (1/3)^2 + 6 \cdot (-1/6)^2 + 6 \cdot (1/2)^2 \right] = 2/9$$

For quark-antiquark?

Color Factor Summary

For QED $e^- q \to e^- q$: $\frac{d\sigma}{dq^2} = \frac{2\pi Q_q^2 \alpha^2}{q^4}\left[1 + \left(1 + \frac{q^2}{s}\right)^2\right]$

For QCD $ud \to ud \longrightarrow$ replace $Q_q \cdot \alpha$ with α_s

$$\Rightarrow \frac{d\sigma}{dq^2} = \frac{4\pi \alpha_s^2}{q^4}\left[1 + \left(1 + \frac{q^2}{s}\right)^2\right]$$

note that $\sqrt{s} \sim M_Z$ $\alpha \sim 1/130$, $\alpha_s \sim \frac{1}{8}$

Also note that $\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \alpha(\mu^2)\frac{1}{3\pi}\ln\left(\frac{q^2}{\mu^2}\right)}$

$$\alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{1 + B\alpha_s(\mu^2)\ln\left(\frac{q^2}{\mu^2}\right)}$$

$B = \frac{11 N_c - 2N_f}{12\pi}$

N_f : number of flavours
N_c : number of colors

Back to Hadron Collider Kinematics

$pp \to \text{jet-jet} + X$

$\to$ Transverse momentum : $p_T = \sqrt{p_x^2 + p_y^2}$

$\to$ 2-jet system is in general boosted

rapidity $\quad y = 1/2 \ln\left(\frac{E+p_z}{E-p_z}\right)$

if $p_z \simeq E\cos\theta$

$$\to \quad y \simeq \frac{1}{2}\ln\left(\frac{1+\cos\theta}{1-\cos\theta}\right) = \frac{1}{2}\ln\left(\cot^2 \theta/2\right)$$

define $\eta \equiv -\ln\left(\tan \theta/2\right)$

Let's consider $pp \to \mu^+\mu^-$, $q\bar{q} \to \gamma \to \mu^+\mu^-$

$$\sigma(q\bar{q} \to \mu^+\mu^-) \approx \frac{1}{N_c} Q_q^2 \frac{4\pi\alpha^2}{3\hat{s}}$$

With u and $\bar{u}$ quarks: (at a $p\bar{p}$ collider)

$$d^2\sigma = Q_u^2 \frac{4\pi\alpha^2}{9\hat{s}} u^p(x_1)\,dx_1\, \bar{u}^{\bar{p}}(x_2)\,dx_2 \quad , \qquad u^p(x) = \bar{u}^{\bar{p}}(x)$$

$$= \frac{4}{9} \cdot \frac{4}{9} \frac{\pi\alpha^2}{\hat{s}} u(x_1)\,u(x_2)\,dx_1\,dx_2$$

centre of mass system: $\hat{s} = (x_1 p_1 + x_2 p_2)^2 = x_1^2 p_1^2 + x_2^2 p_2^2 + 2x_1 x_2\, p_1 p_2$

neglecting masses $(p_1^2 = p_2^2 = 0) \rightarrow \hat{s} \approx x_1 x_2 (2p_1 p_2) = x_1 x_2 s$

$$d^2\sigma = 4/9 \cdot \frac{4\pi\alpha^2}{9 x_1 x_2 s} u(x_1)\,u(x_2)\,dx_1\,dx_2$$

we also need $d\bar{d}$, and sea quarks:

$$d^2\sigma = \frac{4\pi\alpha^2}{9 x_1 x_2 s}\left[4/9 \left[u(x_1)\,u(x_2) + \bar{u}(x_1)\,\bar{u}(x_2) \right] + \frac{1}{9}\left[d(x_1)\,d(x_2) + \bar{d}(x_1)\,\bar{d}(x_2) \right] \right] dx_1\,dx_2$$

Invariant mass of $\mu^+\mu^-$ system: $M^2 = x_1 x_2 s$

Rapidity of the $\mu^+\mu^-$ system: $y = \frac{1}{2}\ln\left(\frac{E_3 + E_4 + p_{3z} + p_{4z}}{E_3 + E_4 - p_{3z} - p_{4z}}\right)$

$$p_q = \frac{\sqrt{s}}{2}(x_1, 0, 0, x_1) \qquad p_{\bar{q}} = \frac{\sqrt{s}}{2}(x_2, 0, 0, -x_2)$$

$$y = \frac{1}{2}\ln\left(\frac{(x_1+x_2) + (x_1-x_2)}{(x_1+x_2) - (x_1-x_2)}\right) = \frac{1}{2}\ln\frac{x_1}{x_2}$$

$$x_1 = \frac{M}{\sqrt{s}} e^{y} \quad , \quad x_2 = \frac{M e^{-y}}{\sqrt{s}}$$

$$dy\, dM = \frac{\partial(y, M)}{\partial(x_1, x_2)} dx_1\, dx_2 = \begin{vmatrix} \frac{\partial y}{\partial x_1} & \frac{\partial y}{\partial x_2} \\ \frac{\partial M}{\partial x_1} & \frac{\partial M}{\partial x_2} \end{vmatrix} dx_1\, dx_2$$

$$dy\, dM = \frac{s}{2M} dx_1\, dx_2$$

see previous page

$$\to d^2\sigma = \frac{4\pi\alpha^2}{9M^2} f(x_1, x_2) \frac{2M}{s} dy\, dM \quad , \quad \boxed{\frac{d^2\sigma}{dy\, dM} = \frac{8\pi\alpha^2}{9Ms} f(x_1, x_2)}$$

JET PRODUCTION AT THE LHC

Consider $qq \to qq$ scattering

We obtained previously: $\frac{d\sigma}{dq^2} = \frac{4\pi\alpha_s^2}{q^4}\left[1 + \left(1 + \frac{q^2}{\hat{s}}\right)^2\right]$

using $Q^2 = -q^2$, $\hat{s} = x_1 x_2 s$, adding PDFs:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha_s^2}{9Q^4}\left[1 + \left(1 - \frac{Q^2}{s x_1 x_2}\right)^2\right] g(x_1, x_2)\, dx_1\, dx_2$$

$$g(x_1, x_2) = \left[u(x_1)\, u(x_2) + u(x_1)\, d(x_2) + d(x_1)\, u(x_2) + d(x_1)\, d(x_2)\right]$$

$$\Rightarrow \frac{d\sigma}{dQ^2\, dx_1\, dx_2} = \frac{4\pi\alpha_s^2}{9Q^4}\left[1 + \left(1 - \frac{Q^2}{s x_1 x_2}\right)^2\right] g(x_1, x_2)$$

$\longrightarrow$ you can rewrite this in terms of p_T, y_3, y_4 *

Note that $qq \to qq$ is just one out of many diagrams that will yield two jets...

*

(26)

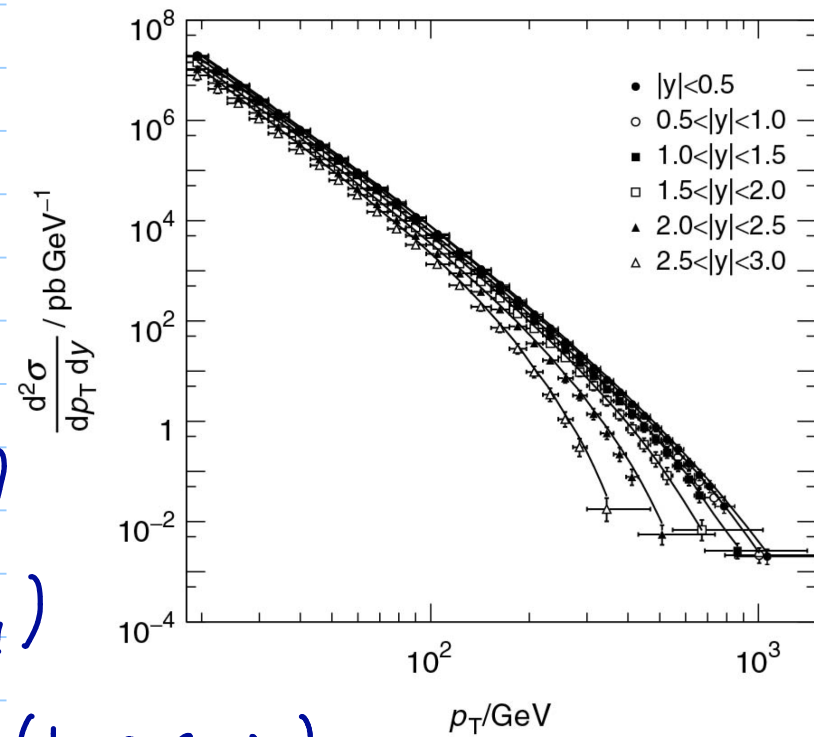
Long Problem Set #2 (cont.)

Problem 3

i) Assuming massless jets and using $E^2 = p_T^2 + p_z^2$ show that:

$$p_3 = (p_T \cosh y_3, +p_T \sin\varphi, +p_T \cos\varphi, p_T \sinh y_3)$$

$$p_4 = (p_T \cosh y_4, -p_T \sin\varphi, -p_T \cos\varphi, p_T \sinh y_4)$$



ii) with $p_1 = \frac{\sqrt{s}}{2}(x_1, 0, 0, x_1)$, $p_2 = \frac{\sqrt{s}}{2}(x_2, 0, 0, -x_2)$

show that $x_1 = \frac{p_T}{\sqrt{s}}(e^{+y_3} + e^{+y_4})$ $x_2 = \frac{p_T}{\sqrt{s}}(e^{-y_3} + e^{-y_4})$

and that $Q^2 = p_T^2(1 + e^{y_4 - y_3})$

Problem 4:

Show that $$\frac{\partial(y_3, y_4, p_T^2)}{\partial(x_1, x_2, q^2)} = \frac{1}{x_1 x_2}$$