

## LECTURE 20 and 21: QCD (Part III, IV)

### Overview:

- Scaling Violations
- Gluon Pair Production
- 3-Jet Events
- Jet Algorithms

(I used Halzen-Martin and Quigg along with talk by Gavin Salam)

# Scaling Violations

(2)

Last lecture we obtained:  $\frac{d\sigma}{dp_T^2} \approx e_i^2 \sigma_0 \frac{1}{p_T^2} \frac{\alpha_s}{2\pi} P_{qq}(z)$

$$\text{with } P_{qq} = \frac{4}{3} \left( \frac{1+z^2}{1-z} \right)$$

What is the contribution of gluon emission to the structure functions?

$$\begin{aligned} \hat{\sigma}(y^+ q \rightarrow qg) &= \int_{\mu^2}^{s/4} dp_T^2 \frac{d\sigma}{dp_T^2} \\ &= e_i^2 \sigma_0 \int_{\mu^2}^{s/4} \frac{dp_T^2}{p_T^2} \frac{\alpha_s}{2\pi} P_{qq}(z) \\ &= e_i^2 \sigma_0 \left( \frac{\alpha_s}{2\pi} P_{qq}(z) \log \frac{Q^2}{\mu^2} \right) \end{aligned}$$

$$\rightarrow P_{qq}^2 \text{ max} = \frac{1}{4} = Q^2 \frac{(1-z)}{4z} \rightarrow \log \frac{1}{4} \approx \log Q^2, \quad Q^2 \text{ large}$$

## Scaling Violations

(3)

→  $\mu$  introduced To regularize  $\beta_1^2 \rightarrow 0$  divergence

$$\frac{F_2(x, Q^2)}{x} = \left| \right|^2 + \left| \right|^2 + \left| \right|^2$$

$$= \sum_q e_q^2 \int_x^1 \frac{dy}{y} q(y) \left( S\left(1 - \frac{x}{y}\right) + \frac{2s}{2\pi} P_{11}\left(\frac{x}{y}\right) \log \frac{Q^2}{\mu^2} \right)$$

$$q(y) \equiv F_1(y)$$

→  $\log Q^2$  Term will introduce scaling violations

→ violation implies gluon emission

Now we will try to obtain quark probability distributions from the above

## Scaling Violations

(4)

$$\begin{aligned} F_2(x, Q^2) &= \sum_f e_f^2 \int_x^1 \frac{dy}{y} (f(y) + \Delta f(y, Q^2)) \Delta\left(1 - \frac{x}{y}\right) \\ &= \sum_f e_f^2 (f(x) + \Delta f(x, Q^2)) \end{aligned}$$

with

$$\Delta f(x, Q^2) = \frac{2f}{2\pi} \log\left(\frac{Q^2}{\mu^2}\right) \int_x^1 \frac{dy}{y} f(y) P_{ff}\left(\frac{x}{y}\right)$$

→ quark densities depend on  $Q^2$

→ our ability to resolve partons increases with  $Q^2$

we can rewrite expression above as:

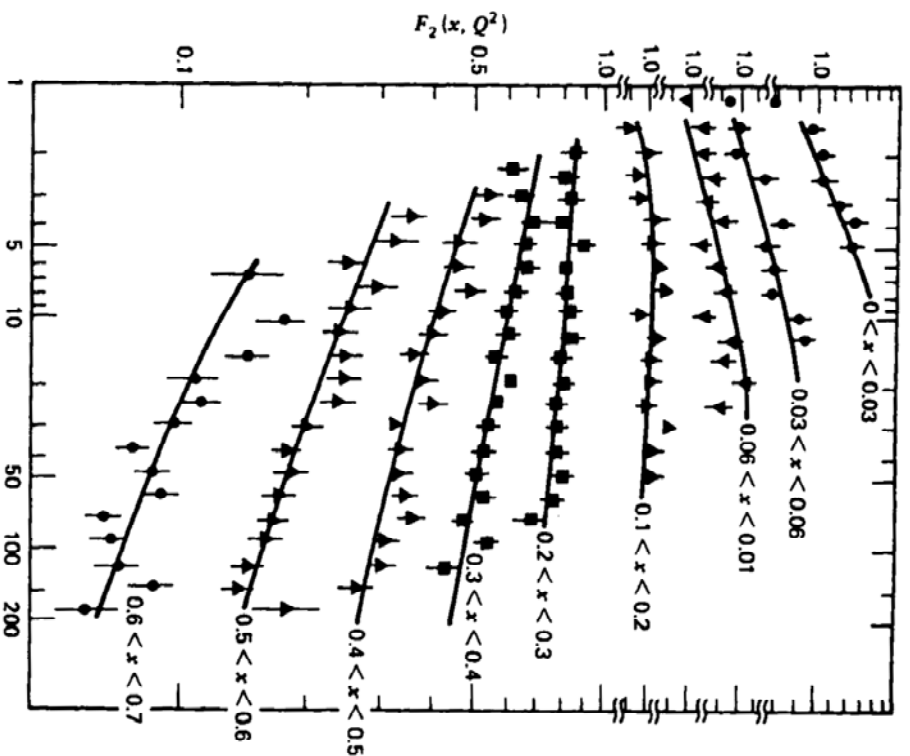
$$\frac{d}{d \log Q^2} f(x, Q^2) = \frac{2f}{2\pi} \int_x^1 \frac{dy}{y} f(y, Q^2) P_{ff}\left(\frac{x}{y}\right)$$

Altarelli-Parisi evolution equation  
→ New "DGLAP"

# Altarelli - Parisi Equation (DGLAP)

(3)

→ a quark with momentum fraction  $x$  could have come from parent quark with momentum  $y$  (which radiated a gluon). This is associated with prob.  $P_{qq}(x/y)$ .



# Gluon Pair Production

$$\rightarrow \gamma^* \gamma \rightarrow q \bar{q}$$

$$\frac{F_2(x, Q^2)}{x} \Big|_{\gamma^* \gamma \rightarrow q \bar{q}} = \int_{\epsilon}^z \dots + \dots \Big|^2$$

$$= \sum_q e_q^2 \int_x^1 \frac{dy}{y} g(y) \frac{dx}{z\pi} P_{q\gamma} \left(\frac{x}{y}\right) \log\left(\frac{Q^2}{\mu_z^2}\right)$$

$g(y)$  : gluon density in proton

$$P_{q\gamma}(z) = (z^2 + (1-z)^2)$$

↳ prob. that produces  $q \bar{q}$  pair with  $q$  having  $z$  fractional momentum.

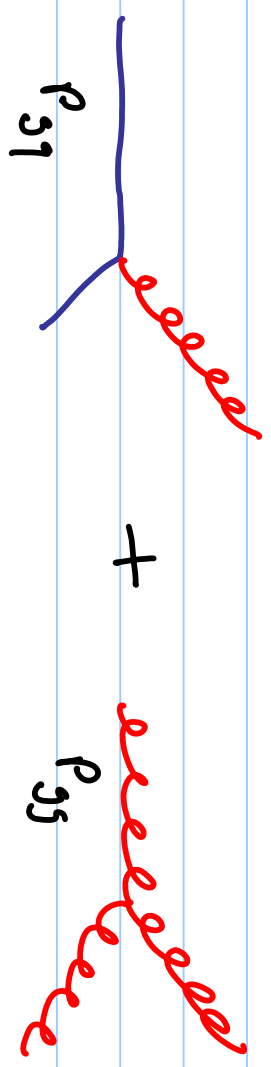
# Evolution Equations

Putting everything together, we get:

$$\frac{dq_i}{d \log Q^2} = \frac{2^i}{2^i \pi} \int_x^1 \frac{dy}{y} \left( q_i(y, Q^2) P_{q_i} \left( \frac{x}{y} \right) + g(y, Q^2) P_{g_i} \left( \frac{x}{y} \right) \right)$$

$$\frac{d}{d \log Q^2} g(x, Q^2) = \sum_i q_i(y, Q^2) + g(y, Q^2)$$

i.e.



$$\frac{dg(x, Q^2)}{d \log Q^2} = \frac{2^i}{2^i \pi} \int_x^1 \frac{dy}{y} \left( \sum_i q_i(y, Q^2) P_{g_i} \left( \frac{x}{y} \right) + g(x, Q^2) P_{g_5} \left( \frac{x}{y} \right) \right)$$

# Evolution Equations (cont.)

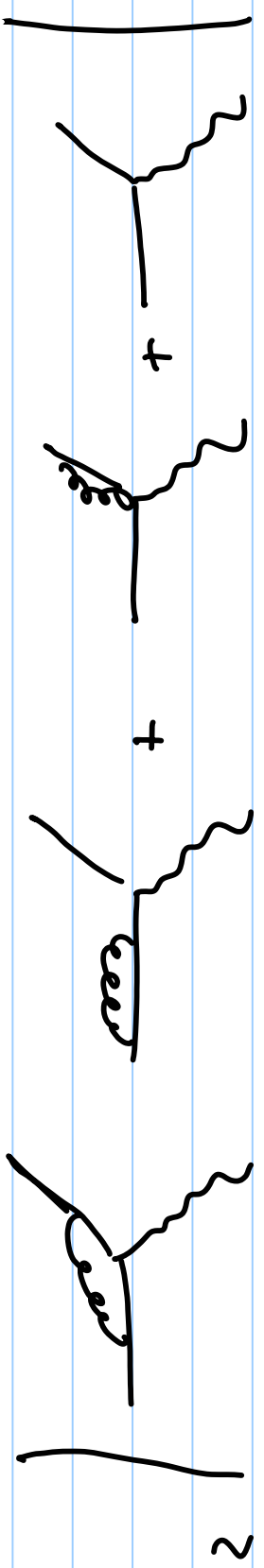
(8)

Recall that:  $\varphi(x, Q^2) + \Delta \varphi(x, Q^2) = \int_0^1 dy \int_0^1 dz \varphi(y, Q^2) \varphi_{11}(z, Q^2) \times \delta(x - zy)$

$$\varphi_{11} \equiv \delta(1-z) + \frac{dz}{2z} P_{11}(z) \log\left(\frac{Q^2}{\mu^2}\right)$$

→ prob. of finding a quark "inside" a quark with momentum fraction  $z$  of parent quark to first order in  $ds$ .

Note that:



Terms cancel divergence at  $z=1$ !



$$e^+e^- \rightarrow q\bar{q} + X$$

(1)

→ Fragmentation Functions

→  $q\bar{q}$  i.e. 3-jet events

Recall that:

$$\begin{aligned}\sigma(e^+e^- \rightarrow \text{hadrons}) &= \sum_1 \sigma(e^+e^- \rightarrow \text{hadrons}) \\ &= 3 \sum_1 e_q^2 \sigma(e^+e^- \rightarrow \mu^+\mu^-)\end{aligned}$$

$$\Rightarrow R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3 \sum_1 e_q^2$$

$$R = \frac{11}{3}$$

$$O(\alpha_s) \quad \text{we'll get} \quad R = 3 \sum_1 e_q^2 \left( 1 + \frac{\alpha_s}{\pi} \left( \frac{16}{3} \right) \right)$$

## FRAGMENTATION

(10)

We can write differential cross section as

$$\frac{d\sigma}{dz} (e^+e^- \rightarrow hX) = \sum_q \sigma (e^+e^- \rightarrow q\bar{q}) [D_q^h(z) + D_{\bar{q}}^h(z)]$$

$$z \equiv \frac{E_h}{E_q} = \frac{2Eh}{Q}$$

$$\sum_h \int_0^1 z D_q^2(z) dz = 1, \quad \sum_q \int_{z_{min}}^1 [D_q^h(z) + D_{\bar{q}}^h(z)] dz = N_h$$

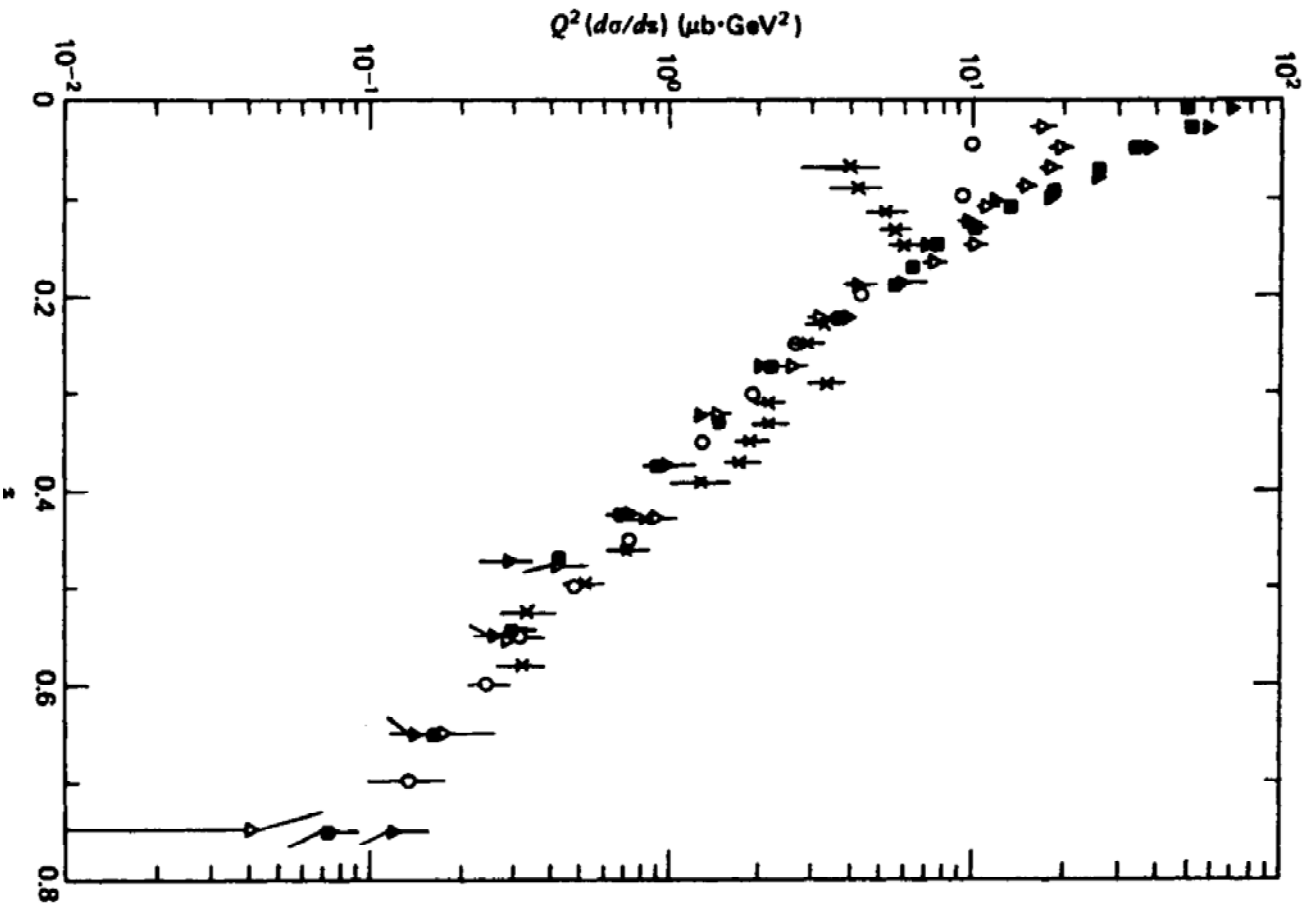
$$z_{min} = z_{min}/Q$$

Using previous results, we get:

$$\frac{1}{\sigma} \frac{d\sigma}{dz} (e^+e^- \rightarrow hX) = \frac{\sum_q e_q^2 [D_q^h(z) + D_{\bar{q}}^h(z)]}{\sum_q e_q^2}$$

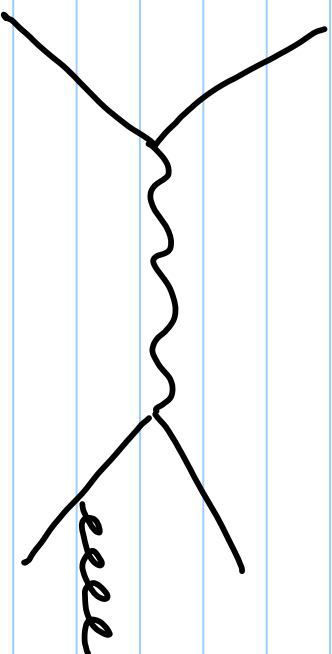
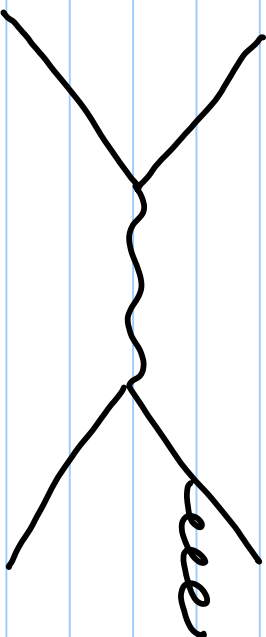
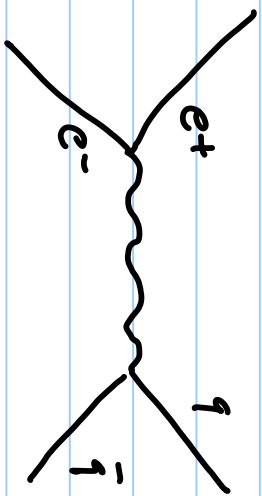
$= \mathcal{F}(z) \rightarrow$  predicted To scale

# Fragmentation (cont.)

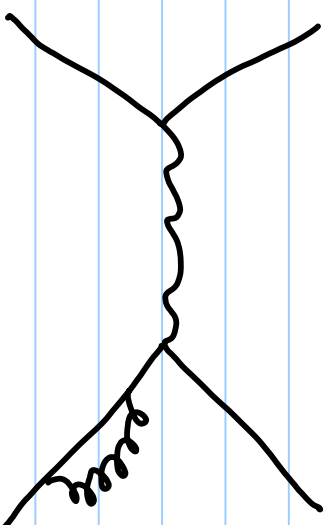
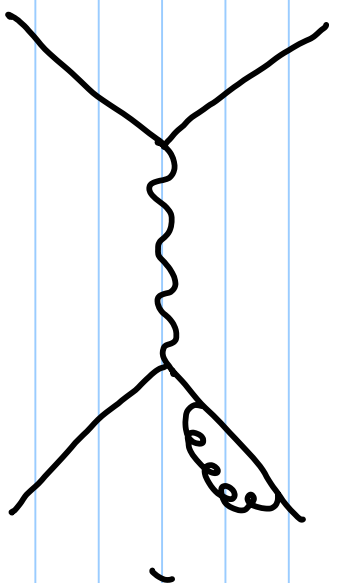
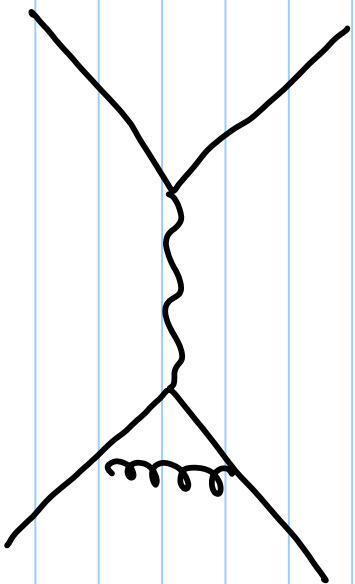


# Three-jet Events

(12)

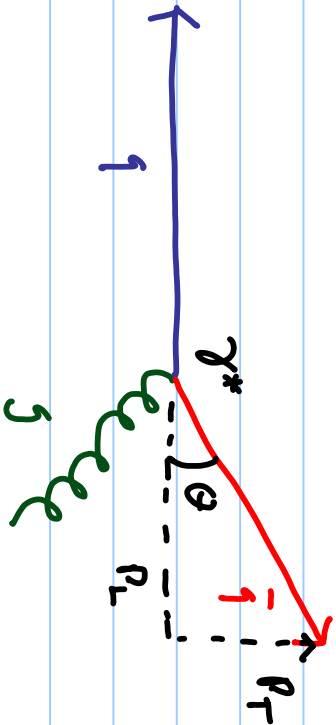


+ other corrections



# Three - Jet Events

(13)



$$x_1 = \frac{2E_1}{Q}, \quad \bar{x}_1 = \frac{2E_{\bar{1}}}{Q}, \quad x_3 = \frac{2E_3}{Q}, \quad x_T = \frac{2p_T}{Q}$$

4 - momentum fractions:

$$(x_1, 0, 0, -x_1)$$

variables ordered relative  
To most energetic jets  
"thrust axis"

$$(x_{\bar{1}}, x_T, 0, x_L)$$

$$(x_3, -x_T, 0, x_1 - x_L)$$

Energy conservation  $\rightarrow x_1 + x_{\bar{1}} + x_3 = 2$

## Three-Jet Events (cont.)

$$x_1^2 - x_f^2 - x_L^2 = 0$$

$$x_3^2 - x_f^2 - (x_L - x_g)^2 = 0$$

$$\rightarrow x_f^2 = \frac{4}{x_1^2} (1 - x_1)(1 - x_1^-)(1 - x_g)$$

We will calculate the cross section for the case where:  $x_1 > x_f > x_g$

$$\frac{d\sigma}{dx_1^- dp_1^2} = \sigma(e^+e^- \rightarrow \gamma\bar{\gamma}) \gamma_{f\bar{f}}(x_1^-, p_f^2)$$

↳ Prob.  $\bar{\gamma}$  emits  $g$  with  
momentum fraction  $(1 - x_1^-)$

$$\sigma(e^+e^- \rightarrow \gamma\bar{\gamma}) = \frac{4\pi\alpha^2}{Q^2} e_f^2, \quad \gamma_{f\bar{f}} = \gamma_{f\bar{f}} = \frac{\alpha_s}{2\pi} \frac{1}{p_f^2} P_{f\bar{f}}(x_1^-)$$

$$\rightarrow \frac{1}{\sigma} \frac{d\sigma}{dx_1^- dp_1^2} = \frac{\alpha_s}{2\pi} \frac{1}{x_1^-} P_{f\bar{f}}(x_1^-)$$

### 3-Jet Events (cont.)

(15)

Using previous result for  $P_{qq}$ :

$$\frac{1}{\sigma} \frac{d\sigma}{dx_T^2} = \frac{2\alpha_s}{2\pi} \frac{1}{x_T^2} \int_{x_T^-}^{x_T^+} dx \frac{4}{3} \left( \frac{1+x^2}{1-x} \right)$$

largest allowed value for  $x_T^-$  corresponds to  $x_T^- = x_T$

$$x_S = x_T$$

$$x_T^- \text{ min} = x_T^- \text{ max} \approx 1 - \frac{x_T}{2}, \quad 1+x^2 \approx 2$$

we get:

$$\frac{1}{\sigma} \frac{d\sigma}{dx_T^2} \approx \frac{4\alpha_s}{3\pi} \frac{1}{x_T^2} \int_{x_T^-}^{1-x_T/2} \frac{dx}{1-x}$$

$$\approx \frac{1}{\sigma} \frac{d\sigma}{dx_T^2} \approx \frac{4\alpha_s}{3\pi} \frac{1}{x_T^2} \log \left( \frac{1}{x_T^2} \right)$$

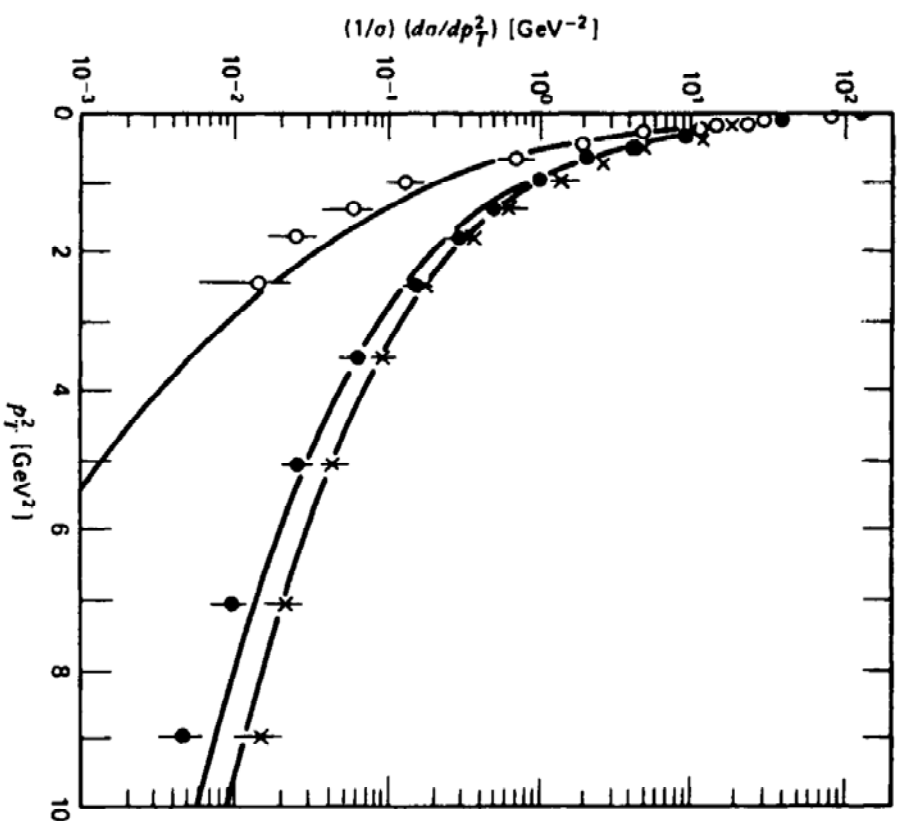
"exact" result:  $\frac{1}{\sigma} \frac{d\sigma}{dx_T dx_T^-} = \frac{2\alpha_s}{3\pi} \frac{x_T^2 + x_T'^2}{(1-x_T)(1-x_T')}$

### 3-Jet Events

$$X_T \equiv \frac{2p_T}{Q}$$

→

$$\frac{1}{\sigma} \frac{d\sigma}{dp_T^2} \sim \alpha_s \frac{1}{p_T^2} \log \left( \frac{Q^2}{4p_T^2} \right)$$





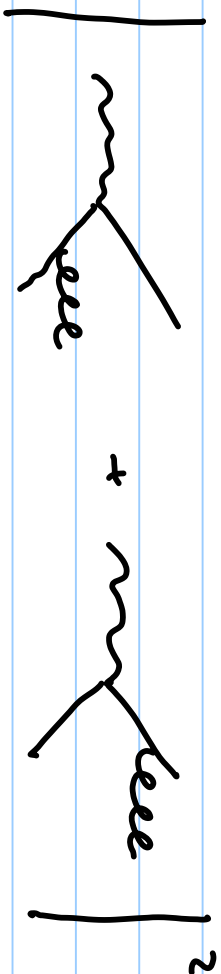
QCD corrections To  $e^+e^- \rightarrow$  hadrons

$$\frac{1}{\sigma} \frac{d\sigma}{dx_1 dx_2} = \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$1-x_1 = \frac{s}{Q^2} = \frac{2\rho\bar{\rho}}{Q^2} = \frac{2}{Q^2} E\bar{E} (1 - \cos\theta_{\bar{f}f})$$

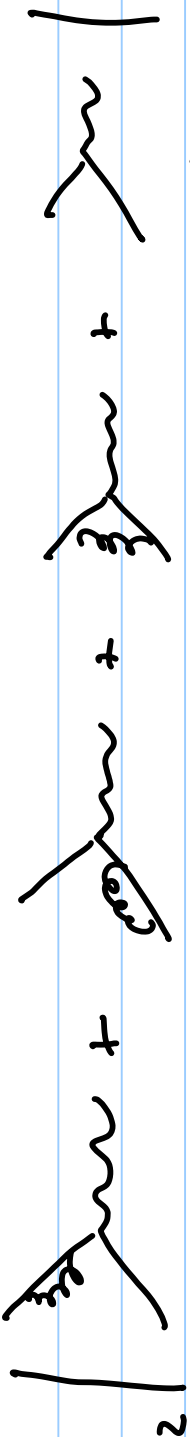
$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3 \sum e_i^2 + \text{correction} \rightarrow \text{diverges...}$$

So:



$$\sigma_R = \sigma_0 \frac{\alpha_s}{2\pi} \frac{4}{3} \left[ \log^2\left(\frac{m_s}{Q}\right) + 3 \log\left(\frac{m_s}{Q}\right) - \frac{\pi^2}{3} + 5 \right]$$

Other Terms:



$$\sigma_V = \sigma_0 \frac{\alpha_s}{2\pi} \frac{4}{3} \left[ -\log^2\left(\frac{m_s}{Q}\right) - 3 \log\left(\frac{m_s}{Q}\right) + \frac{\pi^2}{3} - 7/2 \right]$$

QCD corrections To  $e^+e^- \rightarrow$  hadrons

$$\sigma = \sigma_R + \sigma_V = \sigma_f \frac{d\mathcal{L}_f}{d\Omega}$$

$$\Rightarrow R = 3 \sum_f e_f^2 \left[ 1 + \frac{\alpha_s}{\pi} (Q^2) \right]$$

A word on Jet algorithms (see Sela)