

LECTURE 22: CP Violation (Part I)

Overview:

- Overview
- Meson mixing

(I used Griffiths, Burgess, and Cheng Li as references)

RECAP

②

weak eigenstates \neq mass eigenstates

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

W's couple to :

$$\begin{pmatrix} \nu \\ d' \end{pmatrix}, \begin{pmatrix} e \\ s' \end{pmatrix}$$

$$\begin{pmatrix} \nu \\ d' \end{pmatrix} = \begin{pmatrix} \nu \\ d \cos \theta_c + s \sin \theta_c \end{pmatrix} \quad \begin{pmatrix} e \\ s' \end{pmatrix} = \begin{pmatrix} e \\ -d \sin \theta_c + s \cos \theta_c \end{pmatrix}$$

with 3 generations

$$\begin{pmatrix} \nu \\ d' \end{pmatrix} \begin{pmatrix} e \\ s' \end{pmatrix} \begin{pmatrix} \tau \\ b' \end{pmatrix}, \quad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

First doublet :

$$\begin{pmatrix} \nu \\ V_{ud} d + V_{us} s + V_{ub} b \end{pmatrix}$$

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RECAP (cont.)

If CKM matrix was real, Γ would be orthogonal, couplings would be real.

A general 3×3 unitary matrix can be parameterized by 3 real angles and 6 complex phases.

IF we start with:

$$q_1 = \begin{pmatrix} \nu \\ U_{11}d + U_{12}s + U_{13}b \end{pmatrix}$$

$$U_{11} = R_{11} e^{i\delta}, \quad R_{11} \text{ real}$$

we can redefine u -quark field:

$$u \rightarrow u' = u e^{-i\delta}$$

which gives:

$$q_1 = e^{i\delta} \begin{pmatrix} u' \\ R_{11}d + U'_{12}s + U'_{13}b \end{pmatrix}$$

RECAP (cont.)

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We can factor out complex phases for U_{21} , U_3 , T_{03} for c and T quarks.
We can also absorb two other phases U_{12} and U_{13} by redefining s , b fields

→ down To 13 parameters (9 R_{ij} , 4 phases)

→ normalization of each state reduces this by 3

→ orthogonality of b states reduces this by 6

→ need 3 parameters for 3×3 real orthogonal matrix

→ one independent phase

→ $n(n-1)/2$ angles

→ $(n-1)(n-2)/2$ indep. phases

C and P

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Θ - γ puzzle:

$$\Theta \rightarrow \pi^+ + \pi^0$$

$$P = +1$$

$$\gamma \rightarrow \pi^+ + \pi^0 + \pi^0$$
$$\pi^+ + \pi^+ + \pi^-$$

$$P = -1$$

Θ, γ same particle (K^+)

\rightarrow parity not conserved

C \rightarrow "charge conjugation" changes all internal quantum numbers (charge, baryon and lepton number strangeness, etc.) but leaves mass, spin the same.

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\rightarrow \text{left handed } \mu^+$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

$$\rightarrow \text{no left-handed } \mu^-$$

C and P

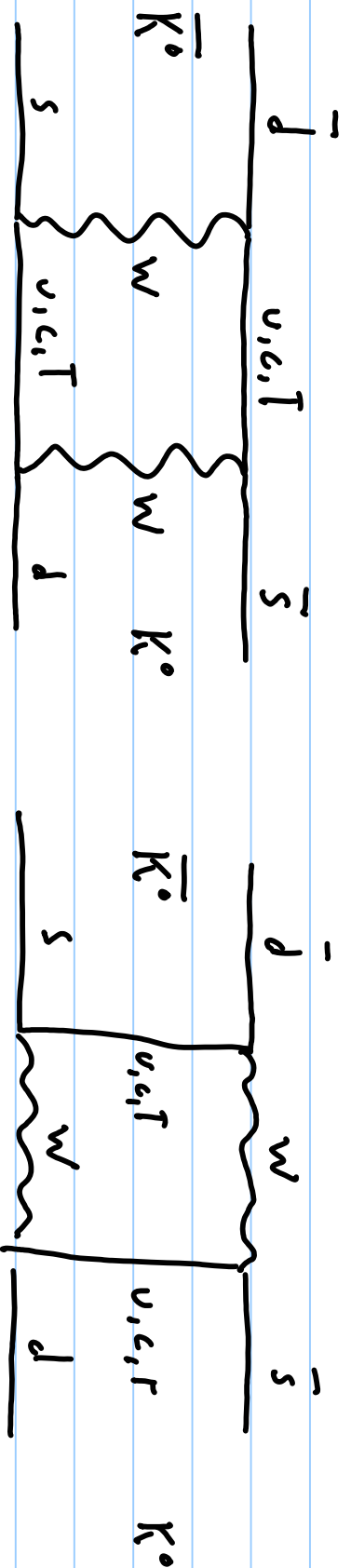
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→ weak interactions not invariant under C either.

→ What about CP?

CP invariance has interesting implications on neutral mesons.

For K mesons:



→ When we turn weak interactions on,

\bar{K}^0 and K^0 will mix

We have to deal with superposition of K^0 and \bar{K}^0

CP Violation

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$$P|K^0\rangle = -|K^0\rangle, \quad P|\bar{K}^0\rangle = -|K^0\rangle$$

$$C|K^0\rangle = |\bar{K}^0\rangle, \quad C|\bar{K}^0\rangle = |K^0\rangle$$

$$CP|K^0\rangle = -|\bar{K}^0\rangle, \quad CP|\bar{K}^0\rangle = -|K^0\rangle$$

→ normalized eigenstates of CP:

$$|K_1\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle), \quad |K_2\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle)$$

$$CP|K_1\rangle = +|K_1\rangle, \quad CP|K_2\rangle = -|K_2\rangle$$

IF CP is conserved in weak interactions:

K_1 will only decay into CP = +1 state

K_2 will only decay into CP = -1 state

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CP Violation (cont.)

→ both states have $C = +1$

K_1 has $P = +1$
 K_2 has $P = -1$

→ Decay To Two pions gives state $P = +1$

→ Decay To three pions gives state $P = -1$

⇒ K_1 will decay To 2 pions and K_2 will decay To 3 pions

⇒ K_1 lifetime $<$ K_2 lifetime

$$\tau_{K_1} = 0.9 \times 10^{-10}$$

$$\tau_{K_2} = 5 \times 10^{-8}$$

$$M_2 - M_1 = 3.5 \times 10^{-6} \text{ eV} !$$

CP Violation (cont.)

If we start with beam of K^0 , weak interactions will turn it into linear combination K^0 and $\bar{K}^0 \rightarrow K_1$ and K_2

$\rightarrow K_1$ component will decay in first few centimetres

$\rightarrow K_2$ component will be the only component left after a few metres

\Rightarrow Decays well away from initial K^0 production should all go to 3 pions (if CP conserved)

Observation: $\sim \frac{1}{500} K_2$ decays to $\pi^0 \pi^0$

\Rightarrow CP not conserved

CP Violation (cont.)

So the long-lived K_{long} is not a perfect CP eigenstate. We should write

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}} (|K_2\rangle + \epsilon |K_1\rangle)$$

→ Note that the short component can be regenerated if K_L been interact with matter

\bar{K}^0 and K^0 components interact differently in material e.g. $\bar{K}^0 \rightarrow \Sigma^0 \pi^+$ \leftrightarrow exchange similar process doesn't exist for K^0

⇒ \bar{K}^0 component can be absorbed

⇒ regenerates K_L !

Meson Mixing (details...)

Mixing changes Flavour by 2 units (G_F^2 process)

Decay rate is also $\propto G_F^2$ so can't be neglected.

So we introduce small imaginary part in meson mass:

$$E_K = \sqrt{K^2 + m^2} \approx \sqrt{K^2 + m^2} - i \frac{\Gamma(K)}{2}$$

$$\Gamma(K) = \frac{m\Gamma}{\sqrt{K^2 + m^2}} \rightarrow \text{decay rate for moving particle}$$

Charge conjugate field \neq complex conjugate field with energy having a complex contribution

$$\bar{\phi}(x) = \int \frac{d^3K}{2\pi^3 2K_0} \left[\bar{a}_K e^{iKx} + a_K^* e^{-iKx} \right]$$

$$\phi(x) = \int \frac{d^3K}{2\pi^3 2K_0} \left[a_K e^{iKx} + \bar{a}_K^* e^{-iKx} \right]$$

$$\bar{\phi}(x) \neq \phi^* \quad (K_x = E_K \cdot T - \vec{K} \cdot \vec{x})$$

Meson Mixing (cont.)

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Most general quadratic Lagrangian for complex scalar field:

$$\mathcal{L} = -\partial_\mu \bar{\phi} \partial^\mu \phi - A \bar{\phi} \phi - \frac{1}{2} (B \phi^2 + C \bar{\phi}^2) \\ = -\partial_\mu \bar{\phi} \partial^\mu \phi - \frac{1}{2} \begin{pmatrix} \phi & \bar{\phi} \end{pmatrix} \begin{pmatrix} B & A \\ A & C \end{pmatrix} \begin{pmatrix} \phi \\ \bar{\phi} \end{pmatrix}$$

- We would have flavour conservation if $C = B = 0$

$$\phi \rightarrow e^{i\alpha} \phi, \quad \bar{\phi} = e^{-i\alpha} \bar{\phi} \quad (\text{U(1) symmetry})$$

- Unitary evolution if $\phi^* = \bar{\phi} \rightarrow A = A^*, B = C^*$

- CP conservation:

$$\text{CP takes } \phi \rightarrow \pm \bar{\phi} \\ \phi^\dagger = \frac{(\phi + \bar{\phi})}{\sqrt{2}}, \quad \phi^- = i \frac{(\bar{\phi} - \phi)}{\sqrt{2}} \quad \text{CP eigenstates}$$

$$\Rightarrow B = C$$

None of the above holds for weak interactions

Meson Mixing (cont.)

The mass matrix can be diagonalized by redefining the field:

$$Q = z\psi, \quad \bar{Q} = z^{-1}\bar{\psi}, \quad z = \left(\frac{C}{B}\right)^{1/4}$$

Propagator eigenstates:

$$\psi_+ = \frac{1}{\sqrt{2}} (\bar{\psi} + \psi) = \frac{1}{\sqrt{2}} (z\bar{Q} + \frac{Q}{z})$$

$$\psi_- = \frac{i}{\sqrt{2}} (\bar{\psi} - \psi) = \frac{i}{\sqrt{2}} (z\bar{Q} - \frac{Q}{z})$$

$$M_{\pm}^2 \approx M_{\mp}^2 - iM_{\mp}\Gamma_{\pm} = A_{\pm} \pm \sqrt{BC}$$

Using $A = |A|e^{-i\alpha}$, $B = |B|e^{-i\beta}$, $C = |C|e^{-i\gamma}$

$$M_{\pm}^2 = |A| \cos \alpha \pm \sqrt{|BC|} \cos \left[\frac{1}{2} (\beta + \gamma) \right]$$

$$M_{\pm}\Gamma_{\pm} = |A| \sin \alpha \pm \sqrt{|BC|} \sin \left[\frac{1}{2} (\beta + \gamma) \right]$$

