LECTURE 23: CP Violation (Part II)

Overview:

- Meson mixing (cont.)
- K meson system
- B meson system

(I used mostly Burgess, and Cheng Li as references, and notes from Frank Wuerthwein)

Meson Mixing (continued from lecture 22)

We define the states $|\varphi\rangle$ and $|\bar{\varphi}\rangle$ (destroyed by the fields $\varphi(x)$ and $\bar{\varphi}(x)$ and define $|\psi_\pm\rangle$ (destroyed by $\psi(x)$ and $\bar{\psi}(x)$). The normalized states are:

$$|\psi_+\rangle = \frac{1}{\sqrt{|p|^2+|q|^2}} \left[p|\varphi\rangle + q|\bar{\varphi}\rangle \right]$$

$$|\psi_-\rangle = \frac{i}{\sqrt{|p|^2+|q|^2}} \left[p|\varphi\rangle - q|\bar{\varphi}\rangle \right]$$

$$\frac{p}{q} = z^2 = \sqrt{\frac{C}{B}}$$

If CP is a symmetry: $C = B \Rightarrow z = 1$

$$\Rightarrow \varphi^+ = \frac{\varphi + \bar{\varphi}}{\sqrt{2}} , \qquad \varphi^- = \frac{i(\bar{\varphi} - \varphi)}{\sqrt{2}}$$

$$M^2_\pm \simeq m^2_\pm - i m \Gamma_\pm = A \pm B$$

Meson Mixing (cont.)

eigenstates:
$$|\psi_+\rangle = \frac{1}{\sqrt{1+|\hat{\varepsilon}|^2}} \left[|\varphi_+\rangle - i\hat{\varepsilon}\, |\varphi_-\rangle \right]$$

$$|\psi_-\rangle = \frac{1}{\sqrt{1+|\hat{\varepsilon}|^2}} \left[|\varphi_-\rangle + i\hat{\varepsilon}\, |\varphi_+\rangle \right.$$

with $\hat{\varepsilon} = \frac{(z^2-1)}{1+z^2} = \frac{p-q}{p+q}$

If we start with state $|\varphi\rangle$:

$$\langle \varphi | \varphi \rangle_t = \frac{1}{2}\left[e^{-iE_+t} + e^{-iE_-t} \right], \quad \langle \bar{\varphi} | \varphi \rangle = \frac{q}{2p}\left[e^{-iE_-t} - e^{-iE_+t} \right]$$

$$P_T\left[\varphi(K) \to \varphi(K)\right] = \frac{1}{4}\left[e^{-\Gamma_+(K)T} + e^{-\Gamma_-(K)T} + 2\, e^{-\left[\frac{\Gamma_+(K)+\Gamma_-(K)}{2}\right]T} \cos\Omega_K T \right]$$

$$P_T\left[\varphi(K) \to \bar{\varphi}(K)\right] = \frac{1}{4}\left|\frac{q}{p}\right|^2 \left[e^{-\Gamma_+(K)T} + e^{-\Gamma_-(K)T} - 2\, e^{-\left[\frac{\Gamma_+(K)+\Gamma_-(K)}{2}\right]T} \cos\Omega_K t \right]$$

$$\Omega_K = \sqrt{K^2+m_+^2} - \sqrt{K^2+m_-^2} \approx m_+ - m_- \quad \text{if } K \ll m$$
$$\approx \frac{m_+^2 - m_-^2}{2K} \quad \text{if } K \gg m$$

$$\rightarrow \quad K\left[\sqrt{1+\frac{m_+^2}{K^2}} - \sqrt{1+\frac{m_-^2}{K^2}}\right]$$

Taylor $\quad (1+x)^{1/2} = 1 + \frac{x}{2}$

$$\Rightarrow K\left[1+\frac{m_+^2}{2K^2} - 1 - \frac{m_-^2}{2K^2}\right] = \frac{m_+^2 - m_-^2}{2K}$$

$K^0 - \bar{K}$ mixing (revisited)

With Kaons we will have $|K_S\rangle = |\psi_+\rangle$

$|K_L\rangle = |\psi_-\rangle$

As we saw, to a good approx. (since $|\hat{\varepsilon}| \ll 1$),

$\Rightarrow \; |K_L\rangle \simeq |K_-\rangle = |q_-\rangle$

$K^0 - \bar{K}^0$ mixing (cont.)

For Kaons, we have:

$m_K = 498$ MeV $\qquad (3 m_\pi = 420$ MeV$)$

$\Gamma_S^{-1} = 9 \times 10^{-11}$ sec , $\Gamma_L^{-1} = 5 \times 10^{-8} \Rightarrow \Gamma_S = 580\, \Gamma_L$

With $\Gamma_+ \gg \Gamma_-$ and assuming slow-moving Kaons:

$$P_t[K(k) \to K(k)] \approx \frac{1}{4} e^{-\Gamma_L(k)T} \left[1 + 2 e^{-[\Gamma_S(k) - \Gamma_L(k)]t/2} \cos(\Delta m t)\right]$$

$$P_t[K(k) \to \bar{K}(k)] \approx \frac{1}{4} e^{-\Gamma_L(k)T} \left|\frac{q}{p}\right|^2 \left[1 - 2 e^{-[\Gamma_S(k) - \Gamma_L(k)]t/2} \cos(\Delta m t)\right]$$

$\Delta m = 3.5 \times 10^{-6}$ eV $\qquad (m_{K_L} - m_{K_S}) > 0$

Frequency comparable to K_S lifetime (how would you measure K^0 vs $\bar{K}^0$?)

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CP violation in Kaon decays

- CP violation can occur through mixing:

$$|K_-\rangle = \frac{1}{\sqrt{1+|\tilde{\varepsilon}|^2}}\left[\, |\varphi_-\rangle + i\tilde{\varepsilon}\, |\varphi_+\rangle \right]$$

With $\mathcal{L}_W$ denoting weak interaction Lagrangian ($\Delta S = \pm 1$) we write $\langle \pi\pi | \mathcal{L}_W | K_+ \rangle$

- CP violation can also occur at the decay:

$\langle \pi\pi | \mathcal{L}_W | K_- \rangle$ i.e. $\mathcal{L}_W$ itself breaks CP (known as "direct" CP violation.

To determine relative size of these contributions we can use the observables:

$$\eta_{00} = \frac{\langle \pi^0 \pi^0 | \mathcal{L}_W | K_L \rangle}{\langle \pi^0 \pi^0 | \mathcal{L}_W | K_S \rangle} \qquad \eta_{+-} = \frac{\langle \pi^+ \pi^- | \mathcal{L}_W | K_L \rangle}{\langle \pi^+ \pi^- | \mathcal{L}_W | K_S \rangle}$$

CP violation in Kaon decays

If mixing is sole source of CP violation: $\eta_{00} = \eta_{+-}$

Note that the Kaons decaying to two pions have either isospin 0 or 2. Amplitudes are:

$$\langle \pi^+ \pi^- | \mathcal{L}_w | K^0 \rangle = A_0 e^{i\xi_0} + A_2 e^{i\xi_2}$$

$$\langle \pi^+ \pi^- | \mathcal{L}_w | \bar{K}_0 \rangle = A_0 e^{-i\xi_0} + A_2 e^{-i\xi_2}$$

A_0, A_2 are CP-conserving strong interaction matrix elements for pion isospin channel, ξ_0, ξ_2 are the CP-violating phases due to $\mathcal{L}_w$ (assuming CP-violating $\mathcal{L}_w$)

$\rightarrow$ Physical decay rates prop. to $|A_0 e^{i\xi_0} + A_2 e^{i\xi_2}|^2$

$\Rightarrow$ relative phase is relevant

CP violation in Kaon decays (cont.)

with $\varepsilon = -\hat{\varepsilon} + \xi_0$

$$\varepsilon' = \left(\frac{A_2}{A_0 + A_2}\right) \boxed{(\xi_2 - \xi_0)}$$

we get : $\eta_{+-} = \varepsilon + \varepsilon'$, $\eta_{00} = \varepsilon - 2\varepsilon'$

Experiments (KTEV, NA48) have confirmed that

$\frac{\varepsilon'}{\varepsilon} \neq 0$, $\mathrm{Re}\left|\frac{\varepsilon'}{\varepsilon}\right| = 1.7 \times 10^{-3}$

$\longrightarrow$ ε' small relative to ε

Measurements with B mesons also have observed direct CP violation

$B-\bar{B}$ Mixing

Similar to $K-\bar{K}$ mixing but b quark mass $\gg$ s quark mass and well above QCD scale:

- Theoretical uncertainties are reduced.
- much larger phase space eliminates (essentially) the lifetime difference. Simplifies expressions ...
- B decays are more CKM suppressed

With $\Gamma_- \simeq \Gamma_+$, we'll denote the two states by "H" for heavy, and "L" for light. Previous oscillation probabilities become (non-relativistic B's):

$$P_T[B^0 \to B^0] = e^{-\Gamma T}\cos^2\left(\frac{\Delta m T}{2}\right)$$

$$P_T[B^0 \to \bar{B}^0] = \left|\frac{q}{p}\right|^2 e^{-\Gamma T}\sin^2\left(\frac{\Delta m t}{2}\right)$$

$B-\bar{B}$ Mixing (cont.)

In our example (non-rel, e.g. CLEO), it is difficult to measure time t elapsed since the B was in a pure B^0 eigenstate. We start from:

$$e^+e^- \rightarrow \gamma^* \rightarrow B\bar{B}$$

$\rightarrow$ relative angular momentum $\ell = 1$

$$CP|B\bar{B}\rangle = -|B\bar{B}\rangle$$

initial state $\quad |B\bar{B}\rangle = \frac{1}{\sqrt{2}}\left[|B(k)\bar{B}(-k)\rangle - |B(-k)\bar{B}(k)\rangle\right.$

We can then "Tag" the flavour of the B using semi-leptonic decay of one B (there are other ways to tag).

By reconstructing decay vertex, we can determine prob. of observing (for instance) same-sign leptons vs distance i.e. versus time

$\longrightarrow$ asymmetric B factories make this easier

$B-\bar{B}$ Mixing (in more detail)

$$|\langle f|H|B^0\rangle|^2 = |\langle f|H|B^0(t)\rangle|^2$$

$$= \frac{1}{4|p|^2}\,|\langle f|B_L(t)\rangle + \langle f|B_H(t)\rangle|^2$$

$$= \frac{1}{4|p|^2}\,|pAe^{(-im_L-\Gamma_L/2)t} + pAe^{(-im_H-\Gamma_H/2)t}|^2$$

$$= \frac{1}{4}|A|^2\left(e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta m t\right)$$

$$|\langle\bar{f}|H|B^0\rangle|^2 = |\langle\bar{f}|B^0(t)\rangle|^2$$

$$= \frac{1}{4|p|^2}\,|\langle\bar{f}|B_L(t)\rangle + \langle\bar{f}|B_H(t)\rangle|^2$$

$$= \frac{1}{4|p|^2}\,|q\bar{A}e^{(-im_L-\Gamma_L/2)t} - q\bar{A}e^{(-im_H-\Gamma_H/2)t}|^2$$

$$= \frac{1}{4}\left|\frac{p}{q}\right|^2|\bar{A}|^2\left(e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_H+\Gamma_L)t/2}\cos\Delta m T\right)$$

$B-\bar{B}$ Mixing (in more detail, cont.)

Remember we have:

$$|B_H\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$$

$$|B_L\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$$

$\rightarrow$ $\langle f|B^0\rangle = A$, $\langle \bar{f}|B^0\rangle = 0$

$\langle \bar{f}|\bar{B}^0\rangle = \bar{A}$, $\langle f|\bar{B}^0\rangle = 0$

$B_H \rightarrow f$ provides $p\,A$

$B_L \rightarrow \bar{f}$ provides $-q\,\bar{A}$

$\rightarrow$ if $A \neq \bar{A}$, CP violation in decay

$\rightarrow$ if $\left|\frac{q}{p}\right| \neq 1$, CP violation in mixing

Without CP violation we would have

$$|\langle f|H|B^0\rangle|^2 + |\langle \bar{f}|H|B^0\rangle|^2 = \frac{1}{2}|A|^2\left(e^{-\Gamma_L t} + e^{-\Gamma_H t}\right)$$

CKM Matrix

In SM, flavour changing and CP-violating physics is due to 4 parameters in unitary CKM matrix.

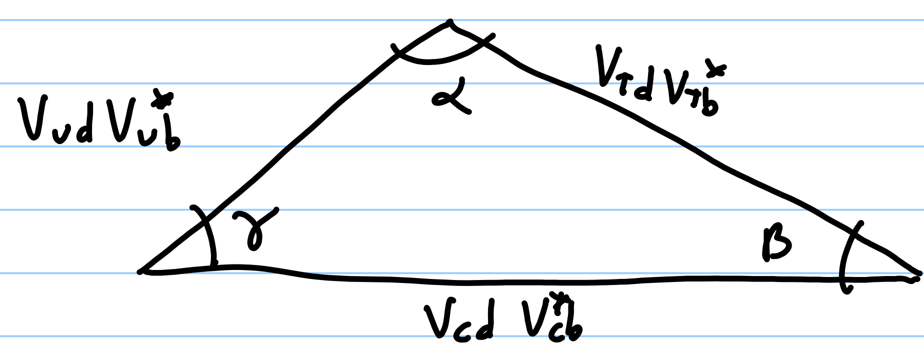
Physics beyond SM could provide new contributions. B physics provides many opportunities of testing SM.

Unitarity conditions $\rightarrow \sum_i V_{in} V_{im}^* = \delta_{mn}$

With B_d mesons, we have $m = b$, $n = d$, which implies:

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

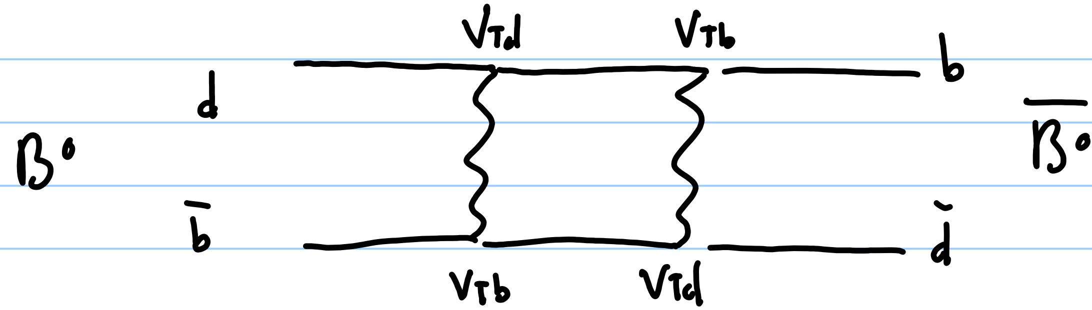
→ sum of 3 complex numbers vanish.
→ can be expressed 3 vectors in complex plane



$$\alpha = \arg\left(-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*}\right)$$

$$\beta = \arg\left(\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*}\right)$$

CKM Matrix and B physics



$$\beta = \arg\left(\frac{V_{cd}V_{cb}^*}{V_{Td}V_{Tb}^*}\right)$$

$B^0 \to \psi K_s$: $V_{Td}V_{Tb}$ from mixing

V_{cb} from $b \to c$

V_{cd} from K^0 mixing

In asymmetric B factories:

- reconstruct one B in CP eigenstate e.g. ψK_s
- reconstruct decay of other B → determine flavour
- measure distance between mesons and convert to proper time

$$\varepsilon = \left(\frac{M_{12}^* - i/_2 \Gamma_{12}^*}{M_{12} - i/_2 \Gamma_{12}} \right)^{1/2} \simeq \frac{M_{12}^*}{|M_{12}|}$$

$$\rightarrow \quad M_{12}^* \propto (V_{Td} V_{Tb}^*)^2 \quad , \quad \varepsilon = \frac{M_{12}^*}{|M_{12}|} = \frac{(V_{Td} V_{Tb}^*)^2}{|(V_{Td} V_{Tb}^*)^2|} = \frac{V_{Td}^2}{|V_{Td}^2|}$$

$$\boxed{V_{Td} = |V_{Td}| e^{-i\beta}}$$

$$|B_L| = \frac{1}{\sqrt{2}} \left(|B^0 \rangle + e^{-i2\beta} |\bar{B}^0 \rangle \right) \quad , \quad |B_H \rangle = \ldots - \ldots$$

$$\Delta m_d = 2|M_{12}| \propto |(V_{Td} V_{Tb}^*)^2| \quad \rightarrow \quad \boxed{\Delta m_d \propto |V_{Td}|^2}$$

Now: $P(\bar{B}^0_{T=0} \rightarrow \bar{B}^0) = e^{-\Gamma T} \cos^2\left(\frac{\Delta m_d T}{2}\right)$

$$P(\bar{B}^0_{T=0} \rightarrow B^0) = \left|\frac{1}{\varepsilon}\right|^2 e^{-\Gamma T} \sin^2\left(\frac{\Delta m_d T}{2}\right)$$

For this system, very few decays are common, interference between decays is small

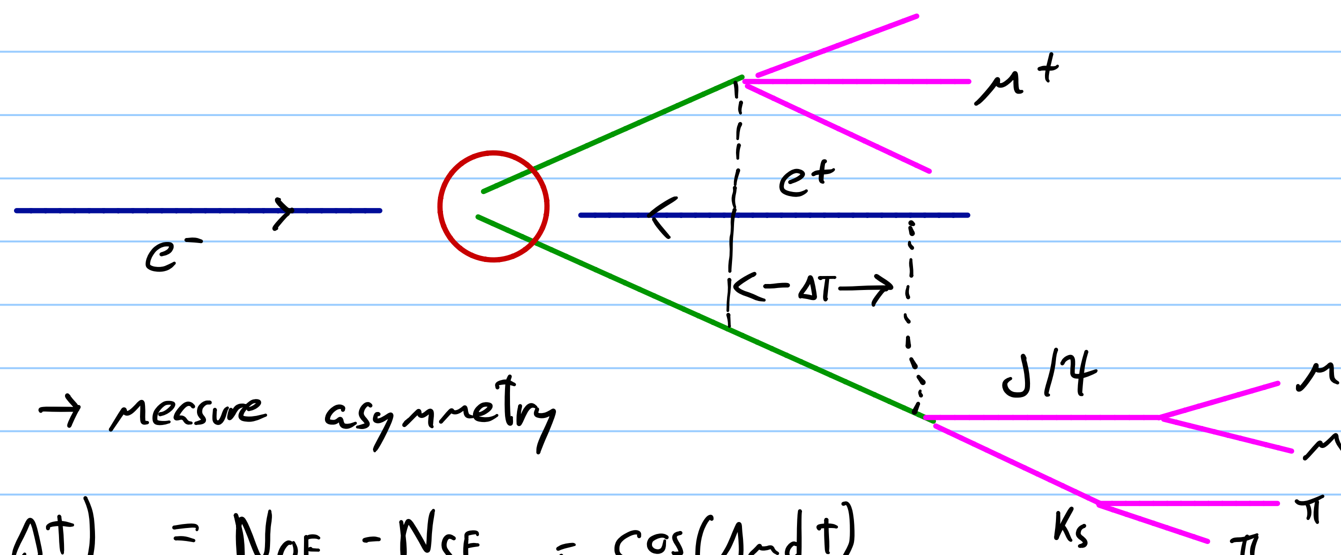
→ Hard to observe CP violation in mixing

What to do?

CP violation can be observed:

- directly : $\Gamma(A \to X) \neq \Gamma(\bar{A} \to \bar{X})$
- through mixing, as we saw for Kaon system
- CP violation in interference between decays

$$B^0 \to F \;,\; B^0 \to \bar{B}^0 \to F$$

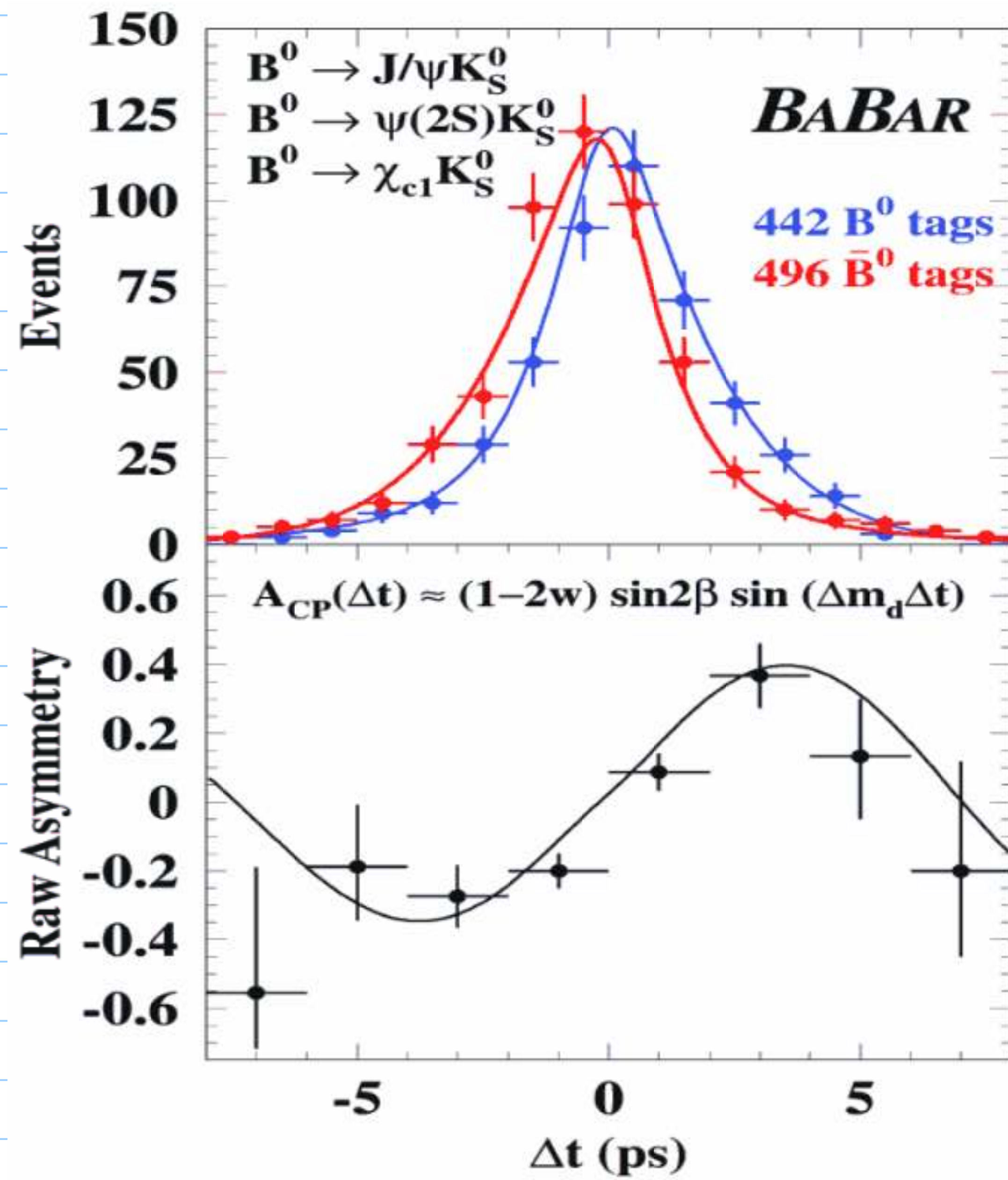


$\Delta m_d \to$ measure asymmetry

$$A(\Delta t) = \frac{N_{OF} - N_{SF}}{N_{OF} + N_{SF}} = \cos(\Delta m_d t)$$

$$A_{CP}^{K_S} = \frac{\Gamma(\bar{B}^0_{T=0} \to J/\psi K_S) - \Gamma(B^0_{T=0} \to J/\psi K_S)}{\text{''} \quad + \quad \text{''}} = \boxed{\sin 2\beta \, \sin \Delta m_d T}$$

16'



$B^0 \to J/\psi K^0_S$
$B^0 \to \psi(2S) K^0_S$
$B^0 \to \chi_{c1} K^0_S$
BABAR
442 B^0 tags
496 $\bar{B}^0$ tags
Events
$A_{CP}(\Delta t) \approx (1-2w)\sin 2\beta \sin(\Delta m_d \Delta t)$
Raw Asymmetry
Δt (ps)

Other sources of CP violation

CP violation due to CKM matrix is too small to explain observed matter anti-matter asymmetry in the Universe.

Are there other potential sources of CP violation in SM?

Yes: - lepton sector (next lecture)

- Strong CP violation (very small if it exists)

Physics beyond SM could also contribute

Strong CP Problem

Most general renorm. Lagrangian involving SM fields includes:

$$\mathcal{L}_{FS} = -\frac{1}{4} G^{\alpha}_{\mu\nu} G^{\alpha\mu\nu} - \frac{1}{4} W^{\alpha}_{\mu\nu} W^{\alpha\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$- \frac{g_3^2 \theta_3}{64\pi^2} \varepsilon_{\mu\nu\lambda\rho} G^{\alpha\mu\nu} G^{\alpha\lambda\rho} \quad - \text{ similar terms for } W, B$$

→ This term is effectively a total derivative and would be expected to have no physical implications

$$\propto \partial^{\mu} K_{\mu}$$

Term determined by change in charge $\int d^3x\, K^0 \equiv N_{CS}$

Change in charge need not be zero in a vacuum to vacuum process → QCD vacuum topologically non-trivial

Axions are a potential solution (need to add scalar field)