LECTURE 24: Neutrinos

Overview:

1- 2-Family Oscillations

2- 3-Family Oscillations

3- Experimental Results

(I used Burgess, and Akhmedov (mainly), C. Giunti as references)

NEUTRINO MIXING

IF NEUTRINOS HAVE MASS, WE CAN HAVE WEAK EIGENSTATES THAT ARE DIFFERENT THAN MASS EIGENSTATES:

WEAK EIGENSTATES $\quad e^- \quad \mu^- \quad \tau^-$
$\qquad\qquad\qquad\qquad\quad \nu_e \quad \nu_\mu \quad \nu_\tau$

MASS EIGENSTATES $\quad e^- \quad \mu^- \quad \tau^-$
$\qquad\qquad\qquad\qquad\quad \nu_1 \quad \nu_2 \quad \nu_3$

MIXING MATRIX
"PMNS"
PONTECORVO, MAKI
NAKAGAWA, SAKATA

$$(\nu_e, \nu_\mu, \nu_\tau) = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

WE'LL START WITH 2-FAMILY MIXING FIRST

2-FAMILY MIXING

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

m_1, m_2 will be masses of two steps

θ mixing angle

- Note that different masses for ν_1 and ν_2 imply different velocities

$$\nu_e = \cos\theta\,\nu_1 + \sin\theta\,\nu_2$$
$$\nu_\mu = -\sin\theta\,\nu_1 + \cos\theta\,\nu_2$$

For STATE OF MASS, ENERGY, MOMENTUM GIVEN BY m_i, E_i, p_i:

$$\nu_i(T,x) = \nu_i(0,0)\, e^{i\varphi_i(T,x)}$$
$$\rightarrow \varphi_i = E_i T - p_i x \quad , \quad i = 1,2$$

with initial state given by: $T = x = 0$ $\quad \nu_e(0) = 1$, $\nu_\mu(0) = 0$

$$\nu_1(0) = \nu_e(0)\cos\theta$$
$$\nu_2(0) = \nu_e(0)\sin\theta$$

2-FAMILY MIXING (CONT.)

As a function of T and x:

$$\nu_e(T,x) = \cos\theta\, \nu_1(T,x) + \sin\theta\, \nu_2(T,x)$$

$$P_{e\to e} = \left|\frac{\nu_e(T,x)}{\nu_e(0,0)}\right|^2 = \left|\cos^2\theta\, e^{i\varphi_1(T,x)} + \sin^2\theta\, e^{i\varphi_2(T,x)}\right|^2$$

$$= 1 - \sin^2 2\theta \sin^2\left(\frac{\varphi_1 - \varphi_2}{2}\right)$$

$$\varphi_1 - \varphi_2 = (E_1 - E_2)T - (p_1 - p_2)x$$

with $T = x\left(\frac{E_1+E_2}{p_1+p_2}\right)$ we get:

$$\varphi_1 - \varphi_2 = \frac{(E_1^2 - E_2^2) - (p_1^2 - p_2^2)}{p_1+p_2} \cdot x = \frac{m_1^2 - m_2^2}{p_1 + p_2} x$$

$$= \frac{\Delta m^2}{2p} x \approx \frac{\Delta m^2 x}{2E}$$

2-FAMILY MIXING (CONT.)

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \, \sin^2\left(\frac{\Delta m^2 x}{4E_\nu}\right)$$

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \, \sin^2 \underbrace{\left(\frac{\Delta m^2 x}{4E_\nu}\right)} \rightarrow \left(\frac{1.27\,\Delta m^2 L}{E_\nu}\right)$$

L in Km
E_ν in GeV
m in eV

Max mixing at $\theta = \frac{\pi}{4}$
and $L = \frac{L_{osc}}{2}$

3-FAMILY MIXING

with 3 generations we will have 3 Δm^2 values but 2 are independent

$$\Delta m_{12}^2 = m_1^2 - m_2^2 \, , \quad \Delta m_{23}^2 = m_2^2 - m_3^2 \, , \quad \Delta m_{31}^2 = m_3^2 - m_1^2$$

PMNS will have 4 indep. parameters, like CKM

use $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$: $\theta_{12}, \theta_{23}, \theta_{13}, \underline{\delta}$

- ★ **Hence the two-flavour oscillation probability is:**

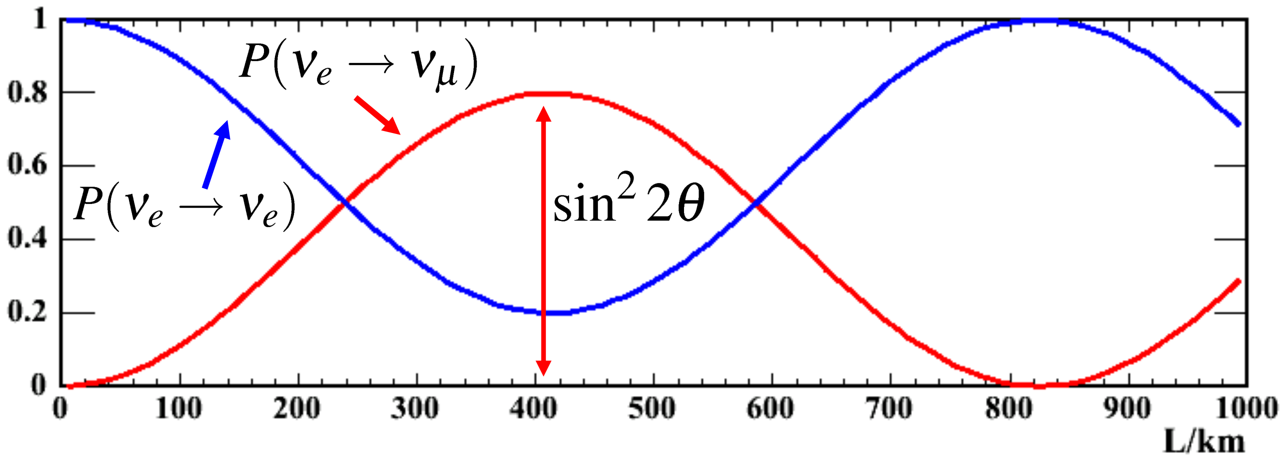
$$\boxed{P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)}$$

with $\boxed{\Delta m_{21}^2 = m_2^2 - m_1^2}$

- ★ **The corresponding two-flavour survival probability is:**

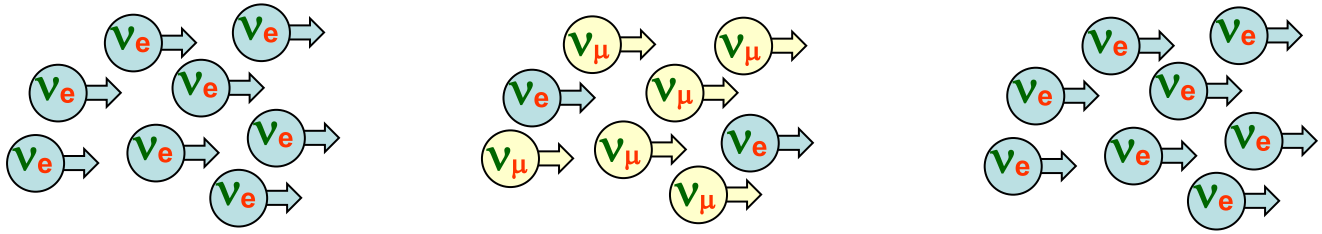
$$\boxed{P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)}$$

- **e.g.** $\Delta m^2 = 0.003\,\mathrm{eV}^2, \qquad \sin^2 2\theta = 0.8, \qquad E_\nu = 1\,\mathrm{GeV}$



- **wavelength**

$$\boxed{\lambda_{\mathrm{osc}} = \frac{4\pi E}{\Delta m^2}}$$



3-FAMILY MIXING (cont.)

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\varphi} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\varphi} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\varphi} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\varphi} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\varphi} & c_{23}c_{13} \end{pmatrix}$$

setting $\varphi = 0$ for now, we break U into 3 rotations:

$$U = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}$$

We'll denote $\nu_{a,b,c}$ as flavour eigenstates
$\nu_{1,2,3}$ or $\nu_{i,j,k}$ as mass eigenstates

We have then: $|\nu_a\rangle = \sum_i U^*_{ai} |\nu_i\rangle$

$$|\nu(0)\rangle = |\nu_a\rangle = \sum U^*_{ai} |\nu_i\rangle$$

$$|\nu(t)\rangle = \sum_i U^*_{ai} e^{-iE_iT} |\nu_i\rangle$$

3-FAMILY MIXING (cont.)

I'll assume Σ's are implicit for what follows:

$$A(\nu_a \to \nu_b; T) = \langle \nu_b | \nu(t) \rangle = U_{ai}^* e^{-iE_iT} \langle \nu_b | \nu_i \rangle$$

$$= U_{bj} U_{ai}^* e^{-iE_it} \langle \nu_i | \nu_j \rangle$$

$$= U_{bj} e^{-iE_jT} U_{aj}^*$$

$$\Rightarrow P(\nu_a \to \nu_b; T) = \boxed{|U_{bj} e^{-iE_jT} U_{aj}^*|^2} =$$

Explicitly:

$$P(\nu_a \to \nu_b; L) = -4\sum_{i>j} (U_{ai}^* U_{bi} U_{aj} U_{bj}^*) \sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right)$$

$$= -2\sum_{i=1} \sum_{j=1,\, j\neq i} (U_{ai} U_{bi} U_{aj} U_{bj}) \sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right)$$

⑧

3-FAMILY MIXING (cont.)

$$= P(\nu_a \to \nu_b) = -4\left[a_{12}\sin^2\left(\ldots\Delta m_{12}^2\right) + a_{13}\sin^2\left(\ldots\Delta m_{13}\right) + \right.$$

$$\left. \underbrace{a_{23}}_{\text{e.g.} \;=\; U_{a2}U_{b2}V_{a3}U_{b3}} \sin^2\left(\cdots\Delta m_{23}^2\right)\right]$$

Experiments Tell us that we need To deal with Two potential hierarchies implied by:

$$|\Delta m_{12}^2| \ll |\Delta m_{13}^2| \simeq |\Delta m_{23}^2|$$

1- $m_1 \ll (\text{or} \lesssim)\; m_2 \ll m_3$

2- $m_3 \ll m_1 \simeq m_2$ (inverted)

$\Rightarrow$ $\Delta m_{13}^2 \simeq \Delta m_{23}^2$ $(\equiv \Delta \hat{m}^2)$ Terms dominate over Δm_{12}^2 Term

3-FAMILY MIXING (cont.)

We can then classify two types of experiments:

1 - small $\frac{L}{E} \Rightarrow \sin^2\left(\Delta m_{12}^2 \ldots\right) \approx 0$ (atmospheric, reactor neutrinos)

We get:

$$P(\nu_e \to \nu_\mu ; L) = 4 |U_{e3}|^2 |U_{\mu 3}|^2 \sin^2\left(\frac{\Delta m_{13}^2 L}{4E}\right)$$

$$= s_{23}^2 \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m_{13}^2 L}{4E}\right)$$

$$P(\nu_e \to \nu_\tau ; L) = 4 |U_{e3}|^2 |U_{\tau 3}|^2 \sin^2\left(\frac{\Delta m_{13}^2 L}{4E}\right)$$

$$= c_{23}^2 \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m_{13}^2 L}{4E}\right)$$

$$P(\nu_\mu \to \nu_\tau) = 4 |U_{\mu 3}|^2 |U_{\tau 3}|^2 \sin^2\left(\frac{\Delta m_{23}^2 L}{4E}\right)$$

$$= c_{13}^4 \sin^2 \theta_{23} \left(\frac{\Delta m_{13}^2 L}{4E}\right)$$

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m_{13}^2 L}{4E}\right)$$

3-FAMILY MIXING (cont.)

2- $\frac{\Delta m_{31}^2}{4E} L \approx \frac{\Delta m_{32}^2}{4E} L \gg 1$ (solar neutrinos)

$\sin^2(\Delta m_{13}^2)$, $\sin^2(\Delta m_{32}^2)$ Terms oscillate very quickly relative To $\sin^2(\Delta m_{12}^2)$. This leads To an averaged value for first Two Terms.

we get $P(\nu_e \to \nu_e) \approx c_{13}^4 P + s_{13}^4$

$$P = 1 - \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m_{12}^2 L}{4E}\right)$$

Finally we consider $|V_{e3}| \ll 1$. We get

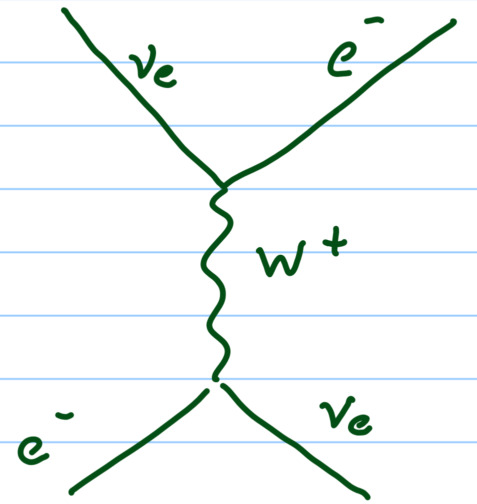
$$P(\nu_e \to \nu_\mu ; L) = c_{23}^2 \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

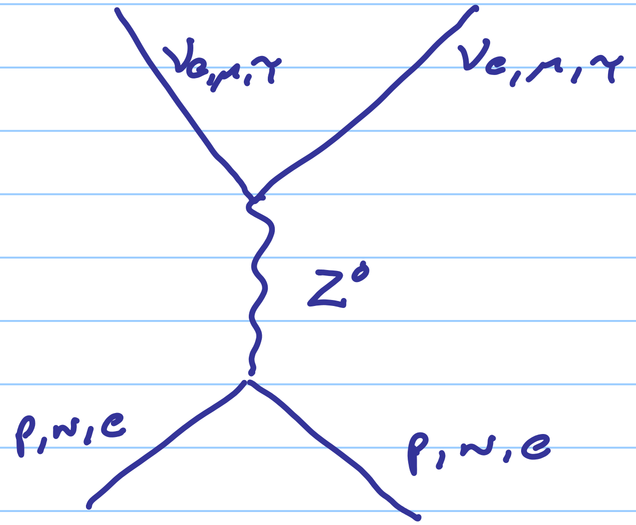
$$P(\nu_e \to \nu_\tau ; L) = s_{23}^2 \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m_{12}^2 L}{4E}\right)$$

$$P(\nu_\mu \to \nu_\tau ; L) = \sin^2\theta_{23} \left(-s_{12}^2 c_{12}^2 \sin^2(\Delta m_{12}^2 \ldots) + s_{12}^2 \sin^2(\Delta m_{13}^2 \ldots) + c_{12}^2 \sin^2(\Delta m_{13}^2 \ldots)\right)$$

In last result, no assumption on mass hierarchy was made.

Neutrino oscillations in MATTER





$$H_{cc} = \frac{G_F}{\sqrt{2}} \left[\bar{e}\gamma_\mu(1-\gamma_5)\nu_e\right]\left[\bar{\nu}_e\gamma^\mu(1-\gamma_5)e\right] \qquad \text{Fierz} \longrightarrow$$

$$= \frac{G_F}{\sqrt{2}} \left[\bar{e}\gamma_\mu(1-\gamma_5)e\right]\left[\bar{\nu}_e\gamma^\mu(1-\gamma_5)\nu_e\right] \qquad \text{(low E neutrinos)}$$

$$H_{eff}(\nu_e) = \langle H_{cc}\rangle_e \equiv \bar{\nu}_e V_e \nu_e$$

↳ integrated over all e variables

Unpolarized medium with zero total momentum, relevant term is $\langle \bar{e}\gamma_0 e\rangle = \langle e^+e\rangle = N_e$ → number density

Neutrino oscillations in MATTER (cont.)

$\rightarrow \quad (V_e)_{CC} \equiv V_{CC} = \sqrt{2}\, G_F N_e$

$(V_e)_{NC} = -G_F N_n/\sqrt{2}$ (protons, electrons cancel off)

$(V_\mu)_{NC} = (V_\tau)_{NC} = -\dfrac{G_F N_n}{\sqrt{2}}$

More convenient to work in flavour basis because effective potentials are diagonal in this basis.

For the two-flavour case, in the absence of matter:

$i\left(\dfrac{d}{dt}\right)|\nu_m\rangle = H_m|\nu_m\rangle$, H_m is diagonal

$i\left(\dfrac{d}{dt}\right)|\nu_F\rangle = H_F|\nu_F\rangle = U H_m U^\dagger |\nu_F\rangle$

$E_i \simeq p + m_i^2/2E$

$$i\left(\frac{d}{dt}\right)\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\left(p+\frac{m_1^2+m_2^2}{4E}\right) - \Delta m^2\cos 2\theta_0 & \frac{\Delta m^2}{4E}\sin 2\theta_0\\ \frac{\Delta m^2}{4E}\sin 2\theta_0 & \left(p+\frac{m_1^2+m_2^2}{4E}\right) - \Delta m^2\cos 2\theta_0\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

Neutrino oscillations in MATTER (cont.)

Extra terms on the diag. can only modify the common phase of the neutrino states ⇒ we can omit them

We get
$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta_0 & \frac{\Delta m^2}{4E}\sin 2\theta_0\\ \frac{\Delta m^2}{4E}\sin 2\theta_0 & \frac{\Delta m^2}{4E}\cos 2\theta_0\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

With matter present:
$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta_0 \boxed{+\sqrt{2}G_F N_e} & \frac{\Delta m^2}{4E}\sin 2\theta_0\\ \frac{\Delta m^2}{4E}\sin 2\theta_0 & \frac{\Delta m^2}{4E}\cos 2\theta_0\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

Note the $G_F N_n$ term is common to both and on diag.
so we can discard it (overall phase)

Neutrino oscillations in MATTER (cont.)

In matter with constant density (N_e = const), diag. of Hamiltonian gives following eigenstates:

$$\nu_A = \nu_e \cos\theta + \nu_\mu \sin\theta$$

$$\nu_B = -\nu_e \sin\theta + \nu_\mu \cos\theta$$

$$\tan 2\theta = \frac{\frac{\Delta m^2}{2E} \sin 2\theta_0}{\frac{\Delta m^2}{2E} \cos 2\theta_0 - \sqrt{2}\, G_F N_e}$$

$$P(\nu_e \to \nu_\mu ; L) = \sin^2 2\theta \sin^2\left(\pi \frac{L}{l_m}\right)$$

l_m (oscillation length in matter $= \dfrac{2\pi}{E_A - E_B}$

$$E_A - E_B = \sqrt{\left(\frac{\Delta m^2}{2E} \cos 2\theta_0 - \sqrt{2}\, G_F N_e\right)^2 + \left(\frac{\Delta m^2}{2E}\right) \sin^2 2\theta_0}$$

Neutrino oscillations in Matter (cont.)

$$P(\nu_e \to \nu_\mu ; L) = \underline{\sin^2 2\theta}\ \sin^2\left(\pi \frac{L}{\ell_m}\right)$$

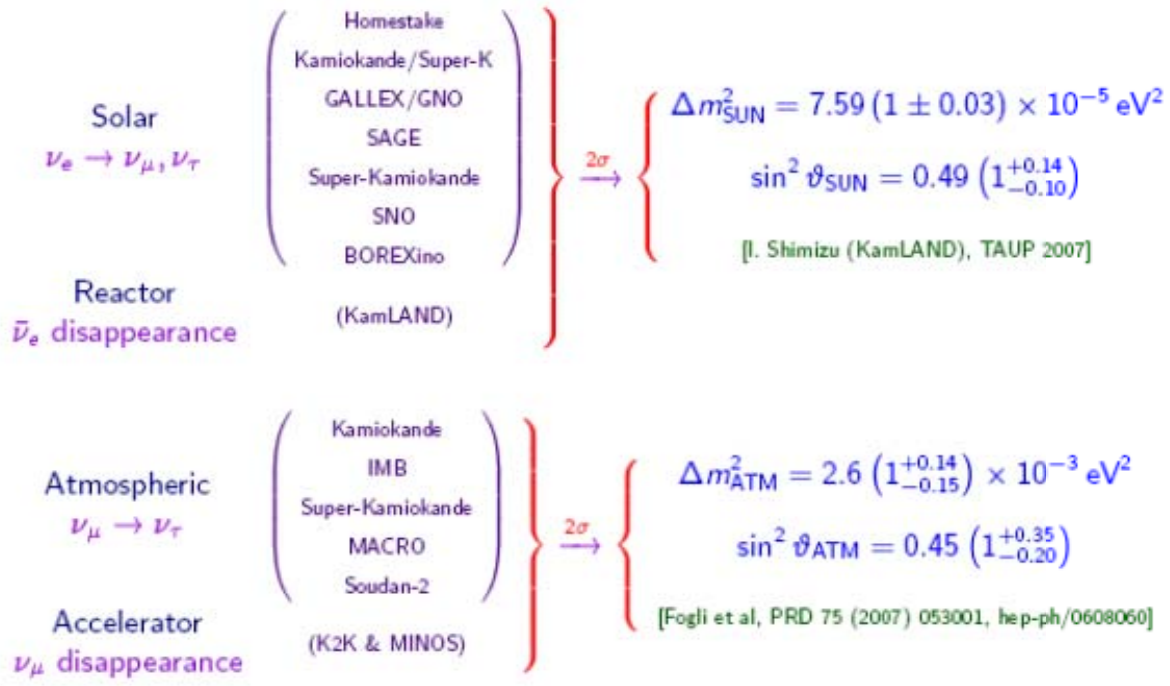
Amplitude of oscillation is maximized when

$$\sqrt{2}\, G_F N_e = \frac{\Delta m^2}{2E} \cos 2\theta_0 \quad \to \text{MSW resonance}$$

Experimental Results

Atmospheric results indicate that ν_μ goes to ν_τ (θ_{23})

Atmos + reactor → θ_{13} is small



Solar $\nu_e \to \nu_\mu, \nu_\tau$ (Homestake, Kamiokande/Super-K, GALLEX/GNO, SAGE, Super-Kamiokande, SNO, BOREXino)

Reactor $\bar{\nu}_e$ disappearance (KamLAND)

$\xrightarrow{2\sigma}$ $\Delta m^2_{\mathrm{SUN}} = 7.59\,(1 \pm 0.03) \times 10^{-5}\,\mathrm{eV}^2$

$\sin^2 \vartheta_{\mathrm{SUN}} = 0.49\,(1^{+0.14}_{-0.10})$

[I. Shimizu (KamLAND), TAUP 2007]

Atmospheric $\nu_\mu \to \nu_\tau$ (Kamiokande, IMB, Super-Kamiokande, MACRO, Soudan-2)

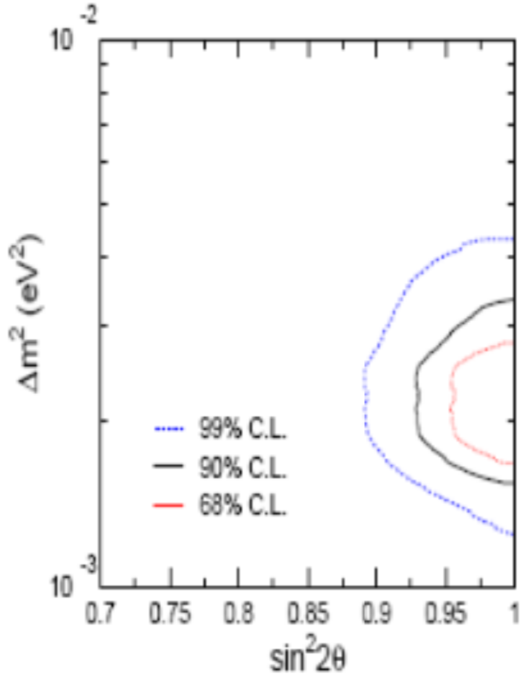
Accelerator ν_μ disappearance (K2K & MINOS)

$\xrightarrow{2\sigma}$ $\Delta m^2_{\mathrm{ATM}} = 2.6\,(1^{+0.14}_{-0.15}) \times 10^{-3}\,\mathrm{eV}^2$

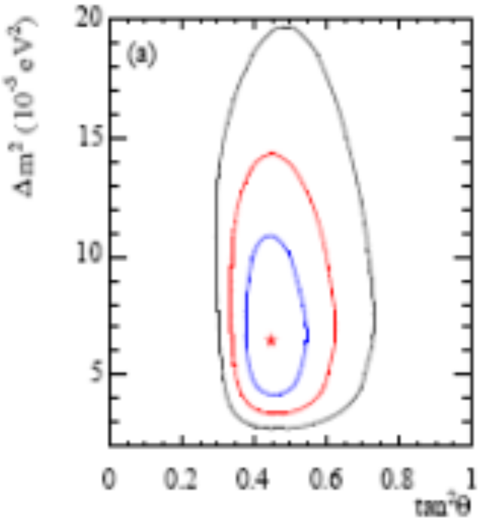
$\sin^2 \vartheta_{\mathrm{ATM}} = 0.45\,(1^{+0.35}_{-0.20})$

[Fogli et al, PRD 75 (2007) 053001, hep-ph/0608060]

Experimental Results



Atmos. ↗



Solar ↑

$$\boxed{\Delta m^2_{21} \ll |\Delta m^2_{31}|}$$

SUN →

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}$$

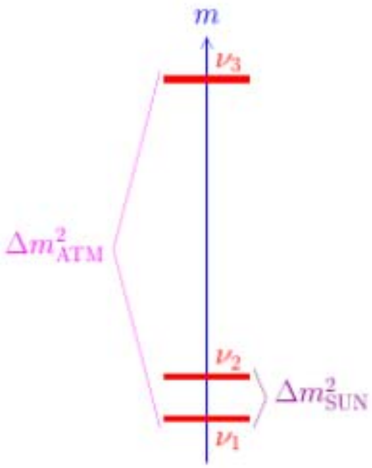
↑ ATM

$$\text{CHOOZ: } \begin{cases} \Delta m^2_{\text{CHOOZ}} = \Delta m^2_{31} = \Delta m^2_{\text{ATM}} \\ \sin^2 2\vartheta_{\text{CHOOZ}} = 4|U_{e3}|^2(1 - |U_{e3}|^2) \end{cases}$$

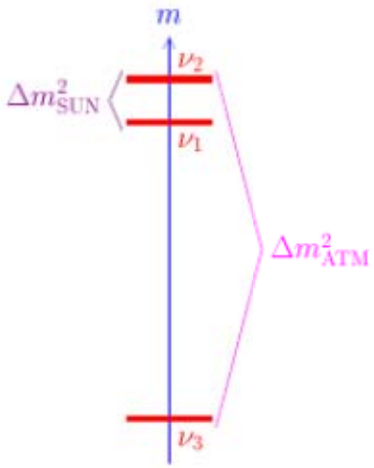
$$\Downarrow$$

$$\boxed{|U_{e3}|^2 \lesssim 5 \times 10^{-2} \text{ for } \Delta m^2 \gtrsim 2 \times 10^{-2}\,\text{eV}^2}$$

Experimental Results



"normal"



"inverted"

$$\Delta m^2_{21} = 7.9^{+0.27}_{-0.28} \left(^{+1.1}_{-0.89}\right) \times 10^{-5}\,\text{eV}^2$$

$$\left|\Delta m^2_{31}\right| = 2.6 \pm 0.2\,(0.6) \times 10^{-3}\,\text{eV}^2$$

$$\theta_{12} = 33.7 \pm 1.3 \left(^{+4.3}_{-3.5}\right)$$

$$\theta_{23} = 43.3^{+4.3}_{-3.8} \left(^{+9.8}_{-8.8}\right)$$

$$\theta_{13} = 0^{+5.2}_{-0.0} \left(^{+11.5}_{-0.0}\right)$$

$$|U|_{90\%} = \begin{pmatrix} 0.81-0.85 & 0.53-0.58 & 0.00-0.12 \\ 0.32-0.49 & 0.52-0.69 & 0.60-0.76 \\ 0.27-0.46 & 0.47-0.64 & 0.65-0.80 \end{pmatrix}$$

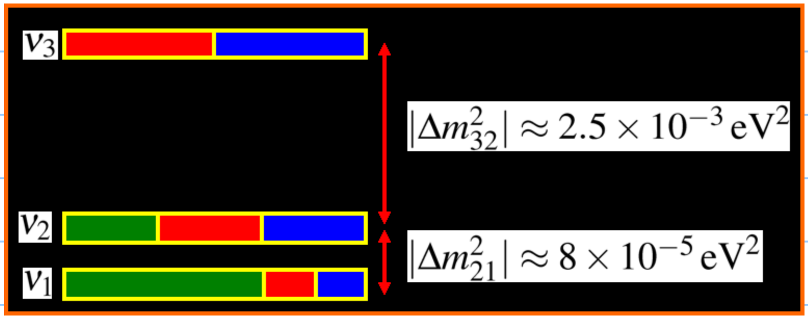
U or PMNS very different than CKM!

Updated Values for $\theta_{12}, \theta_{23}, \theta_{13}$

Parameter	Normal Ordering	Inverted Ordering
θ_{12} (deg)	$33.56\,^{+0.77}_{-0.75}$	$33.56\,^{+0.77}_{-0.75}$
θ_{23} (deg)	$41.6\,^{+1.5}_{-1.2}$	$50.0\,^{+1.1}_{-1.4}$
θ_{13} (deg)	$8.46\,^{+0.15}_{-0.15}$	$8.49\,^{+0.15}_{-0.15}$
δ_{CP} (deg)	$261\,^{+51}_{-59}$	$277\,^{+40}_{-46}$
Δm^2_{21} $(10^{-5}\mathrm{eV}^2)$	$7.50\,^{+0.19}_{-0.17}$	$7.50\,^{+0.19}_{-0.17}$
Δm^2_{3l} $(10^{-3}\mathrm{eV}^2)$	$2.524\,^{+0.039}_{-0.040}$	$-2.514\,^{+0.038}_{-0.041}$

Note updated PMNS:

$$|U| = \begin{pmatrix} 0.800-0.844 & 0.515-0.581 & 0.139-0.155 \\ 0.229-0.516 & 0.438-0.699 & 0.614-0.790 \\ 0.249-0.528 & 0.462-0.715 & 0.595-0.776 \end{pmatrix}$$



← note small values