Neutrino Masses

Chan Gwak[1]

[1]Dept. of Physics
University of Toronto

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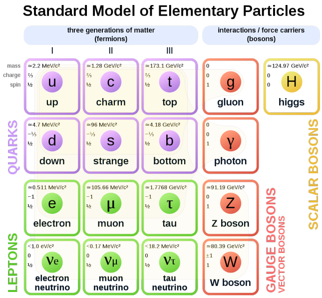
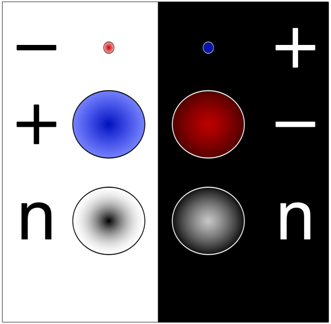
Neutrinos and the SM

- Neutrino oscillations indicate that neutrinos have masses (total < 0.32 eV)[1]
- Basic SM gives massless neutrinos
- Why are neutrinos so much lighter than the other particles?



(https://en.wikipedia.org/wiki/Standard_Model)

Particle-antiparticle conjugation



(https://en.wikipedia.org/wiki/Antiparticle)

- A right-handed version of ν_e does not exist.
- Define a transformation that turns fermions into antifermions:

$$\hat{C} : \psi \to \psi^c = C\bar{\psi}^T, \qquad C = i\gamma^2\gamma^0.$$

- Then $\psi_L \to (\psi_L)^c = {\psi^c}_R$, i.e. left-fermions become right-antifermions.

Majorana particles

- Fermionic mass term: $-m\bar{\psi}\psi = -m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$
- Need ψ_L, ψ_R nonzero.
- Dirac: ψ_L and ψ_R independent
  Four dofs: e.g. e_L^-, e_R^-, e_L^+, e_R^-
- Majorana: $\psi = \psi_L + \eta(\psi^c)_R = \psi_L + \eta(\psi_L)^c$, or
  $\psi_R = (\psi_L)^c = {\psi^c}_R$.
  $\psi^c = \eta^*\psi$, so particle is its own antiparticle.
  Two dofs: e.g. ν_{eL}, $\bar{\nu}_{eR}$

Majorana mass term

- For n flavours, a matrix M can mix them:

$$-\frac{1}{2}(\bar{\psi}_R M \psi_L + \bar{\psi}_L M^\dagger \psi_R).$$

- With $\psi_R = (\psi_L)^c$,

$$= -\frac{1}{2}(\psi_L^T C M \psi_L + \bar{\psi}_L C M^\dagger \bar{\psi}_L^T) = -\frac{1}{2}{\psi_L}^T C M \psi_L + h.c.$$

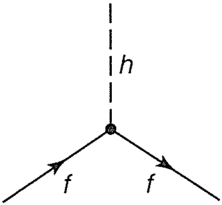
- Kinematically identical to Dirac case[2]
- Majorana mass terms violate all charges of ψ.
(Only neutral particles can be Majorana.)

SM does not have neutrino mass

- In SM, other fermions have couplings to Higgs doublet:

$$\mathcal{L}_{\text{Yukawa}} = -h^u_{ij}\bar{Q}_{Li}u_{Rj}\tilde{H} - h^d_{ij}\bar{Q}_{Li}d_{Rj}H - f^e_{ij}\bar{l}_{Li}e_{Rj}H + h.c.$$

- When EW SSB gives Higgs a VEV v, fermions get Dirac masses like $(m_u)_{ij} = h^u_{ij}v$.
- Neutrinos don't, because there is no ν_{Rj}.



(Griffiths, p. 402)

Including neutrino mass

The Lagrangian must be Lorentz invariant, gauge invariant, and renormalizable, so not every possibility works...

1. Give ν_L a Majorana mass (anomaly: $B - L$ conserved)
2. Introduce a Higgs triplet (wrong Γ_Z or unnatural VEV)

Still, there are many ways that do work:

1. Just add three $SU(2)_L$-singlets ν_R.
2. Left-right symmetric (LR) models, e.g. $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
3. GUTs, quantum gravity,[3] ...

After adding ν_R

- New Higgs coupling $-f^{\nu}_{ij}\bar{l}_{Li}\nu_{Rj}\tilde{H}$
- ν_R are EW singlets; can have Majorana term $-\frac{1}{2}{\nu_R}^T CM\nu_R$
- Lepton number is no longer a symmetry via anomaly ("accidental" symmetry due to Lagrangian conditions and content)
- Zero hypercharge; number of ν_R's flavours need not match number of ν_L's

Mass generation

- The most general mass term would be:

$$-\frac{1}{2}{\nu_L}^T C m_L \nu_L - \bar{\nu}_L m_D^* \nu_R - \frac{1}{2}{\nu_R}^T C m_R^* \nu_R$$

  where m_L is $n \times n$ (sym), m_D is $n \times k$, and m_R is $k \times k$ (sym), and n and k are numbers of left and right flavours.
- Write this as

$$-\frac{1}{2}\begin{pmatrix} {\nu_L}^T & (\nu_R)^{cT} \end{pmatrix} C \begin{pmatrix} m_L & m_D \\ m_D^T & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} \equiv -\frac{1}{2}{n_L}^T C \mathcal{M} n_L.$$

- Change bases $n_L \equiv U\chi_L$ such that $U^T \mathcal{M} U$ is block diagonal:

$$U^T \mathcal{M} U = \begin{pmatrix} \tilde{m}_L & 0 \\ 0 & \tilde{m}_R \end{pmatrix}.$$

Mass generation: One-flavour case

- Basis change can be rotation with $\tan 2\theta = 2m_D/(m_R - m_L)$:

$$n_L \equiv U\chi_L = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \chi_{1L} \\ \chi_{2L} \end{pmatrix}.$$

- The mass eigenvalues are then

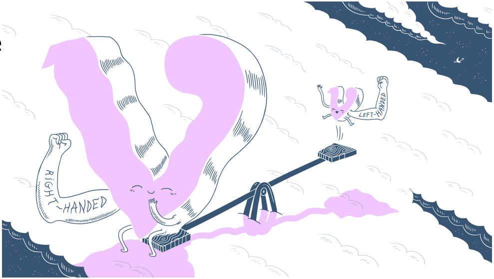
$$\tilde{m}_{R,L} = \frac{m_R + m_L}{2} \pm \sqrt{\left(\frac{m_R - m_L}{2}\right)^2 + m_D^2}.$$

- Write $\chi_i = \chi_{iL} + \eta_i(\chi_{iL})^c$ (Majorana):

$$-\frac{1}{2}|\tilde{m}_L|\bar{\chi}_1\chi_1 - \frac{1}{2}|\tilde{m}_R|\bar{\chi}_2\chi_2$$

Seesaw mechanism: One-flavour case

- ν_R is electroweak singlet, could be quite massive.
- Suppose $m_L \ll m_D \ll m_R$. Then $\tilde{m}_R \simeq m_R$, and $\tilde{m}_L \simeq m_L - m_D^2/m_R$.
- Even if $m_L = 0$, $|\tilde{m}_L| \ll |\tilde{m}_R|$.



(https://www.symmetrymagazine.org/article/neutrinos-on-a-seesaw)

Seesaw mechanism: *n*-flavour case

- Back to matrices, but similar:

$$\tilde{m}_R \simeq m_R, \qquad \tilde{m}_L \simeq m_L - m_D m_R^{-1} m_D^T.$$

- $\tilde{m}_L$ is $n \times n$, mixes n flavours
- n light Majorana neutrinos, k heavy
- If $m_L = 0$, $m_D \sim 174$ GeV (EW scale), and $m_R \sim 10^{15}$ GeV (GUT), then heaviest light neutrino is $\sim 10^{-2}$ eV.
- Smallness of seen neutrino mass is related to other large mass scales. Satisfying!
- ...so how does this affect the physics?

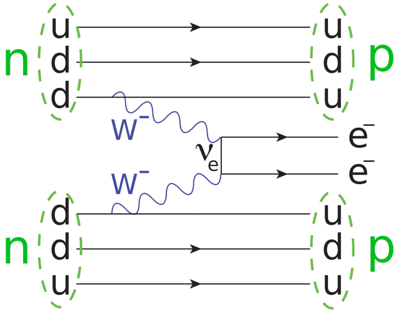
Revisiting oscillations

The time evolution of oscillations depends on $U\,\mathrm{diag}(m_1^2, m_2^2, m_3^3)\;U^\dagger$ where U is the mixing matrix:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}$$
$$\times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\varphi_1} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\varphi_2} \end{pmatrix}$$

but last part is consequential for oscillations.

Neutrinoless double beta decay



(https://en.wikipedia.org/wiki/Double_beta_decay)

- $0\nu\beta\beta$
- Violates lepton number and chirality
- Would imply $m_L, m_R \neq 0$.[4]
- Not yet detected; $T_{1/2}^{0\nu} > 1.07 \times 10^{26}$ yrs, $m_{\beta\beta} < (0.061 - 0.165)$ eV (KamLAND-Zen)[5,6]

Summary

1. Can add ν_R to introduce neutrino masses
2. Models can have Dirac and Majorana masses
3. Seesaw mechanism explains tiny neutrino mass
4. Majorana masses do not affect oscillations
5. Neutrinoless double beta decay would confirm that lepton number is not conserved and that neutrinos have Majorana masses

Thank You!

References

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