

## LECTURE 15: Hadron Structure

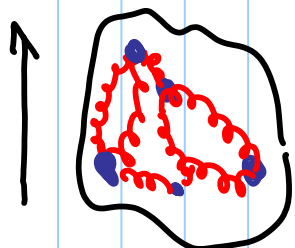
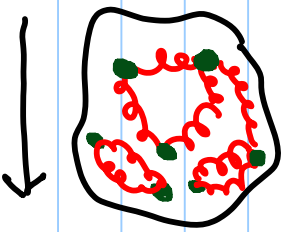
### Overview:

- Formalism for the kinematics
- Elastic Scattering of protons and neutrons
- Deep Inelastic scattering

(I used Quigg and Halzen-Martin as references)

# Hydrogen Structure

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We would like to understand the composition of hydrogens:

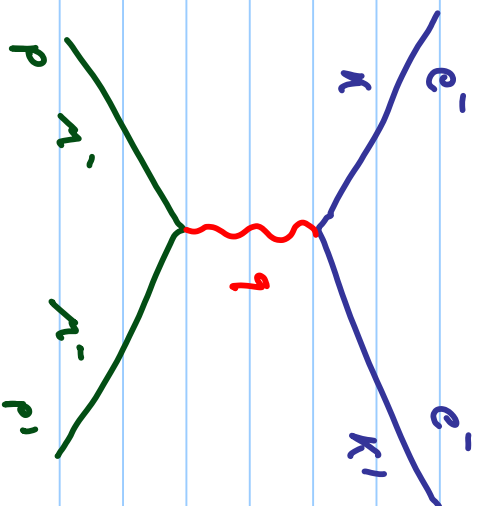
→ determine neutron flux in hydrogen collisions

→ learn more about QCD

We'll start by doing a cross section calculation in the lab frame. This will prove convenient later on.

We go back to  $e^-p$  scattering. We calculated the cross section in the CM frame. Here we switch to the lab frame.

## Electron-muon scattering



we neglect  $m_e$  and obtained:

$$|M|^2 = \frac{8e^4}{q^4} [(k \cdot p)(k \cdot p) + (k \cdot p')(k \cdot p') - m^2 k \cdot k']$$

↪ muon mass

We have:  $q = k - k'$

$$\begin{cases} p \cdot p' = (k - k' + p) \cdot p \\ q^2 \approx -2k \cdot k' \end{cases}, \quad k^2 \approx 0$$

Eliminate  $p'$  ↘

$$\rightarrow |M|^2 = \frac{8e^4}{q^4} \left[ -\frac{1}{2} q^2 (k \cdot p - k' \cdot p) + 2(k' \cdot p)(k \cdot p) + \frac{1}{2} m^2 q^2 \right]$$

# Electron-muon scattering

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Lab frame:

$$\begin{aligned}
 p &= (M, \vec{0}) \\
 k &= (E, \vec{k}) \\
 k' &= (E', \vec{k}') \\
 q &= (\nu, \vec{q})
 \end{aligned}$$

We get  $|\vec{M}'|^2 = \frac{g_e^4}{q^4} \left[ -\frac{1}{2} q^2 M(E-E') + 2EE' M^2 + \frac{1}{2} M^2 q^2 \right]$

$\hookrightarrow$  muon mass  $\nearrow$

$$= \frac{g_e^4}{q^4} 2M^2 E E' \left[ 1 + \frac{q^2}{4EE'} - \frac{q^2}{2M^2} \frac{M(E-E')}{2EE'} \right]$$

We have  $q^2 \approx -2k \cdot k' \approx -2EE'(1 - \cos\theta)$   
 $= -4EE' \sin^2 \frac{\theta}{2}$

$$p+q = p' \rightarrow (p+q)^2 = p'^2 \rightarrow p^2 + q^2 + 2(q \cdot p) = p'^2 \rightarrow q^2 = -2(q \cdot p)$$

$$q^2 = -2\nu M, \quad \nu = E - E' \Rightarrow E - E' = \frac{-q^2}{2M}$$

$$\Rightarrow |\vec{M}'|^2 = \frac{g_e^4}{q^4} 2M^2 E E' \left\{ 1 - \frac{\sin^2 \theta}{2} - \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2} \right\}$$

$\underbrace{\hspace{10em}}_{\cos^2 \theta/2}$

# Electron-neutron scattering

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$$\text{We set: } \frac{8e^4}{q^4} \frac{2m^2 E E'}{2M^2} \left[ \cos^2 \frac{\theta}{2} - \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2} \right]$$

Now, the cross section:

$$\begin{aligned} d\sigma &= \frac{1}{(2E)(2M)} \frac{|M|^2}{4\pi^2} \frac{d^3K'}{2E'} \frac{d^3p'}{2p_0'} \delta^4(p+k-p'-k') \\ &= \frac{1}{4ME} \frac{|M|^2}{4\pi^2} \frac{1}{2} E' dE' d\Omega \frac{d^3p'}{2p_0'} \delta^4(p+q-p') \end{aligned}$$

For the  $\int \frac{d^3p'}{2p_0'}$   $\delta^4(p+q-p')$  integration we get

$$\text{note that } \delta(p_0'^2 - a^2) = \frac{1}{2|a|} (\delta(p_0' - a) + \delta(p_0' + a))$$

$$\Rightarrow \delta((p+q)^2 - M^2) = \delta(2p \cdot q + q^2) = \frac{1}{2M} \delta\left(\nu + \frac{q^2}{2M}\right)$$

# Electron-muon scattering

$$\frac{1}{2M} \delta \left( \nu + \frac{q^2}{2M} \right) = \frac{1}{2M} \delta \left[ E - E' - E E' \left( 1 - \cos \theta \right) / M \right]$$

$$= \frac{1}{2M A} \delta \left( E' - E/A \right)$$

$$A = 1 + \frac{2E}{M} \sin^2 \frac{\theta}{2}$$

Inserting matrix element and using result above, we get:

$$\frac{d\sigma}{dE' d\Omega} = \left( \frac{2 \alpha E'^2}{q^4} \right)^2 \left[ \cos^2 \frac{\theta}{2} - \frac{q^2}{2M} \sin^2 \frac{\theta}{2} \right] \delta \left( \nu + \frac{q^2}{2M} \right)$$

$$\frac{d\sigma}{d\Omega} = \left[ \frac{\alpha^2}{4 E^2 \sin^4 \frac{\theta}{2}} \right] \frac{E'}{E} \left[ \cos^2 \frac{\theta}{2} - \frac{q^2}{2M} \sin^2 \frac{\theta}{2} \right]$$

↳ due to magnetic moment

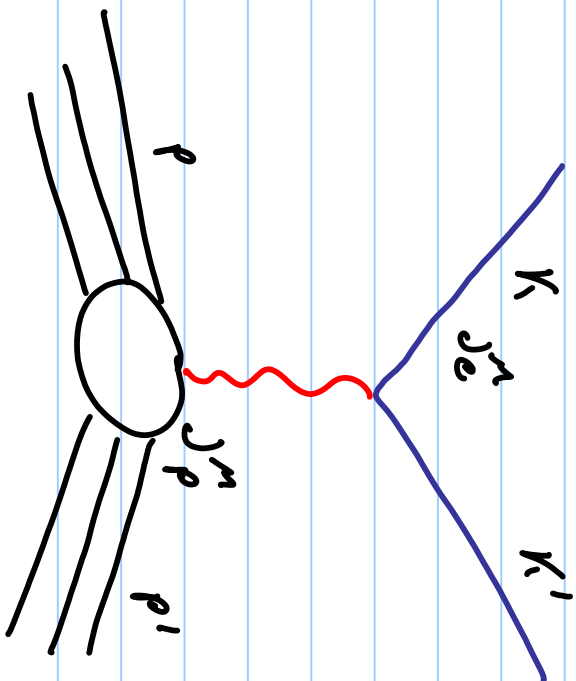
Note that for spinless particles we had vertex factors of the form:  $i e (p_A + p_B)_\mu$  and

$$\bar{u}(p_f) \gamma^\mu v = \frac{1}{2M} \bar{u}(p_f + p_i) \gamma^\mu + i \sigma^{\mu\nu} (p_f - p_i)_\nu v_i$$

# Electron-Proton scattering

If the proton was a point charge with Dirac magnetic moment  $\frac{e}{2m}$ , we would just replace

$M_N \rightarrow M_p$ . But it is not... so:



$$j_e^\mu = -e \bar{u}(k') \gamma^\mu u(k) e^{i(k'-k) \cdot x}$$

$$j_p^\mu = e \bar{u}(p') [ \gamma^\mu ] u(p) e^{i(p'-p) \cdot x}$$

## Electron-Proton scattering

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- $J^{\mu}$  must be a 4-vector
- Two indep. Terms:  $J^{\mu}$ ,  $i\sigma^{\mu\nu}q_{\nu}$
- no  $\gamma^5$   $\rightarrow$  parity is conserved

$$\boxed{?} = \left[ F_1(q^2) \gamma^{\mu} + \frac{\kappa}{2m} F_2(q^2) i\sigma^{\mu\nu} q_{\nu} \right]$$

For  $q^2 \rightarrow 0$  proton currents not resolved. We have a particle of charge  $e$  and magnetic moment  $(1+x)\frac{e}{2m}$

$\hookrightarrow$  dimensionless mag. moment: proton = 1.79, neutron = -1.91

with  $F_1(0) = 1$ ,  $F_2(0) = 1$

The expression similar to  $e_{\mu}$  scattering is:



## ELECTRON-PROTON SCATTERING

$$\frac{d\sigma}{d\Omega} \Big|_{lab} = \left( \frac{q^2}{4E^2 \sin^2 \theta/2} \right) \frac{E'}{E} \left[ \left( F_1^2 - \frac{\kappa^2 q^2}{4M^2} F_2^2 \right) \cos^2 \frac{\theta}{2} - \frac{q^2}{2M} (F_1 + \kappa F_2)^2 \sin^2 \frac{\theta}{2} \right]$$

→ ROSENBLUTH FORMULA

$\kappa = 0$ ,  $F_1 = 1$  recovers muon result

In practice, different factors are used:

$$G_E \equiv F_1 + \frac{\kappa q^2}{4M^2} F_2, \quad G_M \equiv F_1 + \kappa F_2$$

$$\text{we get: } \frac{d\sigma}{d\Omega} = \frac{q^2}{4E^2 \sin^4 \theta/2} \frac{E'}{E} \left( \frac{G_E^2 + \gamma G_M^2}{1 + \gamma} \cos^2 \frac{\theta}{2} + 2\gamma G_M^2 \sin^2 \frac{\theta}{2} \right)$$

$$\gamma = \frac{-q^2}{4M^2}$$

→ no "interference" term

$G_E$  and  $G_M$  RELATED TO PROTON CHARGE AND MAGNETIC MOMENT DIST. IN BREIT FRAME  $\rho' = -\rho$

# ELECTRON-PROTON SCATTERING

$$G_E(q^2) \approx \left(1 - \frac{q^2}{0.71}\right)^{-2} \quad \text{in } \text{GeV}^2$$

$$\langle r^2 \rangle = 6 \left( \frac{dG_E(q^2)}{dq^2} \right)_{q^2=0} = 0.8 \times 10^{-15} \text{ m}$$

$$\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{point}} |F(q)|^2$$

$$F(q) = \int \rho(x) e^{iq \cdot x} d^3x$$

$$\int \rho(x) d^3x = 1$$

$$F(0) = 1$$

$$F(q) = \left( 1 + iq \cdot x - \frac{(q \cdot x)^2}{2} \dots \right) \rho(x) d^3x$$

$$= 1 - \frac{1}{6} |q|^2 \langle r^2 \rangle + \dots$$

