LECTURE 16: Hadron Structure (Part 2)

Overview:

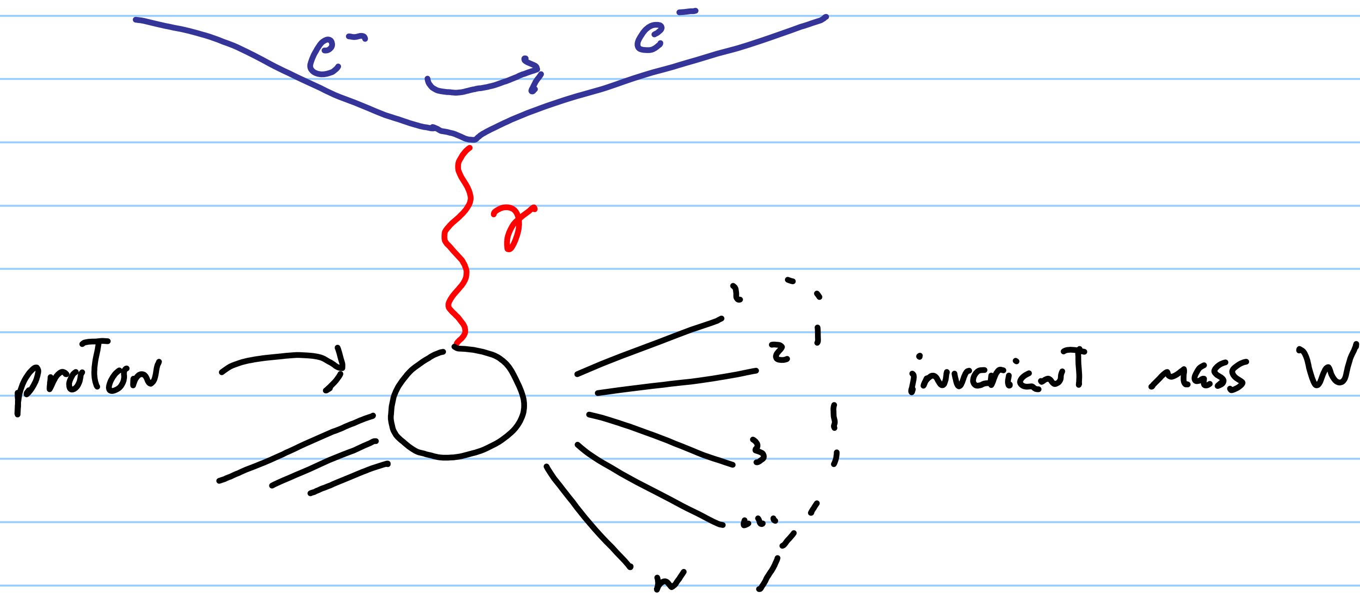
-Inelastic Scattering of protons and electrons

-Contents of the proton and neutron

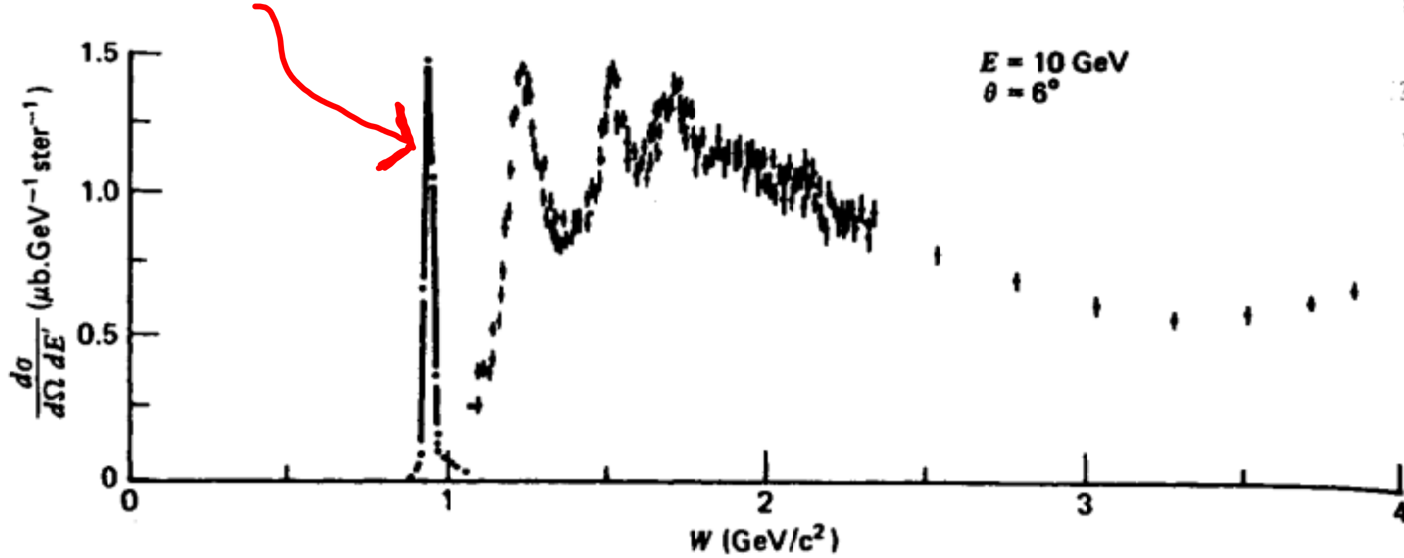
-Parton Distribution Functions

(I used Quigg and mostly Halzen-Martin as references)

INELASTIC ELECTRON - PROTON SCATTERING



elastic contribution



Inelastic Electron-Proton Scattering

We had before for $e\mu$ scattering : $d\sigma \sim L^{e}_{\mu\nu} (L^{\mu})^{\mu\nu}$

We'll Try : $L^{e}_{\mu\nu} W^{\mu\nu}$

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^\mu p^\nu + \frac{W_4}{M^2} q^\mu q^\nu + \frac{W_5}{M^2} (p^\mu q^\nu + p^\nu q^\mu)$$

$L_{\mu\nu}$ is symmetric $\rightarrow$ antisymmetric Terms will vanish

W_3 will be parity violating Term

We use $q^\mu L^{e}_{\mu\nu} = q^\nu L^{e}_{\mu\nu} = 0$ (check this)

We can also show that $q_\mu W^{\mu\nu} = 0$ which follows from $\partial_\mu J^\mu = 0$

$$\Rightarrow \quad W_5 = -\frac{p \cdot q}{q^2} W_2 \quad , \quad W_4 = \left(\frac{p \cdot q}{q^2}\right)^2 W_2 + \frac{M^2}{q^2} W_1$$

$\Rightarrow$ only Two structure functions are indep.

Inelastic Electron-Proton Scattering

$$W^{\mu\nu} = W_1\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + W_2\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}q^\nu\right)\cdot\frac{1}{M^2}$$

We can chose two indep. variables : q^2 , $\frac{p\cdot q}{M} \equiv \nu$

or dimensionless : $x = \frac{-q^2}{2p\cdot q} = \frac{-q^2}{2M\nu}$, $y = \frac{p\cdot q}{p\cdot K}$

Invariant mass : $W^2 = (p+q)^2 = M^2 + 2M\nu + q^2$

In rest frame of proton : $\nu = E - E'$

$$y = \frac{E-E'}{E}$$

We now have:

$$(L^e)^{\mu\nu}W_{\mu\nu} = 4W_1(K\cdot K') + \frac{2W_2}{M^2}\left[2(p\cdot K)(p\cdot K') - M^2 K\cdot K'\right]$$

Inelastic Electron-Proton Scattering

In lab frame:

$$L^{e\,\mu\nu} W_{\mu\nu} = 4EE' \left[\cos^2\frac{\theta}{2}\, W_2(\nu,q^2) + \sin^2\frac{\theta}{2}\, 2W_1(\nu,q^2) \right]$$

$$d\sigma = \frac{1}{4\left((k\cdot p)^2 - m^2M^2\right)^{1/2}} \underbrace{\left[\frac{e^4}{q^4} L^{\mu\nu} W_{\mu\nu} \underbrace{4\pi M}_{\text{normalisation of } W^{\mu\nu}} \right]}_{|M|^2} \frac{d^3k'}{2E'(2\pi)^3}$$

$$\left.\frac{d\sigma}{dE'\,d\Omega}\right|_{lab} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}} \left[W_2(\nu,q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu,q^2)\sin^2\frac{\theta}{2} \right]$$

→ neglected electron mass.

Inelastic Electron-Proton Scattering

Summary of recent results:

For all reactions, the diff. cross section can be written as

$$\boxed{\frac{d\sigma}{dE'\,d\Omega} = \frac{4\alpha^2 E'^2}{q^4}\left[\quad\right]}$$

Point particle (we did muon but we'll deal with quarks...)

1. $$\boxed{[\ ] = \left(\cos^2\frac{\theta}{2} - \frac{q^2}{2m^2}\sin^2\frac{\theta}{2}\right)\delta\left(\nu + \frac{q^2}{2m}\right)}$$

elastic proton-electron:

2. $$\boxed{[\ ] = \left(\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2}\right)\delta\left(\nu + \frac{q^2}{2m}\right), \quad \tau = \frac{-q^2}{4m^2}}$$

inelastic proton-electron:

3. $$\boxed{[\ ] = W_2(\nu, q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu, q^2)\sin^2\frac{\theta}{2}}$$

INELASTIC ELECTRON - PROTON SCATTERING

If point-like spin 1/2 quarks live inside the proton, we should be able to resolve them with photons that have small enough wavelength.

$\rightarrow$ structure functions would become: $Q^2 \equiv -q^2$

$$2W_1 \rightarrow \frac{Q^2}{2m^2}\,\delta\left(\nu - \frac{Q^2}{2m}\right), \qquad W_2 = \delta\left(\nu - \frac{Q^2}{2m}\right)$$

inelastic proton-electron scattering $\rightarrow$ elastic electron-quark scattering

Using $\delta\left(\frac{x}{a}\right) = a\,\delta(x)$:

$$2m\,W_1(\nu, Q^2) = \frac{Q^2}{2m\nu}\,\delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

$$\nu W_2(\nu, Q^2) = \delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

$\rightarrow$ W_1, W_2 now functions of $\frac{Q^2}{2m\nu}$ and not Q^2 and ν independently.

INELASTIC ELECTRON-PROTON SCATTERING

For elastic scattering of ep with $\kappa = 0 \Rightarrow G_E = G_M = G$

$$W_1 = \frac{Q^2}{4M^2} G^2(Q^2)\, \delta\left(\nu - \frac{Q^2}{2m}\right)$$

$$W_2 = G^2(Q^2)\, \delta\left(\nu - \frac{Q^2}{2m}\right)$$

$\hookrightarrow$ reflects size of proton

So for point constituents probed by large Q^2 photons:

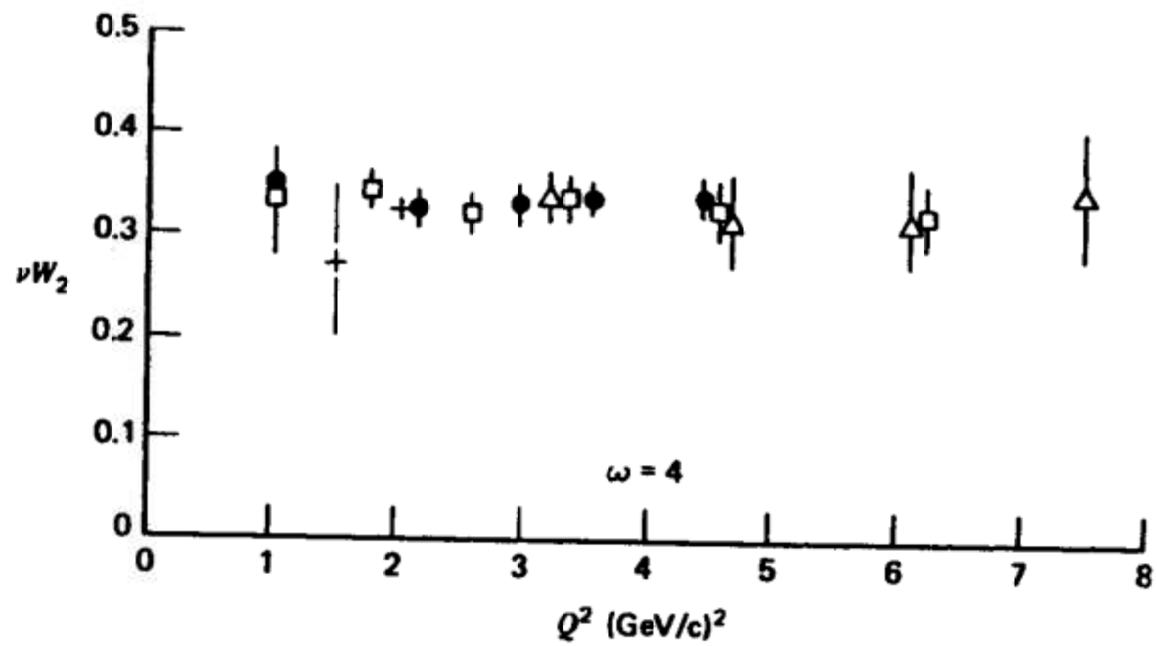
$$\begin{aligned} M W_1(\nu, Q^2) &\to F_1(\omega) \\ \nu W_2(\nu, Q^2) &\to F_2(\omega) \end{aligned} \quad , \quad \omega = \frac{2q \cdot p}{Q^2} = \frac{2M\nu}{Q^2}$$

→ we should see that νW_2 is independent of Q^2 for a given ω

→ evidence for "partons"

→ Bjorken Scaling

Inelastic Electron-Proton Scattering



"PDFs" : parton distribution functions

$f_i(x) = \frac{dP_i}{dx}$ describes probability that a parton will carry a fraction x of the proton's momentum

$$\sum_i \int dx \, x f_i(x) = 1$$

INELASTIC ELECTRON-PROTON SCATTERING

In Terms of x and ω, structure functions given by:

$$F_1(\omega) = \frac{Q^2}{2M\nu x}\,\delta\left(1 - \frac{Q^2}{2M\nu}\right) = \frac{1}{2x^2\omega}\,\delta\left(1 - \frac{1}{x\omega}\right)$$

$$F_2(\omega) = \delta\left(1 - \frac{Q^2}{2M\nu}\right) = \delta\left(1 - \frac{1}{x\omega}\right)$$

$$\Rightarrow \quad F_2(\omega) = \sum_i \int dx\, e_i^2 f_i(x)\, x\, \delta\left(x - \frac{1}{\omega}\right)$$

$$F_1(\omega) = \frac{\omega}{2} F_2(\omega)$$

In Terms of x:

$$\left.\begin{aligned} \nu W_2(\nu, Q^2) &\to F_2(x) = \sum_i e_i^2\, x\, f_i(x) \\ M W_1(\nu, Q^2) &\to F_1(x) = \frac{1}{2x} F_2(x) \end{aligned}\right\} \text{satisfies Bjorken scaling}$$

$$\boxed{x = \frac{1}{\omega} = \frac{Q^2}{2M\nu}}$$

Inelastic Electron-Proton Scattering

We can reexpress the results we obtained in terms of x and

$$y = \frac{\nu}{E} = \frac{p \cdot q}{p \cdot k} \quad , \quad 1-y \simeq \frac{1}{2}(1+\cos\theta)$$

$$dE'\,d\Omega = \frac{\pi}{EE'}\,dQ^2\,d\nu = \frac{2ME}{E'}\,\pi y\,dx\,dy$$

$$M\nu_{max}\frac{d\sigma}{dx\,dy} = \frac{2\pi\alpha^2}{x^2y^2}\left\{ xy^2F_1 + \left[(1-y) - \frac{Mxy}{2\nu_{max}}\right]F_2 \right\}$$

$\nu_{max} = E$ in the lab frame

Parton model predicts:

$$\frac{d\sigma}{dx\,dy} = \frac{2\pi\alpha^2}{Q^4}\,s\left[1 + (1-y)^2\right]\sum_i e_i^2\,x f_i(x)$$

Proton Quark Content

We had:
$$F_2(x) = \sum_i e_i^2 \, x f_i(x)$$
$$F_1(x) = \frac{1}{2x} F_2(x)$$

$$\rightarrow \frac{1}{x} F_2^{ep} = \left(\frac{2}{3}\right)^2 \left[u^p(x) + \bar{u}^p(x)\right] + \left(\frac{1}{3}\right)^2 \left[d^p(x) + \bar{d}^p(x)\right] + \left(\frac{1}{3}\right)^2 \left[s^p(x) + \bar{s}^p(x)\right]$$

$\rightarrow$ neglect other heavy quarks

We have a similar expression for neutrons.

$$\frac{1}{x} F_2^{en} = \left(\frac{2}{3}\right)^2 \left[u^n(x) + \bar{u}^n(x)\right] + \dots$$

$$u^p(x) = d^n(x) \equiv u(x)$$
$$d^p(x) = u^n(x) \equiv d(x)$$
$$s^p(x) = s^n(x) \equiv s(x)$$

proton $u_v \; u_v \; d_v$ (valence)
$u_s \bar{u}_s, \; d_s \bar{d}_s, \; s_s \bar{s}_s$ (sea)

Proton Quark Content

$$u_s(x) = \bar{u}_s(x) = d_s(x) = \ldots = s(x)$$

$$u(x) = u_v(x) + u_s(x)$$

$$d(x) = d_v(x) + d_s(x)$$

$$\int_0^1 [u(x) - \bar{u}(x)]\, dx = 2$$

$$\int_0^1 [d(x) - \bar{d}(x)]\, dx = 1$$

$$\int_0^1 [s(x) - \bar{s}(x)]\, dx = 0$$

$$\Rightarrow \quad \frac{1}{x} F_2^{ep} = \frac{1}{9}[4u_v + d_v] + \frac{4}{3} s$$

$$\frac{1}{x} F_2^{en} = \frac{1}{9}[u_v + 4d_v] + \frac{4}{3} s$$

At low x we expect:

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} \xrightarrow[x \to 0]{} 1$$

At high x we expect

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} \to \frac{u_v + 4d_v}{4u_v + d_v}$$

$\rightarrow$ proton $u_v \gg d_v$: ratio tends towards 0.25

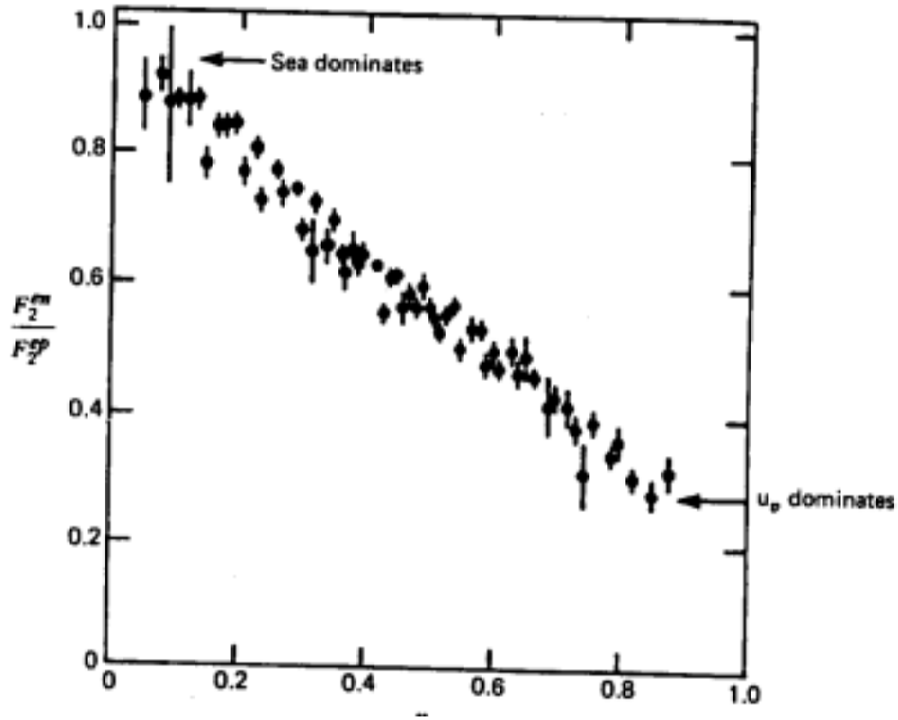
Proton Quark Content

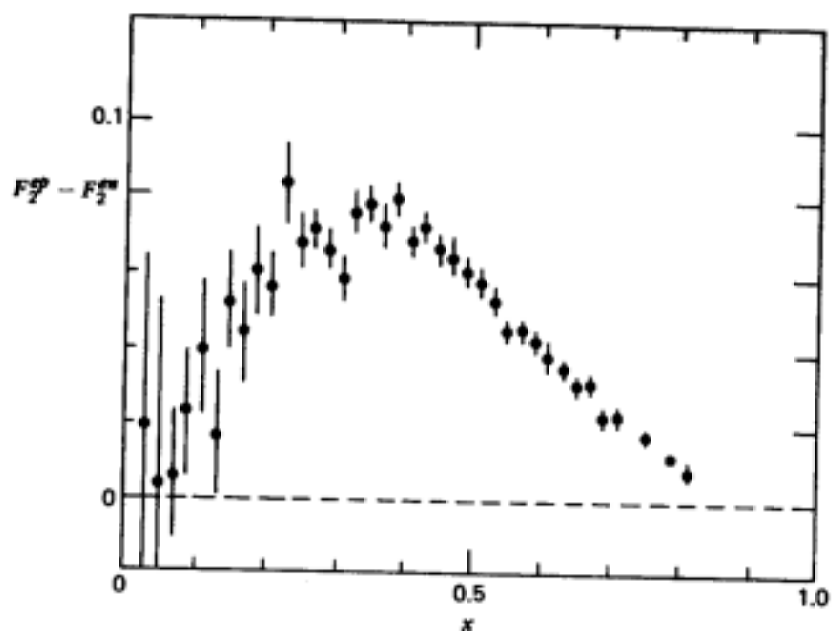
$$\frac{1}{x} F_2^{ep} = \frac{1}{9}\left[4u_v + d_v\right] + \frac{4}{3} S$$

$$\frac{1}{x} F_2^{en} = \frac{1}{9}\left[u_v + 4d_v\right] + \frac{4}{3} S$$

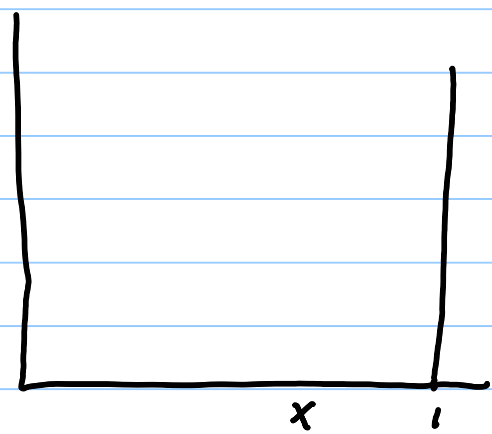
If we subtract the two equations above, we get:

$$\frac{1}{3}\left[u_v(x) - d_v(x)\right]$$





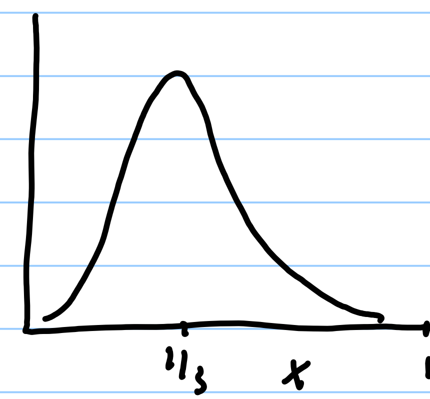
For 1 quark, we should get:

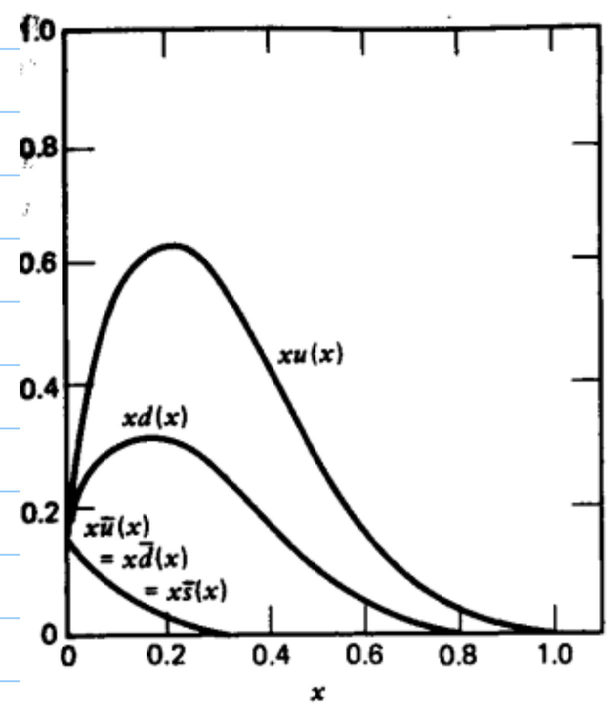


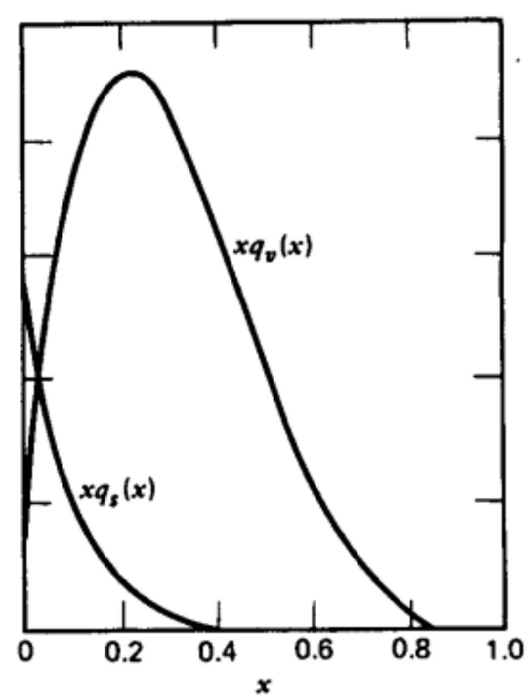
Proton quark content

For 3 quarks:

3 quarks + interactions:





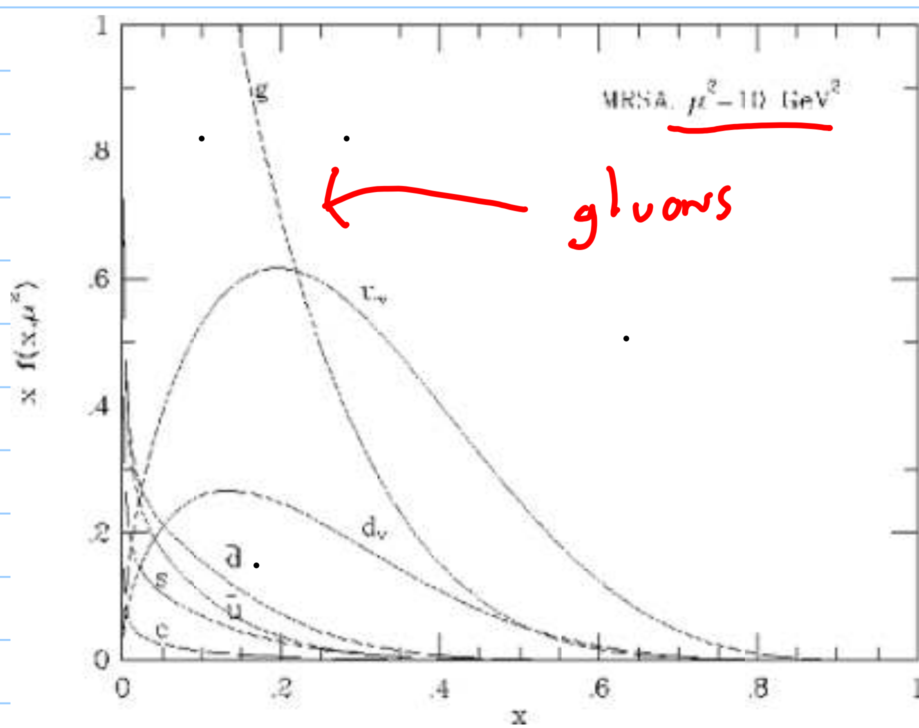


Proton GLUON CONTENT

IF we integrate $\int_0^1 dx\, x[u+\bar{u}+d+\bar{d}+s+\bar{s}] = 1 - F_{gluon}$

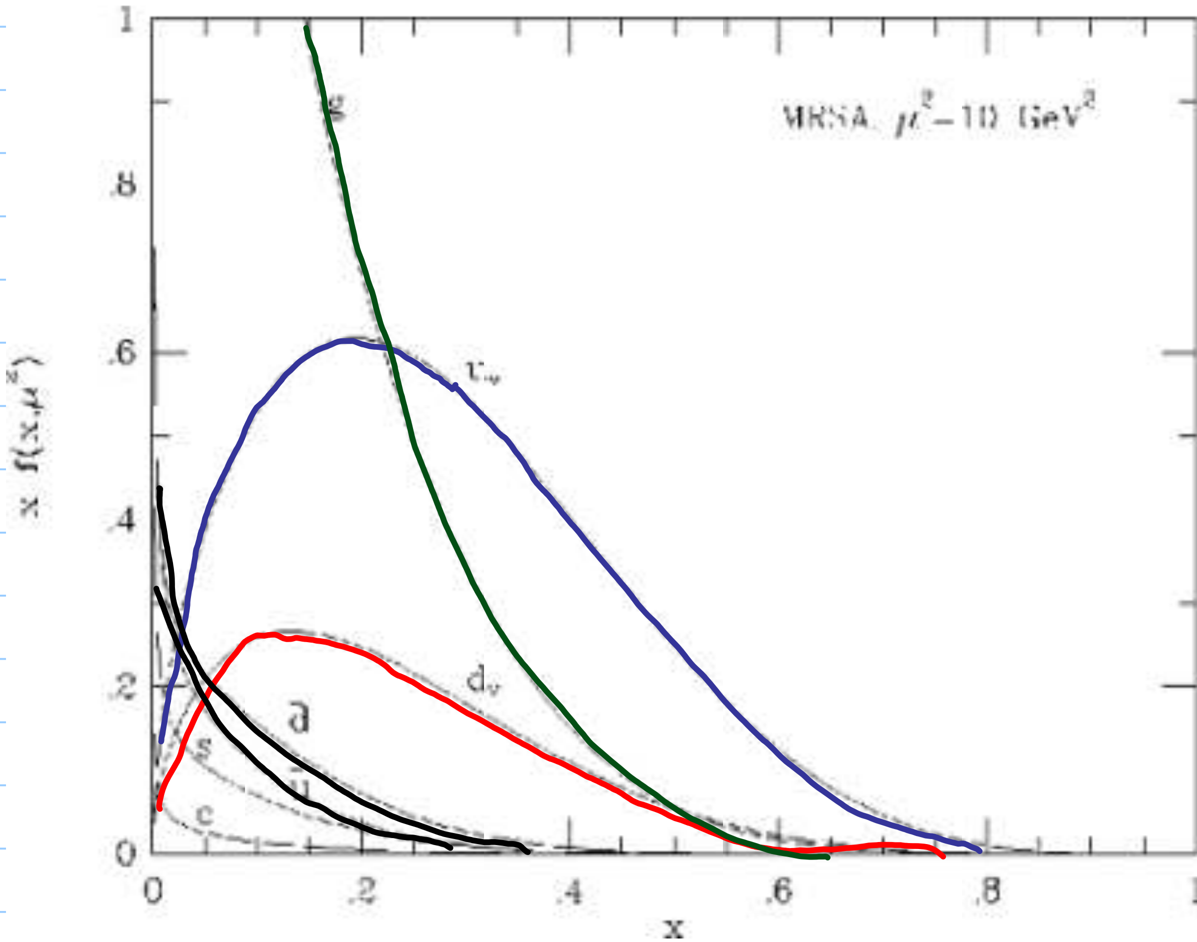
The photon does not interact with the gluon ...

WE GET $F_{gluon} \sim 0.45$!



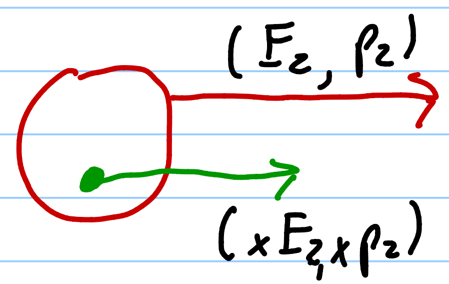
note That these depend on Q^2 ...

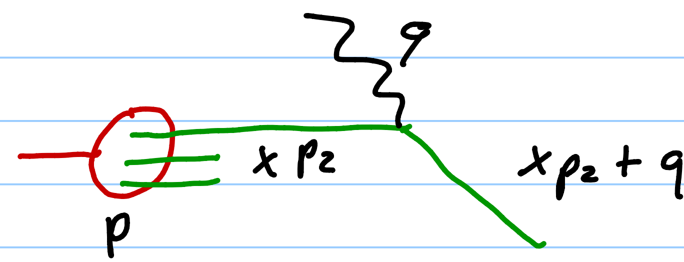
PDFs



Recap $\rightarrow$ Following Thomson

Kinematics





momentum of quark after collision

$$(xp_2 + q)^2 = m_q^2 \approx 0$$

$$\rightarrow \quad x^2 p_2^2 + q^2 + 2x\, p_2 \cdot q = 0$$

$$\hookrightarrow \approx m_q \approx 0$$

$$\Rightarrow \quad x = \frac{-q^2}{2p_2 \cdot q} \equiv \frac{Q^2}{2p_2 \cdot q}$$

$$s = (p_1 + p_2)^2 \approx 2p_1 \cdot p_2 \quad , \quad y = \frac{p_2 \cdot q}{p_2 \cdot p_1} \quad , \quad x = \frac{Q^2}{2p_2 \cdot q}$$

$$s_q = (p_1 + xp_2)^2 = xs \quad , \quad y_q = \frac{xp_2 \cdot q}{x\,p_2 \cdot p_1} = y \quad , \quad x_q = 1$$

Use $e^-\mu^- \to e^-\mu^-$ result for $eq \to eq$

$$\frac{d^2\sigma}{dq^2} = \frac{2\pi\alpha^2 e_q^2}{q^4}\left[1 + \left(1+\frac{q^2}{s_q}\right)^2\right]$$

with $-q^2 = Q^2 = (s_q - m^2)\, x_q y_q \Rightarrow \frac{q^2}{s_q} = -y_q = -y$

$$\frac{d\sigma}{dQ^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4}\left[1 + (1-y)^2\right] = \frac{4\pi\alpha^2 e_q^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right]$$

Introduce partons $q^p(x)\, dx$

$\downarrow$ number of partons of type q with fraction of momentum between x and $x + dx$

$$\rightarrow \frac{d^2\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right]\sum_q e_q^2\, q^p(x)$$

with structure functions: $\frac{d^2\sigma}{dx\, dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]$

Note that $F_2^p(x, Q^2) = 2x F_1^p(x, Q^2) = x \sum_q e_q^2 q^p(x)$

Parton model predicts

- Bjorken scaling $\rightarrow$ $F_1(x, Q^2) \rightarrow F_1(x)$

  $F_2(x, Q^2) \rightarrow F_2(x)$

- Callan - Gross relation $F_2(x) = 2x F_1(x)$

  $\rightarrow$ due to spin 1/2 particles. Fixes electromagnetic and pure magnetic terms with respect to each other

$$F_2^{ep}(x) = x \sum_q e_q^2 q^p(x) = x \left(\tfrac{4}{9} u^p(x) + \tfrac{1}{9} d^p(x) + \tfrac{4}{9} \bar{u}^p(x) + \tfrac{1}{9} \bar{d}^p(x) \right)$$

Hera ep collider

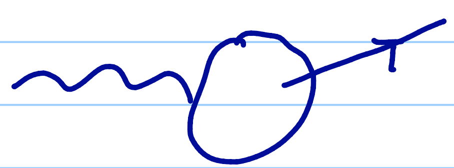
Two main experiments:

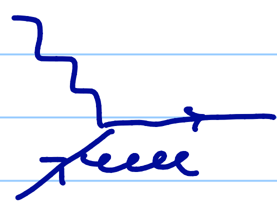
ZEUS and H1

Note behavior of F_2 on the right

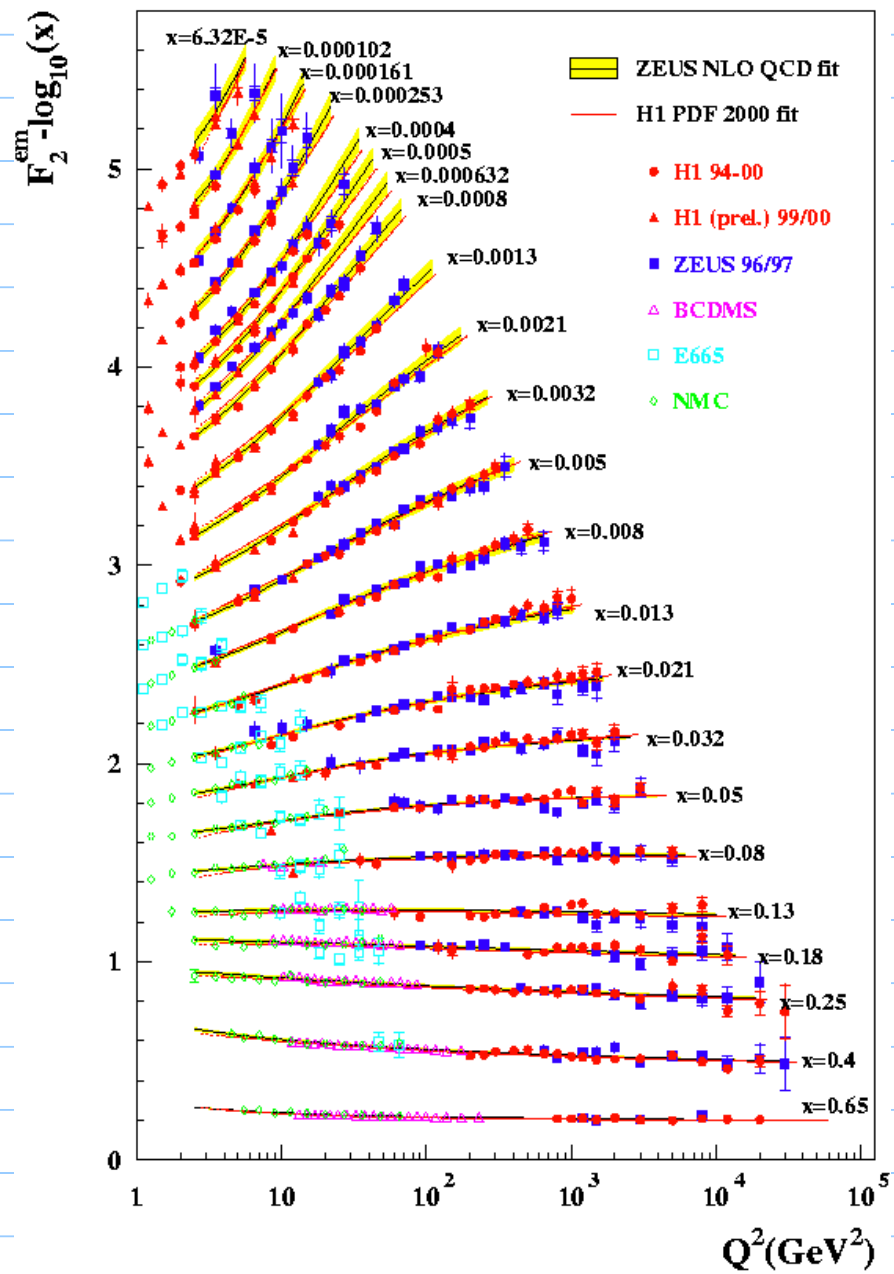
Scaling violations at low x

→ why?





we'll come back to this

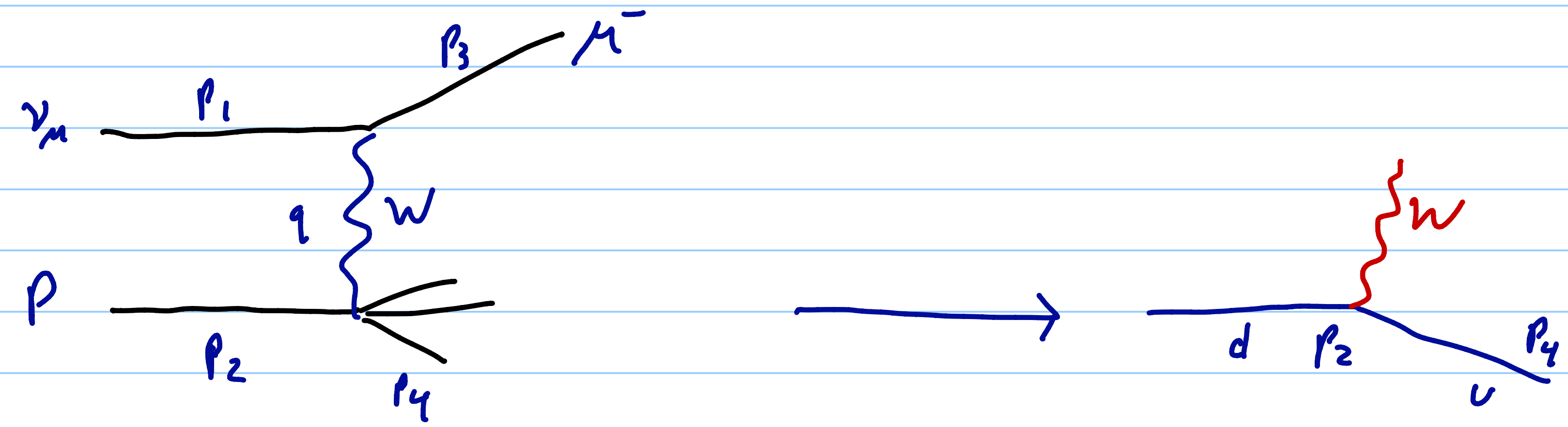


Lecture 17: HADRON STRUCTURE (Part 3)

- Deep Inelastic Scattering
- Deep Inelastic Scattering
- Hadron Collider Kinematics

(I used mainly Thomson)

Neutrino - Quark Scattering



$$\mathcal{M} = \frac{g_W^2}{2m_W^2} g_{\mu\nu} \left[\bar{u}(p_3)\gamma^\mu \tfrac{1}{2}(1-\gamma_5)\, u(p_1) \right] \left[\bar{u}(p_4)\gamma^\nu \tfrac{1}{2}(1-\gamma^5) u(p_2) \right]$$

ultra-relativistic limit $\rightarrow$ neglect masses of fermions
$\rightarrow$ helicity states equivalent to chiral states

$$\Rightarrow \quad \frac{1}{2}(1-\gamma_5)\, u_\uparrow(p_1) = 0 \quad , \quad \frac{1}{2}(1-\gamma_5)\, u_\downarrow(p_1) = u_\downarrow(p_1)$$

$$\rightarrow \mathcal{M} = 0 \quad \text{for} \quad u_\uparrow(p_1)\ ,\ u_\uparrow(p_2)$$

Result: $\mathcal{M} = \dfrac{g_W^2 \hat{s}}{m_W^2}$ *

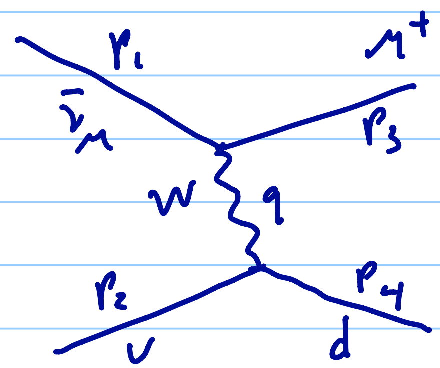
$\hat{s} = (2E)^2$

E: energy of ν, d in CM

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 \hat{s}} |M|^2 = \frac{G_F^2 \hat{s}}{4\pi^2} \qquad G_F = \frac{\sqrt{2}\, g_w^2}{8 m_w^2}$$

$$\sigma_{\nu q} = \frac{G_F^2 \hat{s}}{\pi}$$

Now switch To Antineutrino - Quark Scattering



$$M = \frac{g_w^2}{2 m_w^2} g_{\mu\nu} \left[\bar{v}(p_1) \gamma^\mu \tfrac{1}{2}(1-\gamma^5) v(p_3) \right] \times \left[\bar{u}(p_4) \gamma^\nu \tfrac{1}{2}(1-\gamma^5) u(p_2) \right]$$

only left-handed particles and right-handed anti-particles participate in charged weak current

So:

$$M = \frac{g_w^2}{2 m_w^2} g_{\mu\nu} \left[\bar{v}\uparrow(p_1) \gamma^\mu v\uparrow(p_3) \right] \left[\bar{u}\downarrow(p_4) \gamma^\nu u\downarrow(p_2) \right]$$

$\rightarrow$ spin 1 state

$$\frac{d\sigma_{\bar{\nu}q}}{d\Omega} = \frac{d\nu_q}{d\Omega}\,\frac{1}{4}\,(1+\cos\theta)^2$$

$$\Rightarrow \quad \sigma_{\bar{\nu}q} = \frac{G_F^2\hat{s}}{3\pi} \qquad \Rightarrow \qquad \frac{\sigma_{\bar{\nu}q}}{\sigma_{\nu q}} = \frac{1}{3}$$

and $\quad \sigma_{\nu\bar{q}} = \sigma_{\bar{\nu}q} \quad , \quad \sigma_{\bar{\nu}\bar{q}} = \sigma_{\nu q}$

Let's express the diff. xs in Terms of "y" $\to y = 1/_2\,(1-\cos\theta)$

$$\frac{d\sigma}{dy} = \left|\frac{d\cos\theta}{dy}\right| \frac{d\sigma}{d\cos\theta} = \left|\frac{d\cos\theta}{dy}\right| 2\pi\,\frac{d\sigma}{d\Omega} = 4\pi\,\frac{d\sigma}{d\Omega}$$

$$\Rightarrow \quad \frac{d\sigma_{\nu q}}{dy} = \frac{d\sigma_{\bar{\nu}\bar{q}}}{dy} = \frac{G_F^2\hat{s}}{\pi} \quad \text{and} \quad \frac{d\sigma_{\bar{\nu}q}}{dy} = \frac{d\sigma_{\bar{q}\nu}}{dy} = \frac{G_F^2}{4\pi}(1+\cos\theta)^2\hat{s}$$

$$y = 1/_2\,(1-\cos\theta) \to 1-y = 1/_2\,(1+\cos\theta) \Rightarrow \frac{d\sigma_{\bar{\nu}q}}{dy} = \frac{G_F^2}{\pi}(1-y)^2\hat{s}$$

In the parton model we will use $d^p(x)\,dx$

$\nu_\mu d \to \mu^- u \quad \Rightarrow \quad \dfrac{d\sigma_{\nu p}}{dy} = \dfrac{G_F^2}{\pi}\,\hat{s}\,d^p(x)\,dx$ (for d quarks)

and also $\bar{u}^p(x)\,dx$

$$\frac{d\sigma_{\nu p}}{dy} = \frac{G_F^2}{\pi}\,\hat{s}\,(1-y)^2\,\bar{u}^p(x)\,dx \quad \text{(for } \bar{u} \text{ quarks)}$$

Sum: $$\frac{d^2\sigma^{\nu p}}{dx\,dy} = \frac{G_F^2 s x}{\pi}\left[d^p(x) + (1-y^2)\,\bar{u}^p(x)\right] \quad (1)$$

Antineutrino-proton : $\dfrac{G_F^2 s x}{\pi}\left[(1-y)^2 u^p(x) + \bar{d}^p(x)\right]$

neutrino - neutron : $\dfrac{G_F^2 s x}{\pi}\left[d^n(x) + (1-y^2)\,\bar{u}^n(x)\right]$

Antineutrino-neutron : $\dfrac{G_F^2 s x}{\pi}\left[(1-y)^2 u^n(x) + \bar{d}^n(x)\right]$

Can then define $u^p(x) = d^n(x) \equiv u(x)$

$\bar{u}^p(x) = \bar{d}^n(x) \equiv \bar{u}(x)$

etc.

Can then rewrite expressions above

For isoscalar target (number of protons = number of neutrons)

$$\frac{d^2\sigma^{\nu N}}{dx\,dy} = \frac{1}{2}\left(\frac{d^2\sigma^{\nu p}}{dx\,dy} + \frac{d^2\sigma^{\nu n}}{dx\,dy}\right)$$

$$\simeq \frac{G_F^2}{2\pi} sx \left[u(x) + d(x) + (1-y)^2\left(\bar{u}(x) + \bar{d}(x)\right)\right]$$

Integrate over x: $\quad \frac{d\sigma^{\nu N}}{dy} = \frac{G_F^2}{2\pi} s\left[f_q + (1-y)^2 f_{\bar{q}}\right]$

with $\quad f_q \equiv f_d + f_u = \int_0^1 x\left[u(x) + d(x)\right] dx$

$$\frac{d\sigma^{\bar{\nu} N}}{dy} = \frac{G_F^2 s}{2\pi}\left[(1-y)^2 f_q + f_{\bar{q}}\right]$$

⇒ integrate to get σ: $\quad \sigma^{\nu N} = \frac{G_F^2 s}{2\pi}\left[f_q + 1/3\, f_{\bar{q}}\right]$, $\quad \sigma^{\bar{\nu} N} = \frac{G_F^2 s}{2\pi}\left[1/3\, f_q + f_{\bar{q}}\right]$

no antiquarks in nucleon: $\frac{\sigma^{\nu N}}{\sigma^{\bar{\nu} N}} = 3 \qquad$ with antiquarks: $\frac{\sigma^{\nu N}}{\sigma^{\bar{\nu} N}} \approx 2$

Structure Functions

For QED:

$$\frac{d^2\sigma^{ep}}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^2}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\right]$$

neutrino scattering $\rightarrow$ measure outgoing muon

$E_\mu = E_\nu(1-y)$, use $Q^2 = (s-M^2)xy \approx sxy$

$$\Rightarrow \frac{d^2\sigma}{dx\,dy} = \left|\frac{dQ^2}{dy}\right| \frac{d^2\sigma}{dx\,dQ^2} = sx\frac{d^2\sigma}{dx\,dQ^2}$$

when $s \gg M^2$ $\rightarrow$ $\frac{d^2\sigma^{ep}}{dx\,dy} = \frac{4\pi\alpha^2 s}{Q^4}\left[(1-y)F_2(x,Q^2) + y^2xF_1(x,Q^2)\right)$

For $\nu_\mu p \rightarrow \mu^- X$ we get:

$$\frac{d^2\sigma}{dx\,dy} = \frac{G_F^2 s}{2\pi}\left[(1-y)F_2^{\nu p}(x,Q^2) + y^2xF_1^{\nu p}(x,Q^2) + y(1-y/2) + F_3^{\nu p}(x,Q^2)\right] \quad (2)$$

Assignment $\boxed{\text{due March 11}}$

Using 1 and 2

and assume Bjorken scaling: $F(x, Q^2) \to F(x)$

show that

$$F_2^{\nu p} = 2x F_1^{\nu p} = 2x[d(x) + \bar{u}(x)]$$

$$x F_3^{\nu p} = 2x[d(x) - \bar{u}(x)]$$

$\Rightarrow$ can determine $\bar{u}(x)$ and $d(x)$ separately

this is problem 12.4 in thomson textbook

HADRON-HADRON INTERACTIONS

$$\sigma(a+b \to c+X) = \sum_{i,j} F_i^{(a)}(x_a)\, F_j^{(b)}(x_b)\, \hat{\sigma}(i+j \to c+X')$$

– Invariant mass of i,j system $M = \sqrt{s\tau}$

↳ dimensionless parameter

i,j system longitudinal momentum in hadron-hadron in c.m. :

$$p = x\sqrt{s}/2$$

– $x_{a,b} = \frac{1}{2}\left[(x^2 + 4\tau)^{1/2} \pm x \right]$

DRELL-YAN PRODUCTION

$$a+b \to \ell^+\ell^- + X$$

$$q + \bar{q} \to \gamma \to \ell^+\ell^-$$

invariant mass M

Hadron-Hadron Interactions

diff. cross section given by:

$$\frac{d\sigma}{dM^2\,dx} = \underbrace{\left(\frac{4\pi\alpha^2}{3M^4}\right)}_{} F(\tau, x)$$

$\hookrightarrow$ as in $e^+e^- \to \mu^+\mu^-$

$$F(\tau, x) = \frac{x_a x_b}{(x^2+4\tau)^{1/2}}\; g(x_a, x_b)$$

$$\to g(x_a, x_b) = \frac{1}{3}\sum_i e_i^2 \left[q_i^a(x_a)\,\bar{q}_i^b(x_b) + \bar{q}_i^a(x_a) + q_i^b(x_b) \right]$$