



## LECTURE 18: QCD (Part 1)

### Overview:

-QCD introduction

-Feynman rules

-Calculating colour factors

-quark anti-quark annihilation

(I used Quigg and mostly Giffiths as references)

(See Thomson Chapter 10)

# QCD (I)

Evidence that quarks come in 3 different "colours" include

→ spin-stat. problem for baryons

→ cross section  $e^-e^+ \rightarrow \text{hadrons}$

→  $\Upsilon$  branching ratios

→  $\pi^0$  lifetime

→ anomaly cancellation

Could we use  $U(3)$ ,  $SO(3)$  as candidate gauge groups for the strong interaction?

$SO(3)$  → no asymptotic freedom

→ existence of diquark states, no distinction between quarks and anti-quarks

(3)

## QCD (II)

$U(3)$  we get a singlet gauge boson  $\rightarrow$  long-range interaction.

For the Lagrangian:

$$\mathcal{L} = \bar{\psi} (i\gamma^\mu D_\mu - m)\psi - \frac{1}{2} \text{Tr}(G_{\mu\nu} G^{\mu\nu})$$

$$\psi = \begin{pmatrix} \psi \\ \psi \\ \psi \end{pmatrix}$$

$$D_\mu = \partial_\mu + ig B_\mu$$

$\rightarrow$  3x3 matrix:  $B_\mu = \frac{1}{2} \lambda \cdot b_\mu = \frac{1}{2} \lambda^a b_\mu^a$

$b_\mu$ : 8 colour gauge fields

$$G_{\mu\nu} = \frac{1}{2} G_{\mu\nu} \cdot \lambda = \frac{1}{2} G_{\mu\nu}^a \lambda^a$$

$$= (ig)^{-1} [D_\nu, D_\mu] = \partial_\nu B_\mu - \partial_\mu B_\nu + ig [B_\nu, B_\mu]$$

# QCD (III)

Properties of the  $\lambda$  matrices:

$$\text{Tr}(\lambda^a) = 0$$

$$\text{Tr}(\lambda^k \lambda^l) = 2 \delta^{kl}$$

$$[\lambda^j, \lambda^k] = 2i f^{jkl} \lambda^l$$

$$F_{123} = 1, \quad F_{458} = F_{678} = \sqrt{3}/2$$

$$F_{147} = F_{276} = F_{345} = F_{516} = F_{637} = 1/2$$

$$G_{\mu\nu}^a = \partial_\nu b_\mu^a - \partial_\mu b_\nu^a + g f^{jkl} b_\mu^j b_\nu^k$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

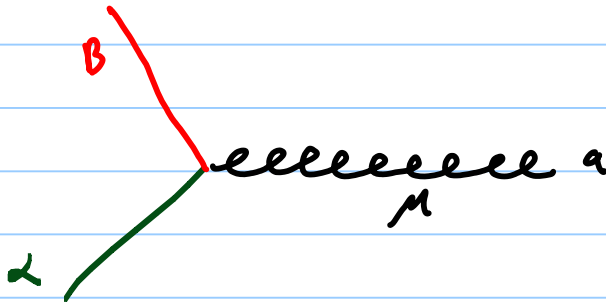
$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

## QCD IV

quark - gluon interaction:

$$\mathcal{L}_{int} = -\frac{g}{2} \bar{\psi} \gamma^\mu \lambda^a \psi$$



$$-\frac{ig}{2} \lambda_{\alpha\beta}^a \delta_\mu$$

→ one gluon exchange force prop.  $\frac{g^2}{4} \sum_a \lambda_{\alpha\beta}^a \lambda_{\gamma\delta}^a$

$$\alpha + \gamma \rightarrow \beta + \delta$$

Gluons are massless particles with spin 1 which we will represent by pol. vector  $\epsilon^\mu$ .

$$\rightarrow \epsilon^\mu p_\mu = 0 \quad (\text{Lorentz Condition})$$

6

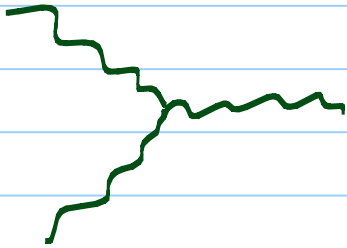
# QCD (V)

We will also use Coulomb gauge:  $\epsilon^0 = 0$   $\vec{\epsilon} \cdot \vec{p} = 0$

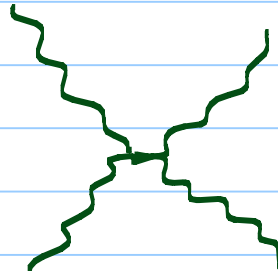
To describe colour state of gluon, we'll use column vector:

$$a = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \equiv |1\rangle \quad \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \equiv |4\rangle$$

Non-Abelian group  $\rightarrow$  gauge mediators interact with themselves:



3-gluon vertex

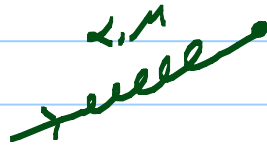


4-gluon vertex

# QCD (VI)

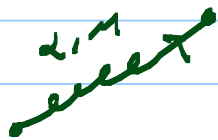
## Feynman Rules

incoming gluon:



$$\epsilon_\mu(p) a^\alpha$$

outgoing gluon



$$\epsilon_\mu^*(p) a^{\alpha*}$$

## Propagators

$q \bar{q}$



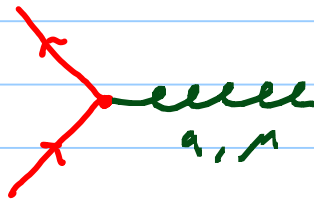
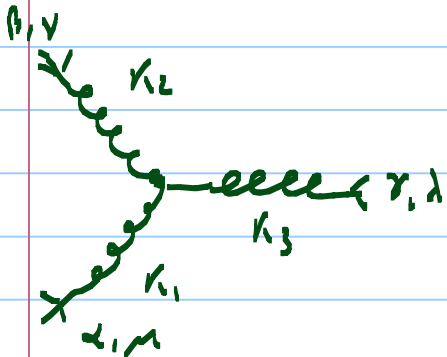
$$\frac{i(\not{q} + m)}{q^2 - m^2}$$

gluon



$$\frac{-ig_{\mu\nu} \int d\Omega}{q^2}$$

## Vertices



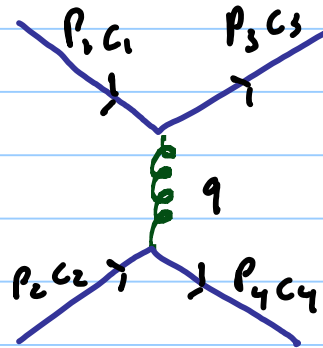
$$\frac{-ig_s}{2} \lambda^a \gamma^\mu$$

$$-g_s f^{abc} \gamma [g_{\mu\nu} (k_1 - k_2)_\lambda + g_{\nu\lambda} (k_2 - k_3)_\mu + g_{\lambda\mu} (k_3 - k_1)_\nu]$$

8

# QCD

Consider  $u + \bar{d} \rightarrow u + \bar{d}$



$$-i\mathcal{M} = [\bar{u}(3) c_3^T] \left[ -i \frac{g_s}{2} \lambda^\alpha \gamma^\mu \right] [u(1) c_1] \left[ \frac{-i g_{\mu\nu} \delta^{ab}}{q^2} \right]$$

$$\times [\bar{v}(2) c_2^T] \left[ -i \frac{g_s}{2} \lambda^b \gamma^\nu \right] [v(4) c_4]$$

$$\mathcal{M} = -\frac{g_s^2}{4} \frac{1}{q^2} [\bar{u}(3) \gamma^\mu u(1)] [\bar{v}(2) \gamma_\nu v(4)] \underbrace{(c_3^T \lambda^\alpha c_1) (c_2^T \lambda^\alpha c_4)}_{\text{colour factor}}$$

→ same as electron-positron scattering with  $g_e \rightarrow g_s$   
but with extra colour factor

Potential similar to EM case  $V \propto -\frac{\alpha_s}{r}$



# QCD

- we'll consider the colour octet case e.g.  $r\bar{b}$

$$c_1 = c_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad c_2 = c_4 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$F = \frac{1}{4} \left[ (1100) \lambda^\alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right] \left[ (0100) \lambda^\alpha \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right] = \frac{1}{4} \lambda_{11}^\alpha \lambda_{22}^\alpha$$

→  $\lambda^3$  and  $\lambda^8$  are the only relevant matrices

$$F = \frac{1}{4} (\lambda_{11}^3 \lambda_{22}^3 + \lambda_{11}^8 \lambda_{22}^8) = -1/6$$

- now we consider the singlet state:  $\frac{1}{\sqrt{3}} (r\bar{r} + b\bar{b} + g\bar{g})$

$$F = \frac{1}{4} \cdot \frac{1}{\sqrt{3}} \left[ \left( c_3^T \lambda^\alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right) [(1100) \lambda^\alpha c_4] + \left( c_3^T \lambda^\alpha \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right) [(0100) \lambda^\alpha c_4] \right. \\ \left. + \left( c_3^T \lambda^\alpha \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right) [(0010) \lambda^\alpha c_4] \right]$$

# QCD

(10)

outgoing state also in colour singlet state

→ we get 9 terms

$$F = \frac{1}{4} \cdot \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} (\lambda_{ij}^{\alpha} \lambda_{ji}^{\alpha}) = \frac{1}{12} \text{Tr}(\lambda^{\alpha} \lambda^{\alpha})$$

$$\rightarrow \text{Tr}(\lambda^{\alpha} \lambda^{\alpha}) = 2 f^{\alpha\beta}$$

$$\rightarrow \text{Tr}(\lambda^{\alpha} \lambda^{\alpha}) = 16$$

$$\Rightarrow F = +\frac{4}{3}$$

$$\rightarrow V_{q\bar{q}}(r) = -\underbrace{\frac{4}{3} \frac{\alpha_s}{r}}_{\text{singlet}}, \quad V_{q\bar{q}}(r) = \underbrace{\frac{1}{6} \frac{\alpha_s}{r}}_{\text{octet}}$$

# QCD

(11)

quark - quark interaction

$$M = -\frac{g_s^2}{4} \frac{1}{f^2} [\bar{u}(3) \gamma^\mu u(1)] [\bar{u}(4) \gamma^\mu u(2)] (c_3^\dagger \lambda^a c_1) (c_4^\dagger \lambda^a c_2)$$

$$F = \frac{1}{4} (c_3^\dagger \lambda^a c_1) (c_4^\dagger \lambda^a c_2)$$

we get a Triplet:

$$\left. \begin{array}{l} (rb - br) / \sqrt{2} \\ (bg - gb) / \sqrt{2} \\ (gr - rg) / \sqrt{2} \end{array} \right\} \text{anti-sym combinations}$$

and a sextet:

$$rr, gg, bb \\ + \text{symmetric combinations}$$

# QCD

(12)

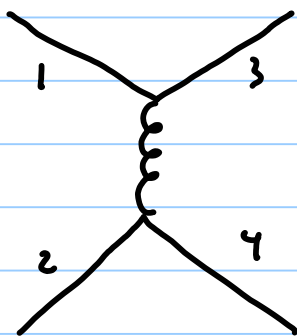
sextet :  $c_1 = c_2 = c_3 = c_4 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$  (for rr)

$$F = \frac{1}{4} \left[ (100) \lambda^a \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right] \left[ (100) \lambda^a \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right] = \frac{1}{4} \lambda_{ii}^a \lambda_{ii}^a$$

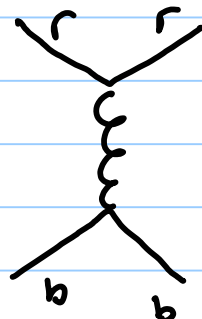
$$= \frac{1}{4} \left[ \lambda_{ii}^3 \lambda_{ii}^3 + \lambda_{ii}^8 \lambda_{ii}^8 \right] = \frac{1}{4} \left[ 1 + \frac{1}{3} \right] = \frac{1}{3}$$

For Triplet:

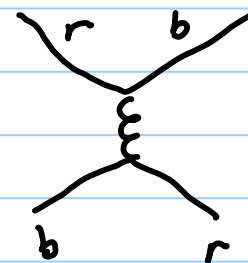
For example  $(rb-br)/\sqrt{2}$



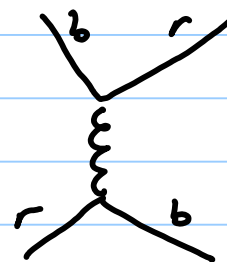
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# QCD

(13)

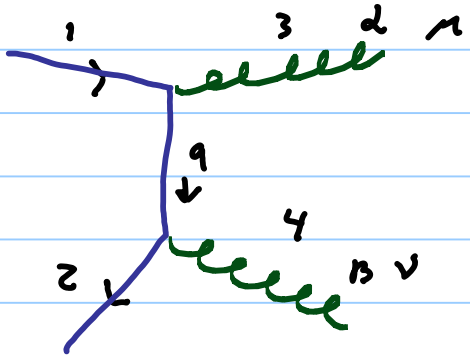
We get:

$$\begin{aligned} F &= \frac{1}{4} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \left[ \left[ (100) \lambda^a \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right] \left[ (010) \lambda^a \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right] \right. \\ &\quad - \left[ (010) \lambda^a \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right] \left[ (100) \lambda^a \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right] \\ &\quad + \left[ (010) \lambda^a \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right] \left[ (100) \lambda^a \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right] \\ &\quad \left. - \left[ (100) \lambda^a \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right] \left[ (010) \lambda^a \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right] \right] \\ &= \frac{1}{8} \left[ \lambda_{11}^a \lambda_{22}^a - \lambda_{21}^a \lambda_{12}^a + \lambda_{22}^a \lambda_{11}^a - \lambda_{12}^a \lambda_{21}^a \right] \\ &= \frac{1}{4} \left[ \lambda_{11}^a \lambda_{22}^a - \lambda_{12}^a \lambda_{21}^a \right] \\ &= \frac{1}{4} \left[ \lambda_{11}^3 \lambda_{22}^3 + \lambda_{11}^8 \lambda_{22}^8 - \lambda_{12}^1 \lambda_{21}^1 - \lambda_{12}^2 \lambda_{21}^2 \right] = -\frac{2}{3} \end{aligned}$$

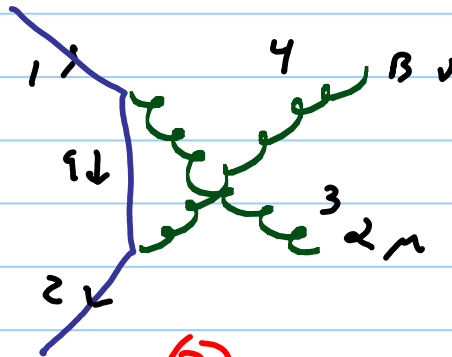
## QCD

$$\left. \begin{aligned} V_{gg}(r) &= -\frac{2}{3} \frac{2s}{r} && \text{Triplet} \\ V_{gg}(r) &= \frac{1}{3} \frac{2s}{r} && \text{sextet} \end{aligned} \right\} \text{not seen...}$$

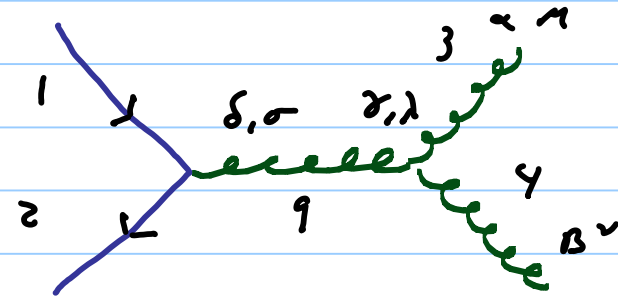
## PAIR ANNIHILATION



①



②



③

$$\begin{aligned} \textcircled{1}: -iM_1 &= \bar{v}(2) c_2^\dagger \left[ -i \frac{g_s}{2} \lambda^B \gamma^\nu \right] \left[ \epsilon_{4\nu}^\alpha a_4^{\alpha*} \right] \left[ \frac{i(\not{q} + m)}{q^2 - m^2} \right] \\ &\times \left[ -i \frac{g_s}{2} \lambda^A \gamma^\mu \right] \left[ \epsilon_3^\beta a_3^{\beta*} \right] v(1) c_1 \end{aligned}$$

QCD

(15)

$$q = p_1 - p_3 \rightarrow$$

$$q^2 - m^2 = p_1^2 - 2 p_1 \cdot p_3 + p_3^2 - m^2 = -2 p_1 \cdot p_3$$

$$M_1 = -\frac{g_s^2}{8} \frac{1}{p_1 \cdot p_3} \bar{v}(2) \left[ \cancel{\not{p}_4} (\cancel{\not{p}_1} - \cancel{\not{p}_3} + \cancel{m}) \cancel{\not{p}_3} \right] u(1) \\ \times a_3^A a_4^B (c_2^T \lambda^A \lambda^B c_1)$$

Diagram 2: switch  $3 \leftrightarrow 4$

colour factor:  $a_3^A a_4^B (c_2^T \lambda^A \lambda^B c_1)$

$$(3): -iM_3 = \bar{v}(2) c_2^T \left[ -i \frac{g_s}{2} \lambda^d \cancel{\not{p}_4} \right] u(1) c_1 \left[ -i g^{\alpha\lambda} \frac{\cancel{\not{p}_3} \cancel{\not{p}_1}}{q^2} \right]$$

$$\cdot \left[ -g_s F^{AB} \left[ g_{\mu\nu} (-p_3 + p_4)_\lambda + g_{\nu\lambda} (-p_4 - q)_\mu + g_{\lambda\mu} (q + p_3)_\nu \right] \right]$$

$$\cdot \epsilon_3^A c_3^A \epsilon_4^B a_4^B$$

(A)

QCD

(16)

$$q = p_3 + p_4, \quad q^2 = 2p_3 \cdot p_4, \quad \epsilon_3 \cdot p_3 = 0 = \epsilon_4 \cdot p_4$$

$$M_3 = \frac{ig_s^2}{4} \frac{1}{p_3 \cdot p_4} \bar{v}(2) \left[ (\epsilon_3 \cdot \epsilon_4) (p_4 - p_3) + 2(p_3 \cdot \epsilon_4) \epsilon_3 - 2(p_4 \cdot \epsilon_3) \epsilon_4 \right] u(1) \\ \times \text{Feynman } a_3^a a_4^b (c_2^+ \lambda^c c_1) \quad (13)$$

PROBLEM SET #2, problem 1: show how we get (13) from (A) (Griffiths)

To simplify things, we'll assume that the particles are at rest.

$$p_1 = p_2 = (m, 0), \quad p_3 = (E, p), \quad p_4 = (E, -p)$$

$$p_1 \cdot p_3 = p_1 \cdot p_4 = m^2, \quad p_3 \cdot p_4 = 2m^2$$

$$p_3 \cdot \epsilon_4 = -\vec{p} \cdot \epsilon_4 = -p_4 \cdot \epsilon_4 = 0 \quad (\text{same for } p_4 \cdot \epsilon_3)$$



## QCD

We now get a Total amplitude :

$$M = -\frac{g_s^2}{8m^2} a_3^{\lambda} a_4^{\beta} \bar{v}(2) c_2^{\dagger} \left[ \not{\epsilon}_3 \not{\epsilon}_4 \not{p}_4 \lambda^{\lambda} \lambda^{\beta} + \not{\epsilon}_4 \not{\epsilon}_3 \not{p}_3 \lambda^{\beta} \lambda^{\lambda} - i (\epsilon_3 \cdot \epsilon_4) (\not{p}_4 - \not{p}_3) F^{\lambda\beta\gamma} \lambda^{\gamma} \right] c_1 v(1)$$

orient coordinates with  $\vec{p}$  along  $z$  :

$$p_3 = m(\gamma^0 - \gamma^3), \quad p_4 = m(\gamma^0 + \gamma^3), \quad (p_4 - p_3) = 2m\gamma^3$$

$$\text{using } \begin{aligned} \epsilon_3 \epsilon_4 &= -(\epsilon_3 \cdot \epsilon_4) - i (\epsilon_3 \times \epsilon_4) \cdot \Sigma \\ \epsilon_4 \epsilon_3 &= -(\epsilon_3 \cdot \epsilon_4) + i (\epsilon_3 \times \epsilon_4) \cdot \Sigma \end{aligned} \quad \left. \begin{array}{l} \vdots \\ \vdots \end{array} \right\} \Sigma = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix}$$

$$\text{we get } M = \frac{g_s^2}{8m} a_3^{\lambda} a_4^{\beta} \bar{v}(2) c_2^{\dagger} \left[ (\epsilon_3 \cdot \epsilon_4) \{ \lambda^{\lambda}, \lambda^{\beta} \} \gamma^0 + i (\epsilon_3 \times \epsilon_4) \cdot \Sigma ( [ \lambda^{\lambda}, \lambda^{\beta} ] \gamma^0 + \{ \lambda^{\lambda}, \lambda^{\beta} \} \gamma^3 ) \right] c_1 v(1)$$

→ same as QED result with  $\lambda=1$ , no colour states

## QCD

Now we put the quarks in singlet state (spin 0)

$$M = (M_{\uparrow\downarrow} - M_{\downarrow\uparrow}) / \sqrt{2}$$

For  $M_{\uparrow\downarrow}$  we have:  $\bar{v}(2) \gamma^0 u(1) = \bar{v}(2) \sum \gamma^0 u(1) = 0$

$$\bar{v}(2) \sum \gamma^3 u(1) = -2m \hat{z}$$

$$M_{\downarrow\uparrow} = -M_{\uparrow\downarrow}$$

$$M = -i\sqrt{2} \frac{g_s^2}{4} (\epsilon_3 \times \epsilon_4)_z a_3^A a_4^B (c_2^{\dagger} \{ \lambda^A, \lambda^B \} c_1)$$

$$\text{colour factor} = \frac{1}{8} a_3^A a_4^B (c_2^{\dagger} \{ \lambda^A, \lambda^B \} c_1)$$

Problem set 2, problem # 2

if 3 quarks occupy colour singlet state  $\frac{1}{\sqrt{3}} (r\bar{r} + b\bar{b} + g\bar{g})$

Find the colour factor.

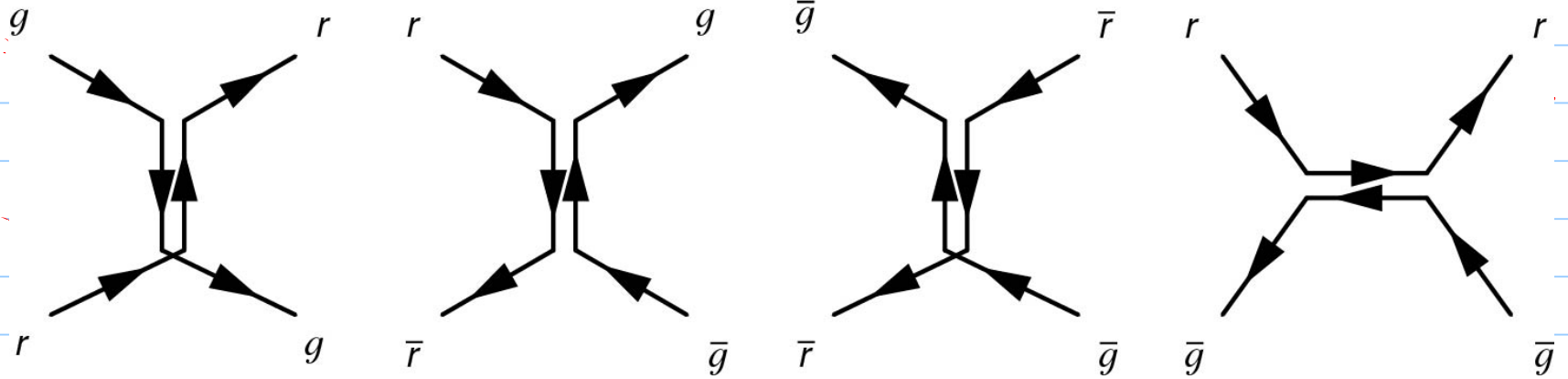
# QCD

(19)

$$M = -4 \cdot \sqrt{2/3} g_s^2$$

$$\rightarrow \frac{d\sigma}{d\Omega} = \left[ \frac{1}{8\pi(E_1 + E_2)} \right]^2 \frac{|p_f|}{|p_i|} |M|^2$$

$$\Rightarrow \sigma \propto \frac{\alpha_s^2}{S} *$$



# Colour Factor Summary

(20)

From slide 5: colour factor  $C(\alpha\beta \rightarrow \delta\gamma) = \frac{g_s^2}{4} \sum_{a=1}^8 \lambda_{\alpha\beta}^a \lambda_{\delta\gamma}^a$

For quark-quark:  $C(rr \rightarrow rr) = 1/3 = C(gg \rightarrow gg) = \text{etc.}$

$$C(rb \rightarrow rb) = -1/6 = C(rg \rightarrow rg) \\ = C(gr \rightarrow gr) = \text{etc.}$$

$$C(rg \rightarrow gr) = 1/2 = C(rb \rightarrow br) = \text{etc.}$$

note that  $C(rb \rightarrow bg) = \frac{1}{4} \sum_{a=1}^8 \lambda_{31}^a \lambda_{23}^a = 0$

why?

Averaged colour factor for quark-quark:

$$\langle |M|^2 \rangle = \frac{1}{9} \sum_{\alpha\beta\delta\gamma=1}^3 |M(\alpha\beta \rightarrow \delta\gamma)|^2$$

$$\langle |C|^2 \rangle = \frac{1}{9} \sum_{\alpha\beta\delta\gamma=1}^3 |C(\alpha\beta \rightarrow \delta\gamma)|^2$$

3 x rr-Type with  $C = 1/3$   
6 x rb  $\rightarrow$  rb-Type with  $C = -1/6$   
6 x rb  $\rightarrow$  br-Type with  $C = 1/2$

$$\Rightarrow \langle |C|^2 \rangle = \frac{1}{9} \cdot [3 \cdot (1/3)^2 + 6 \cdot (-1/6)^2 + 6 \cdot (1/2)^2] = 2/9$$

For quark-anti-quark?

# Colour Factor Summary

(21)

$$\text{For QED } e^- q \rightarrow e^- q \quad : \quad \frac{d\sigma}{dq^2} = \frac{2\pi Q_f^2 \alpha^2}{q^4} \left[ 1 + \left( 1 + \frac{q^2}{s} \right)^2 \right]$$

For QCD  $ud \rightarrow ud \rightarrow$  replace  $Q_f \cdot 2$  with  $\alpha_s$

$$\Rightarrow \frac{d\sigma}{dq^2} = \frac{4\pi \alpha_s^2}{q^4} \left[ 1 + \left( 1 + \frac{q^2}{s} \right)^2 \right]$$

note that  $\sqrt{s} \sim M_Z$        $\alpha \sim 1/130$       ,       $\alpha_s \sim \frac{1}{8}$

$$\text{Also note that } \alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \alpha(\mu^2) \frac{1}{3\pi} \ln\left(\frac{q^2}{\mu^2}\right)}$$

$$\alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{1 + \beta \alpha_s(\mu^2) \ln\left(\frac{q^2}{\mu^2}\right)}$$

$$\beta = \frac{11 N_c - 2 N_f}{12\pi}$$

$N_f$  : number of flavours  
 $N_c$  : number of colours

# Back to Hadron Collider Kinematics

(22)

$pp \rightarrow \text{jet-jet} + X$

→ Transverse momentum :  $p_T = \sqrt{p_x^2 + p_y^2}$

→ 2-jet system is in general boosted

rapidity  $y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right)$

if  $p_z \approx E \cos \theta$

$$\rightarrow y \approx \frac{1}{2} \ln \left( \frac{1 + \cos \theta}{1 - \cos \theta} \right) = \frac{1}{2} \ln \left( \cot^2 \theta/2 \right)$$

define  $\eta \equiv -\ln \left( \tan \theta/2 \right)$

Let's consider  $pp \rightarrow \mu^+ \mu^-$  ,  $q \bar{q} \rightarrow \gamma \rightarrow \mu^+ \mu^-$

$$\sigma(q \bar{q} \rightarrow \mu^+ \mu^-) = \frac{1}{N_c} \alpha_q^2 \frac{4\pi \alpha^2}{3s}$$

With  $v$  and  $\bar{u}$  quarks: (at a  $p\bar{p}$  collider)

$$d^2\sigma = Q_v^2 \frac{4\pi\alpha^2}{9\hat{s}} v^p(x_1) \bar{u}^{\bar{p}}(x_2) dx_1 dx_2, \quad v^p(x) = \bar{u}^{\bar{p}}(x)$$

$$= \frac{4}{9} \cdot \frac{4}{9} \frac{\pi\alpha^2}{\hat{s}} v(x_1) v(x_2) dx_1 dx_2$$

centre of mass system:  $\hat{s} = (x_1 p_1 + x_2 p_2)^2 = x_1^2 p_1^2 + x_2^2 p_2^2 + 2x_1 x_2 p_1 p_2$

neglecting masses ( $p_1^2 = p_2^2 = 0$ )  $\rightarrow \hat{s} \approx x_1 x_2 (2 p_1 p_2) = x_1 x_2 s$

$$d^2\sigma = \frac{4}{9} \cdot \frac{4\pi\alpha^2}{9x_1 x_2 s} v(x_1) v(x_2) dx_1 dx_2$$

we also need  $d\bar{d}$ , and sea quarks:

$$d^2\sigma = \frac{4\pi\alpha^2}{9x_1 x_2 s} \left[ \frac{4}{9} \left[ v(x_1) v(x_2) + \bar{u}(x_1) \bar{u}(x_2) \right] + \frac{1}{9} \left[ d(x_1) d(x_2) + \bar{d}(x_1) \bar{d}(x_2) \right] \right] dx_1 dx_2$$

(24)

Invariant mass of  $\mu^+\mu^-$  system:  $M^2 = x_1 x_2 S$

Rapidity of the  $\mu^+\mu^-$  system:  $y = \frac{1}{2} \ln \left( \frac{E_3 + E_4 + p_{3z} + p_{4z}}{E_3 + E_4 - p_{3z} - p_{4z}} \right)$

$$p_1 = \frac{\sqrt{s}}{2} (x_1, 0, 0, x_1) \quad p_2 = \frac{\sqrt{s}}{2} (x_2, 0, 0, -x_2)$$

$$y = \frac{1}{2} \ln \left( \frac{(x_1 + x_2) + (x_1 - x_2)}{(x_1 + x_2) - (x_1 - x_2)} \right) = \frac{1}{2} \ln \frac{x_1}{x_2}$$

$$x_1 = \frac{M}{\sqrt{s}} e^y, \quad x_2 = \frac{M e^{-y}}{\sqrt{s}}$$

$$dy dM = \frac{\partial(y, M)}{\partial(x_1, x_2)} dx_1 dx_2 = \begin{vmatrix} \frac{\partial y}{\partial x_1} & \frac{\partial y}{\partial x_2} \\ \frac{\partial M}{\partial x_1} & \frac{\partial M}{\partial x_2} \end{vmatrix} dx_1 dx_2$$

$$dy dM = \frac{s}{2M} dx_1 dx_2$$

see previous page

$$\rightarrow d^2\sigma = \frac{4\pi\alpha^2}{9M^2} f(x_1, x_2) \frac{2M}{s} dy dM, \quad \boxed{\frac{d^2\sigma}{dy dM} = \frac{8\pi\alpha^2}{9Ms} F(x_1, x_2)}$$



# JET PRODUCTION AT THE LHC

(25)

Consider  $qq \rightarrow qq$  scattering

We obtained previously:  $\frac{d\sigma}{dq^2} = \frac{4\pi\alpha_s^2}{q^4} \left[ 1 + \left( 1 + \frac{q^2}{\hat{s}} \right)^2 \right]$

using  $Q^2 = -q^2$ ,  $\hat{s} = x_1 x_2 S$ , adding PDFs:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha_s^2}{9Q^4} \left[ 1 + \left( 1 - \frac{Q^2}{Sx_1x_2} \right)^2 \right] g(x_1, x_2) dx_1 dx_2$$

$$g(x_1, x_2) = \left[ u(x_1)u(x_2) + u(x_1)d(x_2) + d(x_1)u(x_2) + d(x_1)d(x_2) \right]$$

$$\Rightarrow \frac{d\sigma}{dQ^2 dx_1 dx_2} = \frac{4\pi\alpha_s^2}{9Q^4} \left[ 1 + \left( 1 - \frac{Q^2}{Sx_1x_2} \right)^2 \right] g(x_1, x_2)$$

→ you can rewrite this in terms of  $p_T, y_3, y_4$  \*

Note that  $qq \rightarrow qq$  is just one out of many diagrams that will yield two jets...

\*

# Lang Problem Set #2 (cont.)

## Problem 3

i) Assuming massless jets and using  $E^2 = p_1^2 + p_2^2$  show that:

$$p_3 = (p_T \cosh y_3, +p_T \sinh \varphi, +p_T \cos \varphi, p_T \sinh y_3)$$

$$p_4 = (p_T \cosh y_4, -p_T \sinh \varphi, -p_T \cos \varphi, p_T \sinh y_4)$$

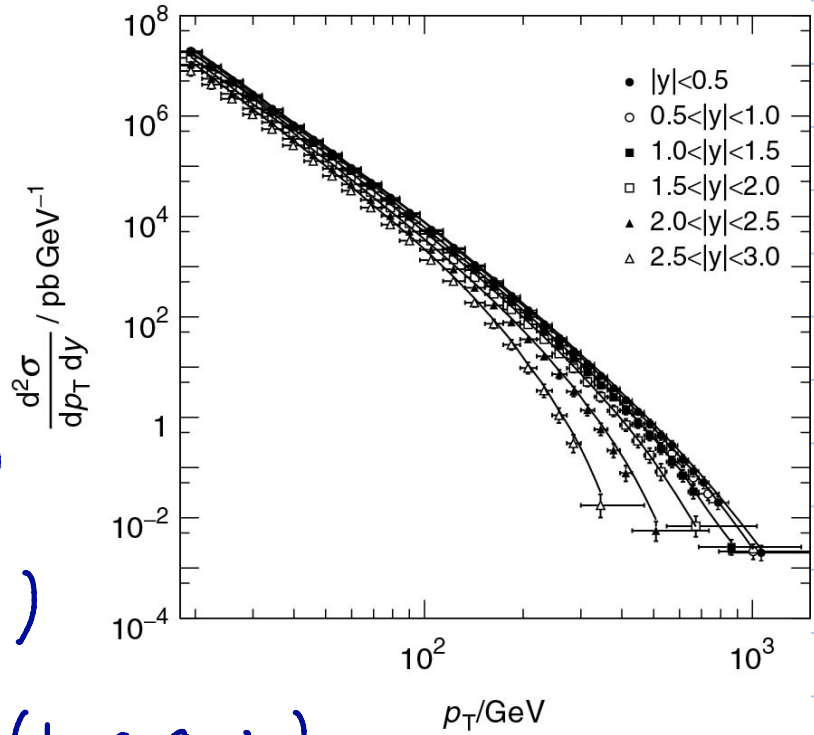
ii) with  $p_1 = \frac{\sqrt{s}}{2} (x_1, 0, 0, x_1)$ ,  $p_2 = \frac{\sqrt{s}}{2} (x_2, 0, 0, -x_2)$

show that  $x_1 = \frac{p_T}{\sqrt{s}} (e^{+y_3} + e^{+y_4})$      $x_2 = \frac{p_T}{\sqrt{s}} (e^{-y_3} + e^{-y_4})$

and that  $Q^2 = p_T^2 (1 + e^{y_4 - y_3})$

## Problem 4:

Show that  $\frac{d(y_3, y_4, p_T^2)}{d(x_1, x_2, q^2)} = \frac{1}{x_1 x_2}$



Lang problem set #2 is due: April 24<sup>th</sup>