

## LECTURE 19: QCD (Part II)

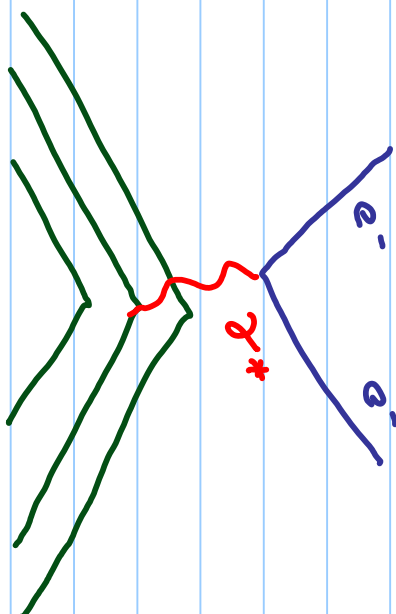
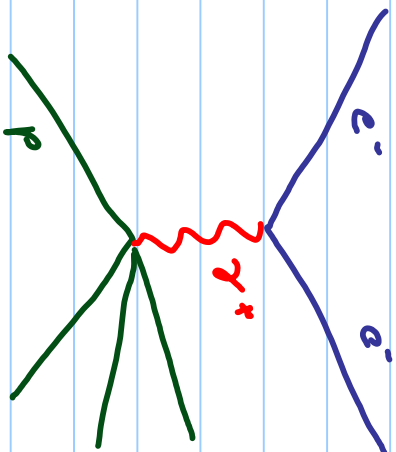
### Overview:

- QCD corrections to DIS
- Gluon emission

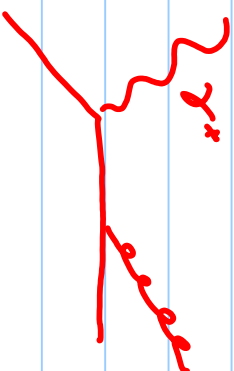
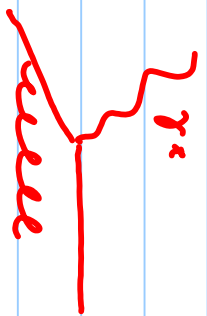
(I used Halzen-Martin and Quigg as a reference)

# DIS and QCD

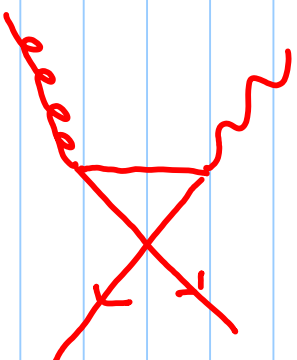
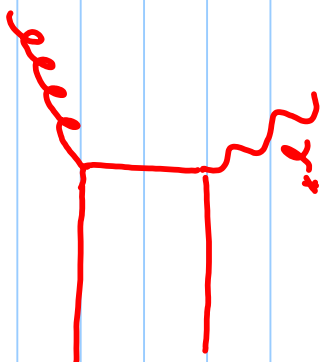
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Adding QCD corrections will change previous result :



+



## DIS and QCD (cont.)

Corrections will change previous results:

→ scaling will not be true

→ quark direction not colinear with photon

We had:

$$F_1 = MW_1(\nu, Q^2)$$

$$F_2 = \nu W_2(\nu, Q^2)$$

$$\nu \equiv \frac{P \cdot q}{M}, \quad Q^2 = -q^2$$

→  $F_1, F_2$  no longer scale since they will be functions of  $\nu$  both  $\nu$  and  $Q^2$  instead of the ratios:  $x = \frac{Q^2}{2M\nu}$

# DIS and QCD (cont.)

We can interpret the structure functions in terms of virtual photon - proton cross sections. In DIS

$$2F_1 = \frac{\sigma_T}{\sigma_0}$$

$$\sigma_0 \equiv \frac{4\pi^2\alpha}{2M_N} \approx \frac{4\pi^2\alpha}{S}$$

$$F_2/x = \frac{\sigma_T + \sigma_L}{\sigma_0}$$

*-> xS for long. and transversely polarized virtual photons.*

We need to express the above as virtual photon - proton xS:

$$\gamma^* - \text{proton} \quad \gamma^* - \text{parton}$$

$$p \quad p_i = y p$$

$$x = \frac{Q^2}{2p \cdot q} \quad z = \frac{Q^2}{2p_i \cdot q} = \frac{x}{y}$$

DIS and QCD (cont.)

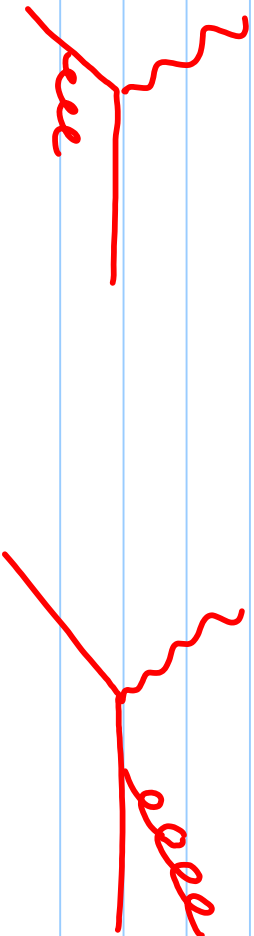
(5)

$$\left( \frac{\sigma_T}{\sigma_0} (x, Q^2) \right)_{\gamma^* p} = \sum_i \int_0^1 dz \int_0^1 dy F_i(y) \delta(x - zy) \left( \frac{\hat{\sigma}_T}{\sigma_0} (z, Q^2) \right)_{\gamma^* i}$$

after integration:  $\frac{\sigma_T}{\sigma_0} (x, Q^2) = \sum_i \int_x^1 \frac{dy}{y} F_i(y) \left( \frac{\hat{\sigma}_T}{\sigma_0} \left( \frac{x}{y}, Q^2 \right) \right)$

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Gluon emission cross section  $\gamma^* q \rightarrow qg$



→ similar to Compton scattering:  $\gamma^* e \rightarrow e\gamma$

$$|\overline{M}|^2 = 32\pi^2 \alpha^2 \left( -\frac{v}{s} - \frac{s}{v} + 2 \frac{TQ^2}{sv} \right)$$

↳ virtual photon result

# DIS and QCD (cont.)

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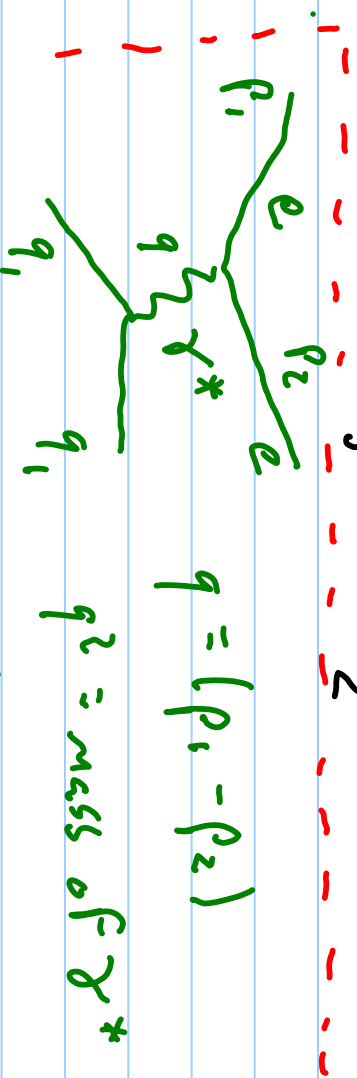
For  $\gamma^* q \rightarrow qg$ , we substitute

-  $2^2 \rightarrow e_i^2 d\Omega$

- colour Factor  $(4/3) \rightarrow \frac{1}{3} \cdot 8 \cdot \frac{1}{2}$

-  $\nu \leftrightarrow t$

$\rightarrow$  convention



$q = (p_1 - p_2)$

$q^2 = \text{mass of } \gamma^*$

$Q^2 \equiv -q^2$

Some Kinematics:

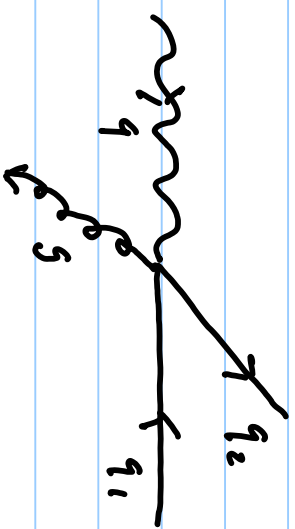
$\gamma^* q_1 \rightarrow q_2 g$

$q (q_0, 0, 0, K)$

$q_2 (K', K' \sin \theta, 0, K' \cos \theta)$

$q_1 (K, 0, 0, -K)$

$g (K', -K' \sin \theta, 0, K' \cos \theta)$



$\vec{S} = (q + q_1)^2 = (q_2 + g)^2$

$\vec{t} = (q - q_2)^2 = (g - q_1)^2$

$\vec{u} = (q_1 - q_2)^2$

DIS and QCD (cont.)

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$$-\hat{f} - \hat{v} = Q^2 + 2K'q_0 + 2KK' = Q^2 + 4K'^2 = Q^2 + S \quad ]$$

$$q_0 = 2K' - K \quad ]$$

$$S = (q_0 + K)^2 = q_0^2 + K^2 + 2q_0K$$

$$(q_0^2 - K^2) = -Q^2$$

$$\Rightarrow S = 2K^2 + 2q_0K - Q^2 \quad ]$$

$$S = (q + q_2)^2 = 4K'^2 \quad ]$$

$$\hat{f} = (q - q_2)^2 = -2KK' (1 - \cos\theta) \quad ]$$

$$\hat{v} = -2KK' (1 + \cos\theta) \quad ]$$

$$P_T = K' \sin\theta \quad ]$$

$$S \hat{f} \hat{v} = 4K'^2 (2KK')^2 \sin^2\theta = (4K'K)^2 P_T^2 = (\hat{f} + \hat{v})^2 P_T^2 = (S + Q^2)^2 P_T^2 \quad ]$$

# DIS and QCD (cont.)

when  $-f \ll \xi \rightarrow -\vec{v} = \hat{s} + Q^2$

$$\Rightarrow p_T^2 = \frac{\xi f v}{(\xi + Q^2)^2} \rightarrow p_T^2 = \frac{\xi(1-f)}{(\xi + Q^2)^2}$$

$$p_T = K' \sin \theta, \quad p_T^2 = K'^2 \sin^2 \theta$$

$$dp_T^2 = 2K'^2 \sin \theta \cos \theta d\theta = \frac{\xi}{2} \sin \theta \cos \theta d\theta$$

$d\Omega = \sin \theta d\theta d\phi$ , after  $\phi$  int.  $\therefore d\Omega = 2\pi \sin \theta d\theta$

$$\cos \theta \approx 1 \rightarrow d\Omega = 2\pi dp_T^2 \cdot \frac{2}{\xi} = \frac{4\pi}{\xi} dp_T^2$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} p_f^2 |M|^2$$

$$\Rightarrow \frac{d\sigma}{dp_T^2} = \frac{4\pi}{\xi} \frac{1}{64\pi^2 s} = \frac{1}{16\pi \xi^2} |M|^2$$



DIS and QCD (cont.)

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$$|M|^2 = 32 \pi^2 (e_i^2 \alpha_s) \frac{4}{3} \left( -\frac{1}{3} - \frac{1}{3} + 2 \frac{2}{3} Q^2 \right)$$

with  $-f \ll s$

$$\frac{d\sigma}{dQ^2} = \frac{8\pi e_i^2 \alpha_s}{3} \cdot \left( -\frac{1}{3} \right) \left[ \frac{1}{3} - 2 \frac{2}{3} Q^2 \right]$$

$$-\nu = \frac{1}{3} + Q^2$$

$$\rightarrow \frac{8\pi e_i^2 \alpha_s}{3} \cdot \left( -\frac{1}{3} \right) \left[ \frac{1}{3} + 2 \left( \frac{1}{3} + Q^2 \right) Q^2 \right]$$

$$= \frac{4\pi^2 \alpha_s}{3} \cdot \frac{e_i^2}{2\pi} \cdot \alpha_s \cdot \frac{4}{3} \left[ -\frac{1}{3} + 2 \left( \frac{1}{3} + Q^2 \right) Q^2 \right]$$

$$P_+^2 = \frac{1}{3} \frac{(-1)}{3 + Q^2}$$

$$\frac{4\pi^2 \alpha_s}{3} \cdot \frac{e_i^2}{2\pi} \alpha_s \cdot \frac{4}{3} \left[ \frac{1}{3} + Q^2 \right] \frac{1}{3} \frac{1}{3 + Q^2} + 2 \left( \frac{1}{3} + Q^2 \right) Q^2 \frac{1}{3 + Q^2} \right]$$

# DIS and QCD (cont.)

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$$= 4\pi^2 \frac{\alpha}{s} \frac{e_i^2}{2\pi} \alpha_s \frac{4}{3} \left[ \frac{1}{P_1^2} \frac{s}{(s+Q^2)} + 2 \frac{Q^2}{s P_2^2} \right]$$

$$= 4\pi^2 \frac{\alpha}{s} \frac{e_i^2}{2\pi} \alpha_s \frac{4}{3} \frac{1}{P_2^2} \left[ \frac{s}{(s+Q^2)} + 2 \frac{Q^2}{s} \right]$$

$$\Rightarrow = \frac{s^2 + 2Q^2(s+Q^2)}{s(s+Q^2)} = \frac{s^2 + 2Q^2s + 2Q^4}{s(s+Q^2)}$$

$$Z = \frac{Q^2}{2P_1 \cdot P_2} = \frac{Q^2}{s+Q^2} \quad 1 \quad z^2 = \frac{Q^4}{s^2 + Q^4 + 2sQ^2}$$

$$z^2 + 1 = \frac{Q^4 + s^2 + Q^4 + 2sQ^2}{s^2 + Q^4 + 2sQ^2} = \frac{2Q^4 + s^2 + 2sQ^2}{(s+Q^2)^2}$$

$$1-z = \frac{s+Q^2 - Q^2}{s+Q^2} = \frac{s}{s+Q^2}$$

$$\Rightarrow \frac{z^2+1}{1-z} = P_{qf}(z) \equiv \frac{4}{3} \frac{z^2+1}{1-z}$$

# DIS and QCD (cont.)

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$$\frac{d\hat{\sigma}}{d\rho_T^2} = e_i^2 \hat{\sigma}_0 \frac{1}{\rho_T^2} \frac{d\mathcal{L}}{dz} P_{qf}(z)$$

↳  $\frac{4\pi^2\alpha}{3}$

↳  $\frac{4}{3} \frac{2z+1}{1-z}$

$z=1 \rightarrow$  divergence (soft massless emission)

$\rho_T^2 \rightarrow 0 \rightarrow$  collinear divergence

we used  $-f \ll \xi \rightarrow$  gg dominant

