LECTURE 20 and 21: QCD (Part III, IV)

Overview:

- Scaling Violations
- Gluon Pair Production
- 3-Jet Events
- Jet Algorithms

(I used Halzen-Martin and Quigg along with talk by Gavin Salam)

Scaling Violations

Last lecture we obtained: $\frac{d\hat{\sigma}}{dp_T^2} \simeq e_i^2 \hat{\sigma}_0 \frac{1}{p_T^2} \frac{\alpha_s}{2\pi} P_{qq}(z)$

with $P_{qq} = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$

What is the contribution of gluon emission to the structure functions?

$$\hat{\sigma}(\gamma^* q \to q g) = \int_{\mu^2}^{\hat{s}/4} dp_T^2 \frac{d\hat{\sigma}}{dp_T^2}$$

$$= e_i^2 \hat{\sigma}_0 \int_{\mu^2}^{\hat{s}/4} \frac{dp_T^2}{p_T^2} \frac{\alpha_s}{2\pi} P_{qq}(z)$$

$$= e_i^2 \hat{\sigma}_0 \left(\frac{\alpha_s}{2\pi} P_{qq}(z) \log \frac{Q^2}{\mu^2} \right)$$

$\rightarrow p_{T\,max}^2 = \frac{\hat{s}}{4} = Q^2 \frac{(1-z)}{4z} \rightarrow \log \frac{\hat{s}}{4} \approx \log Q^2$, Q^2 large

Scaling Violations

→ μ introduced to regularize $p_T^2 \to 0$ divergence

$$\frac{F_2(x, Q^2)}{x} =$$

$$= \sum_q e_q^2 \int_x^1 \frac{dy}{y} q(y) \left(\delta\left(1 - \frac{x}{y}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{y}\right) \log \frac{Q^2}{\mu^2} \right)$$

$$q(y) \equiv f_q(y)$$

→ $\log Q^2$ term will introduces scaling violations

→ violation implies gluon emission

Now we will try to obtain quark probability distributions from the above

Scaling Violations

$$\frac{F_2(x,Q^2)}{x} = \sum_q e_q^2 \int_x^1 \frac{dy}{y} \left(q(y) + \Delta q(y,Q^2)\right) \delta\left(1-\frac{x}{y}\right)$$

$$= \sum_q e_q^2 \left(q(x) + \Delta q(x,Q^2)\right)$$

with

$$\Delta q(x,Q^2) = \frac{\alpha_s}{2\pi} \log\left(\frac{Q^2}{\mu^2}\right) \int_x^1 \frac{dy}{y} q(y) P_{qq}\left(\frac{x}{y}\right)$$

→ quark densities depend on Q^2

→ our ability to resolve partons increases with Q^2

we can rewrite expression above as:

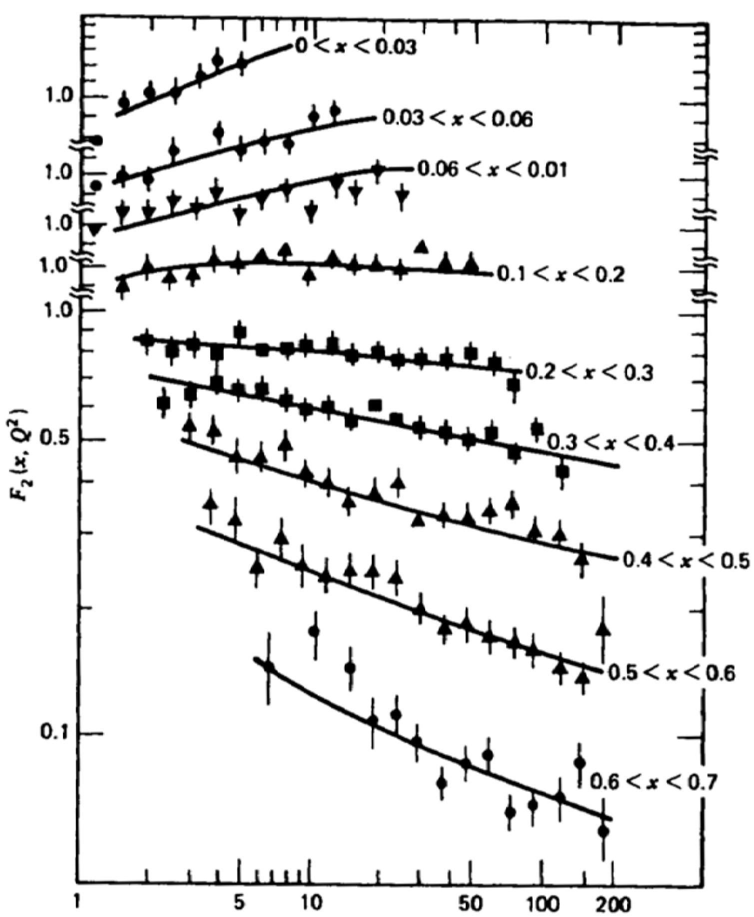
$$\frac{d}{d\log Q^2} q(x,Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} q(y,Q^2) P_{qq}\left(\frac{x}{y}\right)$$

Altarelli-Parisi evolution equation

→ Now "DGLAP"

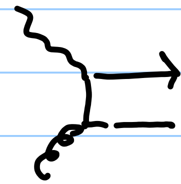
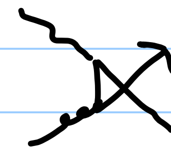
Altarelli-Parisi Equation (DGLAP)

→ a quark with momentum fraction x could have come from parent quark with momentum y (which radiated a gluon). This is associated with prob. $P_{qq}(x/y)$.



Gluon Pair Production

$\rightarrow \gamma^* g \rightarrow q\bar{q}$

$$\frac{F_2(x, Q^2)}{x}\Big|_{\gamma^* g \rightarrow q\bar{q}} = \Big|$$  $$+$$  $$\Big|^2$$

$$= \sum_q e_q^2 \int_x^1 \frac{dy}{y}\, g(y) \frac{\alpha_s}{2\pi} P_{qg}\left(\frac{x}{y}\right) \log\left(\frac{Q^2}{\mu^2}\right)$$

$g(y)$: gluon density in proton

$P_{gq}(z) = \left(z^2 + (1-z)^2\right)$

↳ prob. that produces $q\bar{q}$ pair with q having z fractional momentum.

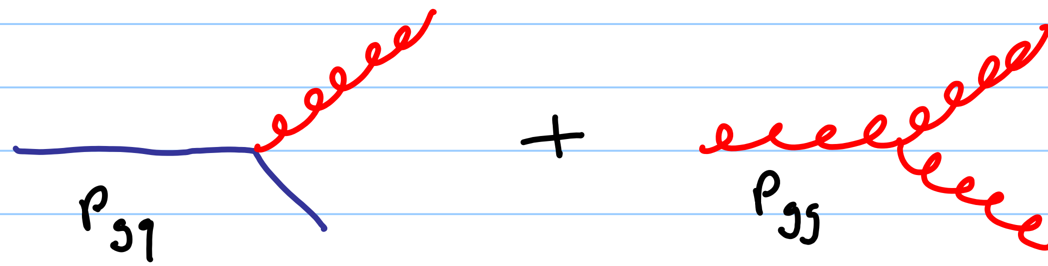
Evolution Equations

Putting everything together, we get:

$$\frac{dq_i}{d\log Q^2} = \frac{\alpha^2}{2\pi}\int_x^1 \frac{dy}{y}\left(q_i(y,Q^2)\,P_{qq}\left(\frac{x}{y}\right) + g(y,Q^2)\,P_{qg}\left(\frac{x}{y}\right)\right)$$

$$\frac{d}{d\log Q^2}\, g(x,Q^2) = \sum_i q_i(y,Q^2) \quad + g(y,Q^2)$$

i.e.



$$\frac{dg(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left(\sum_i q_i(y,Q^2)\,P_{gq}\left(\frac{x}{y}\right) + g(x,Q^2)\,P_{gg}\left(\frac{x}{y}\right)\right)$$

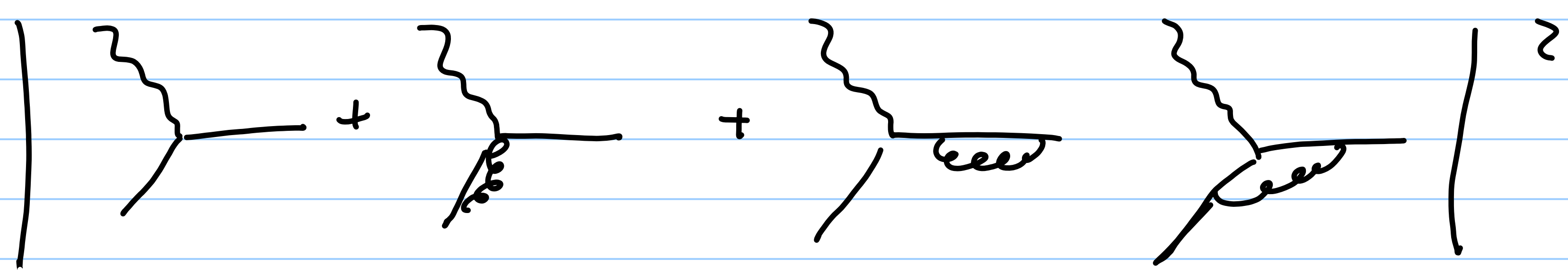
Evolution Equations (cont.)

Recall that: $q(x,Q^2) + \Delta q(x,Q^2) = \int_0^1 dy \int_0^1 dz \; q(y,Q^2)\, \mathcal{P}_{qq}(z,Q^2) \times \delta(x - zy)$

$$\mathcal{P}_{qq} \equiv \delta(1-z) + \frac{\alpha_s}{2\pi} P_{qq}(z) \log\left(\frac{Q^2}{\mu^2}\right)$$

→ prob. of finding a quark "inside" a quark with momentum fraction z of parent quark to first order in α_s.

Note that:



Terms cancel divergence at $z = 1$!

$e^+e^- \to q\bar{q} + X$

$\longrightarrow$ Fragmentation functions

$\longrightarrow$ $q\bar{q}g$ i.e. 3-jet events

Recall that:

$$\sigma(e^+e \to \text{hadrons}) = \sum_q \sigma(e^+e^- \to \text{hadrons})$$
$$= 3\sum_q e_q^2 \, \sigma(e^+e^- \to \mu^+\mu^-)$$

$$\Rightarrow R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_q e_q^2$$

$$R = \frac{11}{3}$$

$O(\alpha_s)$ we'll get $\quad R = 3\sum_q e_q^2 \left(1 + \frac{\alpha_s(Q^2)}{\pi}\right)$

FRAGMENTATION

We can write differential cross section as

$$\frac{d\sigma}{dz}(e^-e^+ \to hX) = \sum_q \sigma(e^+e^- \to q\bar{q})\left[D_q^h(z) + D_{\bar{q}}^h(z)\right]$$

$$z \equiv \frac{E_h}{E_q} = \frac{2E_h}{Q}$$

$$\sum_h \int_0^1 z\, D_q^z(z)\, dz = 1 \quad , \quad \sum_q \int_{z_{min}}^1 \left[D_q^h(z) + D_{\bar{q}}^h(z)\right] dz = N_h$$

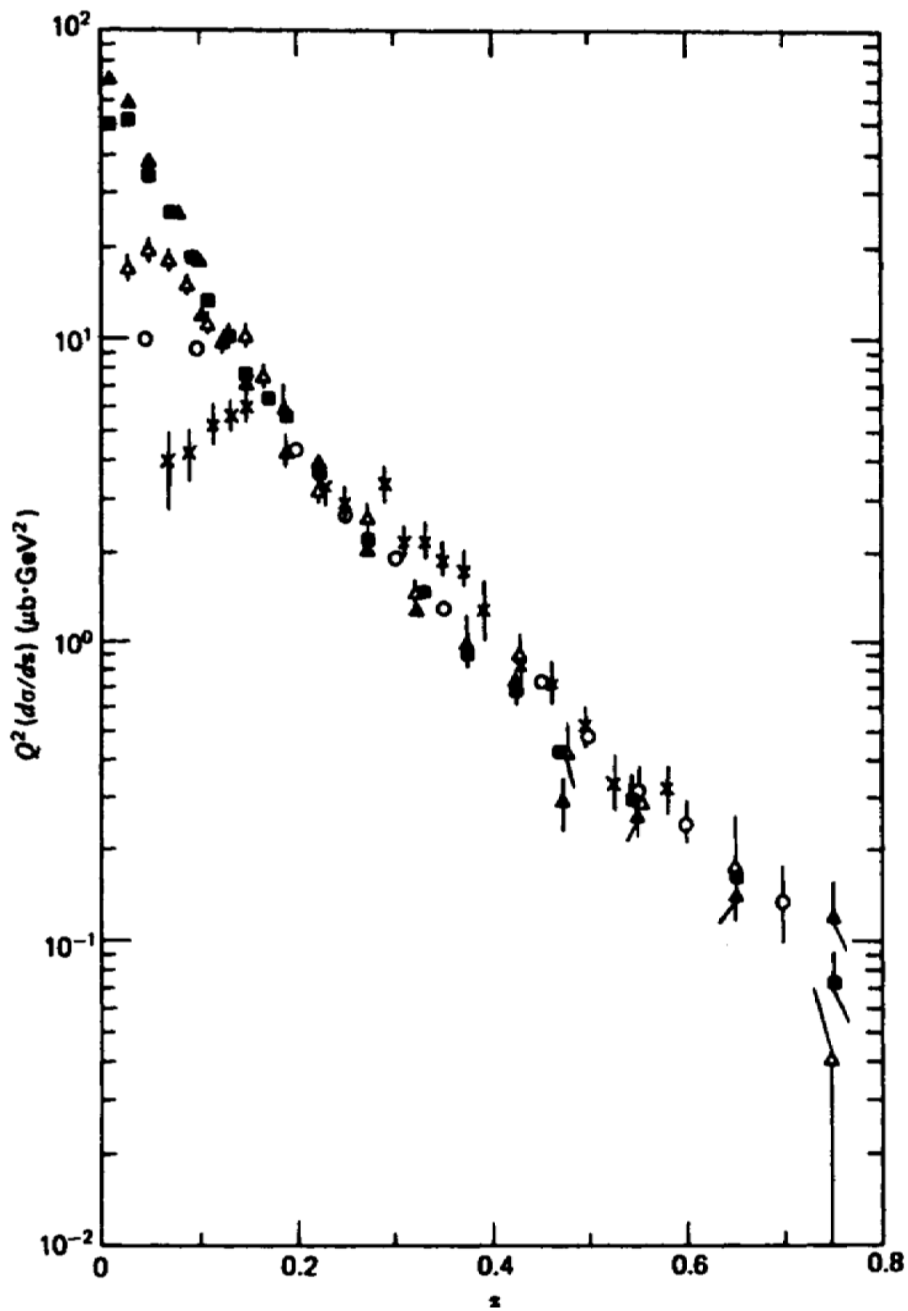
$$z_{min} = 2m_h/Q$$

Using previous results, we get:

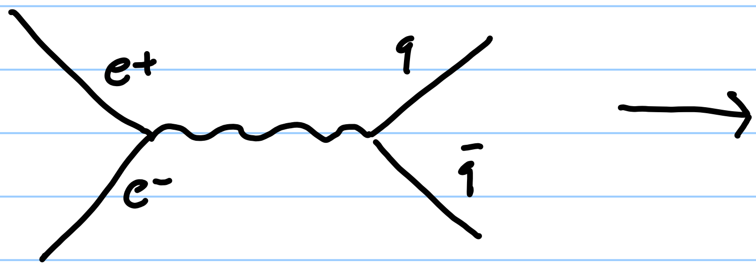
$$\frac{1}{\sigma}\frac{d\sigma}{dz}(e^+e^- \to hX) = \frac{\sum_q e_q^2\left[D_q^h(z) + D_{\bar{q}}^h(z)\right]}{\sum_q e_q^2}$$

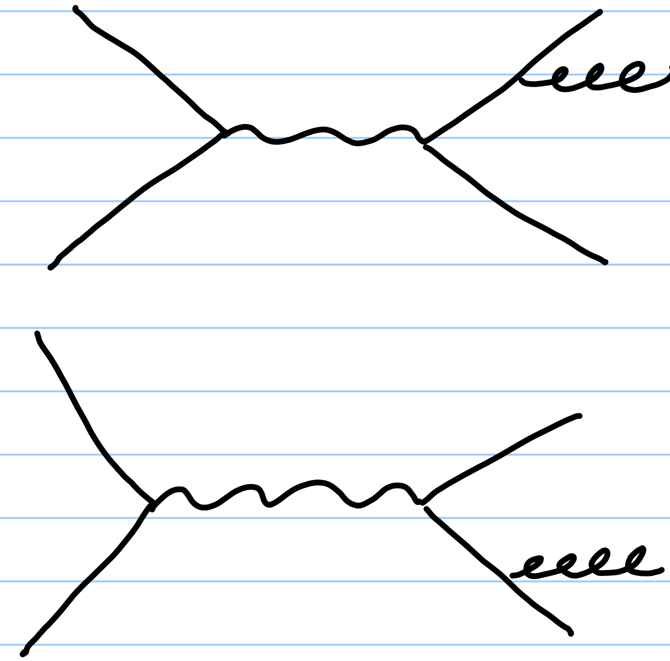
$$= \mathcal{F}(z) \qquad \to \text{predicted to scale}$$

Fragmentation (cont.)

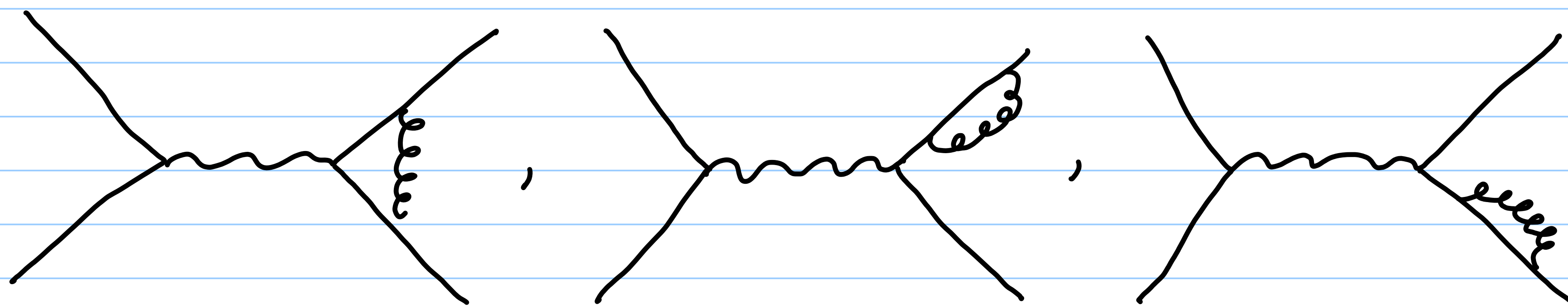


three-jet Events

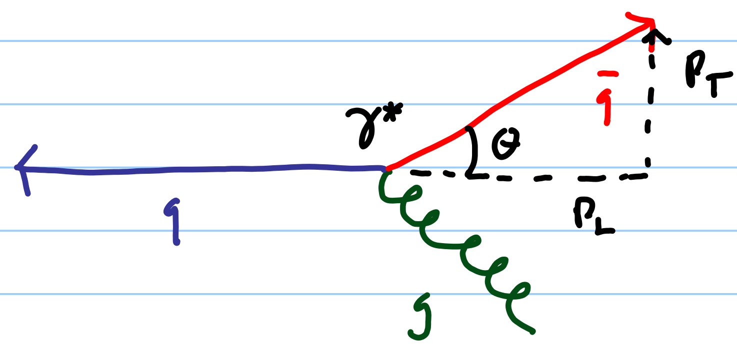




+ other corrections



Three-Jet Events



$$x_q = \frac{2E_q}{Q}, \quad x_{\bar{q}} = \frac{2E_{\bar{q}}}{Q}, \quad x_g = \frac{2E_g}{Q}, \quad x_T = \frac{2p_T}{Q}$$

4-momentum fractions:

$(x_q, 0, 0, -x_q)$

$(x_{\bar{q}}, x_T, 0, x_L)$

$(x_g, -x_T, 0, x_q - x_L)$

variables ordered relative to most energetic jets "thrust axis"

Energy conservation → $x_q + x_{\bar{q}} + x_g = 2$

Three-Jet Events (cont.)

$$x_{\bar{q}}^2 - x_T^2 - x_L^2 = 0$$

$$x_g^2 - x_T^2 - (x_L - x_g)^2 = 0$$

$$\rightarrow \quad x_T^2 = \frac{4}{x_q^2}(1-x_q)(1-x_{\bar{q}})(1-x_g)$$

We will calculate the cross section for the case where: $x_q > x_{\bar{q}} > x_g$

$$\frac{d\sigma}{dx_{\bar{q}}\, dp_T^2} = \sigma(e^+e^- \rightarrow q\bar{q})\, \gamma_{\bar{q}\bar{q}}(x_{\bar{q}}, p_T^2)$$

↳ Prob. $\bar{q}$ emits g with momentum fraction $(1-x_{\bar{q}})$

$$\sigma(e^-e^+ \rightarrow q\bar{q}) = \frac{4\pi\alpha^2}{Q^2} e_q^2 \,, \quad \gamma_{\bar{q}\bar{q}} = \gamma_{qq} = \frac{\alpha_s}{2\pi}\frac{1}{p_T^2} P_{qq}(x_{\bar{q}})$$

$$\rightarrow \quad \frac{1}{\sigma}\frac{d\sigma}{dx_{\bar{q}}\, dx_T^2} = \frac{\alpha_s}{2\pi}\frac{1}{x_T^2} P_{qq}(x_{\bar{q}})$$

3-Jet Events (cont.)

Using previous result for P_{qq}:

$$\frac{1}{\sigma}\frac{d\sigma}{dx_T^2} = \frac{2\alpha_s}{2\pi}\frac{1}{x_T^2}\int_{x_{\bar{q}\,min}}^{x_{\bar{q}\,max}} dx\,\frac{4}{3}\left(\frac{1+x^2}{1-x}\right)$$

largest allowed value for $x_{\bar{q}}$ corresponds to $x_{\bar{q}} = x_q$

$$x_g = x_T$$

$$x_{q\,min} = x_{\bar{q}\,max} \simeq 1 - \frac{x_T}{2} \qquad , \qquad 1+x^2 \simeq 2$$

we get:

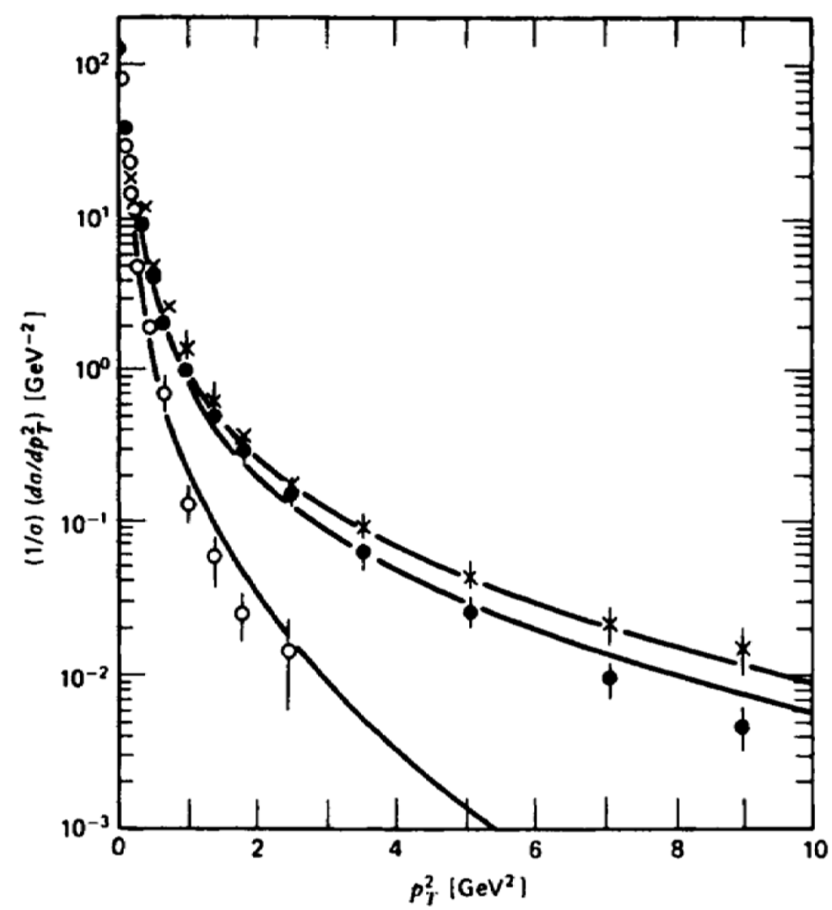
$$\frac{1}{\sigma}\frac{d\sigma}{dx_T^2} \simeq \frac{4\alpha_s}{3\pi}\frac{1}{x_T^2}\int_{x_{\bar{q}\,min}}^{1-x_T/2}\frac{dx}{1-x}$$

$$\simeq \frac{1}{\sigma}\frac{d\sigma}{dx_T^2} \simeq \frac{4\alpha_s}{3\pi}\frac{1}{x_T^2}\log\left(\frac{1}{x_T^2}\right)$$

"exact" result: $\displaystyle \frac{1}{\sigma}\frac{d\sigma}{dx_q\,dx_{\bar{q}}} = \frac{2\alpha_s}{3\pi}\frac{x_q^2 + x_{\bar{q}}^2}{(1-x_q)(1-x_{\bar{q}})}$

3-Jet Events

$$x_T \equiv \frac{2p_T}{Q} \quad \rightarrow \quad \boxed{\frac{1}{\sigma}\frac{d\sigma}{dp_T^2} \sim \alpha_s \frac{1}{p_T^2} \log\left(\frac{Q^2}{4p_T^2}\right)}$$



QCD corrections To $e^+e^- \to$ hadrons

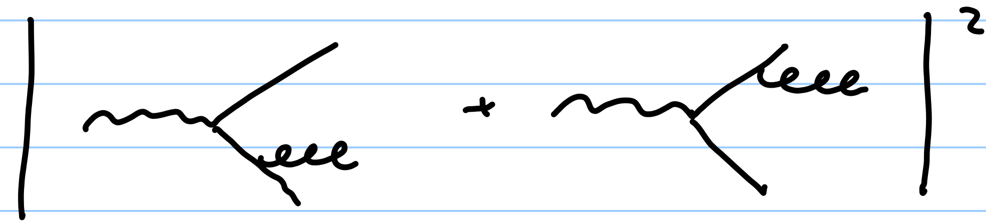
$$\frac{1}{\sigma}\frac{d\sigma}{dx_q\, dx_{\bar{q}}} = \frac{2\alpha_s}{3\pi}\frac{x_q^2 + x_{\bar{q}}^2}{(1-x_q)(1-x_{\bar{q}})}$$

$$1 - x_q = \frac{s}{Q^2} = \frac{2\, p_{\bar{q}}\cdot p_g}{Q^2} = \frac{2}{Q^2} E_{\bar{q}} E_g \left(1 - \cos\theta_{\bar{q}g}\right)$$

$$R \equiv \frac{\sigma(e^-e^+ \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_q e_q^2 + \text{correction}$$

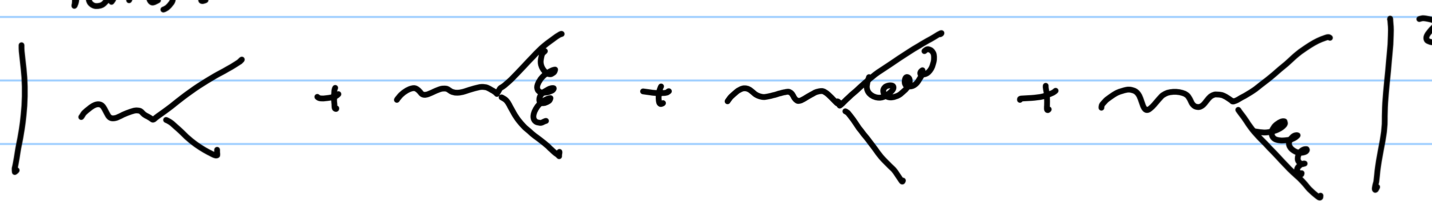
↗ diverges...

So:



$$\sigma_R = \sigma_q \frac{\alpha_s}{2\pi}\frac{4}{3}\left[\log^2\left(\frac{m_g}{Q}\right) + 3\log\left(\frac{m_g}{Q}\right) - \frac{\pi^2}{3} + 5\right.$$

Other Terms:



$$\sigma_v = \sigma_q \frac{\alpha_s}{2\pi}\frac{4}{3}\left[-\log^2\left(\frac{m_g}{Q}\right) - 3\log\left(\frac{m_g}{Q}\right) + \frac{\pi^2}{3} - 7/2\right]$$

QCD corrections to $e^+e^- \to$ hadrons

$$\sigma = \sigma_R + \sigma_V = \sigma_q \frac{\alpha_s}{\pi}$$

$$\Rightarrow \quad R = 3 \sum_q e_q^2 \left[1 + \frac{\alpha_s(Q^2)}{\pi} \right]$$

A word on Jet algorithms (see Salam)

Presentations

Chan Gwak : Neutrino masses
Darij Starko : Quark Gluon Plasma
Tae Park : EFT
Jason Kattan : Supersymetry
Sahibjeet Singh : Top quark phenomenology
Caleb Gemmell : Dark Sectors, Hidden Valley Models

Date: April 21st 13:00 - 17:00
Talk should be 20-25 min → not longer than 25 min.

Evaluation: see email on March 11

Quality of presentation*:	5
Organization of the talk:	15
Questions and answers:	5

* right level for audience, right amount of material, organization and structure of the talk

We will use either vidyo or zoom, please send me your presentations by the morning of April 21st,