

What QCD Tells Us About Nature – and Why We Should Listen *

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I discuss why QCD is our most perfect physical theory. Then I visit a few of its current frontiers. Finally I draw some appropriate conclusions.

1. QCD is our most perfect physical theory

Here's why:

1.1. It embodies deep and beautiful principles.

These are, first of all, the general principles of quantum mechanics, special relativity, and locality, that lead one to relativistic quantum field theory [1]. In addition, we require invariance under the nonabelian gauge symmetry $SU(3)$, the specific matter content of quarks – six spin 1/2 Dirac fermions which are color triplets – and renormalizability. These requirements determine the theory completely, up to a very small number of continuous parameters as discussed below.

Deeper consideration reduces the axioms further. Theoretical physicists have learned the hard way that consistent, non-trivial relativistic quantum field theories are difficult to construct, due to the infinite number of degrees of freedom (per unit volume) needed to construct local fields, which tends to bring in ultraviolet divergences. To construct a relativistic quantum theory, one typically introduces at intermediate stages a cutoff, which spoils the locality or relativistic invariance of the theory. Then one attempts to remove the cutoff, while adjusting the defining parameters, to achieve a finite, cutoff-independent limiting theory. Renormalizable theories are those for which this can be done, order by order in a perturbation expansion around free field theory. This formulation, while convenient for mathematical analysis, obviously begs the question whether the perturbation theory converges (and in practice it never does).

A more straightforward procedure, conceptually, is to regulate the theory as a whole by discretizing it, approximating space-time by a lattice [2]. This spoils the continuous space-time symmetries of the theory. Then one attempts to remove dependence on the discretization by refining it, while if necessary adjusting the defining parameters, to achieve a finite limiting theory that does not depend on the discretization, and does respect the space-time symmetries. The redefinition of parameters is necessary, because in refining

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the discretization one is introducing new degrees of freedom. The earlier, coarser theory results from integrating out these degrees of freedom, and if it is to represent the same physics it must incorporate their effects, for example in vacuum polarization.

In this procedure, the big question is whether the limit exists. It will do so only if the effects of integrating out the additional short-wavelength modes, that are introduced with each refinement of the lattice, can be captured accurately by a re-definition of parameters already appearing in the theory. This, in turn, will occur only in a straightforward way if these modes are weakly coupled. (Another simple possibility is that short-distance modes of different types cancel in vacuum polarization. This is what occurs in supersymmetric theories. Other types of ultraviolet fixed points are in principle possible, but difficult to imagine or investigate.) But this is true, if and only if the theory is asymptotically free.

One can investigate this question, i.e., whether the couplings decrease to zero with distance, or in other words whether the theory is asymptotically free, within weak coupling perturbation theory [3]. One finds that only nonabelian gauge theories with simple matter content, and no non-renormalizable couplings, satisfy this criterion [4]. Supersymmetric versions of these theories allow more elaborate, but still highly constrained, matter content.

Summarizing the argument, only those relativistic field theories which are asymptotically free can be argued in a straightforward way to exist. And the only asymptotically free theories in four space-time dimensions involve nonabelian gauge symmetry, with highly restricted matter content. So the axioms of gauge symmetry and renormalizability are, in a sense, gratuitous. They are implicit in the mere *existence* of non-trivial interacting quantum field theories.

Thus QCD is a member of a small aristocracy: the closed, consistent embodiments of relativity, quantum mechanics, and locality. Within this class, it is among the least affected members.

1.2. It provides algorithms to answer any physically meaningful question within its scope.

As I just discussed, QCD can be constructed by an explicit, precisely defined discretization and limiting procedure. This provides, in principle, a method to compute any observable, in terms that could be communicated to a Turing machine.

In fact marvelous things can be accomplished, in favorable cases, using this direct method. For a stirring example, see Figure 4 below.

The computational burden of the direct approach is, however, heavy at best. When one cannot use importance sampling, as in addressing such basic questions as calculating scattering amplitudes or finding the ground state energy at finite baryon number density, it becomes totally impractical.

For these reasons various improved perturbation theories continue to play an enormous role in our understanding and use of QCD. The most important and well-developed of these, directly based on asymptotic freedom, applies to hard processes and processes involving heavy quarks [5]. It is what is usually called “perturbative QCD”, and leads to extremely impressive results as exemplified by Figures 1-3, below. The scope of these methods is continually expanding, to include additional “semi-hard” processes, as will be discussed in many talks at the Conference. When combined with some additional ideas,

they allow us to address major questions regarding the behavior of the theory at high temperatures and large densities, as I'll touch on below.

Chiral perturbation theory [6], which is based on quite different aspects of QCD, is extremely useful in discussing low-energy processes, though it is difficult to improve systematically. A proper discussion of this, or of many other approaches each of which offers some significant insight (traditional nuclear physics, bag model, Reggeism, sum rules, large N, Skyrme model, ...) would not be appropriate here.

I would however like to mention a perturbation theory which I think is considerably underrated, that is strong coupling perturbation theory [7]. It leads to a simple, appealing, and correct understanding of confinement [8], and even its existing, crude implementations provide a remarkably good caricature of the low-lying hadron spectrum. It may be time to revisit this approach, using modern algorithms and computer resources.

1.3. Its scope is wide.

There are significant applications of QCD to nuclear physics, accelerator physics, cosmology, extreme astrophysics, unification, and natural philosophy. I'll say just a few words about each, in turn.

Nuclear physics : Understanding atomic nuclei was of course the original goal of strong interaction physics. In principle QCD provides answers to all its questions. But in practice QCD has not superseded traditional nuclear physics within its customary domain. The relationship between QCD and traditional nuclear physics is in some respects similar to the relationship between QED and chemistry. The older disciplines retain their integrity and independence, because they tackle questions that are exceedingly refined from the point of view of the microscopic theories, involving delicate cancellations and competitions that manufacture small net energy scales out of much larger gross ones. QCD offers many insights and suggestions, however, of which we will hear much at this Conference. There is also an emerging field of extreme nuclear physics, including the study of nuclei with hard probes and heavy-ion collisions, where the influence of QCD is decisive.

Accelerator physics : Most of what goes on at high-energy accelerators is described by QCD. This application has been so successful, that experimenters no longer speak of "tests of QCD", but of "QCD backgrounds"! Two- and even three-loop calculations of such "backgrounds" are in urgent demand. What can one add to that sincere testimonial?

Cosmology : Because of asymptotic freedom, hadronic matter becomes not impenetrably complex, but rather profoundly simpler, under the extreme conditions predicted for the early moments of the Big Bang. This stunning simplification has opened up a large and fruitful area of investigation.

Extreme astrophysics : The physics of neutron star interiors, neutron star collisions, and collapse of very massive stars involves extreme nuclear physics. It should be, and I believe that in the foreseeable future it will be, firmly based on microscopic QCD.

Unification and Natural philosophy : See below.

1.4. It contains a wealth of phenomena.

Let me enumerate some major ones: radiative corrections, running couplings, confinement, spontaneous (chiral) symmetry breaking, anomalies, instantons. Much could be said about each of these, but I will just add a few words about the first. The Lamb shift in QED is rightly celebrated as a triumph of quantum field theory, because it shows

quantitatively, and beyond reasonable doubt, that loop effects of virtual particles are described by the precise, intricate rules of that discipline. But in QCD, we probably have by now 50 or so cases where two- and even three-loop effects are needed to do justice to experimental results – and the rules are considerably more intricate!

1.5. It has few parameters ...

A straightforward accounting of the parameters in QCD would suggest 8: the masses of six quarks, the value of the strong coupling, and the value of the P and T violating θ parameter. The fact that there are only a small finite number of parameters is quite profound. It is a consequence of the constraints of gauge invariance and renormalizability (or alternatively, as we saw, existence, by way of asymptotic freedom).

In reality there are not 8 parameters, but only 6. g is eliminated by dimensional transmutation [9]. This means, roughly stated, that because the coupling runs as a function of distance, one cannot specify a unique numerical value for it. It will take any value, at some distance or other. One can put (say) $g(l) \equiv 1$, thereby determining a length scale l . What appeared to be a choice of dimensionless coupling, is revealed instead to be a choice of unit of length, or equivalently (with $\hbar = c = 1$) of mass. Only the dimensionless *ratio* of this mass to quark masses can enter into predictions for dimensionless quantities. So what appeared to be a one-parameter family of theories, with different couplings, turns out to be a single theory measured using differently calibrated meter-sticks.

The θ term is eliminated, presumably, by the Peccei-Quinn mechanism [10]. To assure us of this, it would be very nice to observe the quanta associated with this mechanism, namely axions [11]. In any case, we know for sure that the θ term is very small. For purposes of strong interaction physics, within QCD itself, we can safely set it to zero, invoking P or T symmetry, and be done with it.

1.6. ... or none.

This economy of parameters would already be quite impressive, given the wealth of phenomena described. However if we left it at that we would be doing a gross injustice to QCD, and missing one of its most striking features.

To make my point, let me call your attention to a simplified version of QCD, that I call “QCD Lite”. QCD Lite is simply QCD truncated to contain just two flavors of quarks, both of which are strictly massless, and with the θ parameter set to zero. These choices are natural in the technical sense, since they can be replaced by symmetry postulates. Indeed, assuming masslessness of the quarks is tantamount to assuming exact $SU(2) \times SU(2)$ chiral symmetry, and $\theta = 0$ is tantamount to assuming the discrete symmetries P or T. (Actually, once we have set the quark masses to zero, we can dial away θ by a field redefinition.)

Now there are two especially remarkable things about QCD Lite [12]. The first is that it is a theory which *contains no continuous free parameters at all*. Its only inputs are the numbers 3 (colors) and 2 (flavors). The second is that it *provides an excellent semi-quantitative theory of hadronic matter*.

Indeed, in reality the strange and heavier quarks have very little influence on the structure or masses of protons, neutrons, atomic nuclei, pions, rho mesons, Leaving them out would require us to abridge, but not to radically revise, the Rosenfeld Tables. This is proved by the remarkable quantitative success, at the 5-10% level, of lattice gauge theory

in the ‘quenched’ approximation [13]. For in this approximation, the influence of the heavier quarks on the lighter ones is systematically ignored.

The only major effect of putting the u and d masses to zero is to make the pions, which are already quite light by hadronic standards, strictly massless. The perturbation to the proton mass, for instance, can be related using chiral perturbation theory to the value of the so-called σ term, a directly measurable quantity [14]. When this is done, one finds that the u and d quark masses are responsible for only about 5% of the proton mass.

Thus QCD Lite provides a truly remarkable realization of John Wheeler’s program, “Getting Its From Bits”. For here we encounter an extremely rich and complex class of physical phenomena – including, in principle, nuclear and particle spectra – that can be calculated, accurately and without ambiguity, using as sole inputs the numbers 3 and 2.

1.7. It is true.

I will not waste a lot of words on this, showing instead a few pictures, each worth many thousands of words.

Figure 1 [15] displays graphically that many independent types of experiments at different energy scales have yielded determinations of the strong coupling constant, all consistent with the predicted running. The overall accuracy and consistency of this phenomenology is reflected in the precision with which this coupling is determined, to wit 5% (at the Z mass). A remarkable feature of the theory is that a wide range of possible values for the coupling at relatively small energies focuses down to quite a narrow range at the highest accessible energies. Thus any “reasonable” choice of the scale at which the coupling becomes numerically large leads, within a few per cent, to a unique value of the coupling at the Z mass. Our successful QCD predictions for high energy experiments have no wiggle-room!

While Figure 1 is impressive, it does not do complete justice to the situation. For several of the experimental ‘points’ each represents a summary of hundreds of independent measurements, any one of which might have invalidated the theory, and which display many interesting features. Figures 2 and 3 partially ameliorate the omission. Figure 2 [16] shows some of the experimental data on deep inelastic scattering – all subsumed within the ‘DIS’ point in Figure 1 – unfolded to show the complete Q^2 and x dependence. Our predictions [5] for the pattern of evolution of structure functions with Q^2 – decrease at large x , increase at small x – are now confirmed in great detail, and with considerable precision. Particularly spectacular is the rapid growth at small x . This was predicted [17], in the form now observed, very early on. However, even at the time we realized this rise could not continue forever. The proliferating partons begin to form a dense system, and eventually one must cease to regard them as independent. There is a very interesting many-body problem developing here, which seems ripe for experimental and theoretical investigation, and may finally allow us to make contact between microscopic QCD and the remarkably successful Regge-pole phenomenology.

Figure 3 [18] displays a comparison of the experimental distribution of jet energies, in 3-jet events, with the QCD prediction. Shown is the energy fraction of the second hardest jet, compared to its kinematic maximum. For a detailed explanation, see [18]. The rise at $x_2 \rightarrow 1$ reflects the singularity of soft gluon bremsstrahlung, and matches the prediction of QCD (solid line). For comparison, the predictions for hypothetical scalar gluons are

shown by the dotted line. This is as close to a direct measurement of the core interaction of QCD, the basic quark-gluon vertex, as you could hope to see. The other piece of this Figure displays a related but more sophisticated comparison, using the Ellis-Karliner angle.

Finally Figure 4 [19] shows the comparison of the QCD predictions to the spectrum of low-lying hadrons. Unlike what was shown in the previous two Figures, and most of the points in Figure 1, this tests the whole structure of the theory, not only its perturbative aspect. The quality of the fit is remarkable. Note that only one adjustable parameter (the strange quark mass) and one overall choice of normalization go into the calculation. Otherwise it's pure "Its From Bits". Improvements due to enhanced computing power and to the use of domain wall quarks [20], that more nearly respect chiral symmetry, are on the horizon.

Since there seems to be much confusion (and obfuscation) on the point, let me emphasize an aspect of Figure 4 that ought to be blindingly clear: what you don't see in it. You don't see massless degrees of freedom with long-range gauge interactions, nor parity doublets. That is, confinement and chiral symmetry breaking are simply true facts about the solution of QCD, that emerge by direct calculation. The numerical work has taken us way beyond abstract discussion of these features.

1.8. It lacks flaws.

Finally, to justify the adverb in "*most* perfect" I must briefly recall for you some prominent flaws in our other best theories of physics, which QCD does not share.

Quantum electrodynamics is of course extremely useful – incomparably more useful than QCD – and successful in practice. But there is a worm in its bud. It is not asymptotically free. Treated outside of perturbation theory, or extrapolated to extremely high energy, QED becomes internally inconsistent. Modern electroweak theory shares many of the virtues of QED, but it harbors the same worm, and in addition contains many loose ends and continuous free parameters. General relativity is the deepest and most beautiful theory of all, but it breaks down in several known circumstances, producing singularities that have no meaningful interpretation within the theory. Nor does it mesh seamlessly with the considerably better tested and established framework we use for understanding the remainder of physics. Specifically, general relativity is notoriously difficult to quantize. Finally, it begs the question of why the cosmological term is zero, or at least fantastically small when measured in its natural units. Superstring theory promises to solve some and conceivably all of these difficulties, and to provide a fully integrated theory of Nature, but I think it is fair to say that in its present form superstring theory is not defined by clear principles, nor does it provide definite algorithms to answer questions within its claimed scope, so there remains a big gap between promise and delivery.

2. Breaking New Ground in QCD, 1: High Temperature

The behavior of QCD at high temperature, and low baryon number density, is relevant to cosmology – indeed, it describes the bulk of matter filling the Universe, during the first few seconds of the Big Bang – and to the description of both numerical and physical experiments. There are ambitious experimental programs planned for RHIC, and eventually LHC, to probe this physics. It is also, I think, intrinsically fascinating to ask – what

happens to empty space, if you keep adding heat?

The equilibrium thermodynamics of QCD at finite temperature (and zero chemical potential) is amenable to direct simulation, using the techniques of lattice gauge theory. Figure 5 [21] does not quite represent the current, rapidly evolving, state of the art, but it does already demonstrate some major qualitative points.

The chiral symmetry breaking condensate, clearly present (as previously advertised) at zero temperature, weakens and seems to be gone by $T \sim 150$ Mev. Likewise at these temperatures there is a sizable increase in the value of the Polyakov loop, indicating that the force between distant color sources has considerably weakened. Furthermore the energy density increases rapidly, approaching the value one would calculate for an ideal gas of quarks and gluons. The pressure likewise increases rapidly, but lags somewhat behind the ideal gas value.

All these phenomena indicate that at these temperatures and above a description using quarks and gluons as the degrees of freedom is much simpler and more appropriate than a description involving ordinary hadrons. Indeed, the quarks and gluons appear to be quasi-free. That is what one expects, from asymptotic freedom, for the high-energy modes that dominate the thermodynamics.

It is an interesting challenge to reproduce the pressure analytically [22]. Since the only scale in the problem is the large temperature, if one can organize the calculation so as to avoid infrared divergences, asymptotic freedom will legitimize a weak coupling treatment. Even more interesting would be to do this by a method that also works at finite density, since the equation of state is of great interest for astrophysics and is *not* accessible numerically.

There is no doubt, in any case, that QCD predicts the existence of a quark-gluon plasma phase, wherein its basic degrees of freedom, normally hidden, come to occupy center stage.

While transition to a quark-gluon plasma at asymptotically high temperatures is not unexpected, the abruptness and especially the *precocity* of the change is startling. Below 150 Mev the only important hadronic degrees of freedom are the pions. Why does this rather dilute pion gas suddenly go berserk?

The change is enormous, quantitatively. The pions represent precisely 3 degrees of freedom. The free quarks and gluons, with all their colors, spins, and antiparticles, represent 52 degrees of freedom.

There are many ideas for detecting signals of quark-gluon plasma formation in heavy ion collisions, which you will be hearing much of. I would like to discuss briefly a related but more focused question, on which there has been dramatic progress recently [23]. This is the question, whether there is a true phase transition in QCD accessible to experiment.

One might think that the answer is obviously “yes”, since there are striking differences between ordinary hadronic matter and the quark-gluon plasma. This is not decisive, however. Let me remind you that the dissociation of ordinary atomic gases into plasmas is not accompanied by a phase transition, even though these states of matter are very different (so different, that at Princeton they are studied on separate campuses). Similarly, confinement of quarks is believed to go over continuously, at high temperature, into screening – certainly, no one has demonstrated the existence of an order parameter to distinguish between them.

What about chiral symmetry restoration? For massless quarks, there is a definite

difference between the low-temperature phase of broken chiral symmetry and the quasi-free phase with chiral symmetry restored, so there must be a phase transition. A rather subtle analysis using the renormalization group indicated that for two massless quarks one might have a second-order phase transition, while for three massless quarks it must be first-order [24]. This is the pattern observed in numerical simulations.

The real world has two very light quarks and one (the strange quark) whose mass is neither clearly small nor clearly large compared to basic QCD scales. Here, then, a sharp question emerges. If the strange quark is effectively heavy, and the other quarks are taken strictly massless, we should have a second-order transition. If the strange quark is effectively light, we should have a first-order transition. Which is the case, for the physical value of the strange quark mass? Although this has been a controversial question, there seems to be an emerging consensus among lattice gauge theorists that it is second-order.

Unfortunately, this means that with small but finite u and d quark masses we will not have a sharp phase transition at all, but only a crossover. And not a particularly sharp one, at that. For while we are ordinarily encouraged to treat these masses as small perturbations, they are responsible for the pion masses, which are far from negligible at the temperatures under discussion. So the relevant correlation length never gets very large.

Nevertheless it was interesting to point out [25] that in the $m_s - T$ plane one could naturally connect the first- and second- order behaviors, as in Figure 6. The line of first-order transitions ends at a tricritical point. This is a true critical point, with diverging correlation lengths and large fluctuations. The first-order line, since it is a locus of discontinuities, and therefore the existence of a tricritical point where it terminates, are features which survive the small perturbation due to non-zero light quark masses. All these statements can be tested against numerical simulation.

Stephanov, Rajagopal and Shuryak [23] have brought the subject to a new level of interest, taking off from the simple but brilliant observation that one expects similar behavior in the $\mu - T$ plane. The big advantage of this is, that while m_s is not a control parameter one can vary experimentally, the chemical potential μ is. They have proposed quite specific, characteristic signatures for passage near this transition in the thermal history of a fireball, such as might be obtained in heavy ion collisions [26]. The signatures involve enhanced fluctuations and excess, non-thermal production of low-energy pions.

I believe it ought to be possible to refine the prediction, by locating the tricritical point theoretically. For while it is notoriously difficult to deal with large chemical potentials at small temperature numerically, there are good reasons to be optimistic about high temperatures and relatively small chemical potentials, which is our concern here [27].

If all these strands can be brought together, it will be a wonderful interweaving of theory, experiment, and numerics.

3. Breaking New Ground in QCD, 2: High Density

The behavior of QCD at high baryon number density, and low temperature, is of direct interest for describing neutron star interiors, neutron star collisions, and events near the core of collapsing stars. Unfortunately, it has proved quite difficult to calculate this behavior directly numerically using lattice gauge theory techniques. This is because in

the presence of a chemical potential the functional integral for the partition function is no longer positive definite (or even real) configuration by configuration, so importance sampling fails, and the calculation converges only very slowly.

On the other hand, there has been remarkable progress on this problem over the last year or two using analytical techniques. This has shed considerable new light on many aspects of QCD. We have new, fully calculable mechanisms for confinement and chiral symmetry breaking [28]. Amazingly, we find that two famous, historically influential “mistakes” from the prehistory of QCD – the Han-Nambu [24] assignment of integer charge to quarks, and the Sakurai [30] model of vector mesons as Yang-Mills fields [31] – emerge from the microscopic theory at high density. And we find that in the slightly idealized version of QCD with three degenerate light quarks, there need be no phase transition separating the calculable high density phase from (the appropriate version of) nuclear matter [32]!

At the request of the organizers I will be giving a separate seminar on these developments [33], so here I will be telegraphic.

Why might we expect QCD to become analytically tractable at high density? At the crudest heuristic level, it is a case of asymptotic freedom meets the fermi surface.

Let us suppose, optimistically, that a weak coupling treatment is going to be appropriate, and see where it leads.

If the coupling is weak and the density large, our first approximation to the ground state is large fermi balls for all the quarks. Due to the Pauli exclusion principle, the modes deep within the ball will be energetically costly to excite, and the important low-energy degrees of freedom will be the modes close to the fermi surface. But these modes will have large momentum. Thus their interactions, generically, will either hardly deflect them, or will involve large momentum transfer. In the first case we don’t care, while the second involves a weak coupling, due to asymptotic freedom.

On reflection, one perceives two big holes in this argument. First, it doesn’t touch the gluons. They remain massless, with singular interactions and strong couplings in the infrared that do not appear to be under control. Second, as we learn in the theory of superconductivity, the fermi surface is generically unstable, even at weak coupling. This is because pairs of particles (or holes) of equal and opposite momenta are low-energy excitations which can all scatter into one another. Thus one is doing highly degenerate perturbation theory, and in that circumstance even a small coupling can have large qualitative effects.

Fortunately, our brethren in condensed matter physics have taught us how to deal with the second problem, and its proper treatment also cures the first. There is an attractive interaction between quarks on the opposite sides of the fermi surface, and they pair up and condense. In favorable cases – and in particular, for three degenerate or nearly degenerate flavors – this color superconductivity produces a gap for all the fermion excitations, and also gives mass to all the gluons.

Thus a proper weak-coupling treatment automatically avoids all potential infrared divergences, and our optimistic invocation of asymptotic freedom provides, at asymptotically high density, its own justification.

4. Breaking New Ground in QCD, 3: Unification

The different components of the standard model have a similar mathematical structure, all being gauge theories. Their common structure encourages the speculation that they are different facets of a more encompassing gauge symmetry, in which the different strong and weak color charges, as well as electromagnetic charge, would all appear on the same footing. The multiplet structure of the quarks and leptons in the standard model fits beautifully into small representations of unification groups such as $SU(5)$ or $SO(10)$. There is the apparent difficulty, however, that the coupling strengths of the different standard model interactions are widely different, whereas the symmetry required for unification requires that they share a common value.

The running of couplings suggests an escape from this impasse [34]. Since the strong, weak, and electromagnetic couplings run at different rates, their inequality at currently accessible scales need not reflect the ultimate state of affairs. We can imagine that spontaneous symmetry breaking – a soft effect – has hidden the full symmetry of the unified interaction. What is really required is that the fundamental, bare couplings be equal, or in more prosaic terms, that the running couplings of the different interactions should become equal beyond some large scale.

Using simple generalizations of the formulas derived and tested in QCD, which are none other than the ones experimentally validated in Figure 1, we can calculate the running of all the couplings, to see whether this requirement is met. In doing so one must make some hypothesis about the spectrum of virtual particles. If there are additional massive particles (or, better, fields) that have not yet been observed, they will contribute significantly to the running of couplings once the scale exceeds their mass.

Let us first consider the default assumption, that there are no new fields beyond those that occur in the standard model. The results of this calculation are displayed in Figure 7 [35]. Considering the enormity of the extrapolation this works remarkably well, but the accurate experimental data indicates unequivocally that something is wrong.

There is one particularly attractive way to extend the standard model, by including supersymmetry. Supersymmetry cannot be exact, but if it is only mildly broken (so that the superpartners have masses $\lesssim 1$ Tev) it can help explain why radiative corrections to the Higgs mass parameter, and thus to the scale of weak symmetry breaking, are not enormously large. In the absence of supersymmetry power counting would indicate a hard, quadratic dependence of this parameter on the cutoff. Supersymmetry removes the most divergent contribution, by canceling boson against fermion loops. If the masses of the superpartners are not too heavy, the residual finite contributions due to supersymmetry breaking will not be too large. The minimal supersymmetric extension of the standard model, then, makes definite predictions for the spectrum of virtual particles starting at 1 Tev or so. Since the running of couplings is logarithmic, it is not extremely sensitive to the unknown details of the supersymmetric mass spectrum, and we can assess the impact of supersymmetry on the unification hypothesis quantitatively. The results, as shown in Figure 8 [36], are quite encouraging.

A notable result of the unification of couplings calculation, especially in its supersymmetric form, is that the unification occurs at an energy scale which is enormously large by the standards of traditional particle physics, perhaps approaching 10^{16-17} Gev. From a

phenomenological viewpoint, this is fortunate. The most compelling unification schemes merge quarks, antiquarks, leptons, and antileptons into common multiplets, and have gauge bosons mediating transitions among all these particle types. Baryon number violating processes almost inevitably result, whose rate is inversely proportional to the fourth power of the gauge boson masses, and thus to the fourth power of the unification scale. Only for such large values of the scale is one safe from experimental limits on nucleon instability.

From a theoretical point of view the large scale is fascinating because it brings us, from the internal logic of particle physics, to the threshold of quantum gravity.

I find it quite remarkable that the logarithmic running of couplings, discovered theoretically and now amply verified within QCD, permits a meaningful quantitative discussion of these extremely ambitious and otherwise thinly rooted ideas, and even allows us to discriminate between different possibilities (especially, SUSY vs. non-SUSY).

5. Lessons: The Nature of Nature

Since QCD is our most perfect example of a fundamental theory of Nature, it is appropriate to use it as a basis for drawing broad conclusions about how Nature works, or, in other words, for “natural philosophy”.

Let me do this by listing some adjectives we might use to describe the theory; the implication being, that these adjectives therefore describe Nature herself:

alien : As has been the case for all fundamental physical theories since Galileo, QCD is formulated in abstract mathematical terms. In particular, there are no hints of moral concepts or purposes. Nor, in thousands of rigorous experiments, have we encountered any signs of active intervention in the unfolding of the equations according to permanent laws.

simple : In its appropriate, natural language, QCD can be written in one line, using only symbols that cleanly embody its conceptual basis.

beautiful : That she achieves so much with such economy of means, marks Nature as a skillful artist. She plays with symmetries, creating and destroying them in varied, fascinating ways.

weird : Quantum mechanics is notoriously weird, and QCD incorporates it in its marrow. Less remarkable, but to me no less weird, is the need to define QCD through a limiting procedure. This would seem to be a rather difficult and inefficient way to run a Universe.

comprehensible : QCD wonderfully illustrates Einstein’s remark, “The most incomprehensible thing about Nature is that it is comprehensible.” One hundred years ago people did not know there were such things as an atomic nuclei and a strong interaction; just over fifty years ago the pion and kaon were discovered; just over twenty-five years ago the strong interaction problem still seemed hopelessly intractable. That we, collectively, have got from there to here so quickly, against overwhelming odds, is an extraordinary achievement. It is a tribute to our culture, and to the glory of the human mind. And it is there that where we must locate, for now, the most incomprehensible thing about Nature.

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36. Figure 8 courtesy of K. Dienes, CERN Theory Group.

Figure Captions

Figure 1 Experimental verification of the running of the coupling, as predicted in [4].

The determinations, running from left to right, are from: corrections to Bjorken sum rule, corrections to Gross-Llewellyn-Smith sum rule, hadronic width of τ lepton, $b\bar{b}$ threshold production, prompt photon production in pp and $\bar{p}p$ collisions, scaling violation in deeply inelastic scattering, lattice gauge theory calculations for heavy quark spectra, heavy quarkonium decays, shape variables characterizing jets at different energies (white dots), total e^+e^- annihilation cross section, jet production in semileptonic and hadronic processes, energy dependence of photons in Z decay, W production, and electroweak radiative corrections. From [15].

Figure 2 Evolution of the structure function F_2 , as measured and compared with QCD predictions (solid lines). From [16].

Figure 3 Energy and angular characterization of three-jet events, testing the basic quark-gluon vertex very directly. From [18].

Figure 4 Comparison of the hadronic spectrum with first-principles calculations from QCD, using techniques of lattice gauge theory. From [19].

Figure 5 Top part: Evolution of the energy and of the pressure of 2-flavor QCD as a function of temperature, showing precocious and rapid approach to a quasi-free quark-gluon plasma. Bottom part: Evolution of the chiral condensation order parameter and of the Polyakov loop, which is a measure of the inverse induced mass of an inserted color source, and vanishes in a confined phase. One sees clear signals of chiral symmetry restoration and deconfinement [21].

Figure 6 Connecting the second- and first- order chiral symmetry restoration transitions predicted for two, respectively three, light quark flavors. The end of the first-order line is a tricritical point. A similar diagram may be valid for fixed strange quark mass and varying chemical potential.

Figure 7 Running of the couplings extrapolated toward very high scales, using just the fields of the standard model. The couplings do not quite meet. Experimental uncertainties in the extrapolation are indicated by the width of the lines [35].

Figure 8 Running of the couplings extrapolated to high scales, including the effects of supersymmetric particles starting at 1 TeV. Within experimental and theoretical uncertainties, the couplings do meet [36].

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