

Lecture 22: CP Violation

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- Overview
- Meson mixing
- CP violation in Kaon system

(I used Thomson, Griffiths, Cheng Li)

RECAP

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weak eigenstates \neq mass eigenstates

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

W 's couple to : $\begin{pmatrix} u \\ d' \end{pmatrix}$, $\begin{pmatrix} c \\ s' \end{pmatrix}$

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d \cos \theta_c + s \sin \theta_c \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ -d \sin \theta_c + s \cos \theta_c \end{pmatrix}$$

with 3 generations

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} \quad \begin{pmatrix} t \\ b' \end{pmatrix} \quad , \quad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

First doublet : $\begin{pmatrix} u \\ V_{ud} d + V_{us} s + V_{ub} b \end{pmatrix}$

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RECAP (cont.)

If CKM matrix was real, Γ would be orthogonal, couplings would be real.

A general 3×3 unitary matrix can be parameterized by 3 real angles and 6 complex phases.

If we start with:

$$q_1 = \begin{pmatrix} \nu \\ U_{11}d + U_{12}s + U_{13}b \end{pmatrix}$$

$$U_{11} = R_{11} e^{i\delta}, \quad R_{11} \text{ real}$$

we can redefine u -quark field:

$$u \rightarrow u' = u e^{-i\delta}$$

which gives:

$$q_1 = e^{i\delta} \begin{pmatrix} u' \\ R_{11}d + U'_{12}s + U'_{13}b \end{pmatrix}$$

RECAP (cont.)

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We can factor out complex phases for U_{21} , U_3 , T_{03} for c and T quarks.

We can also absorb two other phases U_{12} and U_{13} by redefining s , b fields

→ down To 13 parameters (9 R_{ij} , 4 phases)

→ normalization of each state reduces this by 3

→ orthogonality of b states reduces this by 6

→ need 3 parameters for 3×3 real orthogonal matrix

→ one independent phase

→ $n(n-1)/2$ angles

→ $(n-1)(n-2)/2$ indep. phases

C and P

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Θ - γ puzzle:

$$\Theta \rightarrow \pi^+ + \pi^0$$

$$P = +1$$

$$\gamma \rightarrow \pi^+ + \pi^0 + \pi^0$$

$$P = -1$$

$$\pi^+ + \pi^+ + \pi^-$$

Θ, γ same particle (K^+)

\rightarrow parity not conserved

C \rightarrow "charge conjugation" changes all internal quantum numbers (charge, baryon and lepton number strangeness, etc.) but leaves mass, spin the same.

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\rightarrow \text{left handed } \mu^+$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

$$\rightarrow \text{no left-handed } \mu^-$$

C and P

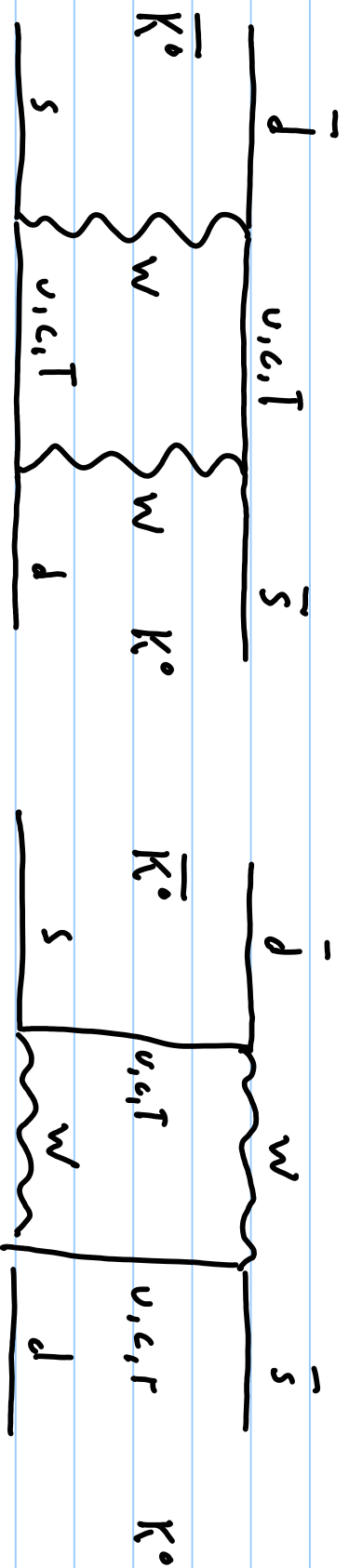
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→ weak interactions not invariant under C either.

→ What about CP?

CP invariance has interesting implications on neutral mesons.

For K mesons:



→ When we turn weak interactions on,

\bar{K}^0 and K^0 will mix

We have to deal with superposition of K^0 and \bar{K}^0

CP Violation

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$$P|K^0\rangle = -|K^0\rangle, \quad P|\bar{K}^0\rangle = -|\bar{K}^0\rangle$$

$$C|K^0\rangle = |\bar{K}^0\rangle, \quad C|\bar{K}^0\rangle = |K^0\rangle$$

$$CP|K^0\rangle = -|\bar{K}^0\rangle, \quad CP|\bar{K}^0\rangle = -|K^0\rangle$$

→ normalized eigenstates of CP:

$$|K_1\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle), \quad |K_2\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle)$$

$$CP|K_1\rangle = +|K_1\rangle, \quad CP|K_2\rangle = -|K_2\rangle$$

IF CP is conserved in weak interactions:

K_1 will only decay into CP = +1 state

K_2 will only decay into CP = -1 state

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CP Violation (cont.)

→ both states have $C = +1$

K_1 has $P = +1$
 K_2 has $P = -1$

→ Decay To Two pions gives state $P = +1$

→ Decay To three pions gives state $P = -1$

⇒ K_1 will decay To 2 pions and K_2 will decay To 3 pions

⇒ K_1 lifetime $<$ K_2 lifetime

$$\tau_{K_1} = 0.9 \times 10^{-10}$$

$$\tau_{K_2} = 5 \times 10^{-8}$$

$$M_2 - M_1 = 3.5 \times 10^{-6} \text{ eV} !$$

CP Violation (cont.)

If we start with beam of K^0 , weak interactions will turn it into linear combination K^0 and $\bar{K}^0 \rightarrow K_1$ and K_2

$\rightarrow K_1$ component will decay in first few centimetres

$\rightarrow K_2$ component will be the only component left after a few metres

\Rightarrow Decays well away from initial K^0 production should all go to 3 pions (if CP conserved)

Observation: $\sim \frac{1}{500} K_2$ decays go to $\pi^0 \pi^0$

\Rightarrow CP not conserved

CP Violation (cont.)

So the long-lived K_{long} is not a perfect CP eigenstate. We should write

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}} (|K_2\rangle + \epsilon |K_1\rangle)$$

→ Note that the short component can be regenerated if K_L been interacts with matter

\bar{K}^0 and K^0 components interact differently in material e.g. $\bar{K}^0 \rightarrow \Sigma^0 \pi^+$ \leftrightarrow ν exchange similar process doesn't exist for K^0

⇒ \bar{K}^0 component can be absorbed

⇒ regenerates K_L !

Time evolution of $K^0 - \bar{K}^0$ system

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Without $K^0 - \bar{K}^0$ mixing we would write

$$|K^0(t)\rangle = |K_0\rangle e^{-\Gamma/2} e^{-i\pi T}$$

$$\rightarrow \frac{d}{dt} |K^0(t)\rangle = \left(m - \frac{i\Gamma}{2}\right) |K^0(t)\rangle$$

\rightarrow effective Hamiltonian

$$H |K^0(t)\rangle = \left(m - \frac{i\Gamma}{2}\right) |K^0(t)\rangle$$

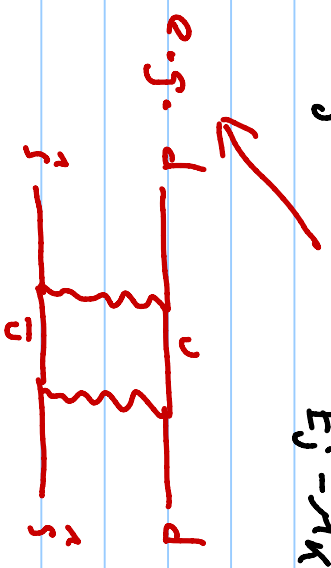
\rightarrow eff. Hamiltonian not Hermitian due to exponential decay term

$$M = M_d + M_s + \langle K^0 | \hat{H}_{\text{acc}} + \hat{H}_{\text{en}} + \hat{H}_w | K^0 \rangle + \sum_j \frac{\langle K^0 | \hat{H}_w | j \rangle \langle j | \hat{H}_w | K^0 \rangle}{E_j - M_K}$$

\hookrightarrow M_{ii} later

$$\Gamma = 2\pi \sum_f |\langle f | \hat{H}_w | K^0 \rangle|^2$$

$f \rightarrow$ final states



with mixing: $|K(t)\rangle = a(t)|K^0\rangle + b(t)|\bar{K}^0\rangle$

$a(t), b(t)$ are amplitudes + phases

we set

$$\begin{pmatrix} M_{11} - i/2 \Gamma_{11} & M_{12} - i/2 \Gamma_{12} \\ M_{21} - i/2 \Gamma_{21} & M_{22} - i/2 \Gamma_{22} \end{pmatrix} \begin{pmatrix} a(t)|K^0\rangle \\ b(t)|\bar{K}^0\rangle \end{pmatrix} = i \frac{d}{dt} \begin{pmatrix} |K^0\rangle \\ |\bar{K}^0\rangle \end{pmatrix}$$

effective Hamiltonian: $H = M - i/2 \Gamma = \begin{pmatrix} M_{11} & \dots \\ \dots & M_{22} \end{pmatrix} - i/2 \begin{pmatrix} \Gamma_{11} & \dots \\ \dots & \Gamma_{22} \end{pmatrix}$

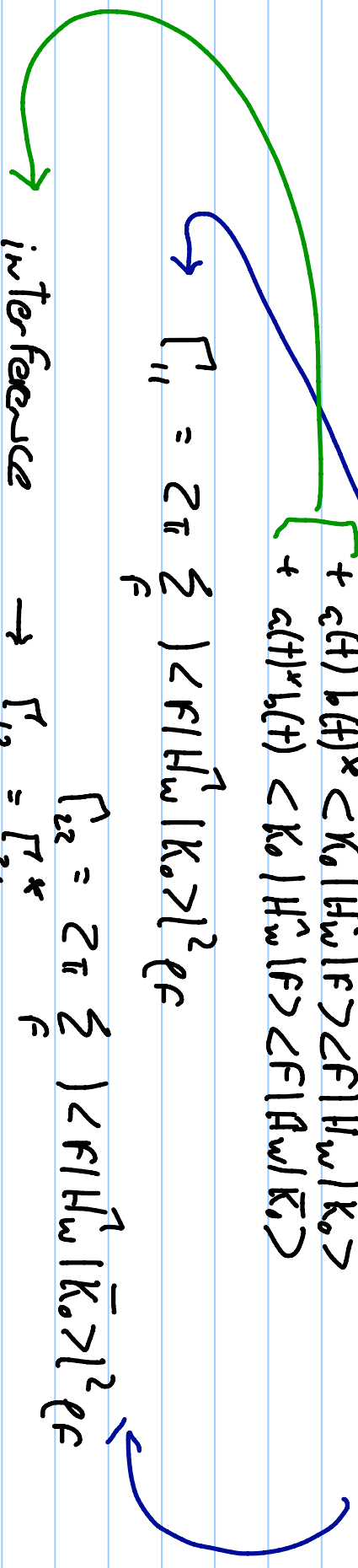
$$\Gamma = 2\pi \sum_f |\langle f | H_w^\dagger | K(t) \rangle|^2 \rho_f$$

$$|\langle f | H_w^\dagger | K(t) \rangle|^2 = |a(t)|^2 |\langle f | H_w^\dagger | K_0 \rangle|^2 + |b(t)|^2 |\langle f | H_w^\dagger | \bar{K}_0 \rangle|^2 + a(t)^* b(t) \langle \bar{K}_0 | H_w^\dagger | f \rangle \langle f | H_w^\dagger | K_0 \rangle + a(t) b(t)^* \langle K_0 | H_w^\dagger | f \rangle \langle f | H_w^\dagger | \bar{K}_0 \rangle$$

$$\Gamma_{11} = 2\pi \sum_f |\langle f | H_w^\dagger | K_0 \rangle|^2 \rho_f$$

$$\Gamma_{22} = 2\pi \sum_f |\langle f | H_w^\dagger | \bar{K}_0 \rangle|^2 \rho_f$$

interference $\rightarrow \Gamma_{12} = \Gamma_{21}^*$



Mass Term M_{11} gives before, M_{22} obtained by swapping $\bar{s} \rightarrow s$, $d \rightarrow \bar{d}$

$$M_{12} = M_{21}^* = \sum_j \frac{\langle \bar{K}_0 | \hat{H}_w | F \rangle \langle F | \hat{H}_w | K_0 \rangle}{E_j - M_K}$$

Note: no $\langle \bar{K}_0 | \hat{H}_w | K_0 \rangle$ term

With CPT we have $M_{11} = M_{22}$, $\Gamma_{11} = \Gamma_{22}$

So we get:
$$\begin{pmatrix} M - i\frac{1}{2}\Gamma & M_{12} - i\frac{1}{2}\Gamma_{12} \\ M_{21} - i\frac{1}{2}\Gamma_{21} & M - i\frac{1}{2}\Gamma \end{pmatrix} \begin{pmatrix} a(t) |K_0^0\rangle \\ b(t) |\bar{K}_0^0\rangle \end{pmatrix} = i\frac{d}{dt} \begin{pmatrix} a(t) |K_0^0\rangle \\ b(t) |\bar{K}_0^0\rangle \end{pmatrix}$$

Eigenstates and Eigenvalues:

$$\begin{pmatrix} |K_+^0\rangle \\ |K_-^0\rangle \end{pmatrix} = \frac{1}{\sqrt{1+|\xi|^2}} \begin{pmatrix} 1 \\ \xi \end{pmatrix} \begin{pmatrix} |K_0^0\rangle \\ |K_0^0\rangle \end{pmatrix} = \frac{1}{\sqrt{1+|\xi|^2}} \begin{pmatrix} |K_0^0\rangle + \xi |K_0^0\rangle \\ |K_0^0\rangle - \xi |K_0^0\rangle \end{pmatrix}$$

$$\begin{pmatrix} |K_+(t)\rangle \\ |K_-(t)\rangle \end{pmatrix} = \frac{1}{\sqrt{1+|\xi|^2}} \begin{pmatrix} |K_0^0\rangle + \xi |K_0^0\rangle \\ |K_0^0\rangle - \xi |K_0^0\rangle \end{pmatrix} \begin{pmatrix} e^{-i\lambda_+ t} \\ e^{-i\lambda_- t} \end{pmatrix}$$

} λ_{\pm} eigenvalues of matrix

Strangeness oscillations

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We will neglect CP violation for now

We start with a K^0

$$|K(t)\rangle = \frac{1}{\sqrt{2}} [\theta_S(t)|K_S\rangle + \theta_L(t)|K_L\rangle]$$

$$\text{with } \theta_S(t) = \exp[-i m_S + \Gamma_S/2)t], \theta_L(t) = \exp[-i m_L + \Gamma_L/2)t]$$

$$\begin{aligned} &\approx \frac{1}{2} [\theta_S (|K^0\rangle + |\bar{K}^0\rangle) + \theta_L (|K^0\rangle - |\bar{K}^0\rangle)] \\ &= \frac{1}{2} (\theta_S + \theta_L) |K^0\rangle + \frac{1}{2} (\theta_S - \theta_L) |\bar{K}^0\rangle \end{aligned}$$

$$\begin{aligned} P(K_{T=0}^0 \rightarrow K^0) &= |\langle K^0 | K(t) \rangle|^2 = \frac{1}{4} |\theta_S + \theta_L|^2 \\ P(K_{T=0}^0 \rightarrow \bar{K}^0) &= \frac{1}{4} |\theta_S - \theta_L|^2 \end{aligned}$$

$$\text{we use: } |\theta_S \pm \theta_L|^2 = |\theta_S|^2 + |\theta_L|^2 \pm 2 \operatorname{Re}(\theta_S \theta_L^*)$$

$$\begin{aligned} &= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2 \operatorname{Re}(e^{-i m_S t} \cdot e^{-\Gamma_S t/2} \cdot e^{i m_L t} \cdot e^{-\Gamma_L t/2}) \\ &= e^{-\Gamma_S t} + e^{-\Gamma_L t} \pm 2 e^{-1/2(\Gamma_S + \Gamma_L)t} \cos(\Delta m t), \text{ with: } \Delta m = m_L - m_S \end{aligned}$$

We get

$$P(K_{T=0}^0 \rightarrow K^0) = \frac{1}{4} e^{-\Gamma_S T} + e^{-\Gamma_T T} + 2 e^{-\frac{1}{2}(\Gamma_S + \Gamma_L)T} \cos(\Delta m t)$$

$$P(K_{T=0}^0 \rightarrow \bar{K}^0) = \dots$$

the period of the oscillation is $\sim 10^{-9}$ s and is about $10 \times$ greater than K_S lifetime

CPLEAR Experiment

beam: $\bar{p}p \rightarrow K^- \pi^+ K^0$ and $\bar{p}p \rightarrow K^+ \pi^- \bar{K}^0$

use decays: $K^0 \rightarrow \pi^+ \ell^- \bar{\nu}$, $\bar{K}^0 \rightarrow \pi^- \ell^+ \nu$

Tag Flavor at production and decay \rightarrow particle id using Cerenkov detector

Decay rates given by R_+ (for positive charge lepton) and R_- (" negative charge lepton)

$$R_+ \propto P(K_{T=0}^0 \rightarrow K^0), \quad R_- \propto P(\bar{K}_{T=0}^0 \rightarrow K^0)$$

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Measure Asymmetry: $A_{\Delta m}(t) = \frac{(R_+ + \bar{R}_-) - (R_- + \bar{R}_+)}{(R_+ + \bar{R}_-) + (R_- + \bar{R}_+)}$

$$A_{\Delta m}(t) = \frac{2e^{-(\Gamma_S + \Gamma_L)t/2} \cos(\Delta m t)}{e^{-\Gamma_S t} + e^{-\Gamma_L t}}$$

→ extract $\Delta m = 3.5 \times 10^{-15} \text{ GeV}$
or $3.5 \times 10^{-6} \text{ eV}$

CP violation in K_{meson} system

We have: $|K_S\rangle = \frac{1}{\sqrt{1+\epsilon}} (|K_1\rangle + \epsilon |K_2\rangle)$

$$|K_L\rangle = \frac{1}{\sqrt{1+\epsilon}} (|K_2\rangle + \epsilon |K_1\rangle)$$

In Terms of Flavor eigenstates:

$$|K_S\rangle = \frac{1}{\sqrt{2(1+\epsilon)}} [(1+\epsilon)|K_0\rangle + (1-\epsilon)|\bar{K}_0\rangle]$$

$$|K_L\rangle = \frac{1}{\sqrt{2(1+\epsilon)}} [(1+\epsilon)|K_0\rangle - (1-\epsilon)|\bar{K}_0\rangle]$$

In Terms of K^0 and \bar{K}^0 :

$$|K^0\rangle = \frac{1}{\sqrt{1+\epsilon}} \sqrt{\frac{1+|\epsilon|^2}{2}} (|K_S\rangle + |K_L\rangle)$$

$$|\bar{K}^0\rangle = \frac{1}{\sqrt{1+\epsilon}} \sqrt{\frac{1+|\epsilon|^2}{2}} (|K_S\rangle - |K_L\rangle)$$

$$|K(t)\rangle = \frac{1}{\sqrt{1+\epsilon}} \sqrt{\frac{1+|\epsilon|^2}{2}} (\theta_S(t)|K_S\rangle - \theta_L(t)|K_L\rangle)$$

In Terms of K_1 and K_2 :

$$\begin{aligned} |K(t)\rangle &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+\epsilon}} \left[\theta_S(t)|K_1\rangle + \epsilon |K_2\rangle + \theta_L(t)|K_1\rangle + \epsilon |K_2\rangle \right] \\ &= \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+\epsilon}} \left[(\theta_S + \epsilon\theta_L)|K_1\rangle + (\theta_L + \epsilon\theta_S)|K_2\rangle \right] \end{aligned}$$

Decay rate to two pions:

$$\Gamma(K_{\pi_0\pi_0}^0 \rightarrow \pi\pi) \propto |\langle K_1 | K(t) \rangle|^2 = \frac{1}{2} \left| \frac{1}{\sqrt{1+\epsilon}} \right|^2 |\theta_S + \epsilon\theta_L|^2$$

$$\left| \frac{1}{1+\varepsilon} \right|^2 \approx 1 - 2\operatorname{Re}(\varepsilon) \quad \rightarrow \text{with } \varepsilon \ll 1$$

$$| \theta_s + \varepsilon \theta_L |^2 = | e^{-i\alpha_s T} - \Gamma_{11/2} + |\varepsilon| e^{i\varphi} e^{-i\alpha_s T - \Gamma_{21/2}} |^2 \quad \rightarrow \text{with } \varepsilon = |\varepsilon| e^{i\varphi}$$

$$\rightarrow \Gamma(K_{T=0}^0 \rightarrow \pi\pi) = \frac{N}{2} (1 - 2\operatorname{Re}(\varepsilon)) \left[e^{-\Gamma_{11} T} |\varepsilon|^2 e^{-\Gamma_{11} T} + 2|\varepsilon| e^{-\Gamma_{21} T} \cos(\Delta\alpha T - \varphi) \right]$$

and:

$$\Gamma(K_{T=0}^0 \rightarrow \pi\pi) = \frac{N}{2} (1 + 2\operatorname{Re}(\varepsilon)) \left[e^{-\Gamma_{11} T} |\varepsilon|^2 e^{-\Gamma_{11} T} - 2|\varepsilon| e^{-\Gamma_{21} T} \cos(\Delta\alpha T - \varphi) \right]$$

the measurement of ε was obtained from the asymmetry:

$$A_{+-} = \frac{\Gamma(K_{T=0}^0 \rightarrow \pi^+\pi^-) - \Gamma(K_{T=0}^0 \rightarrow \pi^-\pi^+)}{\Gamma(K_{T=0}^0 \rightarrow \pi^+\pi^-) + \Gamma(K_{T=0}^0 \rightarrow \pi^-\pi^+)}$$

with ε small we ultimately get:

$$A_{+-} \approx \frac{2\operatorname{Re}|\varepsilon| - 2|\varepsilon| e^{(\Gamma_1 - \Gamma_2)T/2} \cos(\Delta\alpha T - \varphi)}{1 + |\varepsilon|^2 e^{(\Gamma_1 - \Gamma_2)T}}$$

\rightarrow extract $|\varepsilon|$ and φ

Another strategy To measure CP violation is to isolate the

K_L component :

$$|K_L\rangle = \frac{1}{\sqrt{2(1+|\epsilon|^2)}} \left[(1+\epsilon)|K^0\rangle - (1-\epsilon)|\bar{K}^0\rangle \right]$$

with decays : $\pi^- e^+ \nu$ and $\pi^+ e^- \bar{\nu}$

Decay rates :

$$\begin{aligned} \Gamma(|K_L \rightarrow \pi^+ e^- \bar{\nu}\rangle) &\propto |\langle K_0 | K_L \rangle|^2 \propto |1-\epsilon|^2 \approx 1 - 2\text{Re}(\epsilon) \\ \Gamma(|K_L \rightarrow \pi^- e^+ \nu\rangle) &\propto |\langle K_0 | K_L \rangle|^2 \propto |1+\epsilon|^2 \approx 1 + 2\text{Re}(\epsilon) \end{aligned}$$

Experimental measurement again in terms of asymmetry:

$$S = \frac{\Gamma(K_L \rightarrow \pi^- e^+ \nu) - \Gamma(K_L \rightarrow \pi^+ e^- \bar{\nu})}{\Gamma(K_L \rightarrow \pi^- e^+ \nu) + \Gamma(K_L \rightarrow \pi^+ e^- \bar{\nu})} = 2\text{Re}(\epsilon) = 2|\epsilon|\cos\phi$$

$$K_L \rightarrow \pi^+ e^- \bar{\nu} \quad 0.7\% \text{ larger than } K_L \rightarrow \pi^- e^+ \nu$$

Assignment (due March 25th)

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In the neutral Kaon system, we can express time-reversal violation in terms of the asymmetry:

$$A_T = \frac{\Gamma(\bar{K}^0 \rightarrow K^0) - \Gamma(K^0 \rightarrow \bar{K}^0)}{\Gamma(\bar{K}^0 \rightarrow K^0) + \Gamma(K^0 \rightarrow \bar{K}^0)}$$

Show that this is equivalent to:

$$A_T = \frac{\Gamma(\bar{K}_{T=0}^0 \rightarrow \pi^- e^+ \nu_e) - \Gamma(K_{T=0}^0 \rightarrow \pi^+ e^- \bar{\nu}_e)}{\Gamma(\bar{K}_{T=0}^0 \rightarrow \pi^- e^+ \nu_e) + \Gamma(K_{T=0}^0 \rightarrow \pi^+ e^- \bar{\nu}_e)}$$

and therefore:

$$A_T \approx 4|\epsilon| \cos \phi$$

→ problem 14.7 of
THOMSON