LECTURE 6: Spontaneous Symmetry Breaking (Part II)

Overview:

-Recap of Abelian case

-Ginzburg-Laudau

-Higgs Mechanism (non-Abelian case)

(This lecture mostly follows Quigg Chapters 4-5)

Higgs Mechanism (Abelian case recap)

We saw that spontaneous breaking of a continuous symmetry leads to massless bosons (Goldstone bosons). We expect one massless boson per broken generator.

We saw however that in the case of a local gauge theory, the massless
gauge boson and the massless Goldstone boson conspire to give us a massive gauge boson without the massless Goldstone boson.

In the case we studied, we had before symmetry breaking:

2 scalars: 2 degrees of freedom
1 massless vector boson: 2 degrees of freedom

Total =4

After breaking we had (explicit in unitary gauge):

1 massive vector boson: 3 degrees of freedom
1 massive Higgs scalar: 1 degree of freedom Total =4

Ginzburg Landau Superconductivity

ψ : macroscopic wave function describing condensate

Free energy of superconductor can be written as:

$$G_{super}(0) = G_{normal}(0) + \alpha|\psi|^2 + \beta|\psi|^4$$

$$G_{super}(B) = G_{super}(0) + \frac{B^2}{2} + \frac{1}{2m^*}\psi^*(-i\nabla - e^* A)^2 \psi$$

in weak field approx., field equations derived using $G_{super}(B)$ lead to massive photon

Meisner Effect:

- **Cooper pairs form BEC condensate below $T_c \sim 10^0$-10^2 K. Condensate disturbed by EM field**
- **Short range force, attenuation length $\sim 10^{-6}$cm**
- **equivalent to photon acquiring a mass**

Electroweak symmetry breaking:

- **Higgs condenses below $T_c \sim 10^{15}$K. Condensate disturbed by gauge bosons**
- **Short range force, attenuation length $\sim 10^{-18}$cm**
- **W/Z bosons acquire mass**

Higgs Mechanism (non-Abelian case)

We will study an SU(2) doublet of complex scalar fields:

$$\varphi = \sqrt{\tfrac{1}{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix}$$

The Lagrangian is: $(\partial_\mu \varphi)^\dagger (\partial^\mu \varphi) - \mu^2 \varphi^\dagger \varphi - \lambda (\varphi^\dagger \varphi)^2$

The covariant derivative: $D_\mu = \partial_\mu + ig \frac{\tau_a}{2} B_\mu^a$

Under infinitesimal transformation: $\varphi(x)' = (1 + \frac{i}{2} \alpha(x) \cdot \tau) \varphi$

$$B_\mu' = B_\mu - \frac{1}{g} \partial_\mu \alpha - \alpha \times B_\mu$$

We obtain

$$\mathcal{L} = (\partial_\mu \varphi + ig \frac{\tau}{2} \cdot B_\mu \varphi)^\dagger (\partial^\mu \varphi + ig \frac{\tau}{2} \cdot B^\mu \varphi) - V(\varphi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$V(\varphi) = \mu^2 (\varphi^\dagger \varphi) + \lambda (\varphi^\dagger \varphi)^2$$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu - g B_\mu \times B_\nu$$

Higgs Mechanism (non-Abelian case)

minimum of potential at $\varphi^\dagger\varphi = \frac{1}{2}\left(\varphi_1^2+\varphi_2^2+\varphi_3^2+\varphi_4^2\right) = \frac{-\mu^2}{2\lambda}$

we chose minimum around which to do our expansion: $\varphi_3^2 = -\frac{\mu^2}{\lambda} \equiv v^2 \qquad \varphi_1 = \varphi_2 = \varphi_4 = 0$

We parametrize fluctuations from the vacuum $\varphi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix}$ in terms of 4 real scalar fields $\xi_1, \xi_2, \xi_3, \eta$

$$\varphi(x) = e^{i\tau\cdot\xi(x)/v}\frac{1}{\sqrt{2}}\begin{pmatrix}0\\v+\eta\end{pmatrix} \approx \frac{1}{\sqrt{2}}\begin{pmatrix}1+i\xi_3/v & i(\xi_1-\xi_2)/v\\ i(\xi_1+\xi_2)/v & 1-i\xi_3/v\end{pmatrix}\begin{pmatrix}0\\v+\eta\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}\xi_2+i\xi_1\\ v+\eta-i\xi_3\end{pmatrix}, \quad \text{so} \quad \varphi^\dagger\varphi =$$

$$\frac{1}{\sqrt{2}}\left(\xi_2-i\xi_1,\ v+\eta+i\xi_3\right)\frac{1}{\sqrt{2}}\begin{pmatrix}\xi_2+i\xi_1\\ v+\eta-i\xi_3\end{pmatrix} = \xi_1^2+\xi_2^2+\xi_3^2+v^2+\eta^2+2v\eta$$

We know all terms from V will cancel save $|\mu^2|\eta$ → for small oscillations

massive scalar

Higgs Mechanism (non-Abelian case)

Let's move to the unitary gauge right away:

$$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+\eta \end{pmatrix}$$

$$D_\mu \varphi = \frac{1}{\sqrt{2}} \left(\partial_\mu \begin{pmatrix} 0 \\ \eta \end{pmatrix} + \frac{ig}{2} \begin{pmatrix} b^3_\mu & \sqrt{2}\, b^-_\mu \\ \sqrt{2}\, b^+_\mu & -b^3_\mu \end{pmatrix} \begin{pmatrix} 0 \\ v+\eta \end{pmatrix} \right)$$

where $b^\pm_\mu = \frac{1}{\sqrt{2}} \left(b^1_\mu \pm i b^2_\mu \right)$

$$|D_\mu \varphi|^2 = \frac{1}{2} \partial_\mu \eta\, \partial^\mu \eta + \frac{1}{4} g^2 v^2 \left(b^+_\mu b^{\mu -} + \frac{1}{2} b^3_\mu b^{3\mu} \right)$$

$$+ \frac{1}{4} g^2 \eta^2 \left(b^+_\mu b^{-\mu} + \frac{1}{2} b^3_\mu b^{\mu 3} \right) + \frac{1}{2} g^2 v \eta \left(b^+_\mu b^{\mu -} + \frac{1}{2} b^3_\mu b^{\mu 3} \right)$$

$\rightarrow$ 3 bosons with mass $\frac{gv}{2}$

Higgs Mechanism (non-Abelian case)

Summary:

we started with: 4 scalars : 4 dof
3 massless bosons: $3 \times 2 = 6$ dof

$\boxed{\text{Total } = 10 \text{ dof}}$

we end up with: 1 scalar (massive) : 1 dof
3 massive bosons: $3 \times 3 = 9$ dof

$\boxed{\text{Total } = 10 \text{ dof}}$

In the Standard Model we have 3 massive vector bosons (2 charged, one neutral) and one massless boson (neutral).

Future problem set

Problem set 1

Due Date: Wednesday Feb 9^{th}

Problem 1

Analyse the spontaneous breaking of a global SU(2) symmetry for the case of 3 real scalar fields in an SU(2) triplet: $\begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)\cdot(\partial^\mu \varphi) - V(\varphi)$$

$$V(\varphi) = \frac{1}{2}\mu^2 \varphi\cdot\varphi + \frac{1}{4}|\lambda|(\varphi\cdot\varphi)^2$$

Problem 2

Analyse the spontaneous breaking of a local SU(2) symmetry for the case of 3 real scalar fields in an SU(2) triplet: $\begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}$

→ use $\mathcal{L}$ and V from page 4