LECTURE 9: Weak Interactions (Part 3) and the Standard Model (Part 1)

Overview:

- -nu nu -> WW scattering
- -Construction of the Standard Model

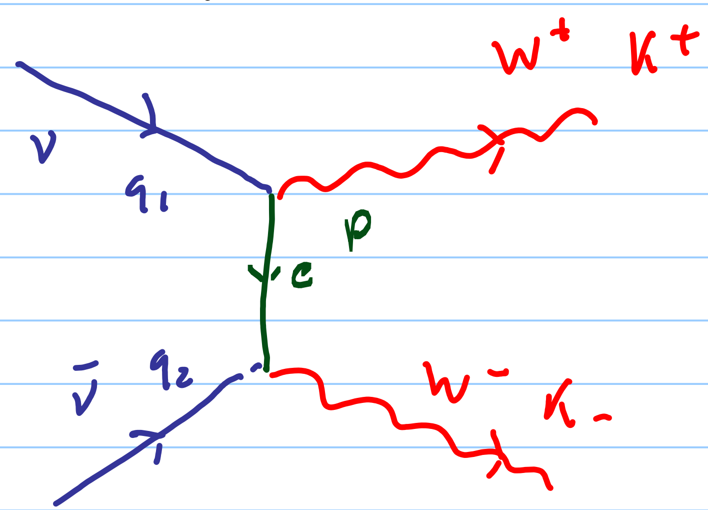
(I used Quigg and Novaes as references)

$\nu\bar{\nu} \to W^+W^-$ scattering

Recall that we had essentially solved our cross section problems at high energies with the introduction of a heavy gauge boson and its associated propagator.

In fact, we have other problems that will require the addition of new particles. Here's an example:

$\nu\bar{\nu} \to W^+W^-$



$$\mathcal{M} = -\,i\,\frac{G_F M_W^2}{\sqrt{2}}\;\bar{v}\,\not{\epsilon}_-^*\,(1-\gamma_5)\,\frac{(\not{p}+m)}{p^2+m^2}\,\not{\epsilon}_+^*\,(1-\gamma_5)\,u$$

$\bar{\nu}\nu \to WW$ scattering continued

$$q_1 \equiv (Q, 0, 0, Q)$$

$$q_2 = (Q, 0, 0, -Q)$$

$$k_+ = (Q, k\sin\theta, 0, k\cos\theta)$$

$$k_- = (Q, -k\sin\theta, 0, -k\cos\theta)$$

$\varepsilon^\mu_\pm = (0, \hat{\varepsilon}_\pm)$ $\to$ W pol. in W rest frame

$\Rightarrow$ in cm frame: $\varepsilon^\mu_\pm = \left[\dfrac{\vec{k}_\pm \cdot \vec{\varepsilon}_\pm}{m_W}, \hat{\varepsilon}_\pm + \dfrac{\vec{k}_\pm (\vec{k}_\pm \cdot \hat{\varepsilon}_\pm)}{m_W (Q + m_W)} \right]$

For longitudinal pol.: $\varepsilon_\mu^\pm = \left(\dfrac{k}{m_W}, \dfrac{Q \hat{k}_\pm}{m_W} \right)$

at very high energies: $Q \simeq k \gg m_W$

$$\to \varepsilon_\pm \approx \frac{k_\pm}{m_W}$$

$\bar{\nu}\nu \to W^+W^-$ scattering (continued)

$$\mathcal{M} = \frac{-iG_F}{\sqrt{2}}\, \bar{v}\, \not{\epsilon}_- (1-\gamma_5) \frac{(\not{p}+m)}{p^2-m^2} \not{\epsilon}_+ (1-\gamma_5)\, u$$

With the Dirac equation we get:

$$\not{q}_1 u = 0 \quad \text{and} \quad \bar{v}\not{q}_2 = 0$$

so we can replace $\not{\epsilon}_+$ with: $\not{\epsilon}_+ - \not{q}_1 = -\not{p}$
and $\not{\epsilon}_-$ with: $\not{\epsilon}_- - \not{q}_2 = \not{p}$

we neglect the mass of the electron (m):

This gives $\frac{iG_F}{\sqrt{2}}\, \bar{v}\, \not{p} (1-\gamma_5) \frac{\not{p}\not{p}}{p^2} (1-\gamma_5)\, u$

$\not{p}\not{p} = p^2 \quad \Rightarrow \quad \mathcal{M} = \sqrt{2}\, iG_F\, \bar{v}\, \not{p}(1-\gamma_5)\, u$

$$|\mathcal{M}|^2 = 2G_F^2\, \mathrm{Tr}\left[\not{q}_2 \not{p} (1-\gamma_5) \not{q}_1 \not{p} (1-\gamma_5)\right]$$

$$= 4G_F^2\, \mathrm{Tr}\left[(1-\gamma_5) \not{q}_2 \not{p} \not{q}_1 \not{p}\right]$$

$\bar{\nu}\nu \to W^+W^-$ scattering (continued)

$$|M|^2 = 16\, G_F^2 \left[(q_2\cdot p)(q_1\cdot p) - (q_2\cdot q_1)(p\cdot p) + (q_2\cdot p)(q_1\cdot p) \right]$$

note that the $\varepsilon_{\mu\nu\sigma\lambda}$ term $= 0$

$$= 16\, G_F^2 \left[2(q_2\cdot p)(q_1\cdot p) - p^2 \underbrace{(q_2\cdot q_1)}_{2Q^2} \right]$$

$p = (0, K\sin\theta, 0, Q - K\cos\theta)$

$q_1\cdot p$: $(Q, 0, 0, Q)\cdot p = + QK\cos\theta - Q^2$

$q_2\cdot p$: $- QK\cos\theta + Q^2$

$2(q_1\cdot p)(q_2\cdot p) = -2Q^2K^2\cos^2\theta \cancel{-2Q^4} + \cancel{4Q^3K^2}\cos^2\theta$

$p^2 = -K^2\sin^2\theta - K^2\cos^2\theta + 2KQ\cos\theta - Q^2$

$= -K^2 - Q^2 + 2KQ\cos\theta$

$p^2\cdot 2Q^2 = -2Q^2K^2 - \cancel{2Q^4} + \cancel{4KQ^3}\cos\theta$

result : $2Q^2K^2(1-\cos^2\theta) = 2Q^2K^2\sin^2\theta$

$$|M|^2 = 32\, G_F^2\, Q^2K^2\sin^2\theta$$

$\bar{\nu}\nu \to W^+W^-$ scattering (continued)

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |M|^2 = \frac{1}{64\pi^2 s} \cdot 32\, G_F^2\, Q^2 K^2 \sin^2\theta$$

integration yields : $2\pi \cdot \frac{4}{3}$ (see W decay, lecture 8)

$$s \simeq 4Q^2 \quad , \quad Q \approx K$$

$$\Rightarrow s^2 = 16\, Q^4$$

$$\sigma = \frac{G_F^2}{64\pi^2 s} \cdot 2 s^2 \cdot 2\pi \cdot \frac{4}{3} = \frac{G_F^2\, s}{12\,\pi}$$

$\frac{G_F^2\, s}{3\pi}$ in Quigg ?

correct...

Again a badly behaved cross section...

We'll see that we need to add the Z boson to tame the high energy behaviour.

The Standard Model

Before we start, a few notes and comments:

$$u_R = \frac{1}{2}(1+\gamma_5)u \quad , \quad u_L = \frac{1}{2}(1-\gamma_5)u$$

$$v(p,s) \equiv C\bar{u}^T(p,s) = i\gamma_2 u^*(p,s)$$

$$v_R = \underbrace{\frac{1}{2}(1-\gamma_5)}_{\equiv L}v \quad , \quad v_L = \underbrace{\frac{1}{2}(1+\gamma_5)}_{\equiv R}v$$

$$\bar{\psi}_L = (L\psi)^\dagger\gamma_0 = \psi^\dagger L^\dagger\gamma_0 = \psi^\dagger L\gamma_0 = \psi^\dagger\gamma_0 R = \bar{\psi}R$$

$$\bar{\psi}_R = \bar{\psi}L$$

note that mass term mixes right and left-handed components. $\bar{\psi}\psi = \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R$

The Standard Model (cont.)

$$\bar\psi\psi = (\bar\psi_R + \bar\psi_L)(\psi_R + \psi_L)$$

$$= (\bar\psi L + \bar\psi R)(R\psi + L\psi)$$

$$= \underbrace{\bar\psi LR\psi}_{\to 0} + \bar\psi LL\psi + \bar\psi RR\psi + \underbrace{\bar\psi RL\psi}_{\to 0}$$

$$= \bar\psi_R\psi_L + \bar\psi_L\psi_R$$

For em interactions:

$$\bar\psi\gamma_\mu\psi = (\bar\psi L + \bar\psi R)\gamma_\mu(L\psi + R\psi)$$

$$= \bar\psi L\gamma_\mu L\psi + \bar\psi L\gamma_\mu R\psi + \bar\psi R\gamma_\mu L\psi + \bar\psi R\gamma_\mu R\psi$$

$$= \underbrace{\bar\psi LR\gamma_\mu\psi}_{=0} + \bar\psi LL\gamma_\mu\psi + \bar\psi RR\gamma_\mu\psi + \underbrace{\bar\psi RL\gamma_\mu\psi}_{=0}$$

$$= \bar\psi_R\gamma_\mu\psi_R + \bar\psi_L\gamma_\mu\psi_L$$

The Standard Model (cont.)

For V-A weak current:

$$\frac{1}{2}\bar{\psi}\gamma^\mu(1-\gamma^5)\psi = \bar{\psi}\gamma^\mu L\psi = \bar{\psi}\gamma^\mu L^2\psi$$
$$= \bar{\psi}R\gamma^\mu L\psi = \bar{\psi}_L\gamma^\mu\psi_L$$

We will now select a gauge group for the em and weak interactions.

→ weak current for lepton ℓ:

$$J_\mu^+ = \bar{\ell}\gamma_\mu(1-\gamma_5)\nu = 2\bar{\ell}_L\gamma_\mu\nu_L$$

We introduce left-handed isospin doublet ($T = 1/2$)

$$L = \begin{pmatrix}\nu\\ \ell\end{pmatrix}_L = \begin{pmatrix}L\nu\\ L\ell\end{pmatrix} = \begin{pmatrix}\nu_L\\ \ell_L\end{pmatrix} \quad \text{with } T_3 = \pm 1/2$$

The Standard Model (cont.)

We'll consider the neutrinos massless for now. So we'll accomodate the right-handed part of the charged lepton in a weak isospin singlet $(T=0)$

$$R\ell = \ell_R$$

The charged weak current can be written as

$$J_\mu^i = \bar{L}\gamma_\mu \frac{\tau^i}{2} L$$

Explicitly:

$$J_\mu^1 = \frac{1}{2}(\bar{\nu}_L\ \bar{\ell}_L)\gamma_\mu \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} = \frac{1}{2}(\bar{\ell}_L\gamma_\mu\nu_L + \bar{\nu}_L\gamma_\mu\ell_L)$$

$$J_\mu^2 = \frac{1}{2}(\bar{\nu}_L\ \bar{\ell}_L)\gamma_\mu \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} = \frac{i}{2}(\bar{\ell}_L\gamma_\mu\nu_L - \bar{\nu}_L\gamma_\mu\ell_L)$$

$$J_\mu^3 = \frac{1}{2}(\bar{\nu}_L\ \bar{\ell}_L)\gamma_\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} = \frac{1}{2}(\bar{\nu}_L\gamma_\mu\nu_L - \bar{\ell}_L\gamma_\mu\ell_L)$$

The Standard Model (cont.)

Charged weak current can be written in terms of J^1 and J^2 : $J^+_\mu = 2(J^1_\mu - iJ^2_\mu)$

$\rightarrow$ will couple to W^-_μ

J^3_μ will involve a neutral current. We define the hyperchange current:

$$J^Y_\mu \equiv -(\bar{L}\gamma_\mu L + 2\bar{R}\gamma_\mu R)$$

$$= -(\bar{\nu}_L \gamma_\mu \nu_L + \bar{l}_L \gamma_\mu l_L + 2\bar{l}_R \gamma_\mu l_R)$$

The em current is given by:

$$J^{em}_\mu = -\bar{l}\gamma_\mu l = -(\bar{l}_L\gamma_\mu l_L + \bar{l}_R\gamma_\mu l_R) = J^3_\mu + 1/2\, J^Y_\mu$$

The Standard Model (cont.)

Note that T_3 and Q do not commute with $T_{1,2}$ but the "charges" associated with J^i and J^Y

$$T^i = \int d^3x\, J_0^i \quad \text{and} \quad Y = \int d^3x\, J_0^Y$$

$$[T^i, T^j] = i\,\varepsilon^{ijk} T^k, \quad [T^i, Y] = 0$$

$$Q = T^3 + \frac{1}{2} Y$$

$\rightarrow$ weak hypercharge of the doublet: $Y_L = -1$

" " of the singlet: $Y_R = -2$

So our gauge group is $SU(2)_L \otimes U(1)_Y$

$$SU(2)_L \rightarrow W_\mu^1,\ W_\mu^2,\ W_\mu^3$$

$$U(1)_Y \rightarrow B_\mu$$

The Standard Model (cont.)

Field strength Tensors:

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\,\varepsilon^{ijk} W^j_\mu W^k_\nu$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

We get $\mathcal{L}_{gauge} = -\frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$

We get $\mathcal{L}_{leptons}$ (free Lagrangian): $\bar{R}\, i \not{\partial} R + \bar{L}\, i \not{\partial} L$

$$= \bar{l}_R\, i \not{\partial}\, l_R + \bar{l}_L\, i \not{\partial}\, l_L + \bar{\nu}_L\, i \not{\partial}\, \nu_L$$

$$= \bar{l}\, i \not{\partial}\, l + \bar{\nu}\, i \not{\partial}\, \nu$$

No mass term. Mixes right and left handed components and would break gauge invariance.

The Standard Model (cont.)

Left and right-handed components transform differently
$\Rightarrow$ will break gauge invariance e.g.:

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L' = \exp\left(-i\alpha \cdot \frac{\tau}{2}\right) \begin{pmatrix} \nu_e \\ e \end{pmatrix}$$

We now introduce fermion-gauge coupling using the gauge-covariant derivative:

$$L : \partial_\mu + \frac{ig}{2} \tau^i W^i_\mu + \frac{ig'}{2} Y B_\mu$$

$$R = \partial_\mu + \frac{ig'}{2} Y B_\mu$$

$g \rightarrow SU(2)$, $Y_{Le} = -1$

$g' \longrightarrow U(1)$, $Y_{Re} = -2$

$$\mathcal{L}_{lep.} = \mathcal{L}_{lep} + \bar{L}\, i\gamma^\mu \left(\frac{ig}{2} \tau^i W^i_\mu + \frac{ig'}{2} Y B_\mu\right) L + \bar{R}\, i\gamma^\mu \left(\frac{ig'}{2} B_\mu\right) R$$