Lecture 2: Behavior of Particles and Waves

Goal of the lecture: give an overview of the wave-partcle nature of matter

I expect you to learn:

- the superposition of waves and interference
- what is the double-slit experiment
- how to calculate the intensity of light on the screen in the double-slit experiment
- What is the electron biprism and how it works

Lecture follows Feynman Vol 3, chapter 1

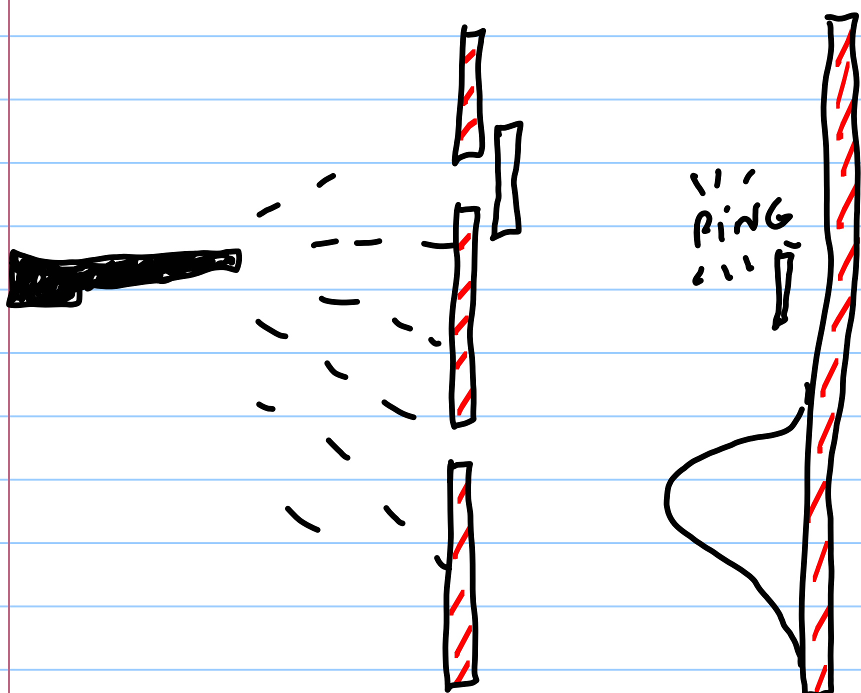
NOTE: website is up. login: phy256student, pass: quantum

Let's start with some observations

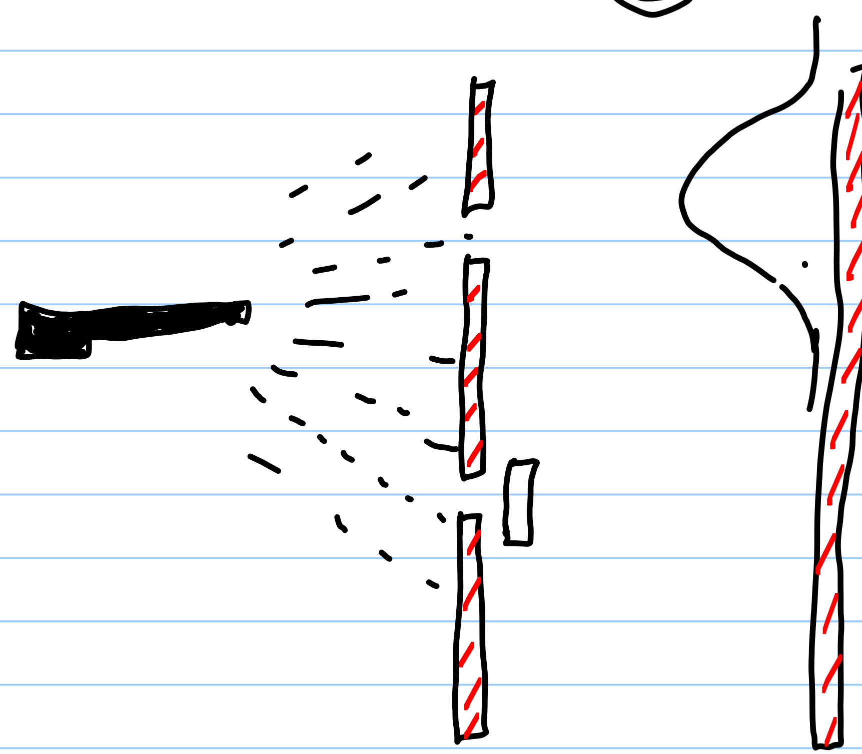
One experiment (A) involves shooting a machine gun through holes in a wall, the other (B), the other involves sending waves of water through holes in a wall.

Experiment A:

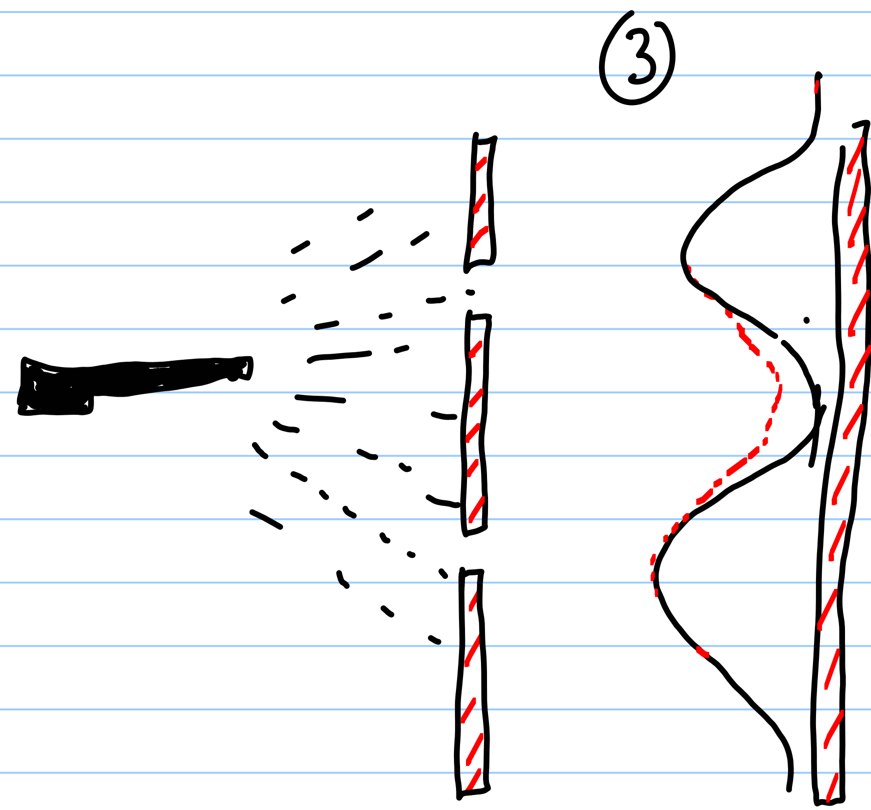
(1)



(2)

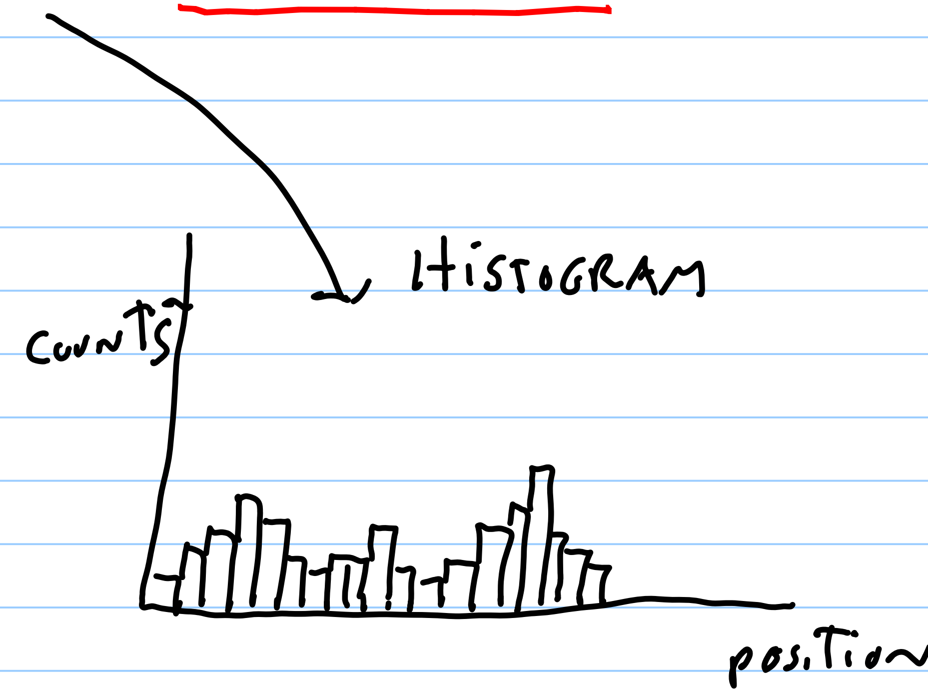


Sum of (1) + (2) =

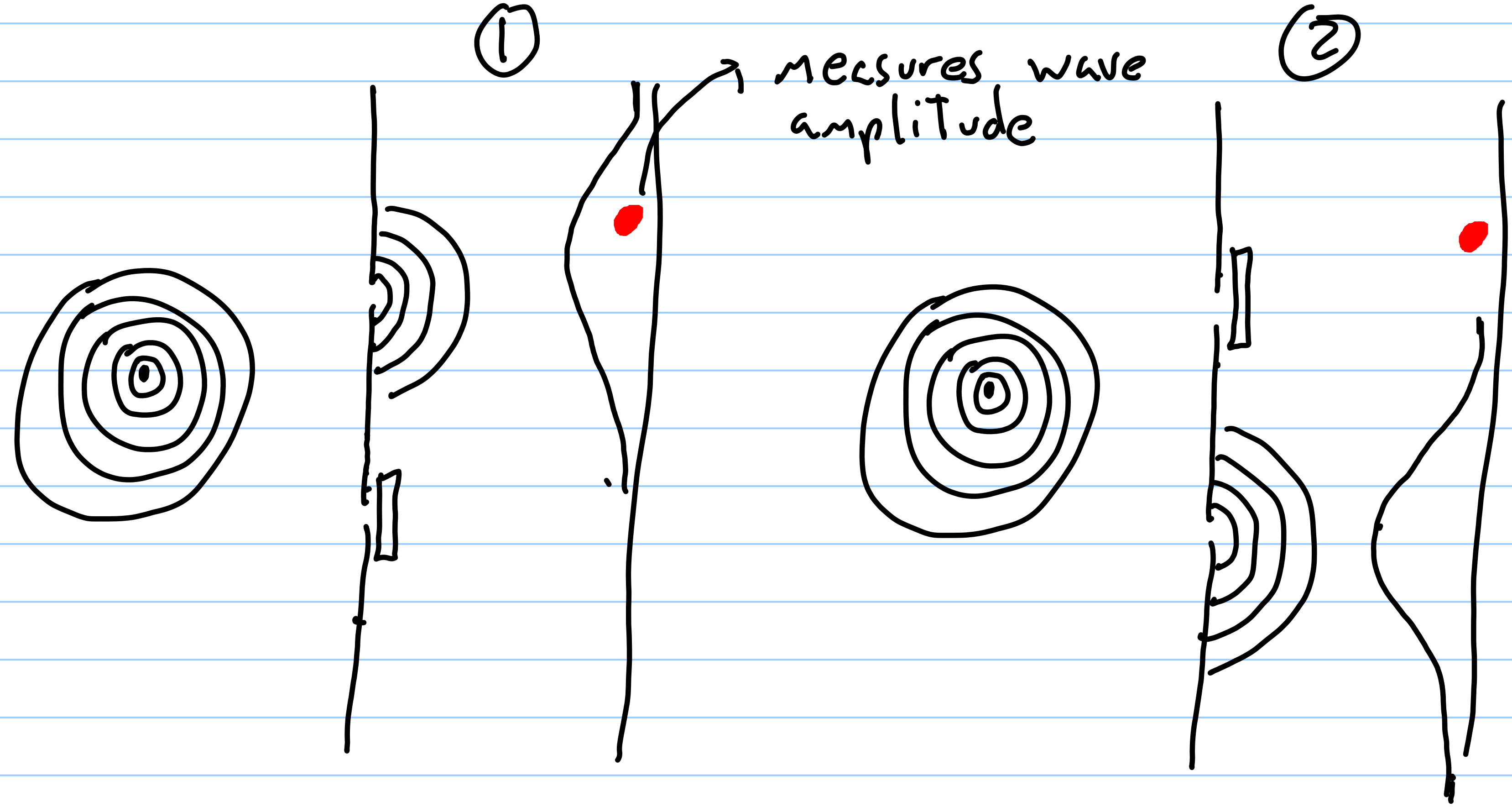


result is a simple sum of case (1) and case (2)

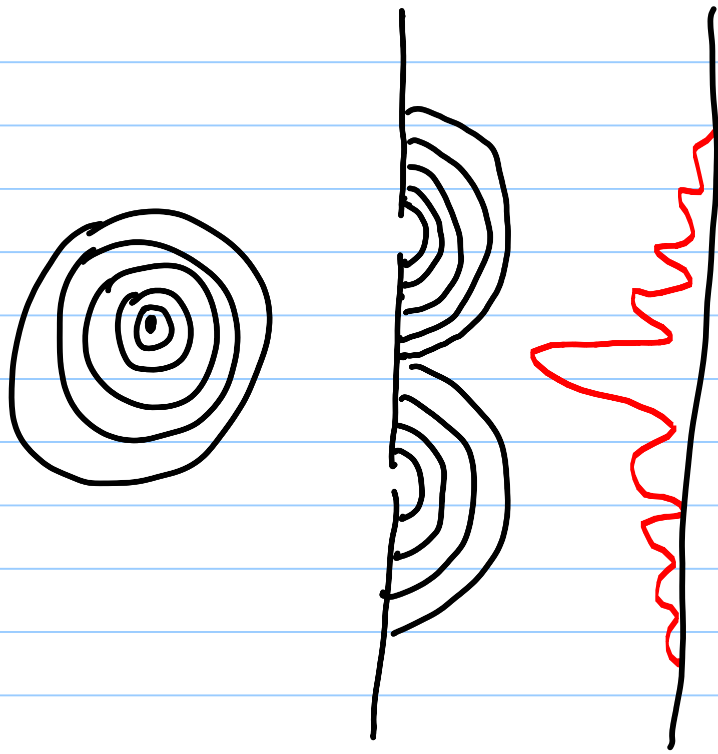
$N_{12} = N_1 + N_2$



Experiment "B"



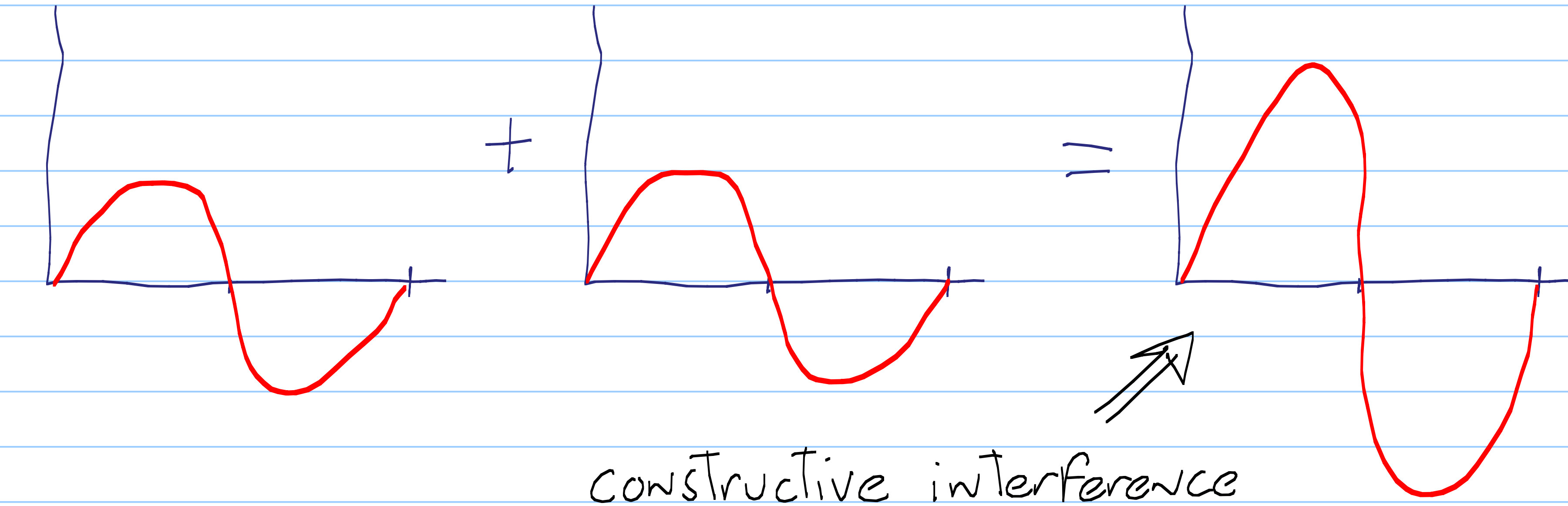
WHAT HAPPENS WHEN WE OPEN BOTH SLITS?



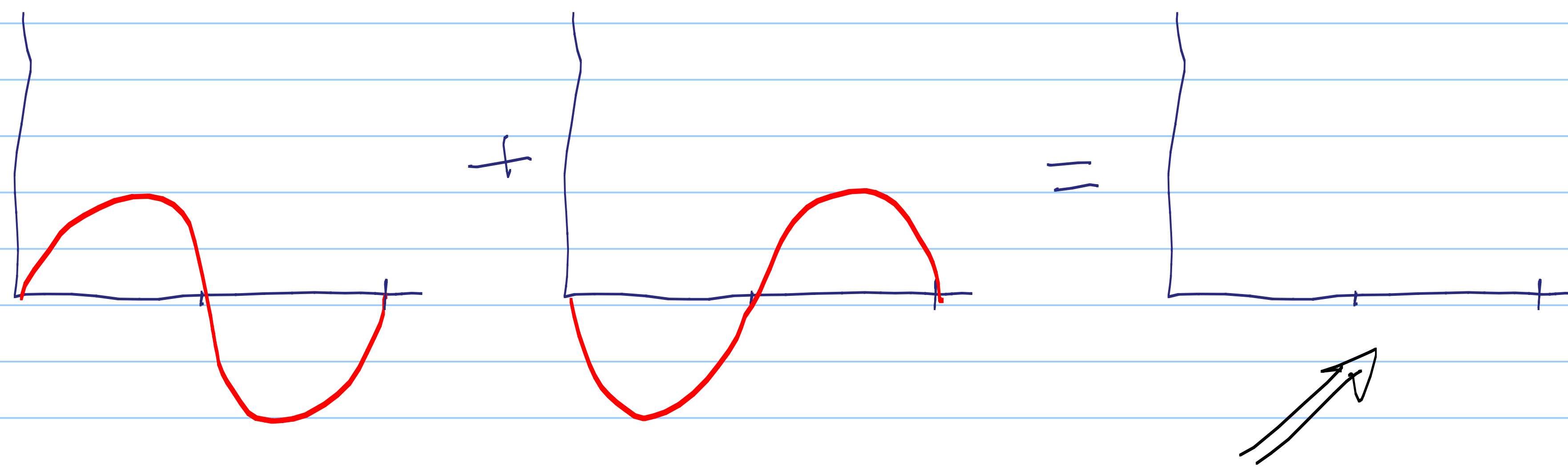
(1) + (2) not a simple sum of case (1) and case (2).

WHAT'S HAPPENING?
INTERFERENCE BETWEEN TWO WAVE FRONTS

INTERFERENCE BETWEEN WAVES:



constructive interference



destructive interference

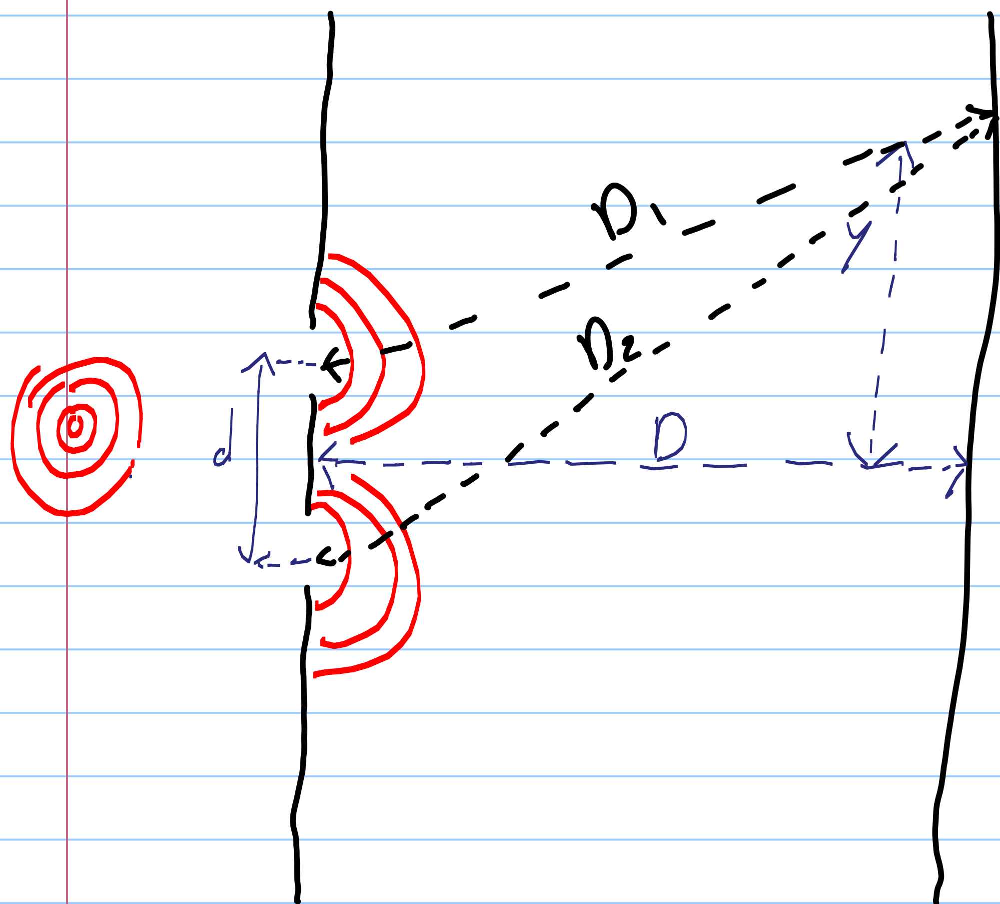
LET'S CALCULATE THE LIGHT INTENSITY ON THE SCREEN

NOTE: $I = A^2$

LET'S SET

$D \gg y$

$D \gg d$



INTERFERENCE WILL DEPEND ON PATH DIFFERENCE

IF PATH DIFFERENCE IS A MULTIPLE OF WAVE LENGTH: CONSTRUCTIVE INTERFERENCE

$$D_1 = \left[D^2 + (y - d/2)^2 \right]^{1/2}$$

$$D_2 = \left[D^2 + (y + d/2)^2 \right]^{1/2}$$

WE HAVE:

$$D_1 = \left[D^2 + (y - d/2)^2 \right]^{1/2}$$

NOTE THAT

$$\left(D + (y - d/2)^2 \right)^2 = D^2 + 2D(y-d/2)^2 + (y-d/2)^4$$

LET'S TRY

$$\left(D + \frac{(y-d/2)^2}{2D} \right)^2 = D^2 + (y-d/2)^2 + \underbrace{\frac{(y-d/2)^4}{4D^2}}_{\text{neglect}}$$

$$\Rightarrow \quad D_1 \approx D + \frac{(y-d/2)^2}{2D}$$

$$D_2 \approx D + \frac{(y+d/2)^2}{2D}$$

PATH DIFFERENCE: $D_2 - D_1 = \boxed{\dfrac{yd}{D}}$

PHASE DIFFERENCE δ : $\boxed{\dfrac{2\pi}{\lambda} \quad \dfrac{yd}{D}}$

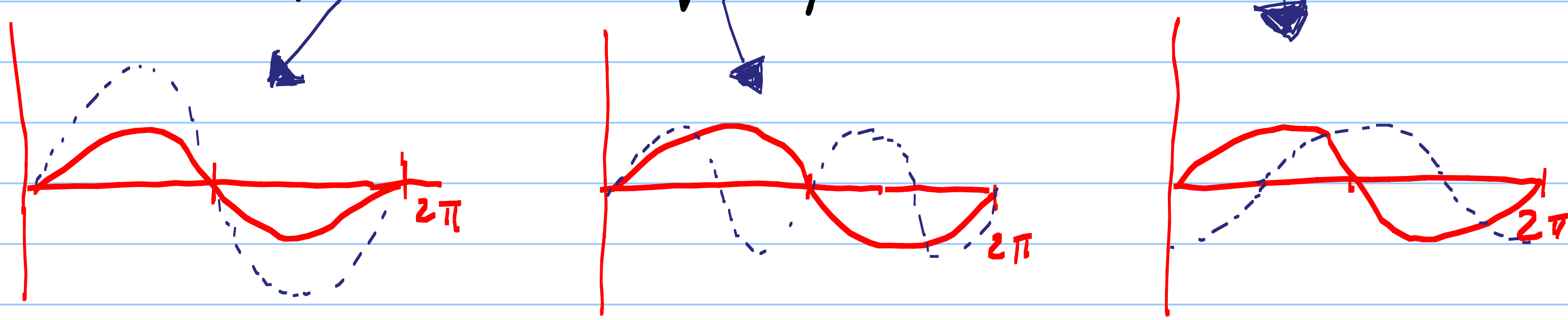
if $\lambda = \dfrac{yd}{D} \Rightarrow \delta = 2\pi$

constructive interference

WE CAN DESCRIBE A PLANE WAVE THIS WAY:

$$y = A\cos(\omega T + \varphi)$$

- A → amplitude
- ω → frequency
- φ → phase



IF WE NEGLECT THE ATTENUATION OF THE INITIAL WAVE AMPLITUDE AT THE SLIT, THE AMPLITUDE AT THE SCREEN WILL BE:

$$A(y) = 2 A_0 \cos \delta/2$$

↓ initial amplitude

$$\delta = \frac{2\pi y d}{D\lambda} \quad , \quad \frac{\delta}{2} = \frac{\pi y d}{D\lambda}$$

$$A(y) = 2 A_0 \cos \left(\frac{\pi d}{D\lambda} \cdot y \right)$$

$$I(y) = 4 A_0^2 \cos^2 \left(\pi d / D\lambda \cdot y \right) = \boxed{4 I_0 \cos^2 \left(\frac{\pi d}{D\lambda} \cdot y \right)}$$

PHASOR NOTATION

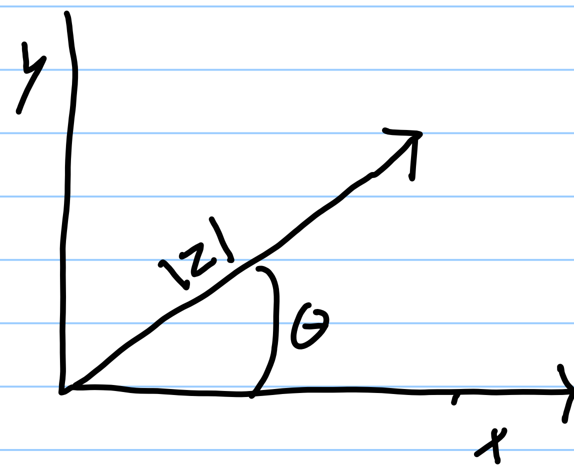
$$i^2 = -1$$

$$z = x + iy$$

$$z = |z| e^{i\theta}$$

$$z^* = |z| e^{-i\theta}$$

$$|z| = \sqrt{z z^*}$$



Euler identity

$$e^{i\theta} = \cos\theta + i \sin\theta$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

Can express plane wave as:

$$A \cos(\omega t + \varphi) = \mathrm{Re}\{ A e^{i(\omega t + \varphi)} \}$$

INTERFERENCE IN PHASOR NOTATION

Amplitude of wave from slit 1

$$A_1 = a_1 e^{i\omega t}$$

Intensity $I_1 = |A_1|^2 = |a_1|^2 \, |e^{i\omega t}|^2$

$$= |a_1|^2 e^{-i\omega t} e^{i\omega t} = |a_1|^2$$

Amplitude of wave from slit 2

$$A_2 = a_2 e^{i(\omega t + \delta)}$$

Intensity $I_2 = |A_2|^2 = |a_2|^2$

PHASOR NOTATION (CONT.)

Amplitude from both slits: $A_{12} = A_1 + A_2$

Intensity from both slits: $I_{12} = |A_{12}|^2 = |A_1 + A_2|^2$

$$I_{12} = |A_1|^2 + |A_2|^2 + A_1 A_2^* + A_1^* A_2$$

$$= |a_1|^2 + |a_2|^2 + a_1 a_2 e^{i\omega t} e^{-i\omega t} e^{-i\delta} + a_1 a_2 e^{-i\omega t} e^{i\omega t} e^{i\delta}$$

$$= \underbrace{|a_1|^2}_{I_1} + \underbrace{|a_2|^2}_{I_2} + a_1 a_2 \left(e^{-i\delta} + e^{i\delta} \right)$$

USE: $\cos\delta = \frac{e^{i\delta} + e^{-i\delta}}{2}$

$$I_{12} = I_1 + I_2 + 2a_1 a_2 \cos\delta$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\delta$$

Show that this gives same result as on slide 10

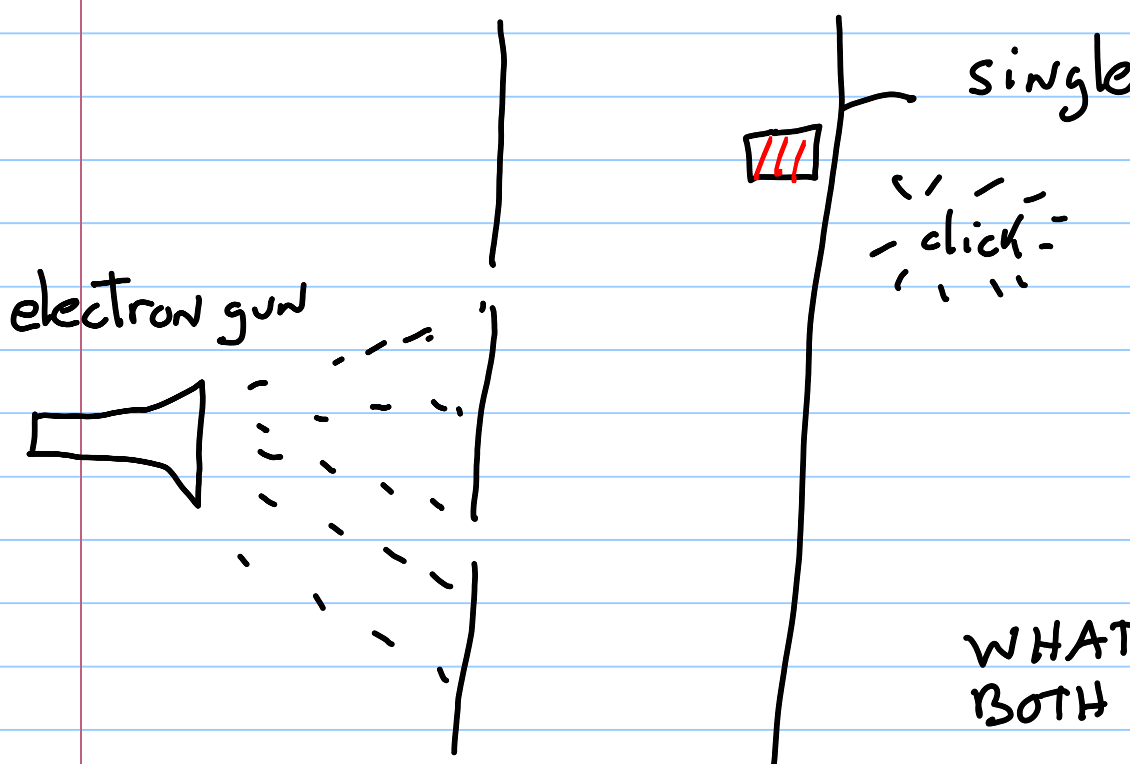
RECAP:

PARTICLES : $N_{12} = N_1 + N_2$

WAVES : $I_{12} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$

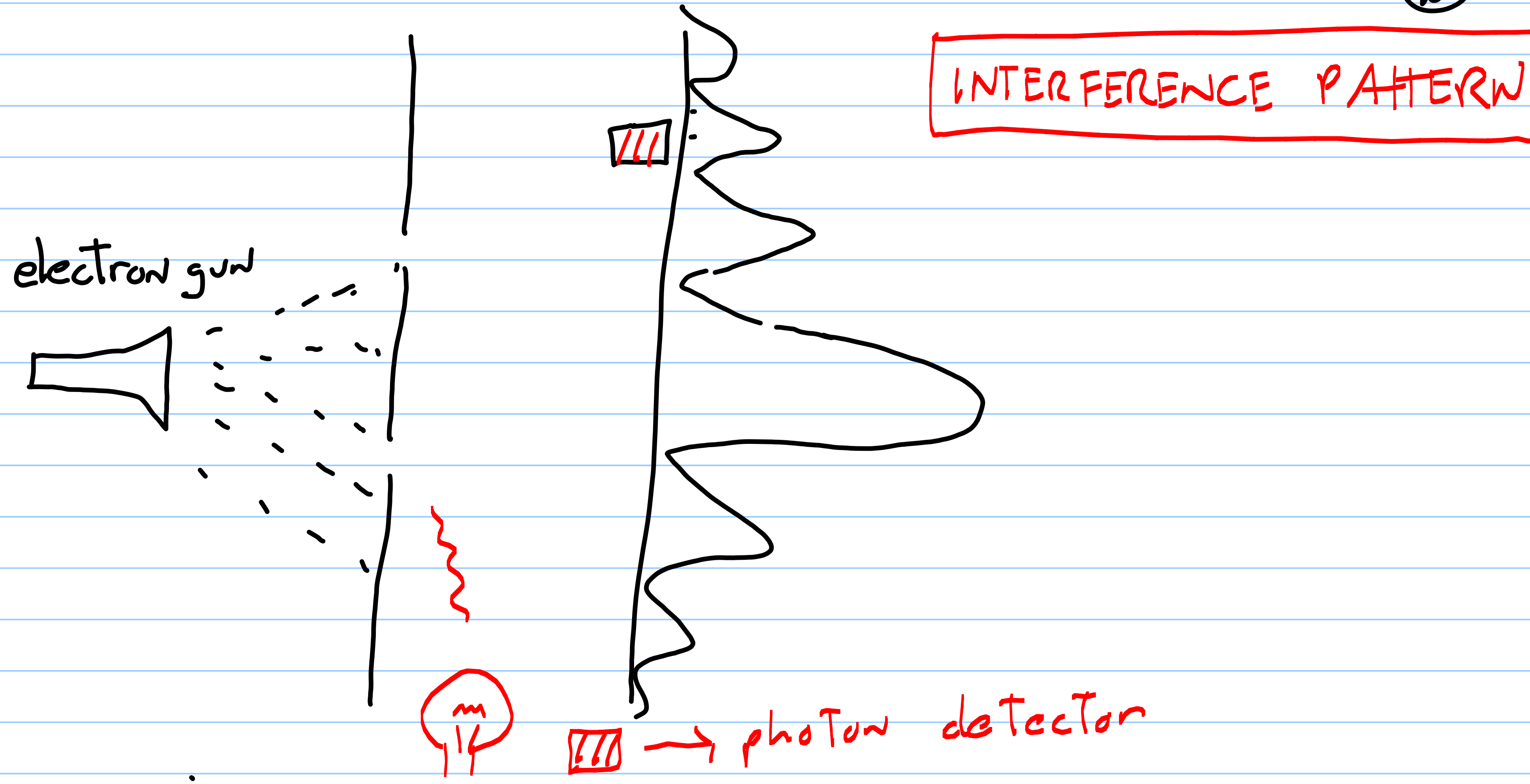
INTERFERENCE TERM

WHAT ABOUT ELECTRONS (MICROSCOPIC PARTICLE)



→ can make histogram of "click" counts

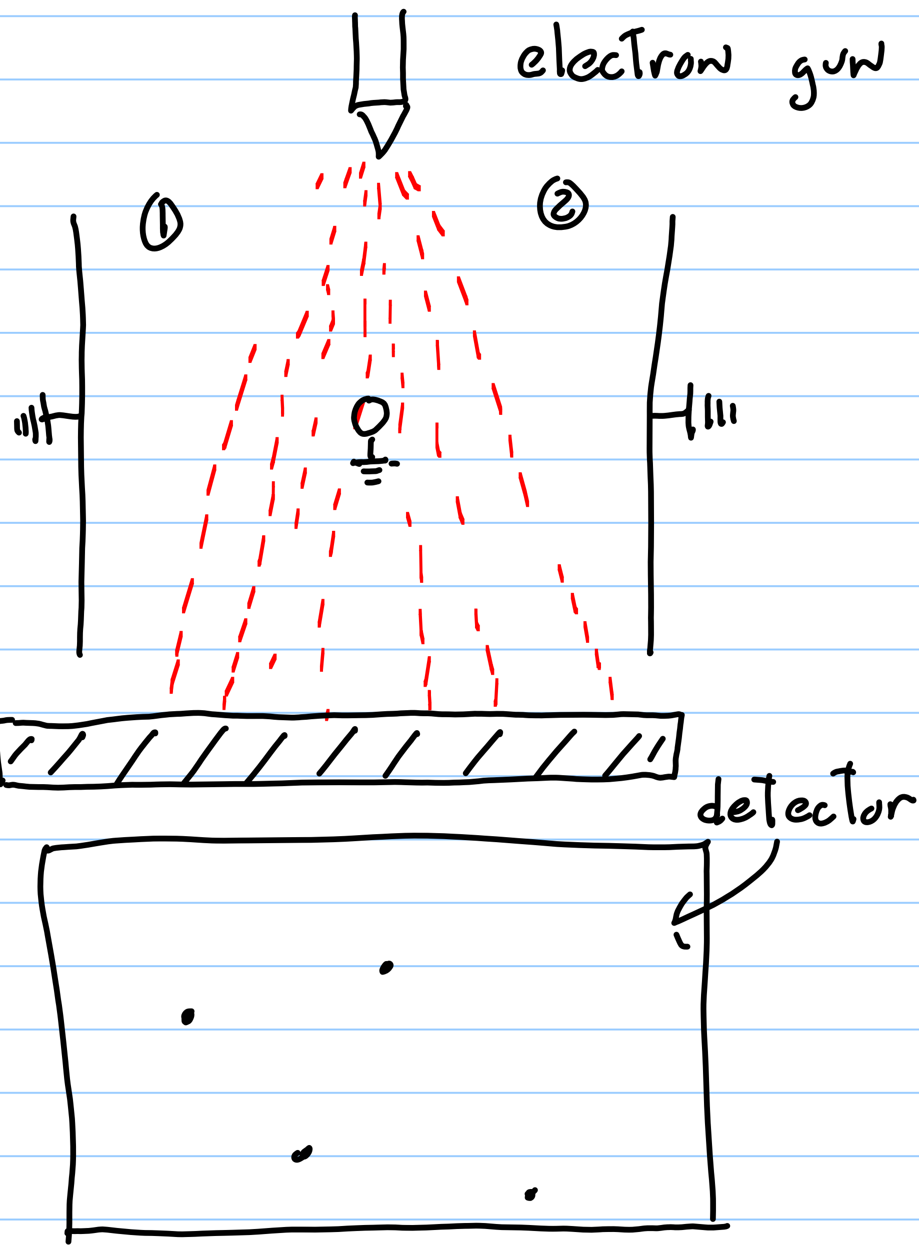
WHAT HAPPENS WHEN BOTH SLITS ARE OPEN?

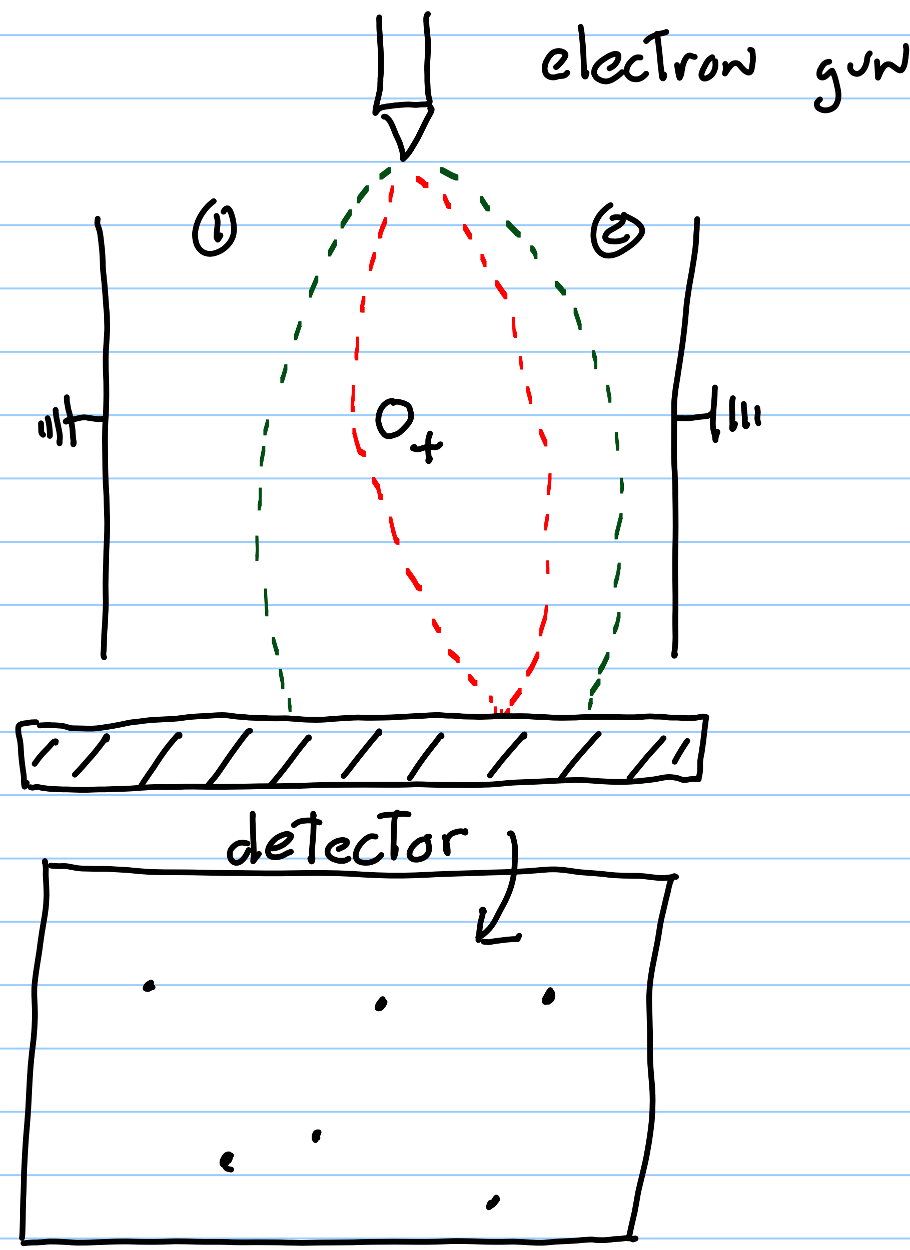


WHAT IF WE LOOK WHERE THE ELECTRON WENT THROUGH?

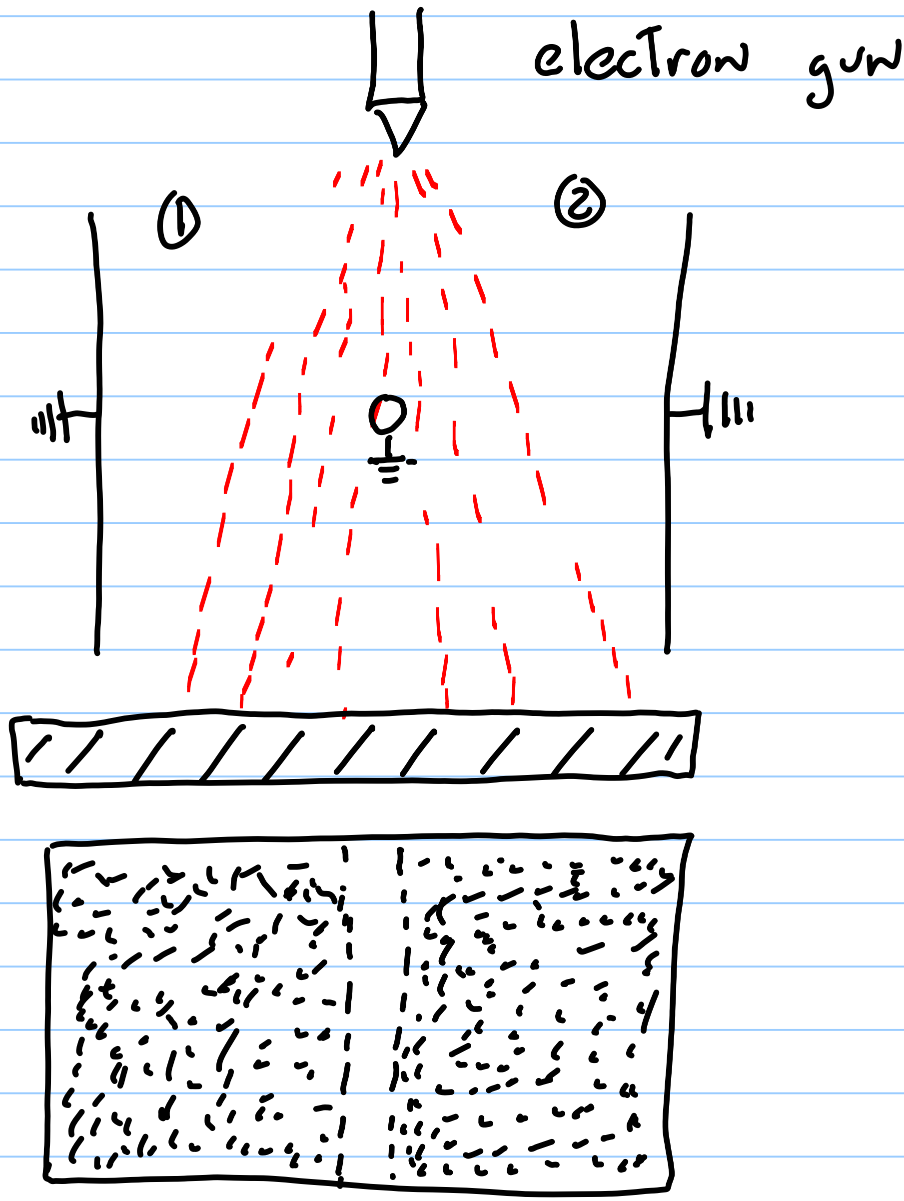
INTERFERENCE PATTERN GOES AWAY!

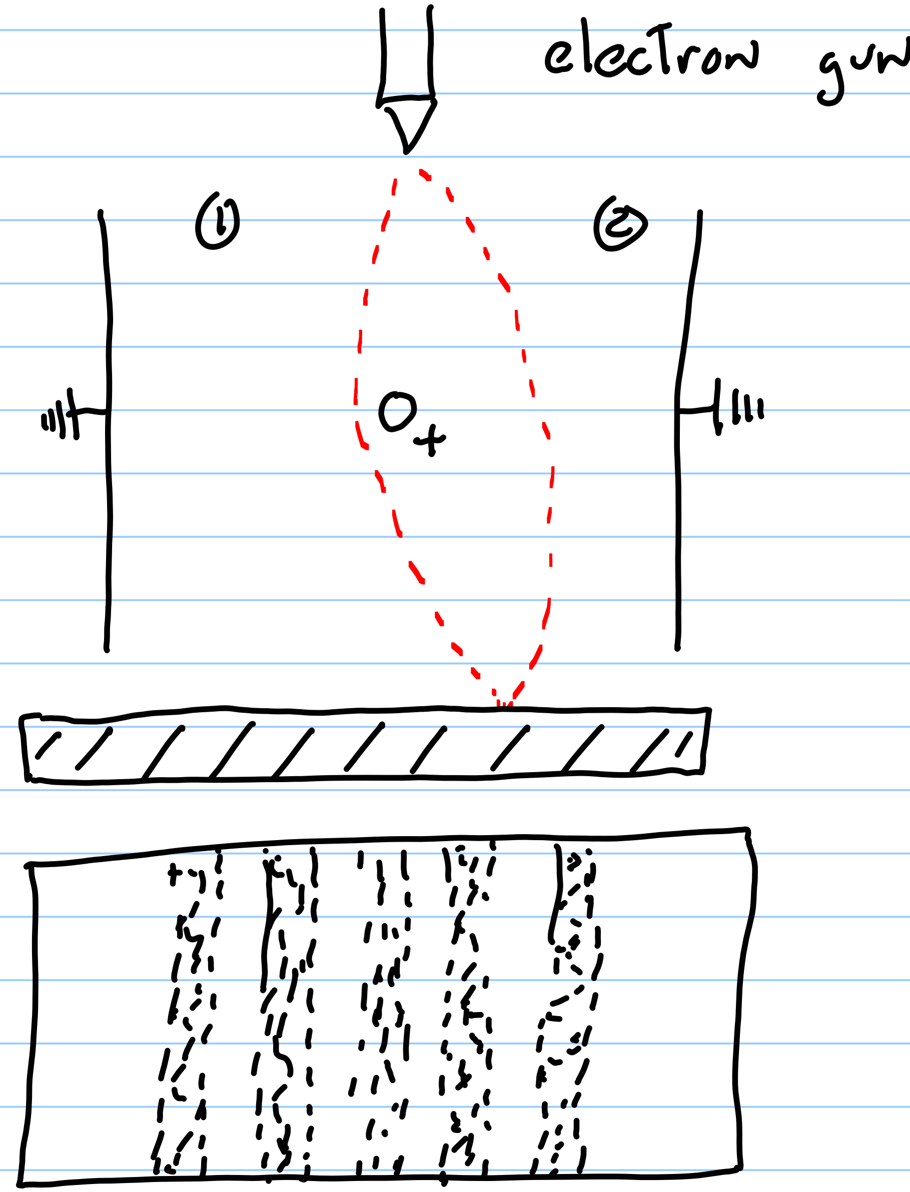
ELECTRON BIPRISM (another double-slit exp.)





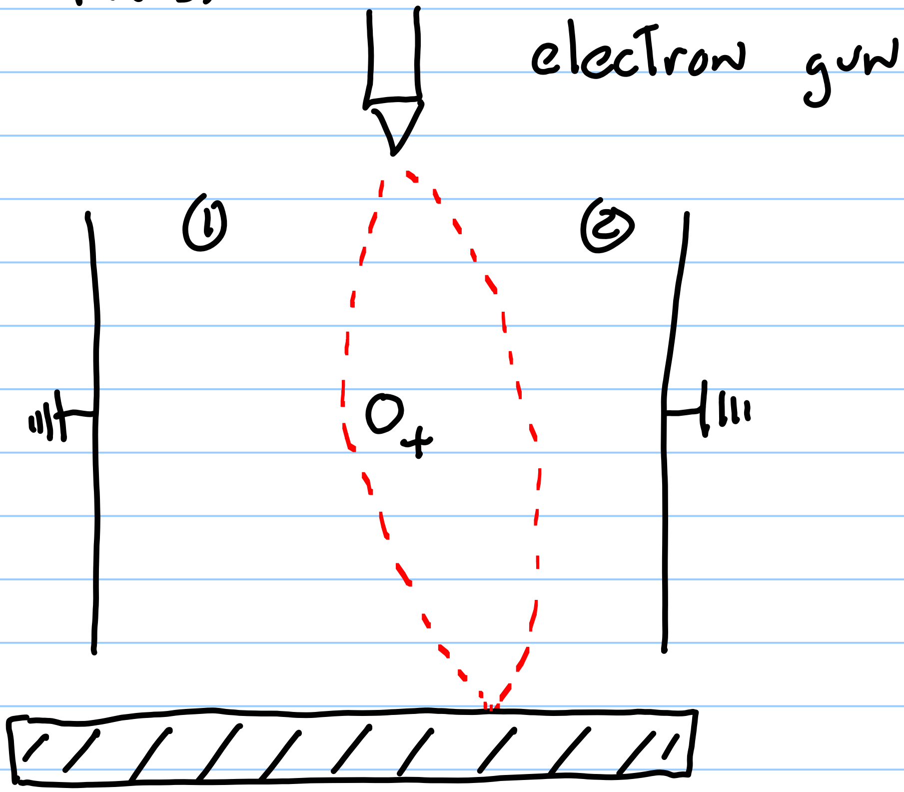
ELECTRON BIPRISM (CONTINUED)





LET'S LOOK AT THIS FOR REAL: TONOMURA'S EXPT. (HITACHI)

BIPRISM CONT.



IF WE CAN TELL IF ELECTRON WENT THROUGH (1) OR (2) THEN WE'LL GET