LECTURE 23: The QM Postulates (part deux)

What I expect you to learn:

- What are the QM postulates
- How to work with the Dirac notation

(Roughly reviews chapter 5 of the textbook)

RECAP OF POSTULATES USING THE INFINITE WELL EXAMPLE:

SUPPOSE WE PUT A PARTICLE (SAY A He^+ ION) IN AN INFINITE WELL.

A- WE USE A "STATE VECTOR" TO DESCRIBE THIS PARTICLE: $|\psi\rangle$. THIS VECTOR CONTAINS ALL THE INFORMATION ABOUT THIS PARTICLE:
- ENERGY
- MOMENTUM
- POSITION
- ANGULAR MOM.
- ETC.

B- IF $|\psi\rangle$ DESCRIBES A He^+ ION IN AN INFINITE WELL AND $|\xi\rangle$ DESCRIBES ANOTHER He^+ ION IN AN INFINITE WELL, THEN:

$|\eta\rangle = c_1|\psi\rangle + c_2|\xi\rangle$ could also

DESCRIBE A He^+ ION IN ANOTHER INFINITE WELL

RECAP. CONT.

③

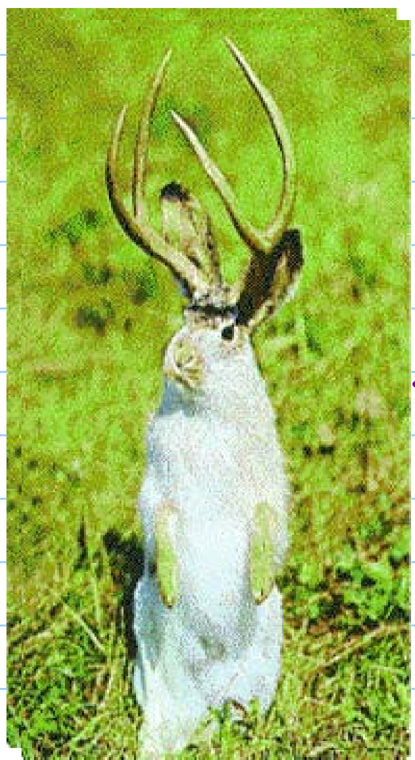
C - TO EVERYTHING WE CAN MEASURE ABOUT THE He^+ ION WE ASSOCIATE AN OPERATOR THAT IS LINEAR AND HERMITIAN

AN OPERATOR TRANSFORMS A STATE VECTOR INTO A NEW STATE VECTOR : $O|\chi\rangle = |\psi\rangle$

IF $|\psi\rangle$ IS AN EIGENVECTOR OF $\hat{O}$, THEN $\hat{O}|\chi\rangle = c|\chi\rangle$

complex number

Initial state



Operator

Final state

D - IF WE MEASURE SOME PHYSICAL QUANTITY A, *
THE RESULT CAN ONLY BE AN EIGENVALUE
OF $\hat{A}$, THE OPERATOR ASSOCIATED WITH THE
QUANTITY A.

→ AND ←

THE SYSTEM WILL BE IN AN EIGENSTATE OF $\hat{A}$
RIGHT AFTER THE MEASUREMENT.

quantum jackalope

$\Rightarrow$ MEASUREMENT $\downarrow$ Operator

rabbiT eigenstate

antelope eigenstate

RECAP. CONT.

E - ONE CAN EXPAND A STATE $|\psi\rangle$ IN TERMS OF THE EIGENSTATES ASSOCIATED WITH THE OPERATOR THAT REPRESENTS A PHYSICALLY MEASURABLE QUANTITY (ENERGY, MOM., POSITION, ANGULAR MOM., ETC).

$$|\psi\rangle = \sum_n A_n |E_n\rangle \quad \text{(discrete basis)}$$

$|E_n\rangle \to$ energy eigenstate

$$|\psi\rangle = \int dx \; c(x) \, |x\rangle \quad \text{(continuous basis)}$$

$|x\rangle \to$ position eigenstate

$c(x) \to \psi(x)$!

TO PERFORM THIS EXPANSION, ONE CAN USE A PROJECTION OPERATOR:

$$\sum_n |E_n\rangle\langle E_n| = 1$$

$$\Rightarrow |\psi\rangle = \sum_n |E_n\rangle\langle E_n|\psi\rangle = \sum_n |E_n\rangle \cdot A_n$$

$$|\psi\rangle = \int dx \; |x\rangle\langle x|\psi\rangle = \int dx \; |x\rangle \, \psi(x)$$

RECAP CONT.

- TO CALCULATE THE PROBABILITY OF FINDING THE PARTICLE WITH AN ENERGY E_n, ONE DOES:

$$|\langle E_n | \psi \rangle|^2 = |\text{complex number}|^2$$

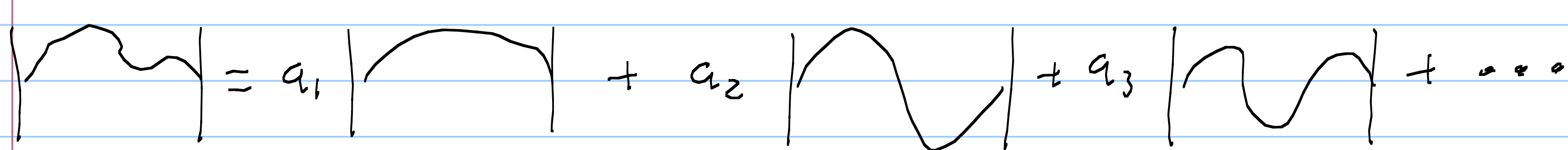
IF WE INSERT THE POSITION EIGENSTATE PROJECTOR:

$$= \left| \int_{-\infty}^{\infty} dx \, \langle E_n | x \rangle \langle x | \psi \rangle \right|^2 = \left| \int_{-\infty}^{\infty} dx \, \psi_n(x) \, \psi(x) \right|^2 \quad (1)$$

$$\psi(x) = \sum_m a_m \psi_m(x) \quad \text{and} \quad \int_{-\infty}^{\infty} \psi_m(x) \psi_n(x) \, dx = \delta_{mn}$$

$$(1) = \left| \int_{-\infty}^{\infty} dx \, \psi_n(x) \sum_m a_m \psi_m(x) \right|^2 = |a_n|^2$$

REMEMBER THE FOURIER SERIES:



RECAP CONT.

BACK TO our He^+ ion in the infinite well described by the state vector $|\psi\rangle$.

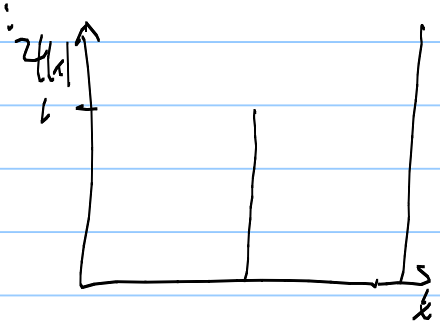
- THE PROBABILITY OF MEASURING THE POSITION OF THE PARTICLE BETWEEN x_1 AND x_2 IS:

$$\int_{x_1}^{x_2} P(x)\, dx = \int_{x_1}^{x_2} |\langle x|\psi\rangle|^2\, dx$$

$$= \int_{x_1}^{x_2} |\psi(x)|^2\, dx$$

$\psi(x)$ For He^+

AFTER THE MEASUREMENT, $\psi(x)$ COULD LOOK LIKE THIS:



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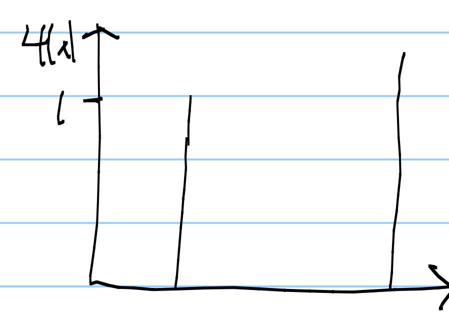

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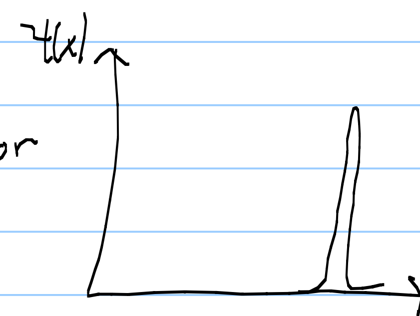
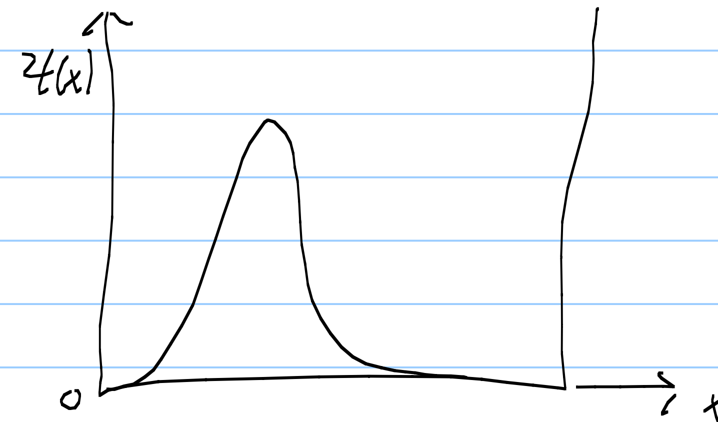


RECAP CONT.

BEFORE THE MEASUREMENT WE HAD FOR $\psi(x)$:



THIS FUNCTION CAN BE EXPANDED USING A FOURIER SERIES

$$\psi(x) \quad \sum_n A_n \psi_n(x) = \sum_n A_n \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$$

NOW WHEN WE MEASURE ENERGY, WE CAN ONLY GET ONE OF THE FOLLOWING EIGENVALUES: $E_n = \frac{\hbar^2 k^2}{2m} = \frac{\hbar \pi^2 n^2}{2ma^2}$

THE PROBABILITY OF MEASURING A GIVEN EIGENVALUE IS:

$$P_n = |\langle E_n | \psi \rangle|^2 = \left| \int_{-\infty}^{\infty} dx \langle E_n | x \rangle \langle x | \psi \rangle \right|^2$$

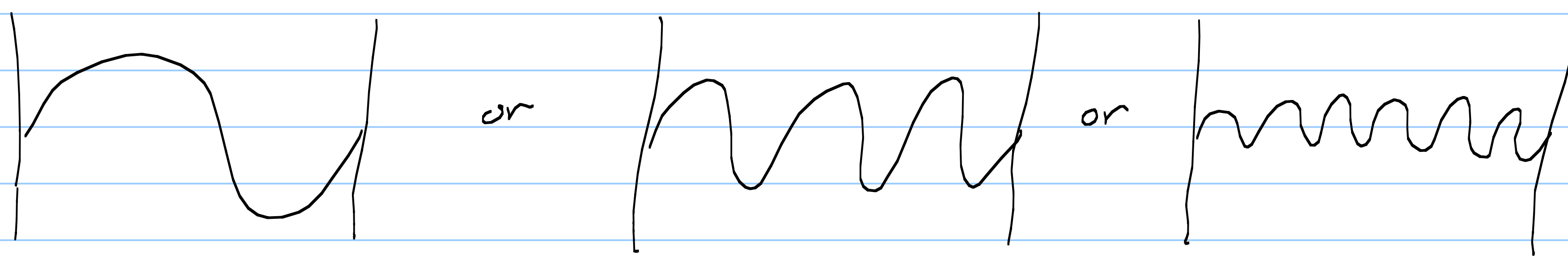
$$= \left| \int_{-\infty}^{\infty} dx\, \psi_n(x)\, \psi(x) \right|^2$$

RECAP CONT.

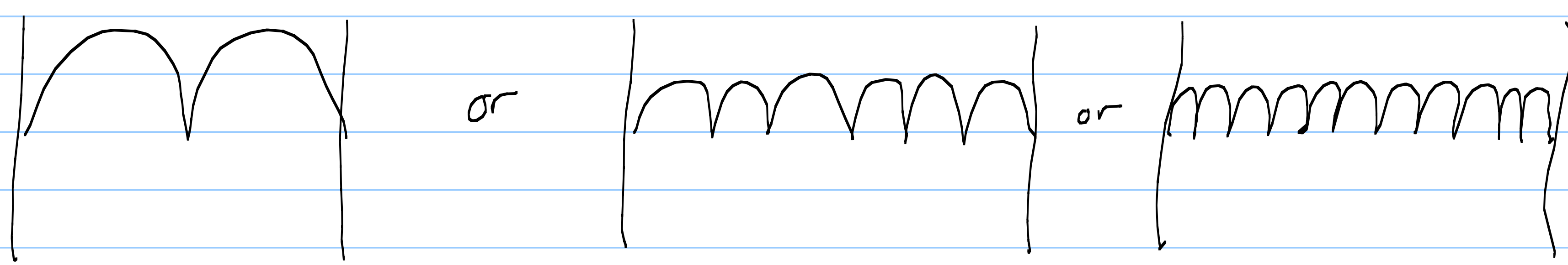
NOTE THAT $E_n = \frac{\hbar^2 k^2}{2m} = \frac{p^2}{2m}$ $(V=0)$

$\Rightarrow$ IF YOU KNOW p, YOU KNOW E. THE OPERATORS ASSOCIATED with $\hat{p}$ AND $\hat{E}$ COMMUTE

AFTER AN ENERGY MEASUREMENT, $\psi(x)$ must be one of the $\psi_n(x)$: eg

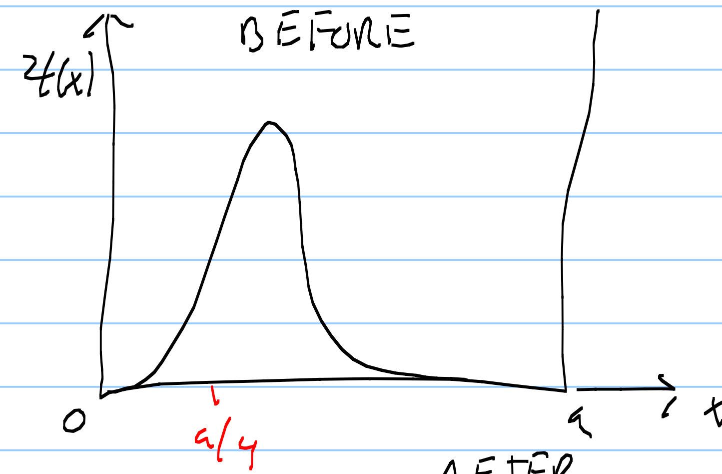


$|\psi_n(x)|^2 =$

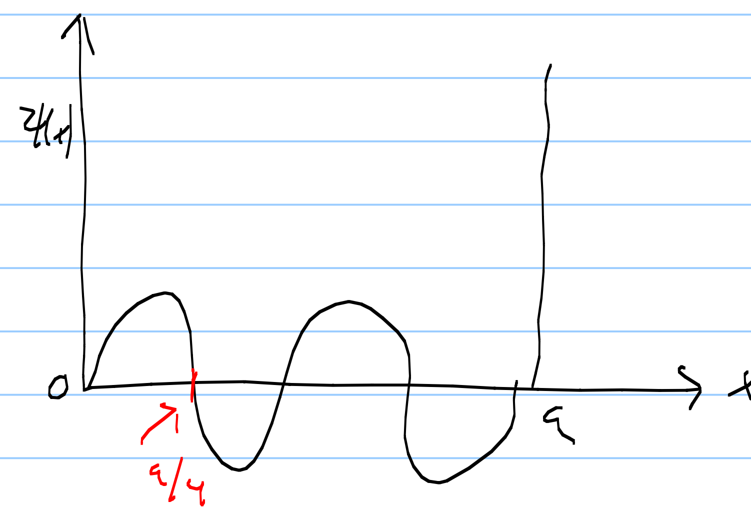


RECAP (CONT)

WE STARTED WITH
$\langle x|\psi\rangle = \psi(x) \longrightarrow$



IF WE MEASURE E (or P) FIRST WE WILL GET ONE OF THE $\psi_n(x)$, SAY: $A_4 \cdot \sqrt{\frac{2}{a}} \sin \frac{4\pi x}{a}$

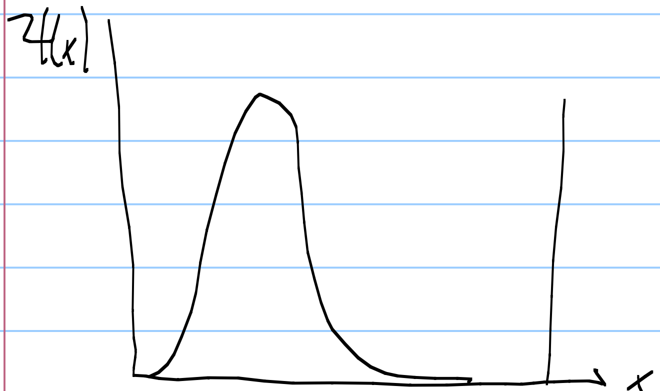


IF I MEASURE THE POSITION NOW, THE PROBABILITY OF FINDING THE PARTICLE AT $x = \frac{a}{4}$ WOULD BE: 0

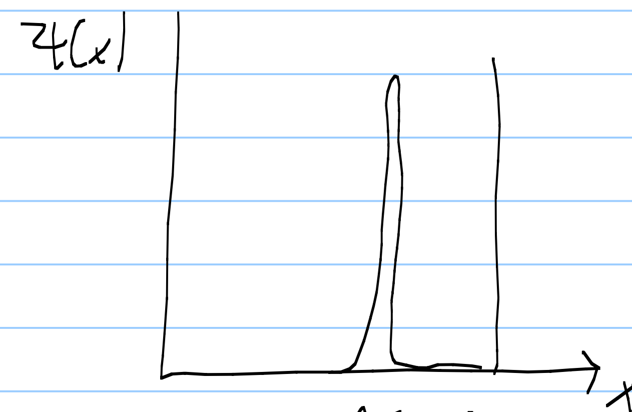
IF YOU MEASURE THE POSITION FIRST, $P(a/4) \neq 0$.

RECAP (cont.)

ALSO $P(E_4) = |A_4|^2$ WHEN ENERGY WAS MEASURED FIRST BUT $P(E_4)$ AFTER A POSITION MEASUREMENT THE A_4 COEFFICIENT WILL HAVE CHANGED:



BEFORE



AFTER

FOURIER SERIES OF "BEFORE" $\neq$ FOURIER SERIES OF "AFTER"

$$\text{BEFORE } \sum_n B_n \sqrt{\frac{2}{a}} \sin\frac{n\pi x}{a} \neq \text{AFTER } \sum_n A_n \sqrt{\frac{2}{a}} \sin\frac{n\pi x}{a}$$

$\Rightarrow B_n \neq A_n \Rightarrow |B_n|^2 \neq |A_n|^2 \Rightarrow$ PROBABILITY OF MEASURING THE ENERGY E_n BEFORE MEASURING THE POSITION $\neq$ PROB. OF MEASURING E_n AFTER POSITION MEASUREMENT.