Lecture 3: Blackbody Radiation

Goal of the lecture: Understand the origin of Planck's constant and the quantisation of energy

I expect you to learn:

- What is a "blackbody"
- What is the emissive power of a blackbody
- What is the Rayleigh-Jeans result and the "Ultraviolet Catastrophe"
- How Planck solved this problem

Lecture follows Section 1.1 of the textbook

Reminder: website login: phy256student, pass: quantum

Origins of Quantum Physics

At the end of the 19th century, attempts at understanding the observed properties of blackbody radiation using classical physics (Newton's laws, Maxwell's laws, laws of thermodynamics)failed.

Planck found a solution but it took time before the conceptual implications of this work were understood. These concepts represented a radical departure from classical physics worldview.

Historical note: textbooks often present the emergence of quantum physics as the result of Planck finding a solution to a "crisis" in classical physics i.e. as a reaction to the "ultraviolet catastrophe" of Rayleigh and Jeans. This is not accurate. See Physics World of December 2000 (physics library 2nd floor) for a more details.

Blackbody Radiation

You have seen the surface of a hot object emit electromagnetic radiation (for example: heating elements in oven or toaster).

The object does not need to be hot to emit radiation: > 0K will do

Two questions we would like to answer in this lecture:

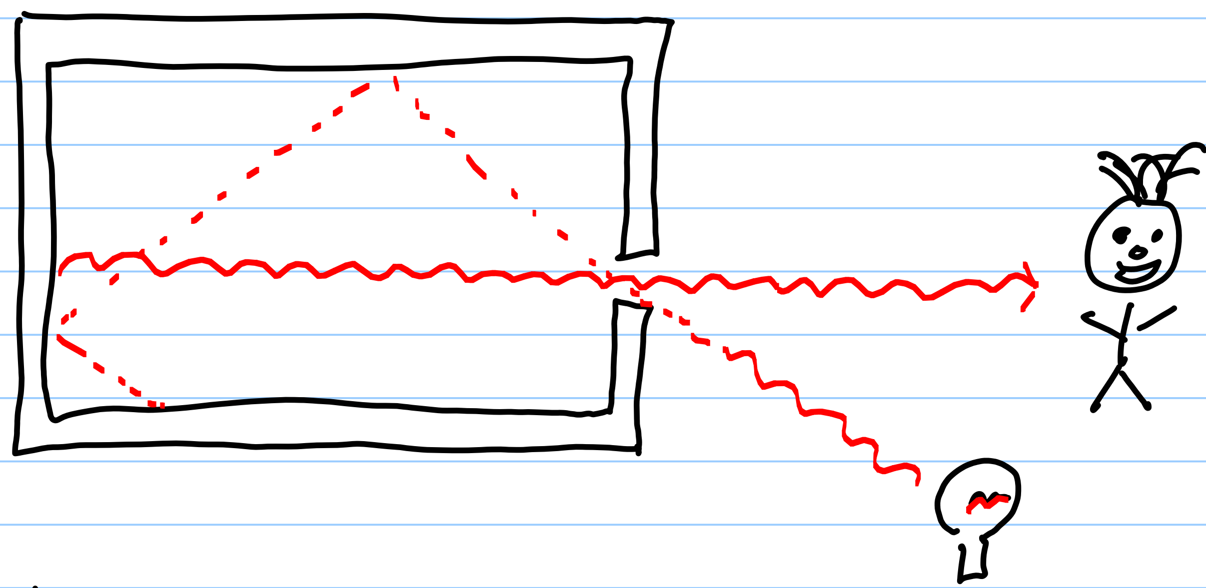
1-how much electromagnetic energy is emitted by a body in thermal equilibrium?

2-how much energy is emitted at a given by that body at a given wavelength?

Blackbody Radiation

So, what do we mean by "blackbody"?

A body that absorbs all radiant energy that falls on it



* the sun is a good "blackbody"

* A black hole Too!!

BLACKBODY RADIATION (CONT.)

SOME DEFINITIONS:

$R(T)$, TOTAL EMISSIVE POWER = $\boxed{\sigma T^4}$

$\sigma = 5.7 \times 10^{-8}$ W m^{-2} K^{-4}

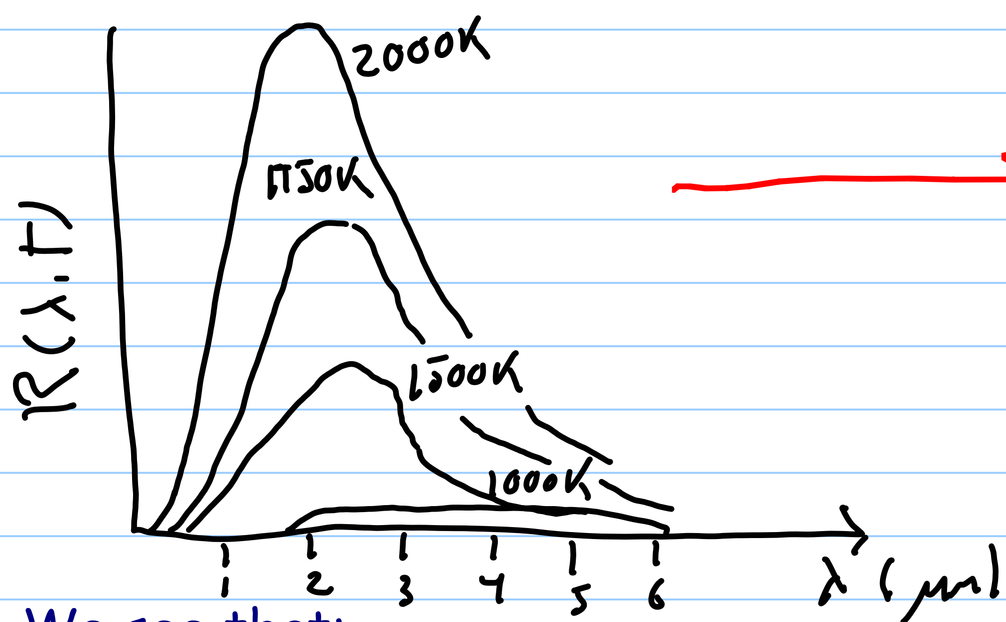
Known as Stefan-Boltzmann law

$$R(T) = \int_0^\infty \underbrace{R(\lambda, T)} \, d\lambda$$

Spectral emittance i.e. The emissive power for a given wavelength λ.

Blackbody Radiation (Continued)

The first accurate measurements of spectral emittance were made by Lummer and Pringsheim in 1899. They observed the distributions below:



→ LET'S LOOK AT A JAVA APPLET OF THIS

We see that:

For fixed λ, $R(\lambda,T) \uparrow$ when $T \uparrow$

For fixed T, $R(\lambda,T)$ has a maximum

→ WIEN'S DISPLACEMENT LAW: $\lambda_{max} T = b$

Example 1 : The "ordinary" lightbulb

Temperature of Tungsten: 3300 K

wavelength of greatest intensity will be: $\lambda = \frac{b}{T}$

$$= \frac{2.9 \times 10^{-3}\ K \cdot m}{3300}$$

$$= 879\ nm$$

↳ infrared!

What about emissive power?

$$R(T) = \sigma T^4 = 5.7 \times 10^{-8}\ W m^{-2} K^{-4} \times (3300)^4$$

$$= 6.7\ \text{Megawatts}/m^2 !$$

Surface of the Sun ~ 70 MW/m^2

BLACKBODY RADIATION (CONT.)

IT'S CONVENIENT TO WORK WITH A QUANTITY RELATED TO $R(\lambda,T)$:

$$\rho(\lambda,T) = \frac{4}{c} R(\lambda,T)$$

↳ monochromatic energy density

→ $\rho(\lambda,T)\,d\lambda$ is then the energy density of radiation in the wavelength interval: $(\lambda, \lambda+d\lambda)$

In 1893, W. Wien showed using thermodynamics that $\rho(\lambda,T) = \frac{f(\lambda T)}{\lambda^5}$ → Wien's Law

Note: can't get $f(\lambda T)$ from thermodynamics

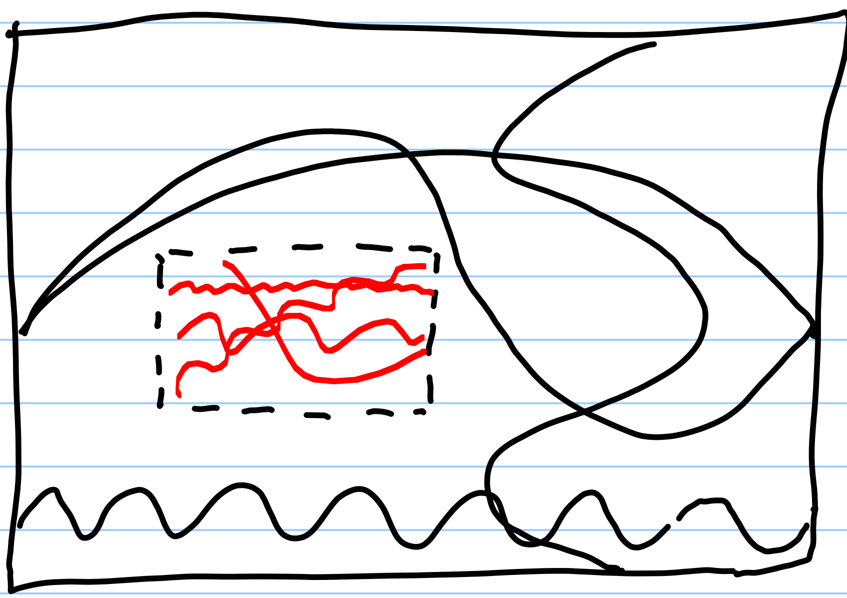
* Can you show that Wien's law includes:
a) Stefan-Boltzmann law and b) Wien's displacement law

Rayleigh-Jeans Law

Rayleigh and Jeans tried to determine $F(\lambda T)$ from electrodynamics.

Note: we will gloss over the details here but you will come back to this material in your thermal physics course. Also, more E&M will help you go through the derivations.

We need to determine how many waves or modes can fit inside a blackbody cavity. The EM waves must be "standing waves" so that there is no dissipation (we've got thermal equilibrium)



how many modes per unit volume per wavelength interval:

$$n(\lambda) = 8\pi / \lambda^4$$

Average energy for mode λ : $\bar{\varepsilon}$

$$\boxed{\Rightarrow \rho(\lambda, T) = \frac{8\pi}{\lambda^4} \cdot \bar{\varepsilon}}$$

Rayleigh-Jeans Law (cont.)

We have:

$$\boxed{\Rightarrow \rho(\lambda,T) = \frac{8\pi}{\lambda^4} \cdot \bar{\varepsilon}}$$

Now we need to determine $\bar{\varepsilon}$

$\rightarrow$ Introduce Boltzmann's probability distribution

In a gas of particles at temperature T, the probability that a particle will have energy ε is proportional to:

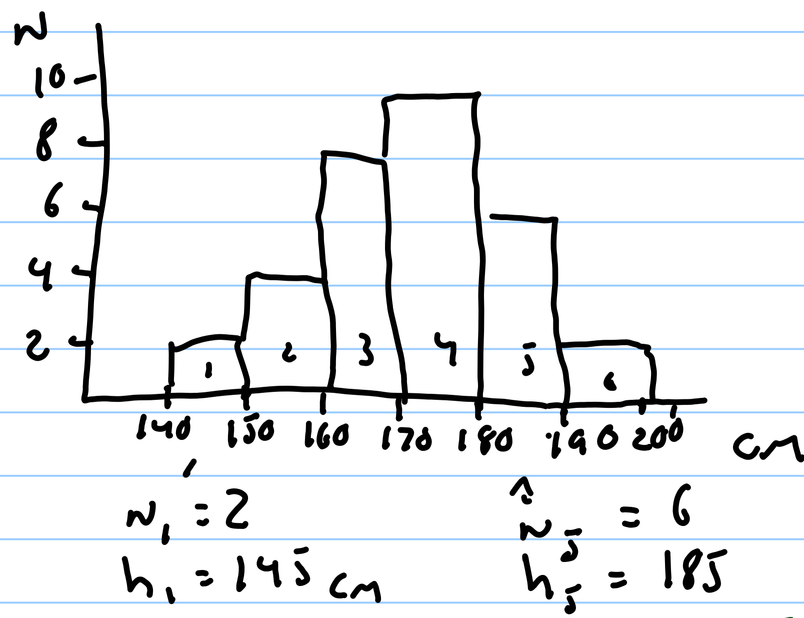
$$e^{(-\varepsilon/kT)}$$

More about this in thermo and stat. mech.

RAYLEIGH-JEANS LAW (cont.)

So we need to calculate the average energy $\bar{\varepsilon}$

Before we do that, let's calculate the average height of a group of people:



$$\bar{h} = \frac{\sum_i h_i N_i}{\sum_i N_i}$$

In a similar way

$$\bar{\varepsilon} = \frac{\int_0^\infty \varepsilon \exp(-\beta\varepsilon)\, d\varepsilon}{\int_0^\infty \exp(-\beta\varepsilon)\, d\varepsilon}$$

with $\boxed{\beta = \frac{1}{KT}}$

Rayleigh-Jeans Law (cont.)

$$\bar{\varepsilon} = \frac{\int_0^\infty \varepsilon \exp(-\beta\varepsilon)\, d\varepsilon}{\int_0^\infty \exp(-\beta\varepsilon)\, d\varepsilon}$$

$$= \frac{d}{d\beta} \log\left[\int_0^\infty \exp(-\beta\varepsilon)\, d\varepsilon \right]$$

do Try this at home

$$= \frac{1}{\beta} = KT$$

(consistent with classical law of equipartition of energy)

So we had: $\rho(\lambda, T) = \frac{8\pi}{\lambda^4} \bar{\varepsilon} = \frac{8\pi}{\lambda^4} KT$

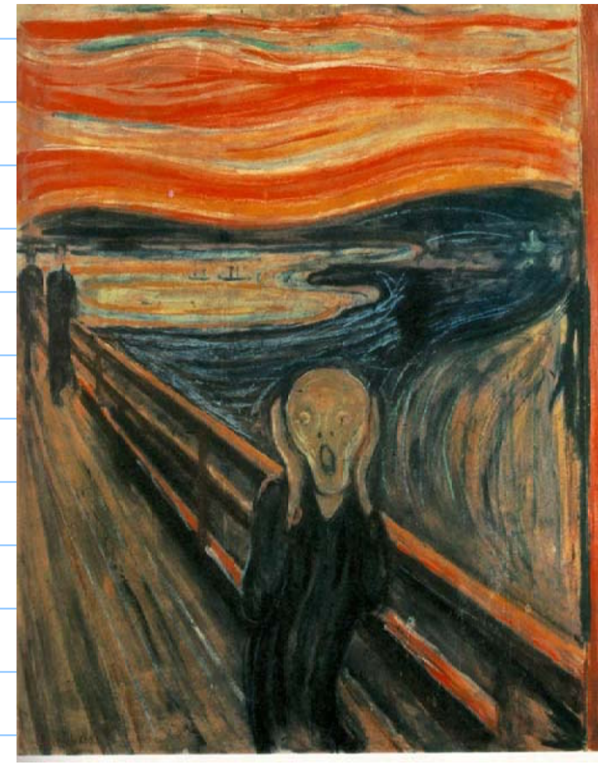
$$\rho(T) = \int_0^\infty \rho(\lambda, T)\, d\lambda = \int_0^\infty \frac{8\pi}{\lambda^4} KT\, d\lambda = ???$$

RAYLEIGH-JEANS LAW (cont.)

$$\int_0^{\infty} \frac{8\pi}{\lambda^4} KT \, d\lambda = \infty \, !$$

Small wavelengths cause this integral to diverge, hence

"ULTRAVIOLET CATASTROPHE"



YET, WE LIVE...

WHAT WENT WRONG?

Standing EM waves are emitted and absorbed by the atoms in cavity. They act as electric dipoles (little harmonic oscillators). The energy of those oscillators depend on the amplitude and frequency of oscillation. The amplitude can take any value therefore the energy of the oscillator can take any value. What if this is wrong?

PLANCK'S QUANTUM THEORY

Energy of oscillator at a given frequency can only take discreet values : $n\varepsilon_0$

ε_0 is a quantum of energy

LET'S GIVE IT A TRY : we need to recalculate $\bar{\varepsilon}$

$$\bar{\varepsilon} = \frac{\sum_{n=0}^{\infty} n\varepsilon_0 \exp(-\beta n\varepsilon_0)}{\sum_{n=0}^{\infty} \exp(-\beta n\varepsilon_0)} = -\frac{d}{d\beta}\left[\log \sum_{n=0}^{\infty} \exp(-\beta n\varepsilon_0)\right]$$

$$= -\frac{d}{d\beta}\left[\log\left(\frac{1}{1-\exp(-\beta\varepsilon_0)}\right)\right] = \frac{\varepsilon_0}{\exp(\beta\varepsilon_0)-1}$$

↳ not KT anymore

$$\boxed{\Rightarrow \rho(\lambda,T) = \frac{8\pi}{\lambda^4}\,\frac{\varepsilon_0}{\exp(\varepsilon_0/k_T)-1}}$$

PLANCK'S QUANTUM THEORY

$$\rho(\lambda,T) = \frac{8\pi}{\lambda^4}\,\frac{\varepsilon_0}{\exp(\varepsilon_0/k_T)-1}$$

→ can you integrate this?

So what is our quantum "ε_0"?

→ To satisfy Wien's Law: $\rho(\lambda,T) = \dfrac{f(\lambda T)}{\lambda^5}$

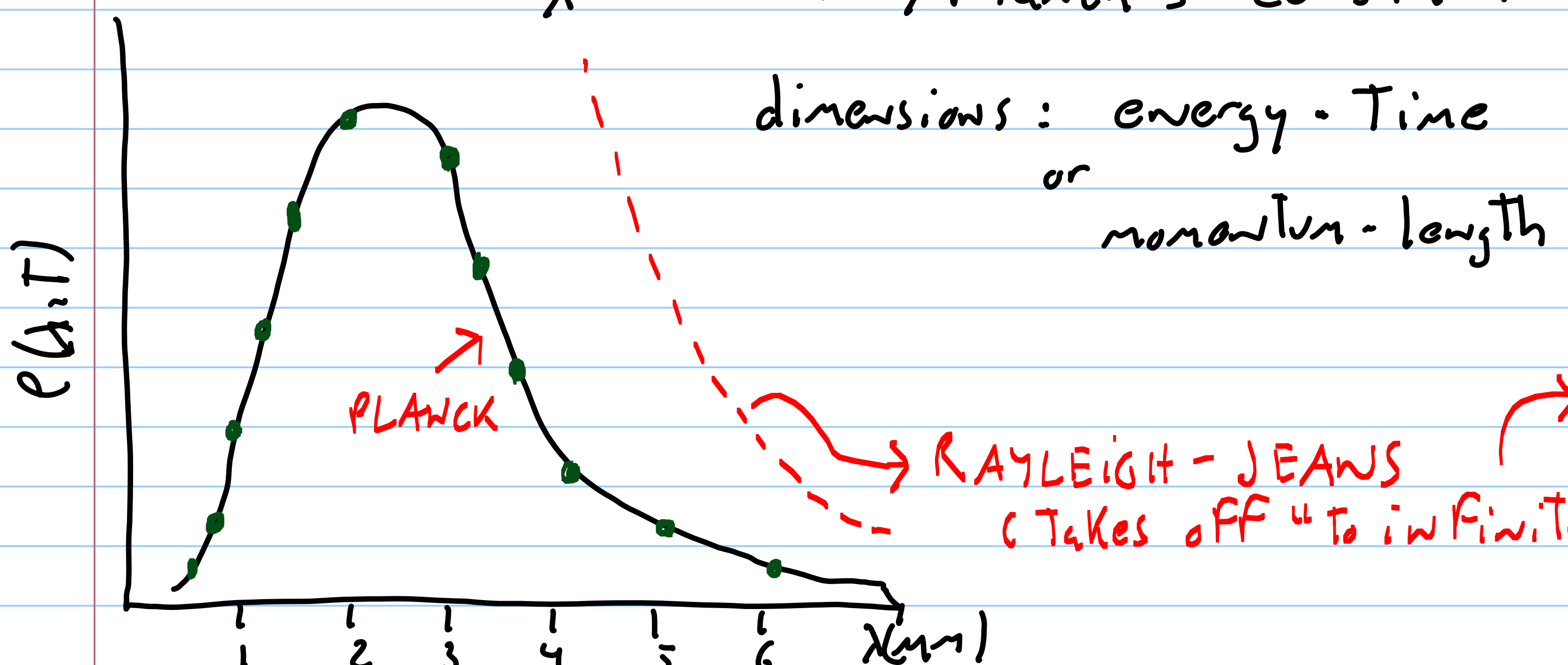
$\Rightarrow \varepsilon_0 = \dfrac{hc}{\lambda} = h\nu$

↳ Planck's constant

dimensions: energy · Time

or

momentum · length



PLANCK'S QUANTUM THEORY

FROM THIS WE CAN EXPRESS "b" = $\lambda_{max} T = b$

as $\dfrac{hc}{4.97K}$

AND "σ" in σT^4 as : $\dfrac{8 \pi^5 K^4}{15 h^3 c^3}$

Note that for long wavelengths, Planck's formula agrees with the one obtained by Rayleigh-Jeans: for long wavelengths, the quantum of energy is small relative to KT so the "discreetness" is less evident and the energies are nearly continously distributed.

Can you prove it mathematically?

(see next page for solution)

Solution:

$$\rho(\lambda,t) = \frac{8\pi}{\lambda^4} \frac{\varepsilon_0}{\exp(\varepsilon_0/KT) - 1}$$

with $\varepsilon_0 = \frac{hc}{\lambda}$, we have $\rho(\lambda,T) = \frac{8\pi}{\lambda^5} \frac{hc}{\exp\left(\frac{hc}{\lambda KT}\right) - 1}$

we use $e^x = \underbrace{1 + x}_{} + \frac{x^2}{2!} + \frac{x^3}{3!}$

$$\exp\left(\frac{hc}{\lambda KT}\right) - 1 \longrightarrow 1 + \frac{hc}{\lambda KT} - 1 = \frac{hc}{\lambda KT}$$

$$\rho(\lambda,T) = \frac{8\pi hc}{\lambda^5} \cdot \frac{\lambda KT}{hc} = \underbrace{\frac{8\pi}{\lambda^4} \cdot KT}_{}$$

rayleigh-jeans' result