Quantum Weirdness

PHY256F

You already know quantum mechanics is weird

- Wave-particle duality
- Imaginary numbers
- Wave-function collapse
- Quantization of energy, momentum etc…

But it gets plenty weirder…

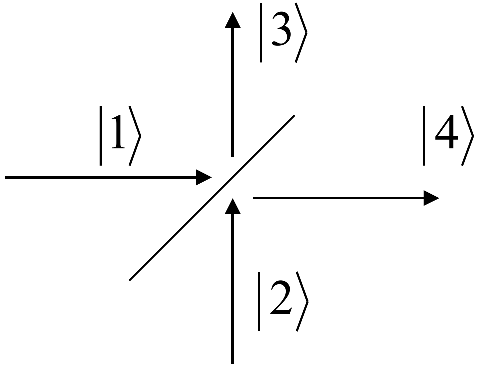
  – Today I want to talk about some ideas from the last fifteen years that really explore just how weird it can get

- What I expect you to learn
  - Interaction-free measurement, Hardy's paradox, the EPR paradox
  - That QM forces us to abandon either reality or locality or both

But first some preliminaries

The beamsplitter

- The action of the beamsplitter is

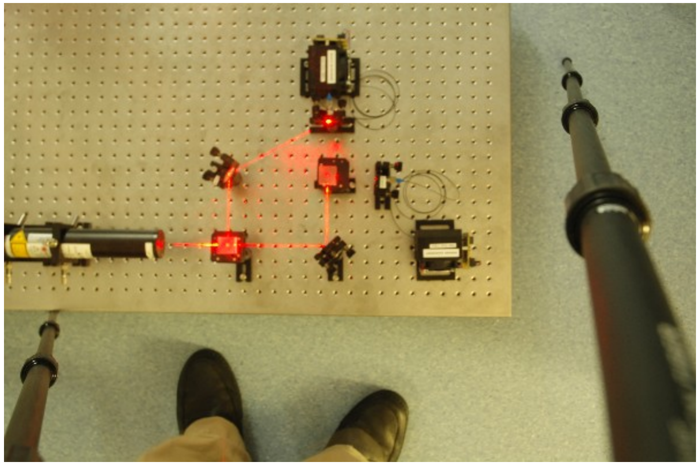


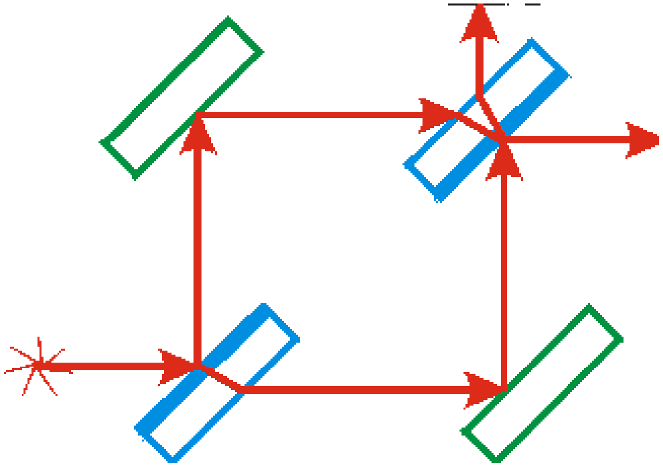
$$|3\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + |2\rangle\right) \qquad |1\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle + |4\rangle\right)$$

$$|4\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle - |2\rangle\right) \qquad |2\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle - |4\rangle\right)$$

The Mach-Zehnder Interferometer

- The MZ interferometer is one of the conceptually simplest interferometers

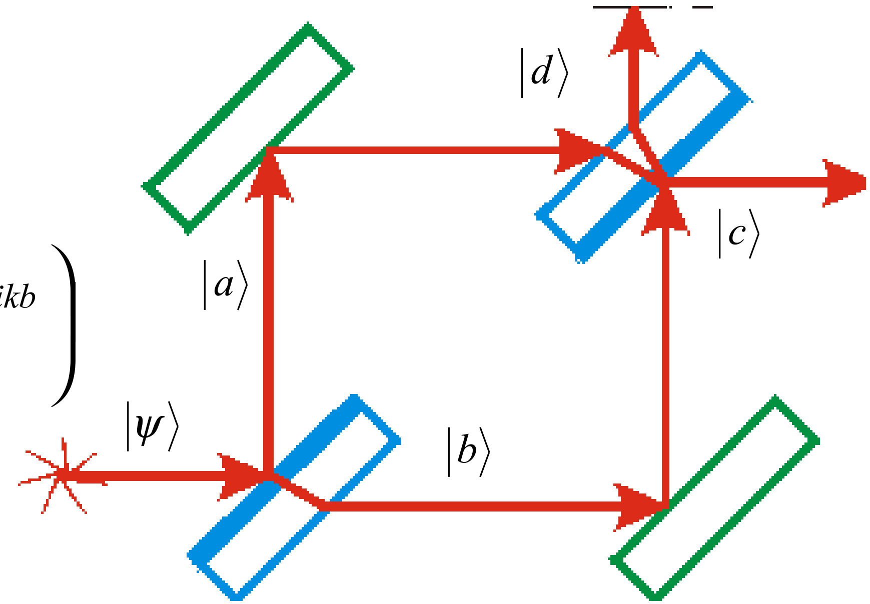




$$|\psi\rangle \Rightarrow \frac{1}{\sqrt{2}}\left(|a\rangle e^{ika} + |b\rangle e^{ikb}\right) \Rightarrow$$

$$\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\left(|c\rangle + |d\rangle\right)e^{ika} + \frac{1}{\sqrt{2}}\left(|c\rangle - |d\rangle\right)e^{ikb}\right)$$

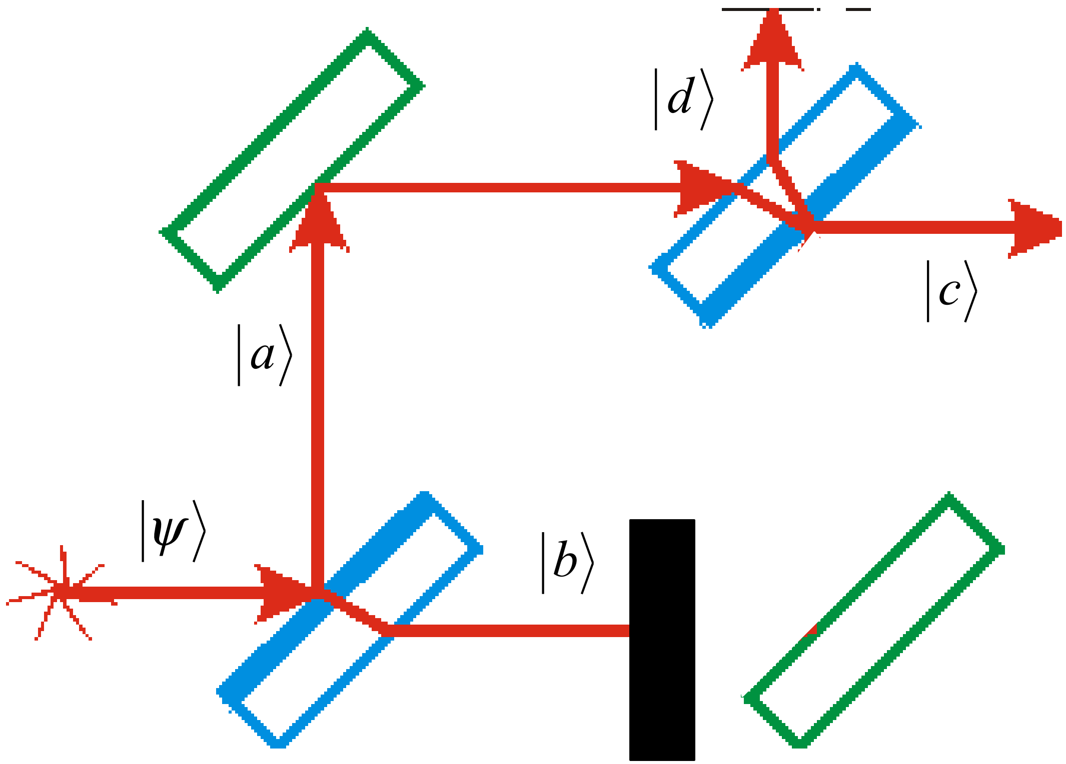
$$= \frac{1}{2}\left(|c\rangle\left(e^{ika} + e^{ikb}\right) + |d\rangle\left(e^{ika} - e^{ikb}\right)\right)$$



$$= \frac{1}{2} e^{ika/2} e^{ikb/2}\left(|c\rangle\left(e^{ik(a-b)/2} + e^{-ik(a-b)/2}\right) + |d\rangle\left(e^{ik(a-b)/2} - e^{-ik(a-b)/2}\right)\right)$$

$$= \frac{1}{2} e^{ika/2} e^{ikb/2}\left(|c\rangle 2\cos\left(k(a-b)/2\right) + |d\rangle 2i\sin\left(k(a-b)/2\right)\right)$$

$$= e^{ika/2} e^{ikb/2}\left(\cos\left(k(a-b)/2\right)|c\rangle + i\sin\left(k(a-b)/2\right)|d\rangle\right)$$



Either

$$|\psi\rangle \Rightarrow \frac{1}{\sqrt{2}}\left(|a\rangle e^{ika} + |b\rangle e^{ikb}\right) \Rightarrow$$

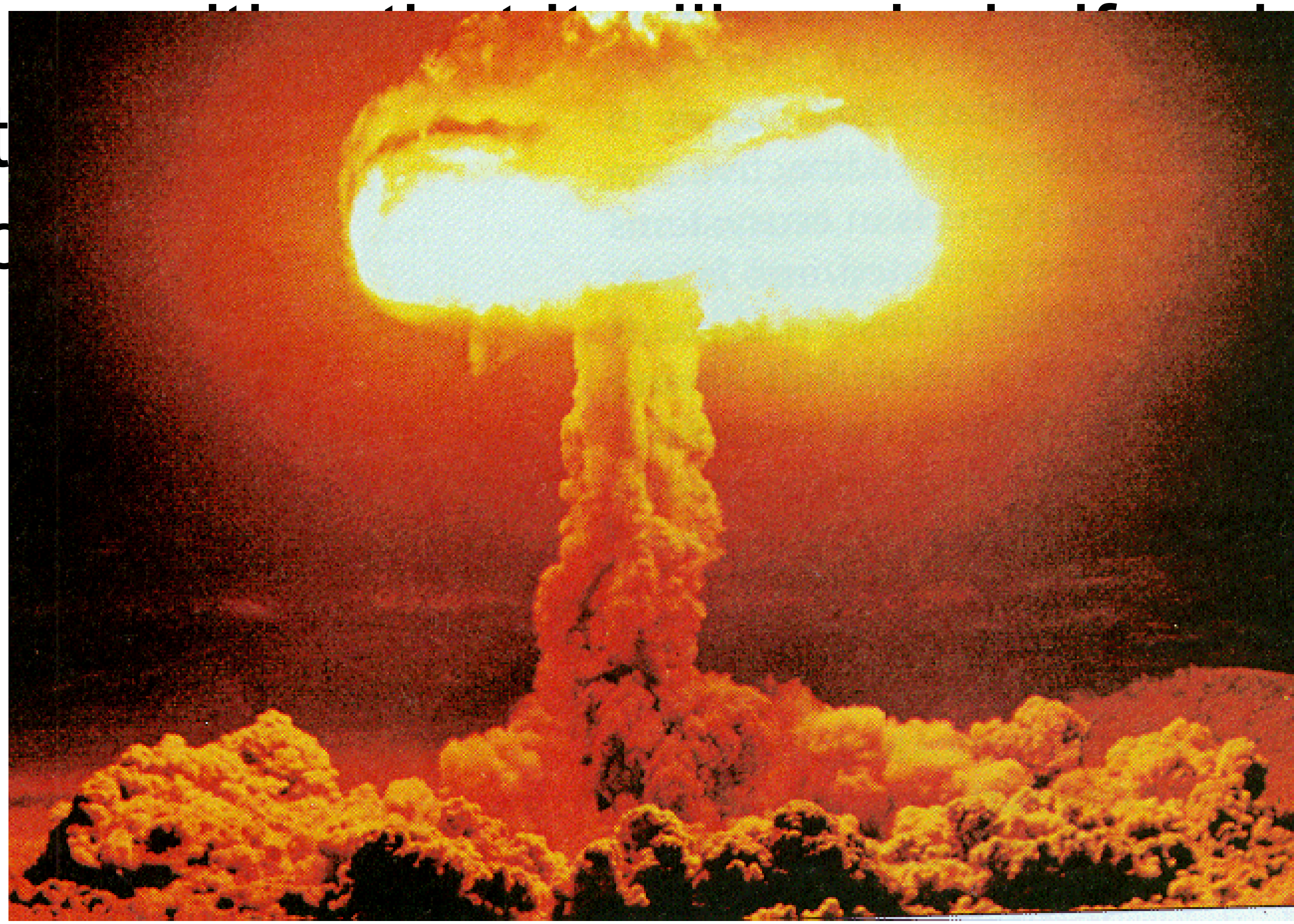
$$|b\rangle e^{ikb} \Rightarrow \frac{1}{\sqrt{2}} e^{ikb}\left(|c\rangle + |d\rangle\right)$$

or

$$|\psi\rangle \Rightarrow \frac{1}{\sqrt{2}}\left(|b\rangle e^{ikb}\right) \Rightarrow$$

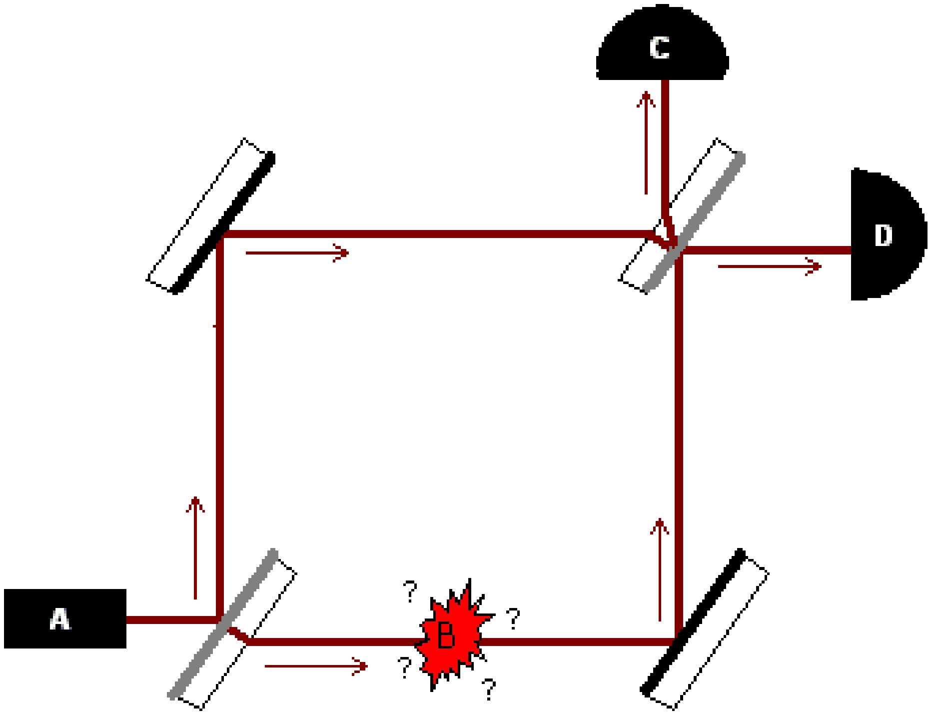
Interaction-free measurement

- Imagine you have a bomb with a trigger on it so … gle phot… b optic…



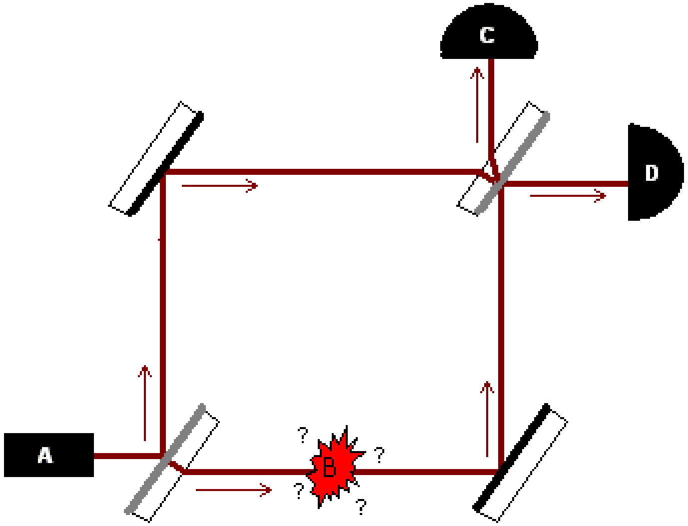
Yes! With Quantum Weirdness!

- Solution: Put the bomb in an MZ interferometer



If there is no bomb…

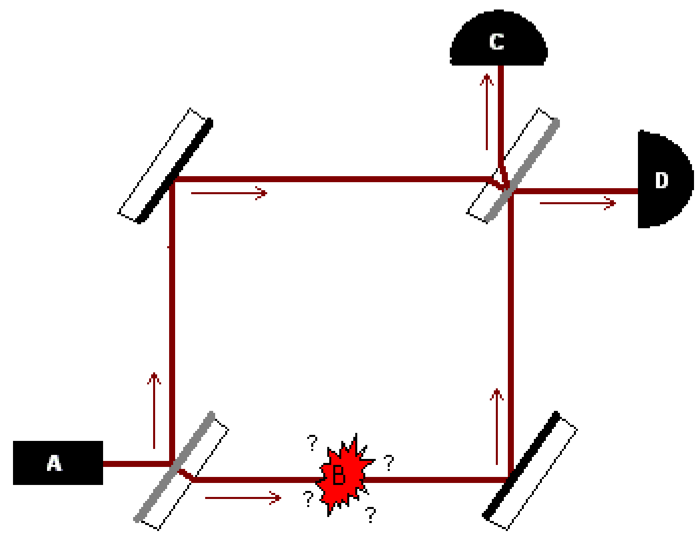
- Then the interference takes place as before.
- By setting k(a-b)/2=2n π we can guarantee that the photons will always go to detector ‘c’ and never to ‘d’.



$$|\psi\rangle = e^{ika/2}e^{ikb/2}\left(\cos\left(k(a-b)/2\right)|c\rangle + i\sin\left(k(a-b)/2\right)|d\rangle\right)$$

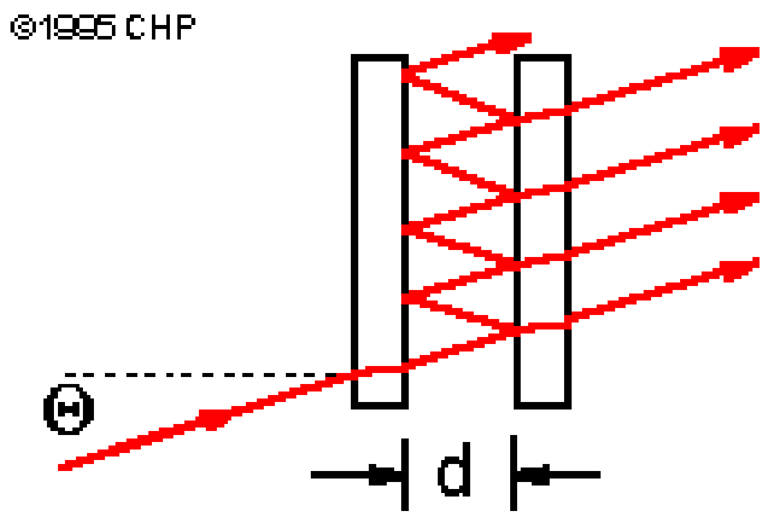
And if there is a bomb

- Then half the time the photon will take path b and hit it and the bomb will go off
- But the other half of the time the photon will take path ‘a’ and not hit the bomb
- If the photon takes path ‘a’ then either detector ‘c’ or ‘d’ can fire
- If detector ‘d’ fires then we know the bomb was there even though the photon never hit it!



Even better..

- Using the same principle in a Fabry-Perot interferometer we can detect the bomb >99% of the time without interacting with it



- This is a fundamental (and maybe useful) manifestation of wave-particle duality

More quantum weirdness:

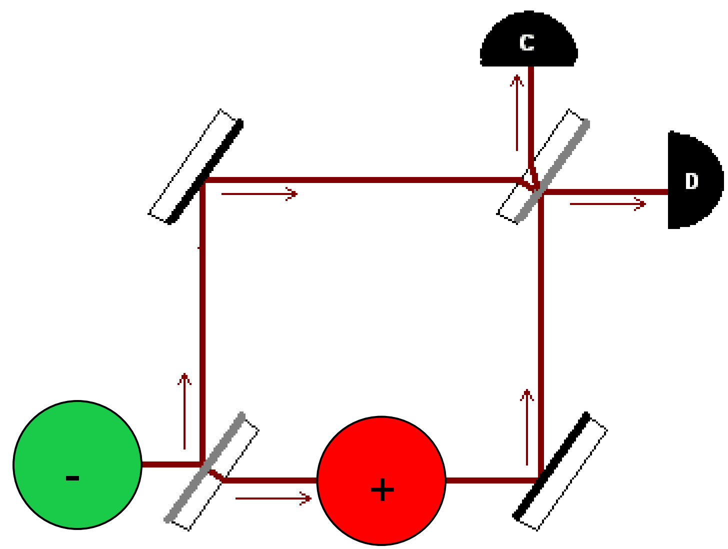
Hardy’s Paradox

But it gets better…

- Imagine now that instead of an MZ interferometer for photons, we make one for electrons
- Instead of a bomb we'll put a positron in the interferometer

Electron - Positron Annihilation

NASA - Goddard Space Flight Center
Scientific Visualization Studio

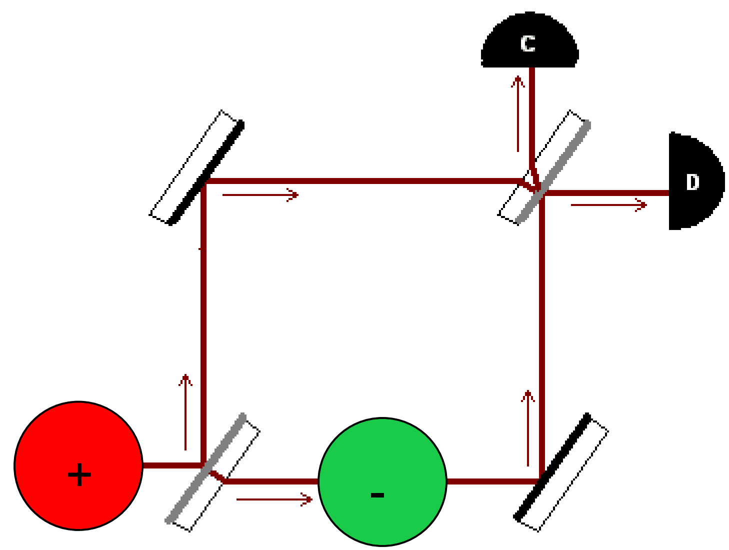


And better…

- We could also build a positron interferometer and use an electron as the “bomb”

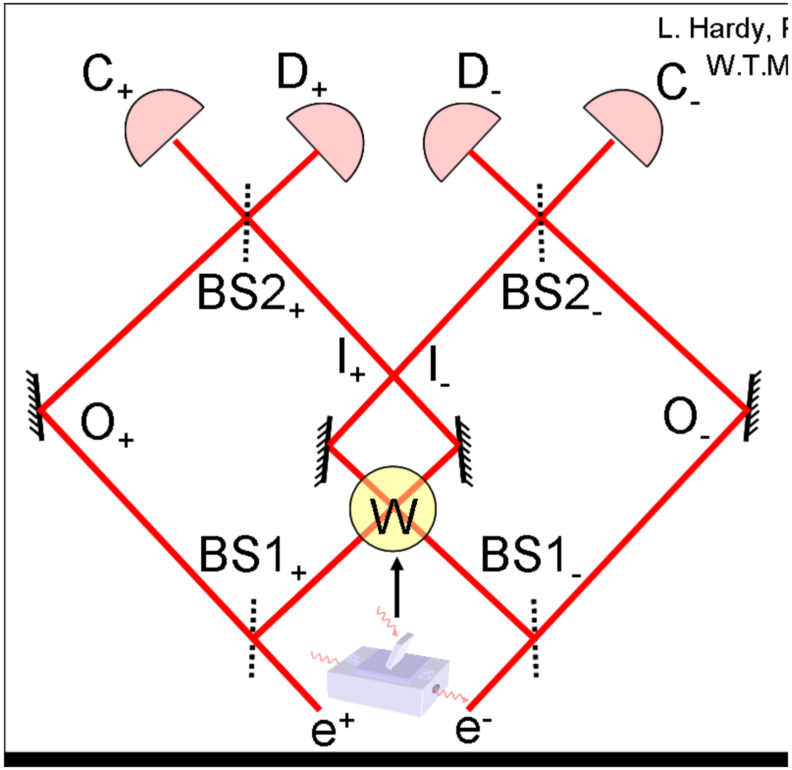
Electron - Positron Annihilation

NASA - Goddard Space Flight Center
Scientific Visualization Studio



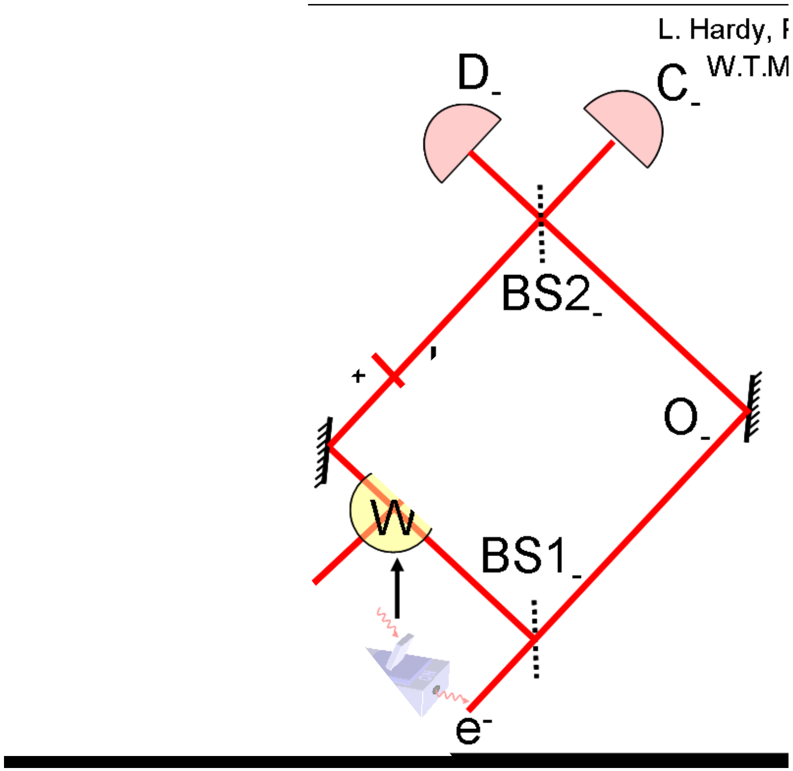
And even better…

- Now let's put both the electron and the positron in their own interferometers



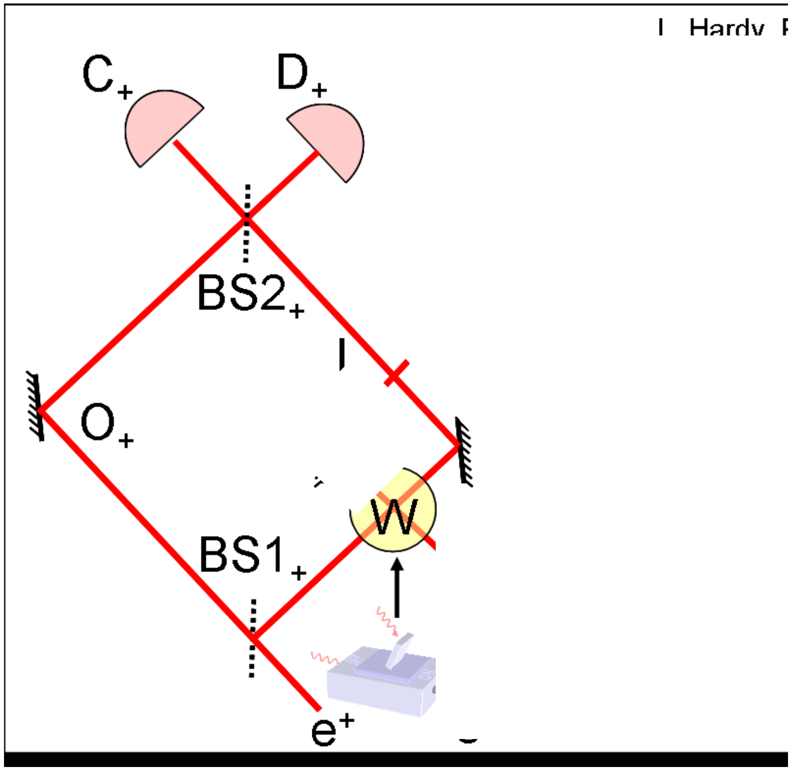
What do the clicks tell us?

- In the electron interferometer
  - If there's no positron then we only get clicks at C_-
  - If a positron is at W, then one-quarter of the clicks are at D_-
  - **Whenever there is a click at D_- we know the positron is at W**

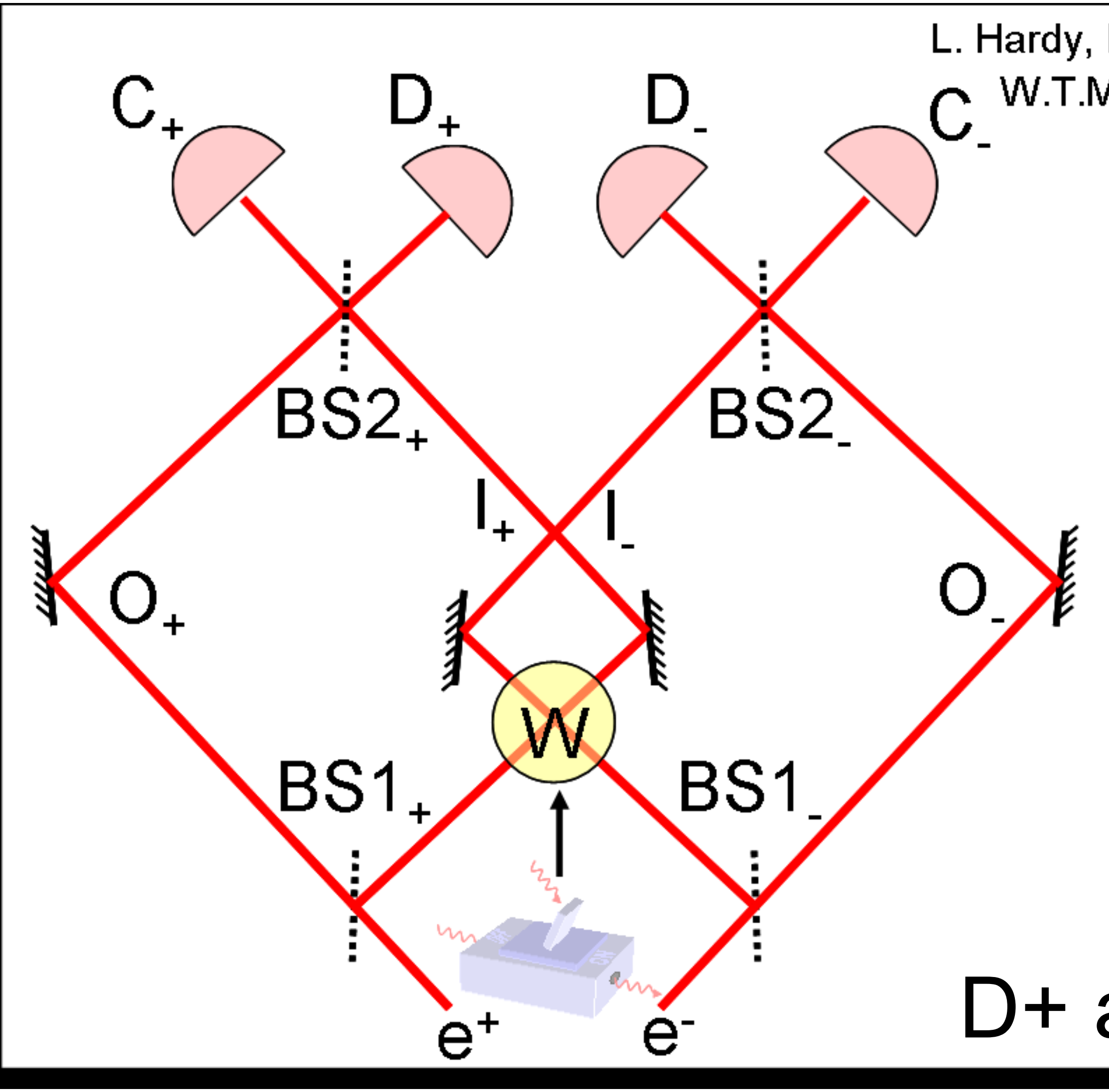


What do the clicks tell us?

- In the positron interferometer
  - If there's no electron then we only get clicks at C_+
  - If an electron is at W, then one-quarter of the clicks are at D_+
  - **Whenever there is a click at D_+ we know that the electron was at W**



And now all together…



- If we get a click at D+ we know that the electron was at W
- If we get a click at D- then we know the positron was at W
- If both the positron and the electron are at W then they go boom
- Therefore…

D+ and D- should never go off together!

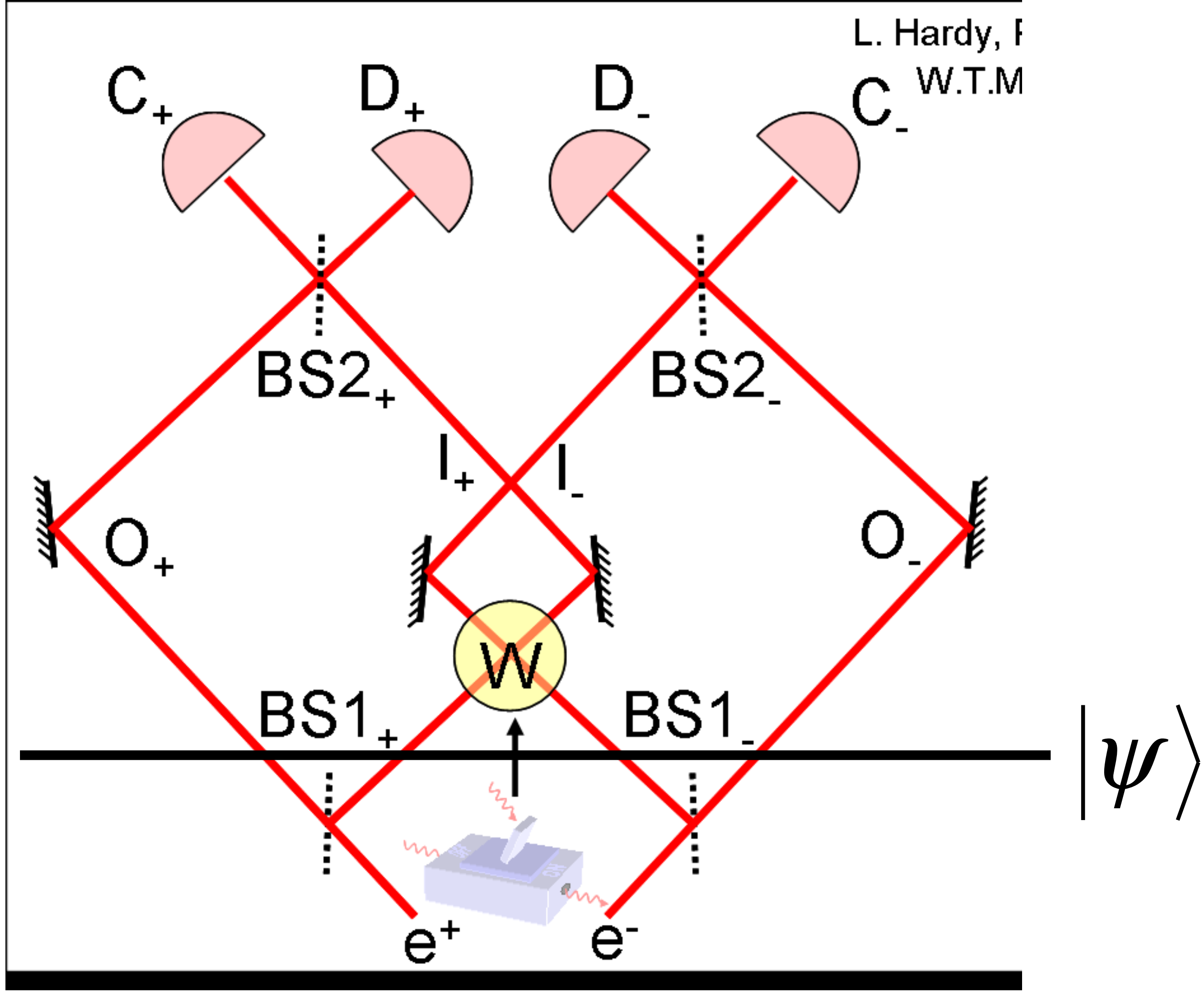
But they do!

The Quantum Café

- I buy a cafe from a guy under two conditions
  - I can't fire his two lazy nephews Mort and Mark
  - I can't look at what happens in the kitchen
- I start having problems, though and I watch my employees come and go.
  - Every time the food comes out of the kitchen too cold Mort is in the kitchen
  - Every time the food comes out of the kitchen overcooked Mark is in the kitchen
- So I decide to make sure I never schedule Mort and Mark to be in the kitchen at the same time
- But when I look in the complaint box I see that customers are complaining that they were served overcooked cold food!

So what happened?

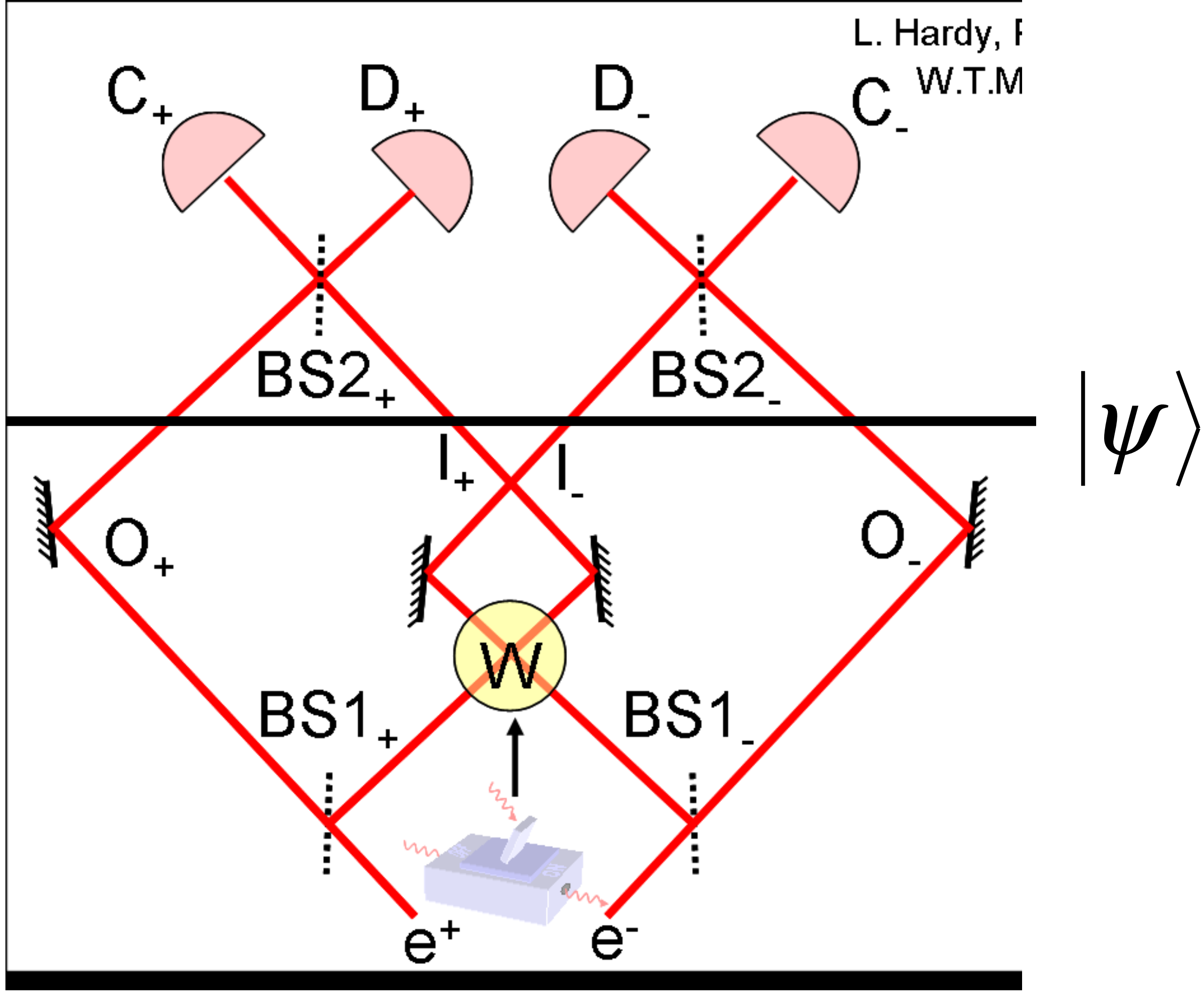
Let's look at Hardy's paradox again and try to describe the quantum state at each step



After BS1+ and BS1-

$$|\psi\rangle = \frac{1}{2}\left(|O_+\rangle + |I_+\rangle\right)\left(|O_-\rangle + |I_-\rangle\right)$$

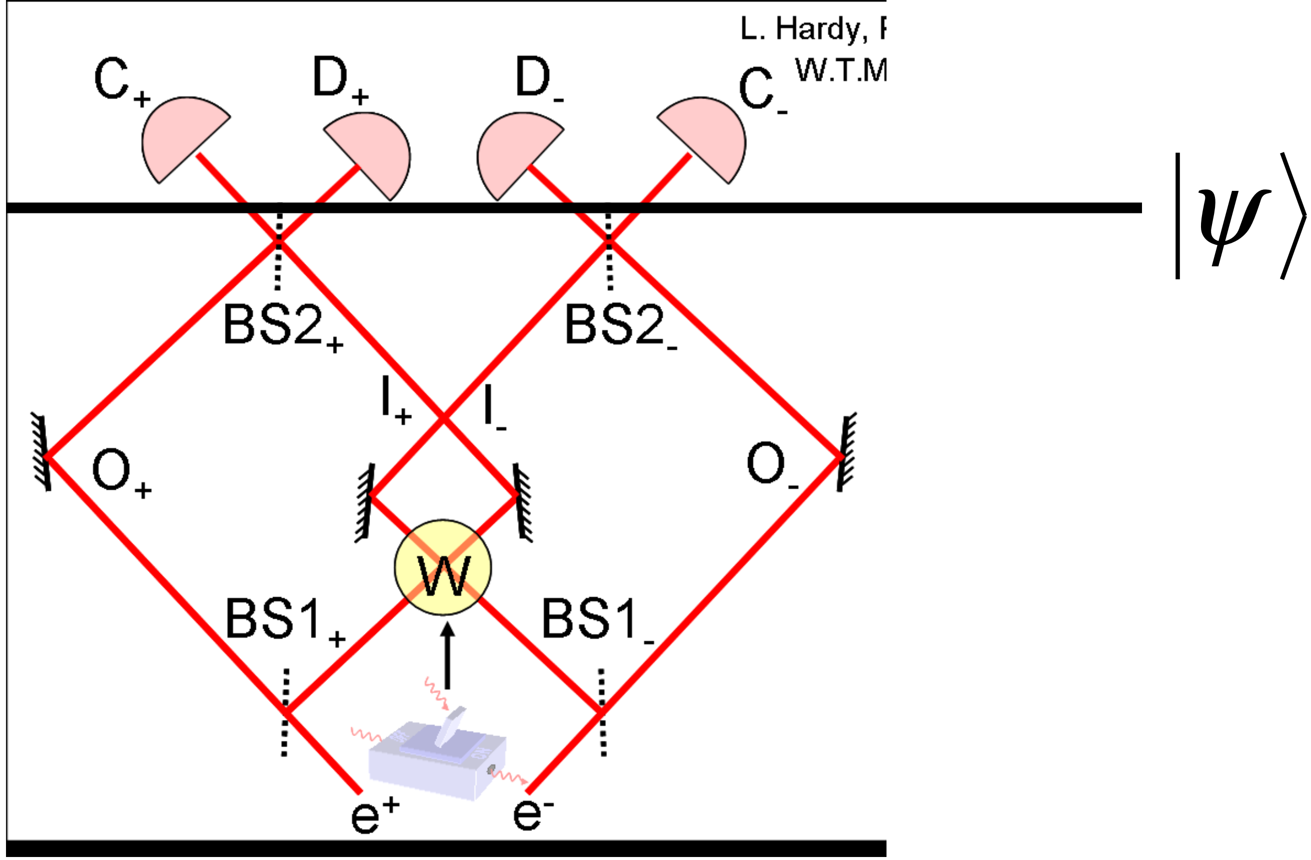
$$|\psi\rangle = \frac{1}{2}\left(|O_+O_-\rangle + |I_+O_-\rangle + |O_+I_-\rangle + |I_+I_-\rangle\right)$$



After W

$$|\psi\rangle = \frac{1}{2}\left(|O_+O_-\rangle + |I_+O_-\rangle + |O_+I_-\rangle + |I_+I_-\rangle\right)$$

$$|\psi\rangle = \frac{1}{\sqrt{3}}\left(|O_+O_-\rangle + |I_+O_-\rangle + |O_+I_-\rangle\right)$$



After BS2+ and BS2-

$$|\psi\rangle = \frac{1}{\sqrt{3}}\left(|O_+O_-\rangle + |I_+O_-\rangle + |O_+I_-\rangle\right)$$

$$\Rightarrow \frac{1}{\sqrt{3}}\Bigg(\frac{1}{\sqrt{2}}\left(|C_+\rangle + |D_+\rangle\right)\frac{1}{\sqrt{2}}\left(|C_-\rangle + |D_-\rangle\right) + \frac{1}{\sqrt{2}}\left(|C_+\rangle - |D_+\rangle\right)\frac{1}{\sqrt{2}}\left(|C_-\rangle + |D_-\rangle\right) +$$

$$\frac{1}{\sqrt{2}}\left(|C_+\rangle + |D_+\rangle\right)\frac{1}{\sqrt{2}}\left(|C_-\rangle - |D_-\rangle\right)\Bigg)$$

$$\Rightarrow \frac{1}{\sqrt{3}}\left(\frac{1}{\sqrt{2}}\left(|C_+\rangle+|D_+\rangle\right)\frac{1}{\sqrt{2}}\left(|C_-\rangle+|D_-\rangle\right)+\frac{1}{\sqrt{2}}\left(|C_+\rangle-|D_+\rangle\right)\frac{1}{\sqrt{2}}\left(|C_-\rangle+|D_-\rangle\right)+\right.$$

$$\left.\frac{1}{\sqrt{2}}\left(|C_+\rangle+|D_+\rangle\right)\frac{1}{\sqrt{2}}\left(|C_-\rangle-|D_-\rangle\right)\right)$$

$$\Rightarrow \frac{1}{2\sqrt{3}}\left(|C_+C_-\rangle+|D_+C_-\rangle+|C_+D_-\rangle+|D_+D_-\rangle+\right.$$

$$|C_+C_-\rangle-|D_+C_-\rangle+|C_+D_-\rangle-|D_+D_-\rangle+$$

$$\left.|C_+C_-\rangle+|D_+C_-\rangle-|C_+D_-\rangle-|D_+D_-\rangle\right)$$

$$=\frac{1}{2\sqrt{3}}\left(3|C_+C_-\rangle+|D_+C_-\rangle+|C_+D_-\rangle-|D_+D_-\rangle\right)$$

All coincidences between detectors are possible, even the ones that classical logic says are impossible

So what’s wrong with our logic?

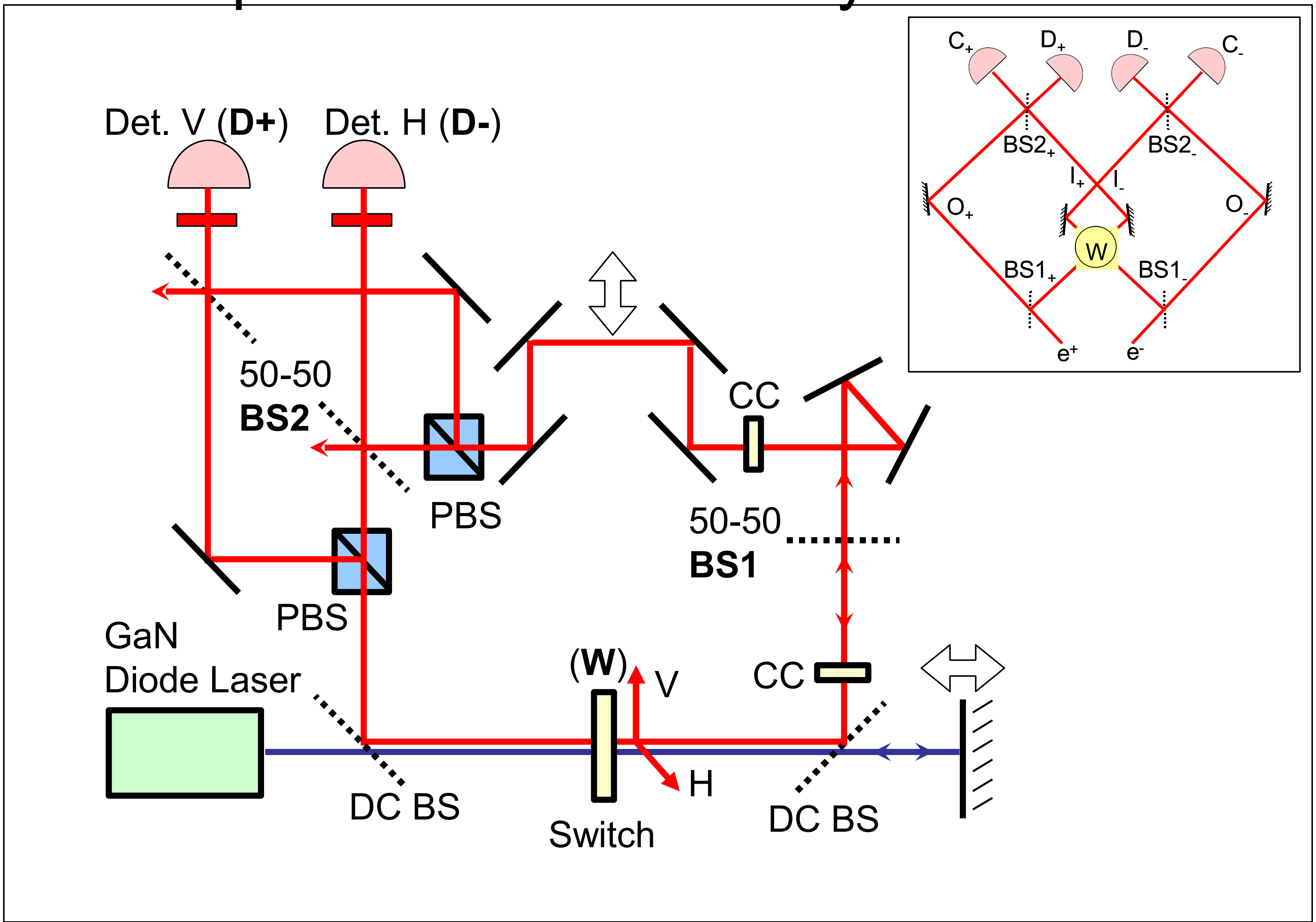
- In order to make an inference about where the electron is from which positron detector fires
  - We have to think about the electron as a “particle”, “localized” in one arm
  - We have to think about the positron as a “wave”, split among the two arms
- Same for making inferences about “where” the positron is from the electron detectors
- Problem when you want to think about “where” both the electron and the positron are *at the same time*

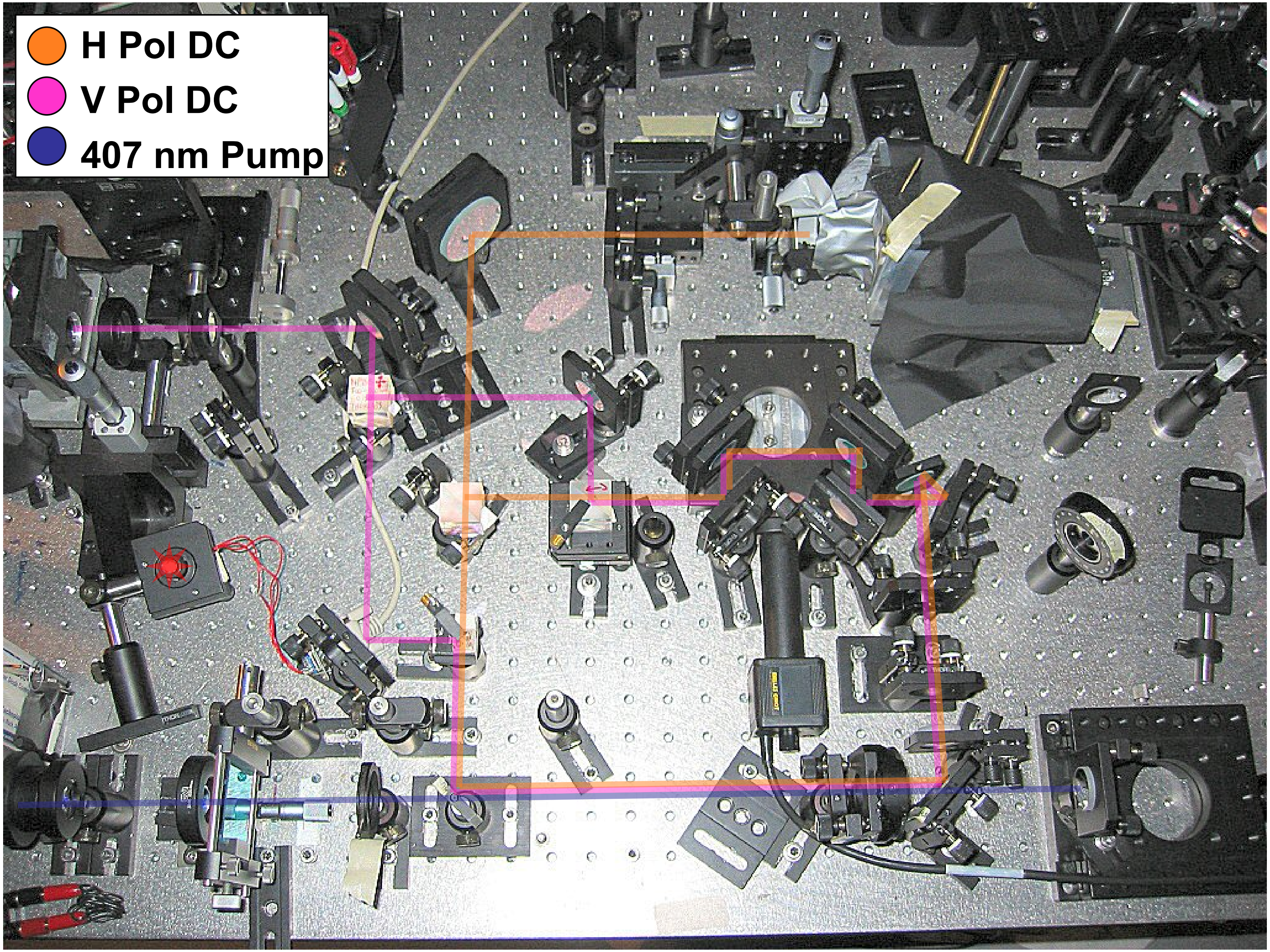
er to

they
aren't a concern of
physics

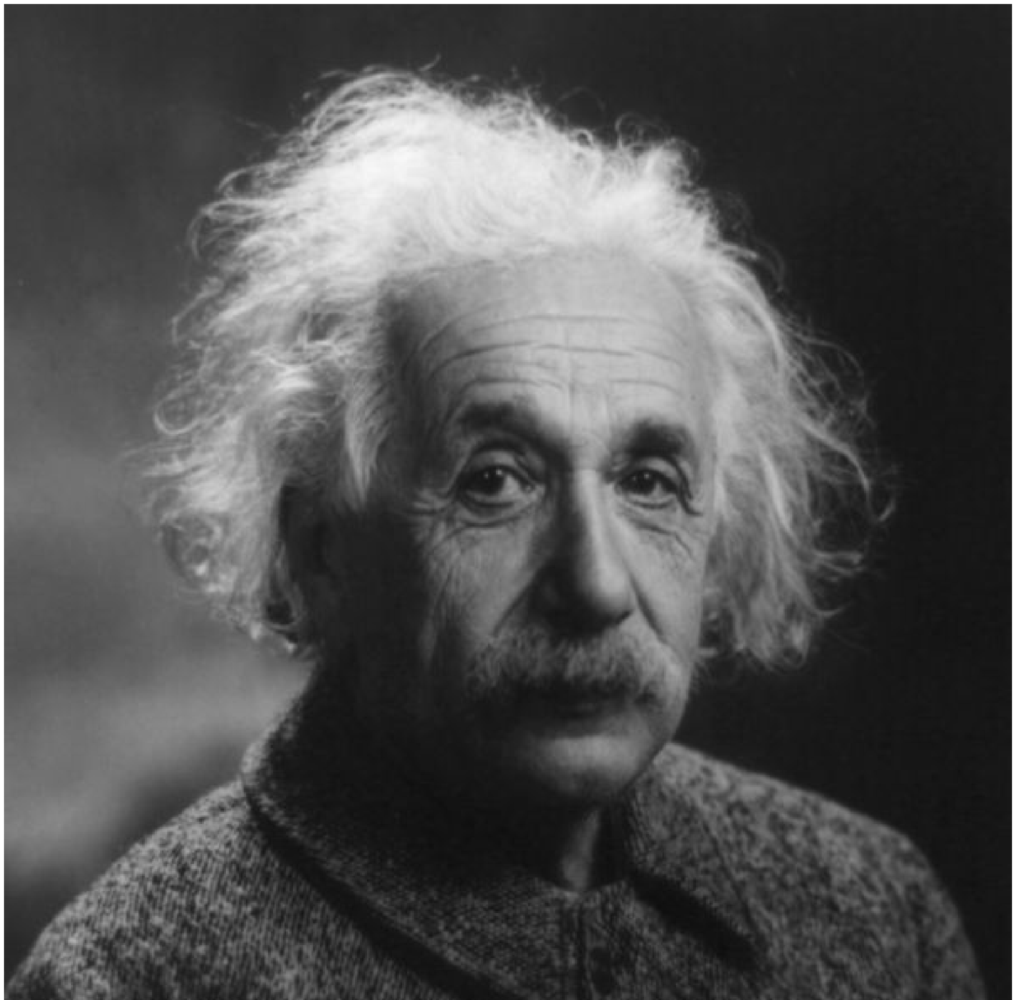
- Einstein's was to say to quantum mechanics must be fundamentally flawed

- U of T's Jeff Lundeen did this experiment with photons in 2005 in my lab




H Pol DC
V Pol DC
407 nm Pump

The EPR paradox



- Einstein was uncomfortable with the departure from realism that QM seemed to require
- In 1935 he came up with a thought experiment that he believed proved that QM was incomplete

The EPR paradox

- Imagine two photons in a polarization state

$$\frac{1}{\sqrt{2}}\left(\left|H\right\rangle_1\left|V\right\rangle_2-\left|V\right\rangle_1\left|H\right\rangle_2\right)$$

- Locations 1 and 2 can be very far from each other, even in space-like separated regions

- The person at 1 does a measurement,
  - If he obtains H, the photon at 2 “collapses” to V instantaneouly
  - If he obtains V, the photon at 1 “collapses” instantaneously to H
- But is this so surprising?
  - Let’s say I work at a shoe factory and I have to unpackage shoeboxes and send the two shoes in them to locations 1 and 2
  - If I receive a right shoe at location 1 then I immediately “collapse” the shoe at location 2 to be left

But quantum mechanics has some extra features…

- I can make another measurement. Define:

$$|D\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle + |V\rangle\right)$$

$$|A\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle - |V\rangle\right)$$

- So that

$$|H\rangle = \frac{1}{\sqrt{2}}\left(|D\rangle + |A\rangle\right)$$

$$|V\rangle = \frac{1}{\sqrt{2}}\left(|D\rangle - |A\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(|H\rangle_1|V\rangle_2-|V\rangle_1|H\rangle_2\right)$$

$$=\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\left(|D\rangle_1+|A\rangle_1\right)\frac{1}{\sqrt{2}}\left(|D\rangle_2-|A\rangle_2\right)-\frac{1}{\sqrt{2}}\left(|D\rangle_1-|A\rangle_1\right)\frac{1}{\sqrt{2}}\left(|D\rangle_2+|A\rangle_2\right)\right)$$

$$=\frac{1}{2\sqrt{2}}\left(\left(|D\rangle_1|D\rangle_2+|A\rangle_1|D\rangle_2-|D\rangle_1|A\rangle_2-|A\rangle_1|A\rangle_2\right)-\left(|D\rangle_1|D\rangle_2-|A\rangle_1|D\rangle_2+|D\rangle_1|A\rangle_2-|A\rangle_1|A\rangle_2\right)\right)$$

$$=\frac{1}{\sqrt{2}}\left(|A\rangle_1|D\rangle_2-|D\rangle_1|A\rangle_2\right)$$

So if the person at 1 does a measurement that tells between D and A, the state at 2 collapses to either A or D

Recipe for communicating faster than the speed of light

1. Make the state

$$\frac{1}{\sqrt{2}}\left(|H\rangle_A |V\rangle_B - |V\rangle_A |H\rangle_B\right) = \frac{1}{\sqrt{2}}\left(|D\rangle_A |A\rangle_B - |A\rangle_A |D\rangle_B\right)$$

3. Send to two spacelike separated parties A and B
4. Have A send 0 by measuring his photon in H/V basis, 1 by measuring in the A/D basis
5. Have B detect whether his photon is A or D or H or V

What's wrong with this picture?

Heisenberg to the rescue…

- Measurement in A/D and H/V don't commute, so B can't distinguish A/D from H/V
- If his photon is A or D and he measures H/V then he gets 50/50 random outcomes, same as if
- If his photon is H and he measures A/D then he gets 50/50 random outcomes
- Measurements outcomes between A and B are perfectly correlated, but no information is sent between them

Causality is OK!

Summary

- Quantum mechanics isn't just hard, it's weird
- The QM state description cannot be reconciled with "local realism" which would make Einstein cry
- This leads to results that violate logic based on any classical notion of what happens in an experiment

“If quantum mechanics hasn't profoundly shocked you, you haven't understood it yet.”

--- Niels Bohr