LECTURE 34: Quantum Computing and Quantum Teleportation

What I expect you to learn:

- What are qubits
- Examples of quantum gates and how they manipulate states
- What is quantum teleportation

(PROBLEM SET 5 DEADLINE EXTENDED: NOW DUE DEC. 6th)

"Classical" digital computing based on manipulation of bits

→ state is stored as on or off
1 or 0

DRAM : Transistor + capacitor
→ capacitor charge determines state

Quantum computer based on Qubits (Quantum bits).

→ superposition of "0" and "1"

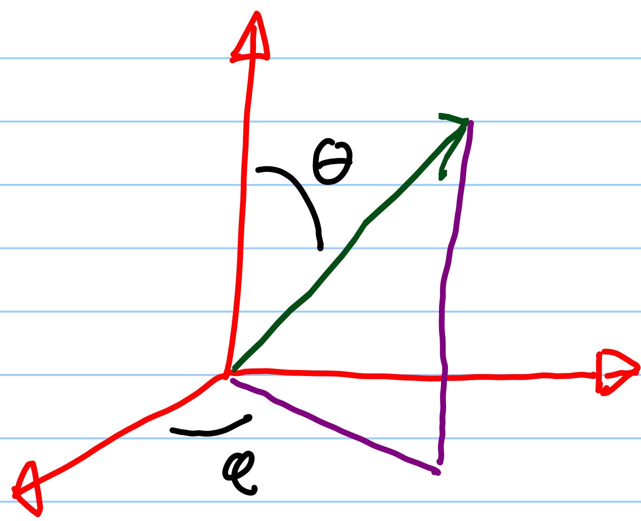
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$

or $|\psi\rangle = \cos\theta\,|0\rangle + e^{i\varphi}\sin\theta\,|1\rangle$

$|\psi\rangle = \cos\theta\,|0\rangle + e^{i\varphi}\sin\theta\,|1\rangle$

can be represented by a sphere (Bloch Sphere)



A lot more information in a qubit...

Need 2^N numbers for N qubits

Two qubits:

$$\alpha|\uparrow\rangle_1 + \beta|\downarrow\rangle_1$$
$$\gamma|\uparrow\rangle_2 + \delta|\downarrow\rangle_2$$

or: $a_1|\uparrow\uparrow\rangle + a_2|\uparrow\downarrow\rangle + a_3|\downarrow\uparrow\rangle + a_4|\downarrow\downarrow\rangle$

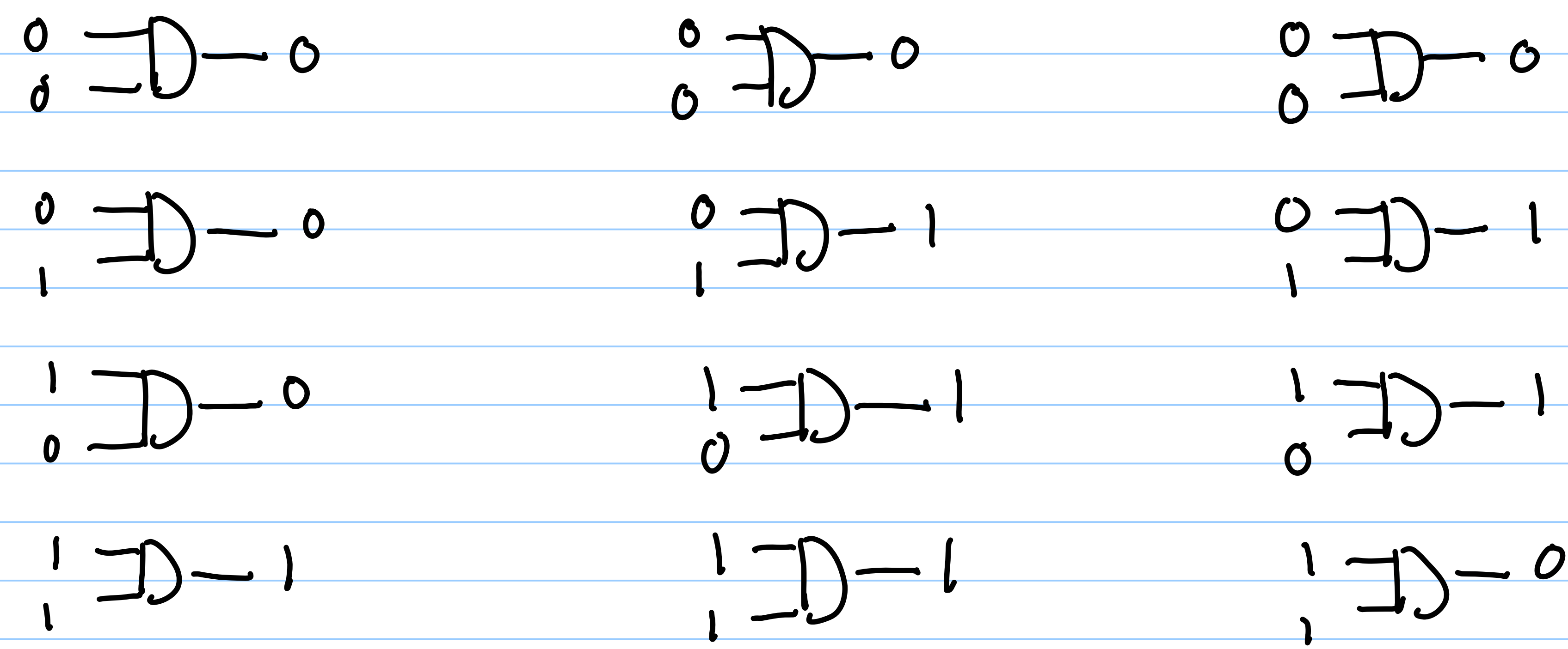
we can associate these vectors with:

$\downarrow\downarrow$: $\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$ $\quad$ $\downarrow\uparrow$ $\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$

$|\uparrow\downarrow\rangle$: $\begin{pmatrix}0\\0\\1\\0\end{pmatrix}$ $\quad$ $\uparrow\uparrow$ $\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$

Classical operations (BOOLEAN GATES)

AND OR XOR



HOW DOES XOR LOOK LIKE IN MATRIX FORM?

$$U_{XOR} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$U_{xor}|\downarrow\downarrow\rangle = |\downarrow\downarrow\rangle$

$U_{xor}|\downarrow\uparrow\rangle = |\downarrow\uparrow\rangle$

$U_{xor}|\uparrow\downarrow\rangle = |\uparrow\uparrow\rangle$

$U_{xor}|\uparrow\uparrow\rangle = |\uparrow\downarrow\rangle$

→ we read the 2nd state

note: if we apply U_{xor} Twice, we restore the initial state

note: we have 2 outputs instead of 1.

Quantum gates

Take $|\uparrow\rangle$ as "True" or $\begin{pmatrix}1\\0\end{pmatrix}$

$|\downarrow\rangle$ as "false" or $\begin{pmatrix}0\\1\end{pmatrix}$

$U_{NOT} = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ i.e True $\rightarrow$ False, False $\rightarrow$ True

↳ classical gate

$U_{\sqrt{NOT}} = \frac{1}{2}\begin{pmatrix}1-i & 1+i\\1+i & 1-i\end{pmatrix}$ $\rightarrow$ a quantum gate

$U_{\sqrt{NOT}} \cdot U_{\sqrt{NOT}} = U_{NOT}$

$\frac{1}{2}\begin{pmatrix}1-i & 1+i\\1+i & 1-i\end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix}1-i & 1+i\\1+i & 1-i\end{pmatrix}$ e.g. First row, column:

$1 - i - i - 1 + 1 + i + i - 1 = 0$

First row, 2nd column:

$1 + i - i + 1 + 1 + 1 - i + i = \frac{4}{4} = 1$

Quantum Gates (cont.)

Another interesting gate:

$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$, $\quad U|0\rangle : \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} =$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

$$U \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

→ apply U once and measure, get 50% $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and 50% $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

→ apply U twice and then measure, get $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ 100% of the time.

How are quantum computers built:

a Key application (Factoring prime numbers) was realised in 2001. Used 5 Fluorine-19 atoms and Two carbon-13 atoms with nuclei spins acting as qubits

bits manipulated using <u>NMR</u>

Factored number 15.

$\Rightarrow$ Public Key cryptography depends on our inability To Factor large numbers (quickly)

Shor's algorithm and a working quantum computer with 2n qubits could do iT

Exercise:

We start with the state: $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle\right)$

And apply U_{XOR}. Exlain the result.

$$|\psi\rangle = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\downarrow\rangle\right)$$

note that we can reverse the result by applying U_{XOR} again.

A couple of Theorems:

1- no accurate measure:

For an arbirary state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ we can't determine what α and β are.

2- no cloning

given $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, There is no operator A and state $|a\rangle$ such that

$$A(|\psi\rangle \otimes |a\rangle) = |\psi\rangle \otimes |\psi\rangle$$

→ can't clone a state

QUANTUM TELEPORTATION

WHAT DO WE MEAN BY TELEPORTATION?

- IN STAR TREK, PEOPLE GET "DISEMBODIED" FROM A LOCATION AND REASSEMBLED SOMEWHERE ELSE.
  → very hard to do...
- WHAT ABOUT TELEPHONES AND FAX MACHINES?
  → making an imperfect macroscopic copy
- Can we transmit the state of a system?
  → what about the no-cloning theorem?

Quantum Teleportation (cont.)

TELEPORTING A QUANTUM STATE:

WE WANT TO SEND $|\psi\rangle$ TO JANE

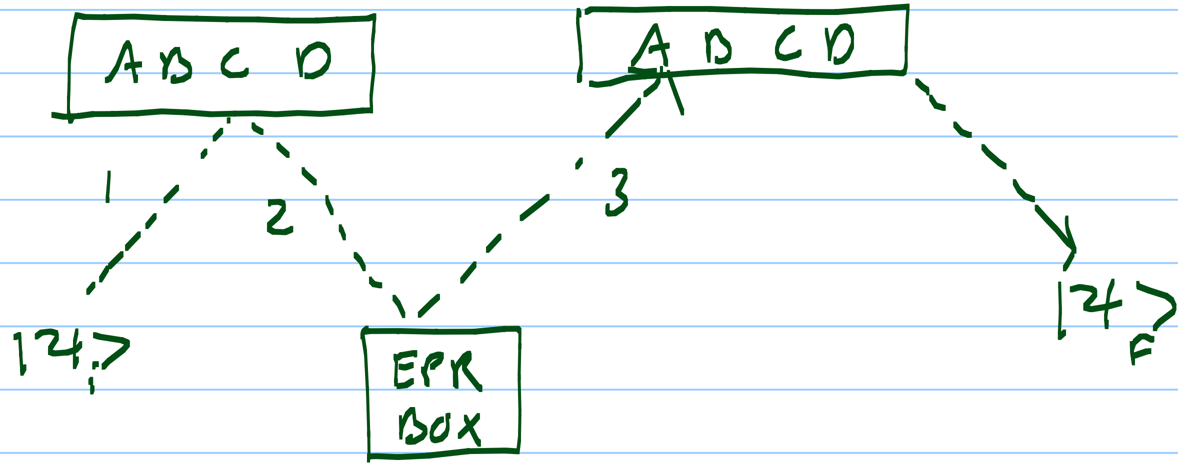
1- we can transport it there

2- we can't measure it and prepare that state at JANE'S HOUSE

3- WE CAN'T ANALYSE $|\psi\rangle$ AND SEND THAT INFO TO JANE

$$|\psi_i\rangle = \alpha|0\rangle_i + \beta|1\rangle_i$$

WE SHARE A MAX. ENTANGLED PAIR OF STATES WITH JANE AND WE ENTANGLE OUR STATE WITH THE ALREADY ENTANGLED STATE.



$$\left.\begin{array}{l} |\phi_A\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|1\rangle - |1\rangle|0\rangle\right) \\ |\phi_B\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|1\rangle + |1\rangle|0\rangle\right) \\ |\phi_C\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle - |1\rangle|1\rangle\right) \\ |\phi_D\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle|0\rangle + |1\rangle|1\rangle\right) \end{array}\right\} \text{BELL STATES}$$

ENTANGLED STATE WITH $|\psi_i\rangle\ |\phi_A\rangle_{23}$: (regroup terms, ignore phase factors)

$$\tfrac{1}{2}\Big[|\phi_A\rangle_{12}\left(a|0\rangle_3 + b|1\rangle_3\right) + |\phi_B\rangle_{12}\left(a|0\rangle_3 - b|1\rangle_3\right)$$

$$-|\phi_C\rangle_{12}\left(a|1\rangle_3 + b|0\rangle_3\right) + |\phi_D\rangle_{12}\left(a|1\rangle_3 - b|0\rangle_3\right)\Big]$$

WE MEASURE 1 AND 2 IN THE BELL BASIS OBTAINING A OR B OR C OR D

Note that we destroy our initial state $|\psi_i\rangle$

WE GIVE THIS INFO TO JANE:

$$\frac{1}{2}\Big[|\phi_A\rangle_{12}\,(a|0\rangle_3 + b|1\rangle_3) + |\phi_B\rangle_{12}\,(a|0\rangle_3 - b|1\rangle_3)$$
$$-|\phi_C\rangle_{12}\,(a|1\rangle_3 + b|0\rangle_3) + |\phi_D\rangle_{12}\,(a|1\rangle_3 - b|0\rangle_3)\Big]$$

FOR INSTANCE IF WE MEASURE ϕ_A, THEN WE RECOVER $|\psi_i\rangle$. with ϕ_B, we get $a|0\rangle_3 - b|1\rangle_3$

→ we can apply a Transformation and recover $|\psi_i\rangle$

→ no violation of cloning theorem
→ info on measurement Transmitted at speed $\leq c$