Lecture 4: The Photoelectric Effect and the Compton Effect

Goal of the lecture: to learn about two key experiments that provided evidence for the particle nature of light

I expect you to learn:

- basic description of the experiments
- experimental results and observations
- why classical physics did not explain the observed results
- to derive Compton scattering formula (i.e. wavelength shift versus angle)

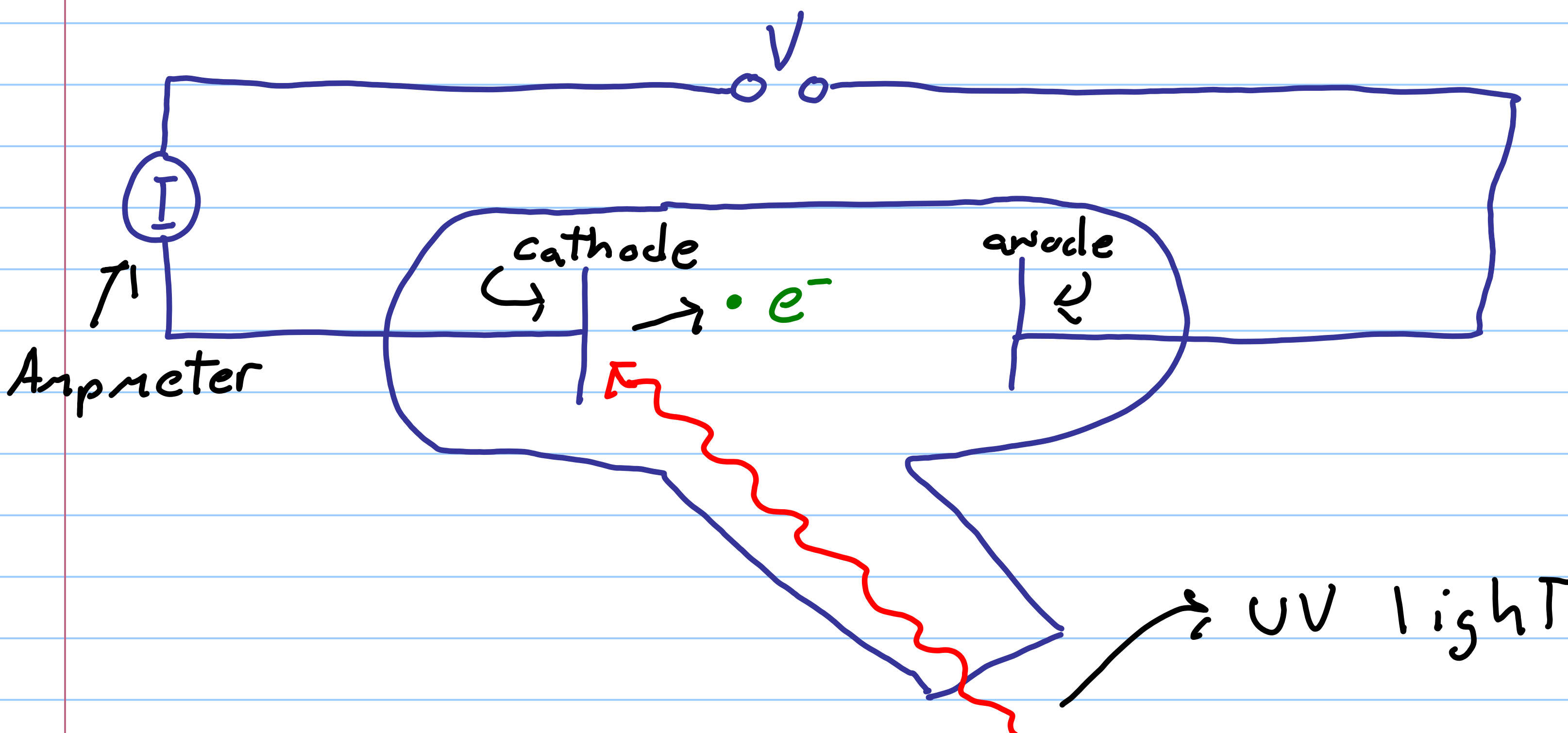
Lecture corresponds to section 1.2 and 1.4 of textbook

Note: the photoelectric effect is a PHY225 lab. Try it out! calculate h/e!

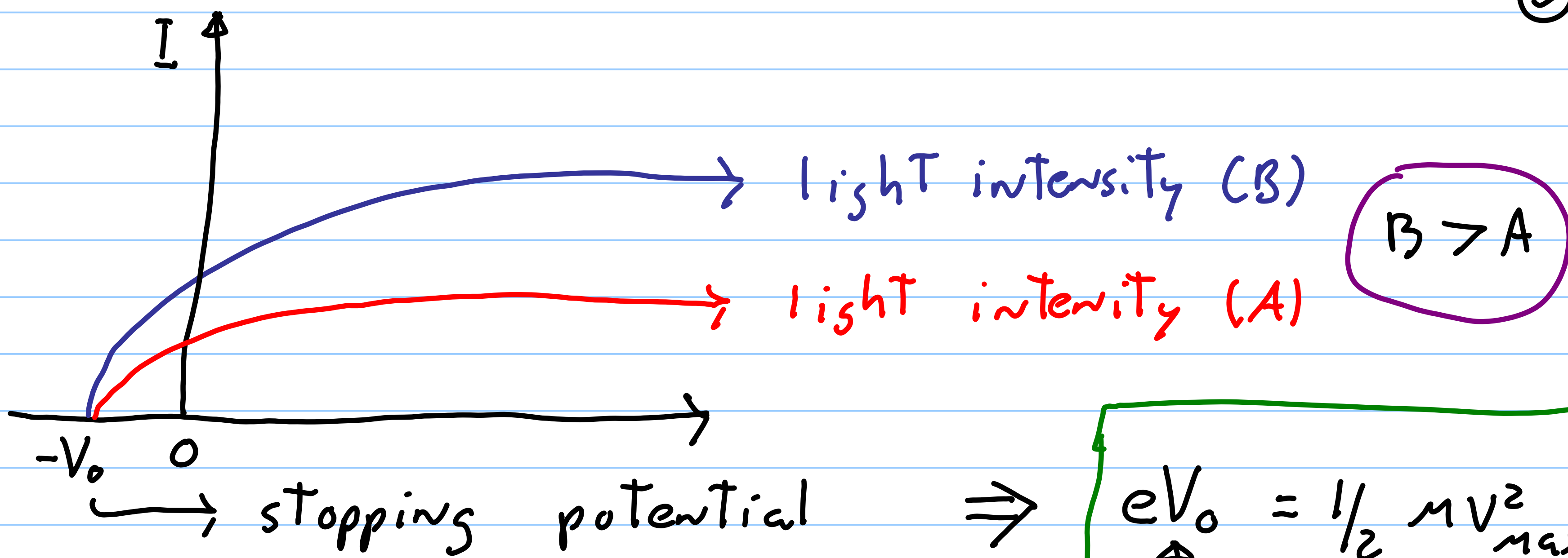
THE PHOTOELECTRIC EFFECT

In 1887, Hertz performed experiments that produced and detected em waves. He also observed that UV light falling on an electrode faciltates spark production.

Further work by Hallwachs, Stoletov, Lenard and others showed that charged particles are emitted from metallic surfaces by em waves.



Results:



$$\Rightarrow \quad eV_0 = \frac{1}{2} m v^2_{max} \quad (1)$$

Experimental data that could not be explained by classical EM theory:

1-There is a minimum frequency below which no electrons are emitted independent of the intensity
2-The stopping potential depends on frequency, not intensity
3- No time delay observed (it should take more time to emit electrons when the intensity is reduced)

So: the physics of the time could not explain the observations made by photoelectric experiments.

Einstein used the idea of quanta of energy to explain the observations. What are the implications of radiation being quanta of energy $E = h\nu = hc/\lambda$. The whole quanta of energy can be absorbed by one atom.

So we have:

$$\frac{1}{2} m v_{max}^2 = h\nu - W \qquad (2)$$

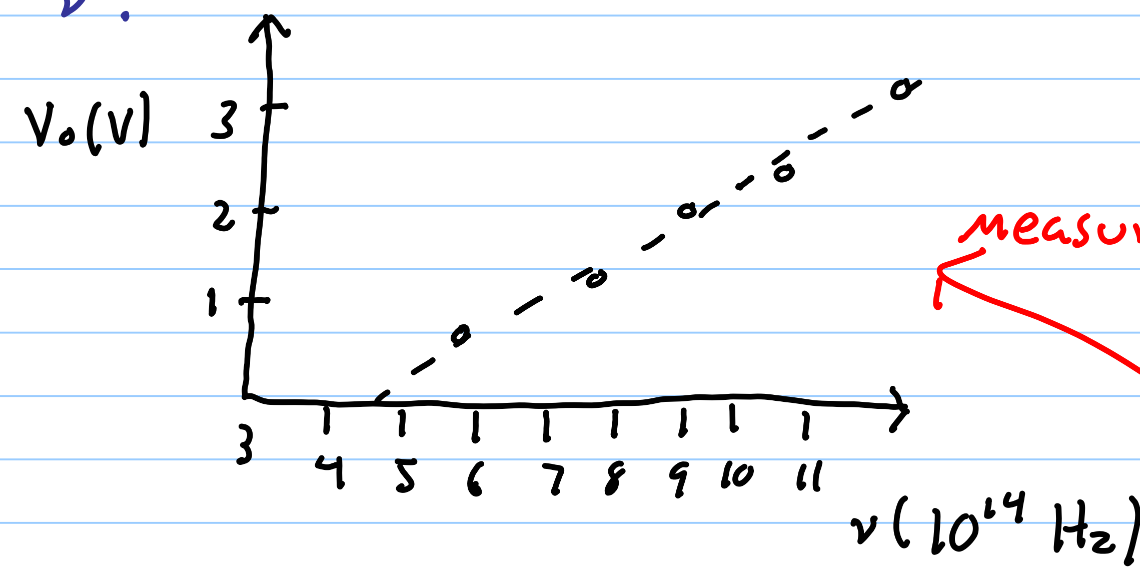
W → work function

v_{max} → max velocity of electrons

To figure out W, make sure $v_{max} = 0$ and determine ν_T (threshold frequency):

$$h\nu_T = W \qquad (3)$$

Millikan (~1915) measured V_0 as a function of ν.



measure slope, get h/e → PHY 225 Lab

Using (1) + (2) we get

$$V_0 = \frac{h}{e}\nu - \frac{W}{e} \qquad (4)$$

Knowing charge of electron (oil drop experiment) Millikan got h which agreed with Planck's blackbody results.

Example:

Two UV beams ($\lambda_1 = 280$ nm, $\lambda_2 = 490$ nm) produce photoelectrons of energies 8.57 and 6.67 eV, respectively. Estimate h with the data above.

Kinetic energies of electrons:

$$E_1 = \frac{hc}{\lambda_1} - W$$

$$E_2 = \frac{hc}{\lambda_2} - W$$

To get rid of W, we take the difference

$$E_1 - E_2 = \frac{hc}{\lambda_2 \lambda_1}(\lambda_2 - \lambda_1)$$

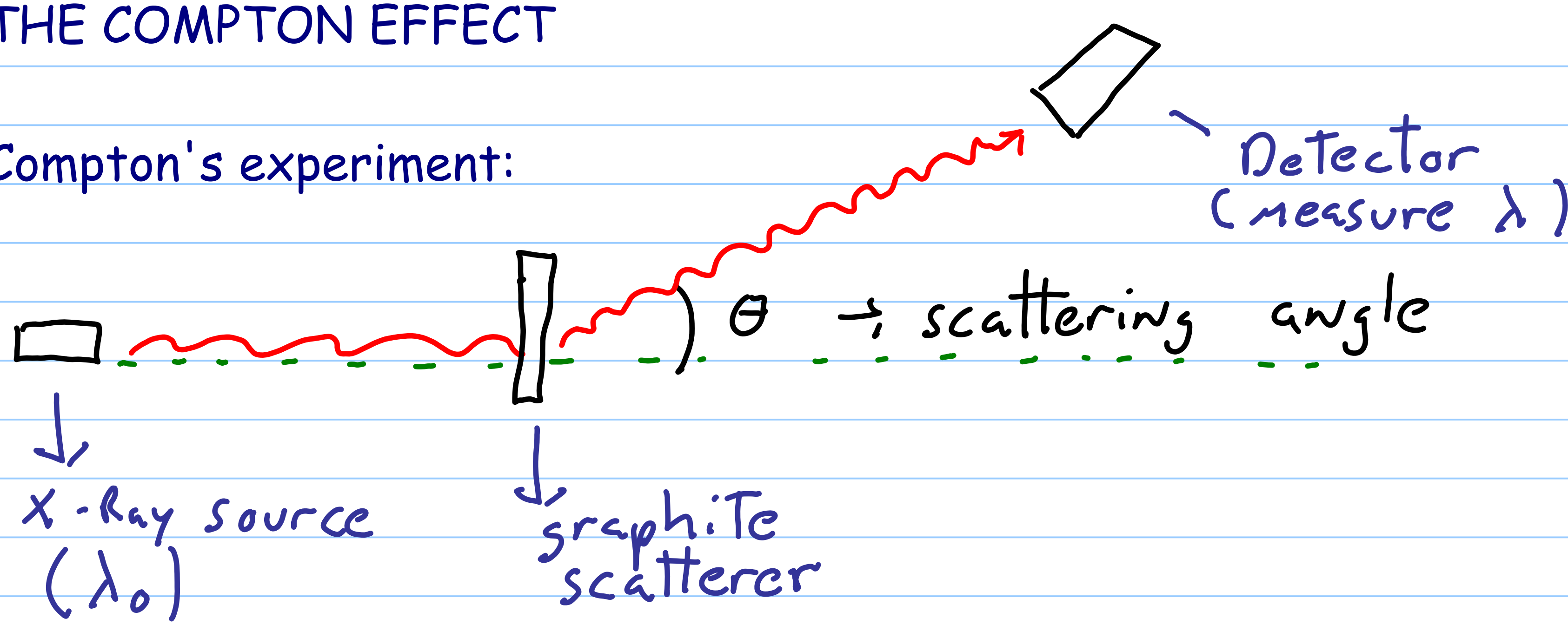
$$\Rightarrow \quad h = \frac{(E_1 - E_2)}{(\lambda_2 - \lambda_1)} \frac{\lambda_2 \lambda_1}{c}$$

Using $1\,\text{eV} = 1.6 \times 10^{-19}$ J, plug the numbers in:

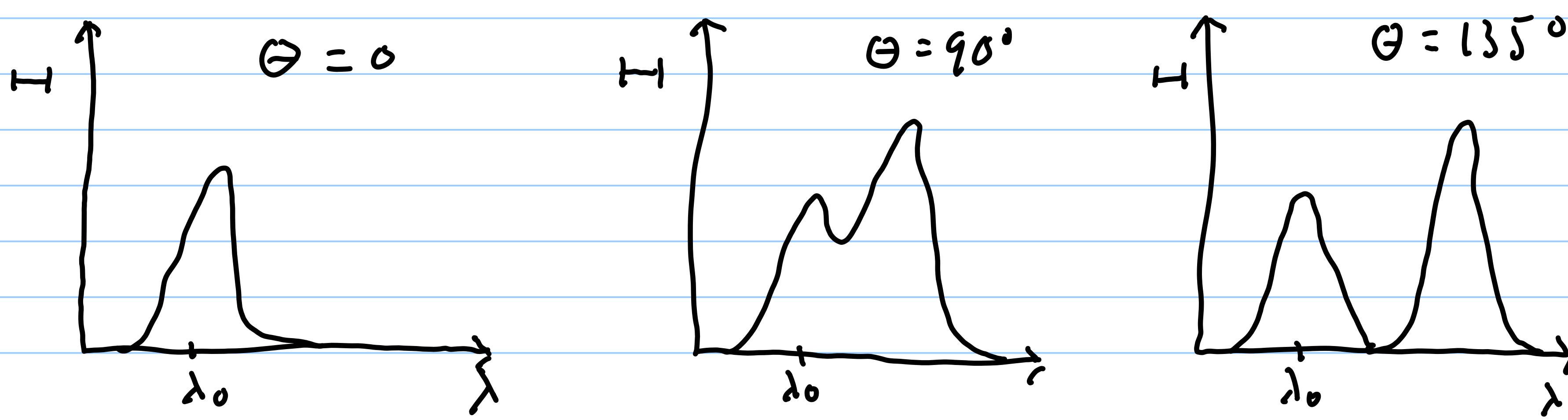
$$\approx 6.62 \times 10^{-34} \text{ J}\cdot\text{s}$$

THE COMPTON EFFECT

Compton's experiment:



With classical physics we expect the scattered radiation to be at the same wavelength as the incident radiation (Thomson scattering). What is observed is:



To understand what is going on, we need special relativity.

Recall : $E = \gamma m c^2$ (1) $\rightarrow \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

Kin. E : $T = E - mc^2$ (2)

momentum: $p = \gamma m v$ (3)

$E^2 = m^2 c^4 + p^2 c^2$ (4)

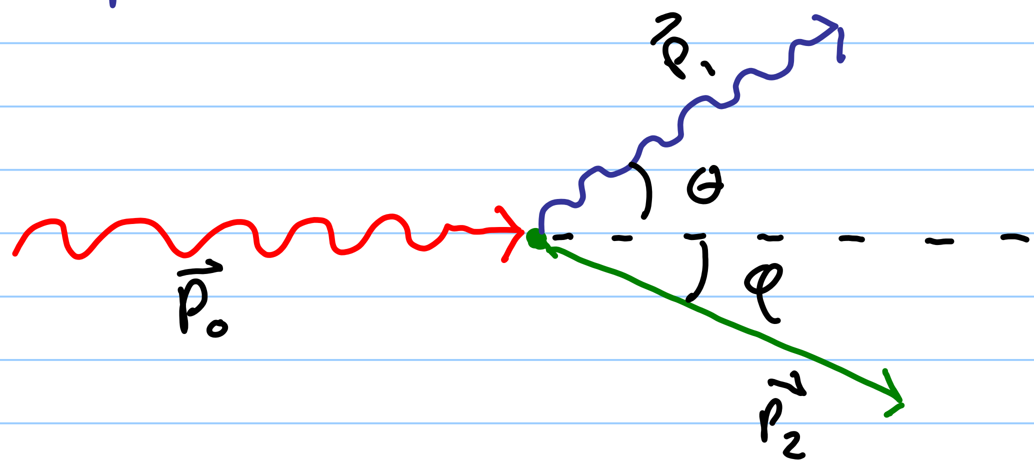
What is the momentum of a photon?

From equations above, $m_\gamma = 0$

From (4) $\Rightarrow$ $$p = E/c = \frac{h\nu}{c} = \frac{h}{\lambda}$$ (5)

Remember this

Compton Effect (cont.)



θ : angle of scattered rad.
φ : " of recoil e

by conservation of momentum: $\vec{p}_0 = \vec{p}_1 + \vec{p}_2$

$$p_0 = p_1 \cos\theta + p_2 \cos\varphi \quad (1)$$
$$0 = p_1 \sin\theta - p_2 \sin\varphi \quad (2)$$

let's get rid of φ

(4) $p_2^2 \cos^2\varphi = (p_0 - p_1 \cos\theta)^2 = p_0^2 + p_1^2 \cos^2\theta - 2p_0 p_1 \cos\theta$

(5) $p_2^2 \sin^2 = p_1^2 \sin^2\theta$

Add (4), (5) $\rightarrow$ $$p_2^2 = p_0^2 + p_1^2 - 2p_0 p_1 \cos\theta \quad (6)$$

Compton Effect (cont.)

Conservation of energy yields:

$$E_0 + mc^2 = E_1 + \sqrt{m^2c^4 + p_2^2c^2} \qquad (7)$$

E_0: initial energy of photon

mc^2: electron rest energy

Kinetic energy of electron must be equal to energy loss of photon:

$$E_k = E_0 - E_1 = c(p_0 - p_1) \qquad (8)$$

So we have:

$$c(p_0 - p_1) = \sqrt{m^2c^4 + p_2^2c^2} - mc^2 \qquad (9)$$

$$mc^2 + c(p_0 - p_1) = \sqrt{m^2c^4 + p_2^2c^2} \qquad (10)$$

$$\cancel{m^2c^4} + c^2(p_0 - p_1)^2 + 2mc^3(p_0 - p_1) = \cancel{m^2c^4} + p_2^2c^2 \qquad (11)$$

$$p_2^2 = (p_0 - p_1)^2 + 2mc(p_0 - p_1) \qquad (12)$$

$$\boxed{= p_0^2 + p_1^2 - 2p_0p_1 + 2mc(p_0 - p_1)} \qquad (13)$$

Compton Effect (cont.)

Use (6) (cons. of momentum) and
(13) (cons. of energy) to get

$$mc(p_0 - p_1) = p_0 p_1 (1 - \cos\theta) \quad (14)$$

$$= 2 p_0 p_1 \sin^2(\theta/2) \quad (15)$$

We need to express this formula in terms of λ_0, λ_1: $\lambda_0 = h/p_0$, $\lambda_1 = h/p_1$

multiply both sides by: $\frac{h}{mc p_0 p_1}$

$$\frac{h}{p_1} - \frac{h}{p_0} = 2 \frac{h}{mc} \sin^2(\theta/2) \quad (16)$$

$$\boxed{\lambda_1 - \lambda_0 = 2 \lambda_c \sin^2(\theta/2)} \quad (17)$$

λ_c : Compton wavelength

How does one explain unmodified λ_0?

Compton Effect (cont.)

Examples: γ rays with energies $\gg m_e c^2$ are backscattered from electron target. Calculate wavelength shift

$$\Delta\lambda = \lambda_1 - \lambda_0 = \frac{2h}{mc}\sin^2(\theta/2)$$

backscattered $\Rightarrow \theta = \pi \Rightarrow$ $\boxed{\Delta\lambda = \frac{2h}{mc}}$

Let's show that $E_1 = \frac{m_e c^2}{2}$ for any E_0

$$E_1 = \frac{hc}{\lambda_1} \quad \text{and} \quad \lambda_1 - \lambda_0 = \frac{2h}{m_e c}, \quad \lambda_1 = \lambda_0 + \frac{2h}{m_e c}$$

$$\rightarrow \quad E_1 = \frac{hc}{\lambda_0 + \frac{2h}{m_e c}} \quad \left(\text{mult. by } m_e c/h \,/\, m_e c/h\right)$$

$$E_1 = \frac{m_e c^2}{\frac{\lambda_0 m_e c}{h} + 2} = \frac{m_e c^2}{\frac{hc}{E_0}\frac{m_e c}{h} + 2} = \frac{m_e c^2}{\frac{m_e c^2}{E_0} + 2}$$

$$E_0 \gg m_e c^2 \Rightarrow \boxed{E_1 = m_e c^2/2}$$