LECTURE 5: BOHR MODEL OF THE HYDROGEN ATOM

Goal of the lecture: to understand how Bohr explained the spectrum of the hydrogen atom using Rutherford's model, Planck's quanta and Einstein's photons

What I expect you to learn:

- Thomson's atomic model
- Rutherford's model
- why Rutherford's model is incompatible with classical physics
- how to derive hydrogen spectrum from Bohr's quantisation of of angular momentum
- why Carlsberg is a respectable brewing company

Corresponds to section 1.4 of textbook

What did people know about the atom and when did they know it??

I will not go through the history of the concept of the atom. I will just mention 4 atomic models and spend most of the time on Bohr's model.

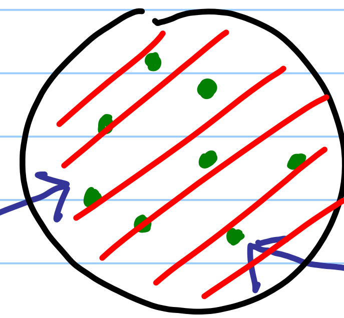
Democritus first proposed the idea of the atom (meaning indivisible) in the 5th century B.C. In this model, matter is composed of tiny indivisible particles in constant motion. Aristotle opposed the theory and it took > 1500 years for the idea to regain popularity.

Modern atomic theory began 200 years ago with the work of Dalton: all atoms of an element are of the same size and weight. Faraday's electrolysis experiments supported Dalton's theory.

In 1897, J.J Thomson discovered the electron and proceeded to develop a theoretical model of the atom

The Thomson model:

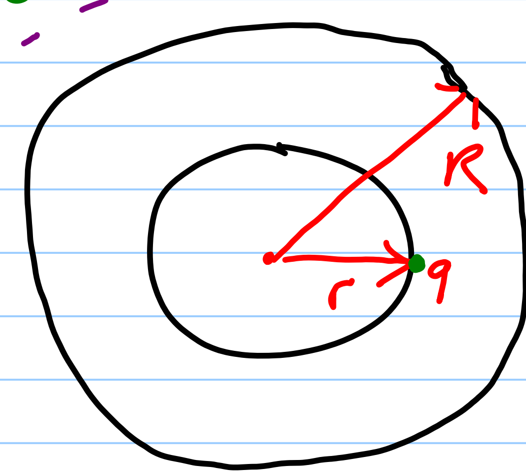
BALL OF POSITIVELY CHARGED MATERIAL



ELECTRONS EMBEDDED

The one electron Thomson atom:

by Gauss's Law, electron feels only the positive charge up to radius r



e experiences force from

$$\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} \cdot Q$$

total positive charge $= Q$

R : atomic radius, known at the time

SO : HOW WOULD AN ELECTRON MOVE IN THIS MODEL? WHAT EM WAVES WOULD IT EMIT?

One electron thomson atom (cont.)

$F = ma$

Coulomb's law:

$$\boxed{F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}}$$

1. $F = \frac{1}{4\pi\varepsilon_0} q \cdot \frac{r^3}{R^3} Q \cdot \frac{1}{r^2}$

$\Rightarrow m\frac{d^2r}{dT^2} = \frac{qQr}{4\pi\varepsilon_0 R^3}$ (2), $Q = e$, $q = -e$

we have: $\frac{d^2r}{dT^2} = \frac{-e^2 r}{m 4\pi\varepsilon_0 R^3}$ (3) MATH INTERLUDE

A solution: $r = A\sin 2\pi\nu T$

(3) becomes $-4\pi^2\nu^2 A\cancel{\sin 2\pi\nu T} = -\frac{e^2 A\cancel{\sin 2\pi\nu T}}{m 4\pi\varepsilon_0 R^3}$

(4) $\boxed{\nu = \frac{1}{2\pi}\sqrt{\frac{e^2}{4\pi\varepsilon_0 m R^3}}}$, plug numbers $\rightarrow$ $\nu = 2\times 10^{15}\,\text{Hz}$

$\lambda = 150\text{ nm}$

Green: $\sim 500\text{ nm}$

MATH INTERLUDE...

An equation you should know (or learn) how to solve:

$$\frac{d^2r}{dt^2} + K^2 r = 0 \quad \text{or} \quad \frac{d^2r}{dt^2} = -K^2 r$$

Solution: $r = A\sin(KT)$

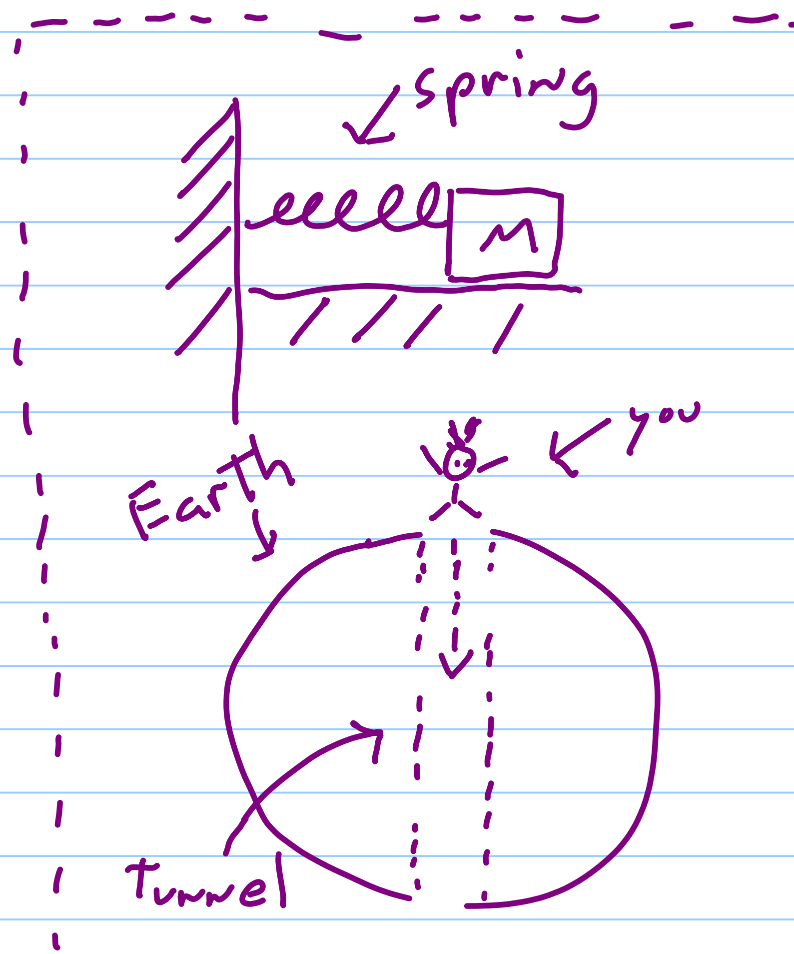
let's try it:

$$\frac{dr}{dT} = AK\cos(Kt)$$

$$\frac{d^2r}{dT} = -AK^2\sin(Kt)$$

$$\underbrace{-AK^2\sin(KT) = -K^2 A\sin(KT)}$$

amplitude arbitrary



THOMSON'S ATOM (CONT.)

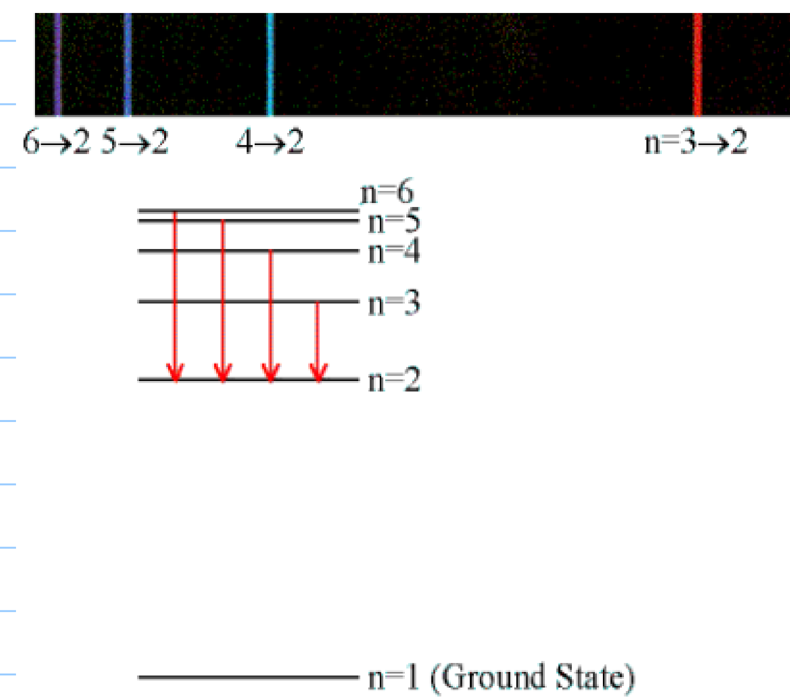
We got $\lambda \sim 150$ nm (not too far off)

But, does not explain **atomic spectra**.

ATOMIC SPECTRA

LET'S TAKE A LOOK AT THE HYDROGEN ATOM SPECTRUM:

BALMER SERIES:



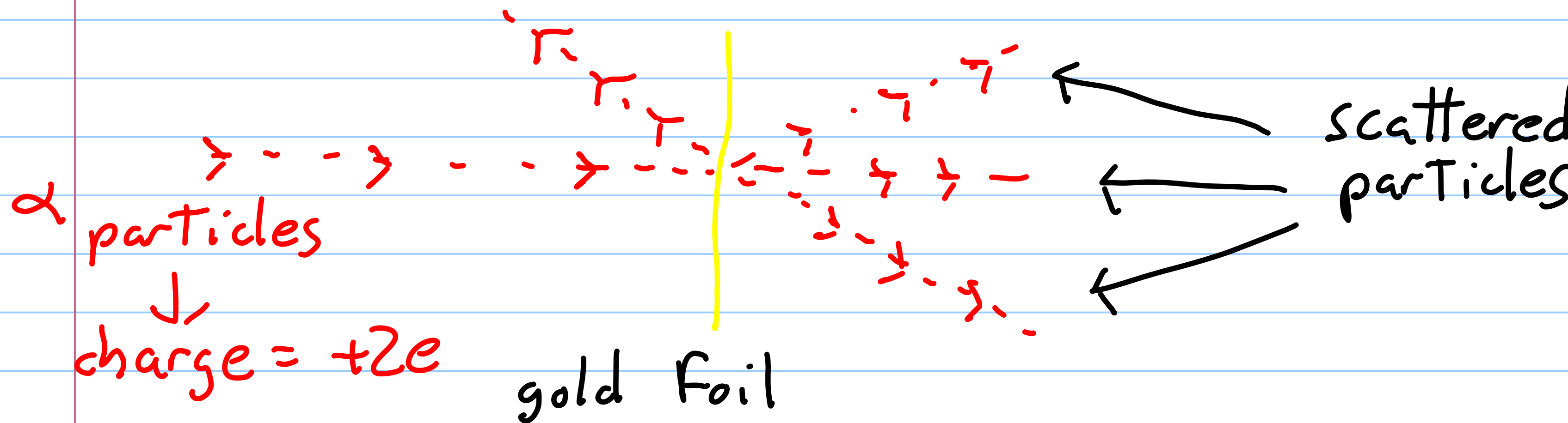
Rydberg constant

$$\frac{1}{\lambda_n} = R_H \left(\frac{1}{4} - \frac{1}{n^2} \right)$$

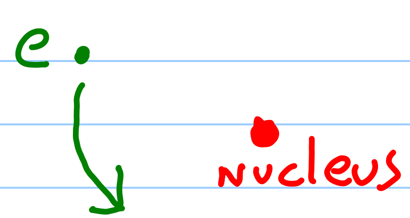
$n = 3, 4, 5, \ldots$

THE RUTHERFORD ATOM

GEIGER - MARSDEN experiment ~ 1909



RUTHERFORD POSTULATED THAT ALL THE POSITIVE CHARGE OF THE ATOM IS CONCENTRATED IN A NUCLEUS OF VERY SMALL DIMENSION. ALMOST ALL THE MASS RESIDES IN NUCLEUS:



$\boxed{\text{"Planetary" model implies electron should radiate and collapse on nucleus in < 1 ns}}$

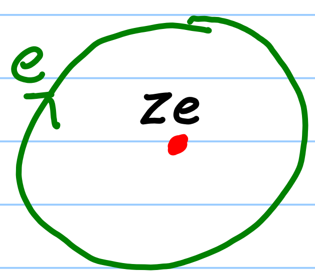
oops...

BOHR'S MODEL

- → ASSUME CIRCULAR ORBITS
- → ASSUME ONLY CERTAIN ORBITS ARE ALLOWED
- → ASSUME ELECTRON DOES NOT RADIATE IN ORBIT, RADIATION OCCURS WHEN ELECTRON CHANGES ORBIT
- → IF PHOTON IS ABSORBED BY ATOM, THEN

$$h\nu = E_i - E_f$$

initial state — final state of the atom



$$\boxed{\frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}} \quad (5)$$

Quantised Angular momentum: $\boxed{mvr = n\hbar}$ (6)

$n = 1, 2, 3 \ldots$

BOHR'S MODEL (cont.)

$$\boxed{\frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}} \quad (5) \qquad \boxed{mvr = n\hbar} \quad (6)$$

From (5) AND (6) WE GET:

$$v = \frac{Ze^2}{(4\pi\varepsilon_0)\hbar n} \quad (7) \qquad \text{AND} \qquad r = \frac{4\pi\varepsilon_0 \hbar^2 n^2}{Zme^2} \quad (8)$$

Energy of ELECTRON : $E = T + V$

Kinetic ↵ ↳ Potential

$$T = \frac{mv^2}{2} = \frac{m}{2\hbar^2}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2 \frac{1}{n^2} \quad (9)$$

$$V = \left(\frac{-Ze^2}{4\pi\varepsilon_0}\right)\frac{1}{r} = -\frac{m}{\hbar^2}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2 \frac{1}{n^2} \quad (10)$$

$$\boxed{E_n = -\frac{m}{2\hbar^2}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2 \frac{1}{n^2}} \quad (11) \qquad n = 1, 2, 3, \ldots$$

BOHR'S MODEL (cont)

PHOTON ENERGIES DETERMINDED BY

$$h\nu = E_b - E_a \qquad n_b > n_a$$

$$= -\frac{m}{2\hbar^2}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2 \frac{1}{n_b^2} - -\frac{m}{2\hbar^2}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2 \frac{1}{n_a^2} \quad (12)$$

$$= \frac{m}{2\hbar^2}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2\left(\frac{1}{n_a^2} - \frac{1}{n_b^2}\right)$$

$$\nu = \frac{m}{4\pi\hbar^3}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2\left(\frac{1}{n_a^2} - \frac{1}{n_b^2}\right) \quad (13)$$

$$\frac{1}{\lambda} = \boxed{\frac{m}{c4\pi\hbar^3}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2}\left(\frac{1}{n_a^2} - \frac{1}{n_b^2}\right) \quad (13)$$

Recall Balmer's Formula:

$$\frac{1}{\lambda} = \boxed{R_H}\left(\frac{1}{4} - \frac{1}{n^2}\right)$$

BOHR'S MODEL (CONT.)

HOW MUCH ENERGY IS REQUIRED TO IONISE H?

$Z = 1$, $n_a = 1$ $n_b = \infty$

(14) $$E_{\text{ionise}} = \frac{m}{2\hbar^2}\left(\frac{Z e^2}{4\pi\varepsilon_0}\right)^2 = \boxed{13.6 \text{ eV}}$$

RADIUS of electron orbit in H (ground state)

$n = 1$

(15) $$a_0 = \frac{(4\pi\varepsilon_0)\hbar^2}{m e^2} = 5.3 \times 10^{-11}$$

VELOCITY OF ELECTRON IN H (GROUND STATE)

(16) $$v_0 = \frac{e^2}{(4\pi\varepsilon_0)\hbar} = \alpha c \rightarrow \boxed{\alpha = \frac{e^2}{(4\pi\varepsilon_0)\hbar c}} \simeq 1/137$$

↳ no dimensions

$$\simeq \frac{c}{137} \rightarrow \text{non-relativistic}$$

BOHR'S ATOM (CONT.)

WE HAVE NEGLECTED THE NUCLEAR MASS UP UNTIL NOW... WE REMEDY THE SITUATION:

$$L = \mu v r = n\hbar \quad , \quad \mu = \frac{mM}{m+M} \quad \text{(reduced mass)}$$

$$\Rightarrow \quad E_n = -\frac{\mu}{2\hbar^2}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)\frac{1}{n^2}$$

$$r = \frac{(4\pi\varepsilon_0)\hbar^2 n^2}{Z\mu e^2} = \frac{n^2 m}{Z\mu} a_0 = \frac{n^2}{Z} a_\mu$$

Rydberg's constant $\rightarrow \frac{\mu}{m} R_\infty = \frac{1}{1+\frac{m}{M}} R_\infty$

THE BOHR ATOM (cont.)

Bohr's correspondance principle: quantum theory results must tend asymptotically to those obatined from classical physics in the limit of large quantum numbers.

EXAMPLE:

RADIATION EMITTED BY ONE-ELECTRON ATOM IN A TRANSITION FROM E_n TO E_{n-1} where n is large. FROM 13 WE HAVE:

$$\nu = \frac{m}{4\pi\hbar^3}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

For $n \gg 1$, we have

$$\frac{1}{(n-1)^2} - \frac{1}{n^2} = \frac{n^2 - (n-1)^2}{n^2(n-1)^2} = \frac{n^2 - n^2 - 1 + 2n}{n^2(n-1)^2}$$

$$= \frac{(2n-1)}{n^2(n-1)^2} = \frac{n(2 - 1/n)}{n^2 \cdot n^2(1-1/n)^2} \simeq \boxed{\frac{2}{n^3}}$$

BOHR ATOM (CONT.)

$$\boxed{\nu \simeq \frac{m}{2\pi\hbar^3}\left(\frac{Ze^2}{4\pi\varepsilon_0}\right)^2 \frac{1}{n^3}}$$

WITH CLASSICAL PHYSICS WE HAVE:

ν_{CL} = FREQUENCY OF ROTATION

$= \boxed{\dfrac{v}{2\pi r}}$, USING (7) AND (8) FOR v AND r

WE GET: $\boxed{\dfrac{m}{2\pi\hbar^3}\left(\dfrac{Ze^2}{4\pi\varepsilon_0}\right)\dfrac{1}{n^3}}$

→ WORKS WHEN n is large

cow enjoying sunset

large n system

LIMITATIONS OF BOHR's MODEL:

-It does not explain why electrons do not radiate in their stationary orbits

-Quantisation of angular momentum is introduced in ad hoc fashion

-Model cannot be extended to atoms with more than one electron

-No method to calculate rate of transitions

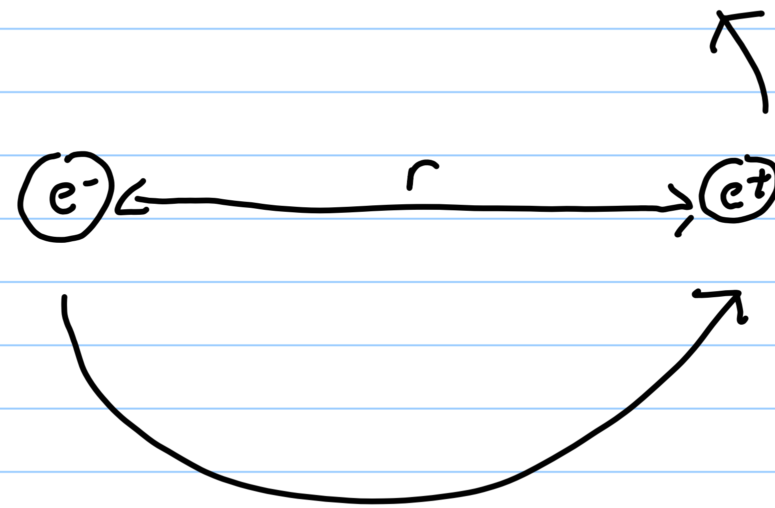
We will need quantum mechanics to address the model's shortcomings

More on Bohr's model in next lecture. We'll also cover Franck Hertz experiement and De Broglie's hypothesis

Example: Calculate the energy levels of positronium by using Bohr's Model of the atom

What's positronium? an electron and it's antimatter partner the positron in a bound state

We need To use μ ...



$$\mu = \frac{Mm}{m+M} = \frac{m_e^2}{2m_e} = \frac{m_e}{2}$$

$$\boxed{L = \mu v r = n\hbar} \quad (A)$$

BALANCE OF FORCES

$$\frac{\mu v^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \Rightarrow \mu v^2 r = \frac{e^2}{4\pi\varepsilon_0}$$

USING (A) : $v = \frac{e^2}{4\pi\varepsilon_0} \cdot \frac{1}{n\hbar}$

$$r = \frac{e^2}{4\pi\varepsilon_0 \mu v^2}$$

Example (cont.)

USE (A) again

$$r = \frac{e^2}{4\pi\varepsilon_0 \mu} \cdot \frac{\mu^2 r^2}{n^2 \hbar^2} \Rightarrow r = \frac{n^2 \hbar^2 \cdot 4\pi\varepsilon_0}{\mu e^2}$$

TOTAL ENERGY: $E = T + V$

$$T = \tfrac{1}{2}\mu v^2 \qquad V = \frac{-e^2}{4\pi\varepsilon_0 r}$$

$$E = \tfrac{1}{2}\mu \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{1}{n^2\hbar^2} - \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \cdot \frac{\mu}{n^2\hbar^2}$$

$$= -\tfrac{1}{2}\mu \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{1}{n^2\hbar^2} = \boxed{-\frac{m_e}{4}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \cdot \frac{1}{n^2\hbar^2}}$$