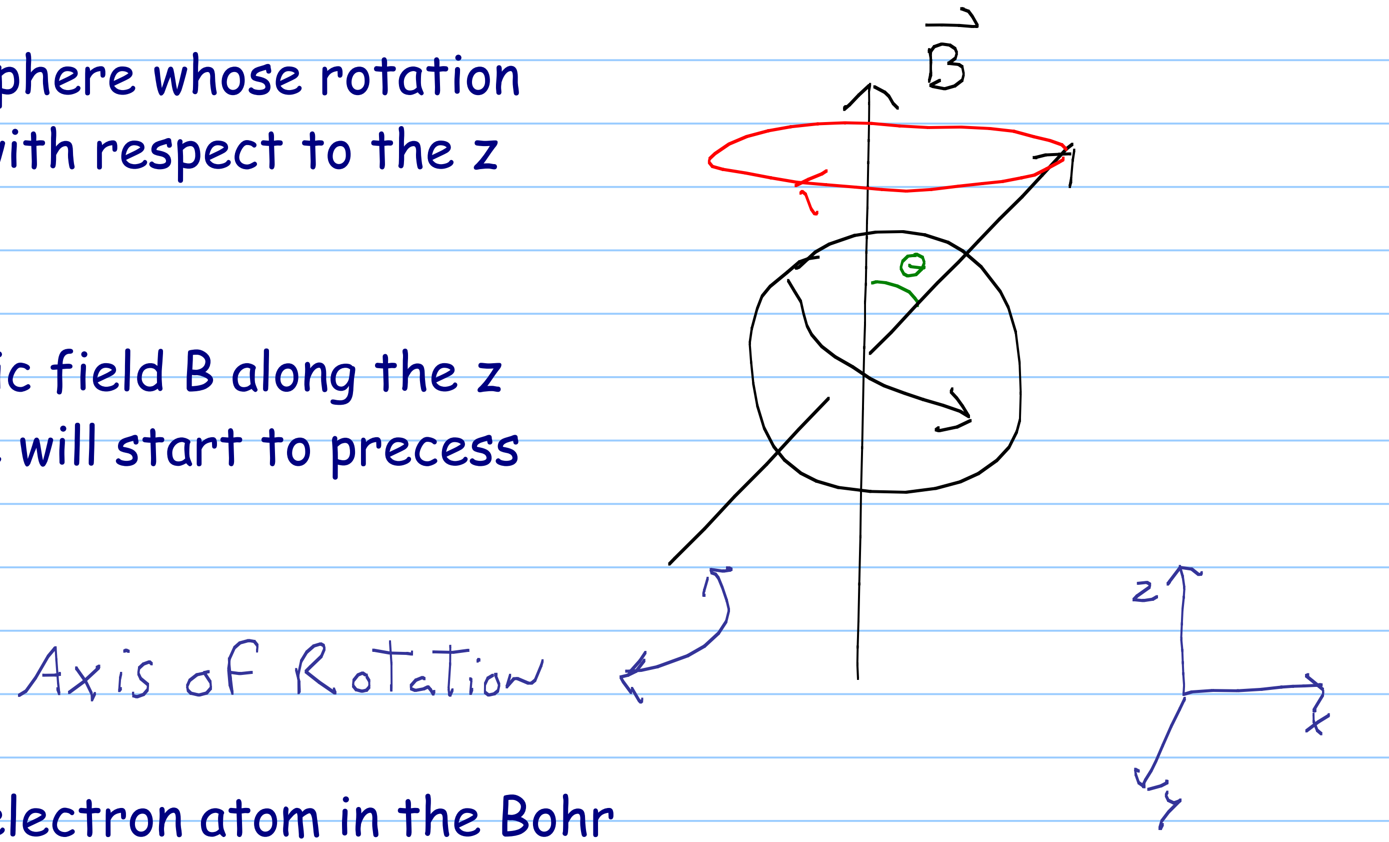
LECTURE 7: The Stern-Gerlach Experiment and Recap of Origins of Quantum Theory

Goals of the lecture:
- finish and recap what we have learned so far regarding the origins of quantum theory
- begin discussion of quantum mechanics with the introduction of the wave function.

What I expect you to learn:
- how to describe Stern-Gerlach experiment
- What are the results and why classical physics does not explain them
- What is the wave function, and how we interpret the wave function

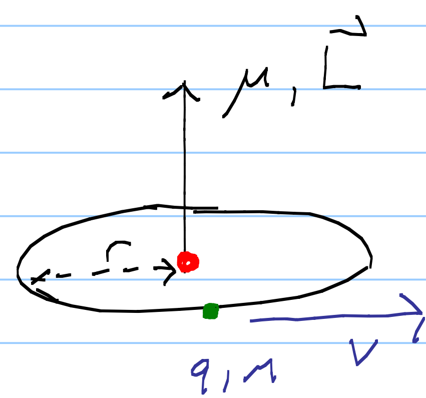
Corresponds to sections 1.5, 2.1, and 2.2 of textbook

Consider a charged sphere whose rotation axis is at an angle θ with respect to the z axis

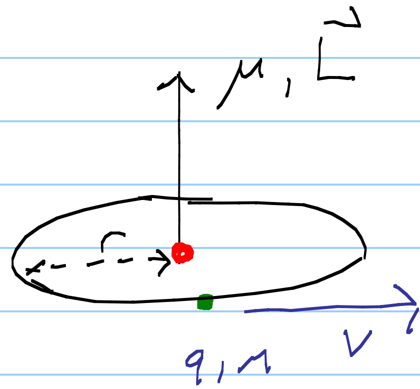


By applying a magnetic field B along the z direction, the sphere will start to precess about the z axis

Now consider a one-electron atom in the Bohr Model:



q : charge of electron = -e
m : mass of electron = m_e
r : radius of orbit
v : velocity of electron
μ : magnetic moment of atom
L : angular momentum



How long does it take for the electron to orbit?

$$\rightarrow \frac{2\pi r}{v} = t_{orbit}$$

$$\Rightarrow \text{current } I = q/T_{orbit} = \frac{qv}{2\pi r}$$

You'll see in E and M that the magnetic moment for a dipole is

$$\mu = I \cdot \text{area} = \pi r^2 I$$

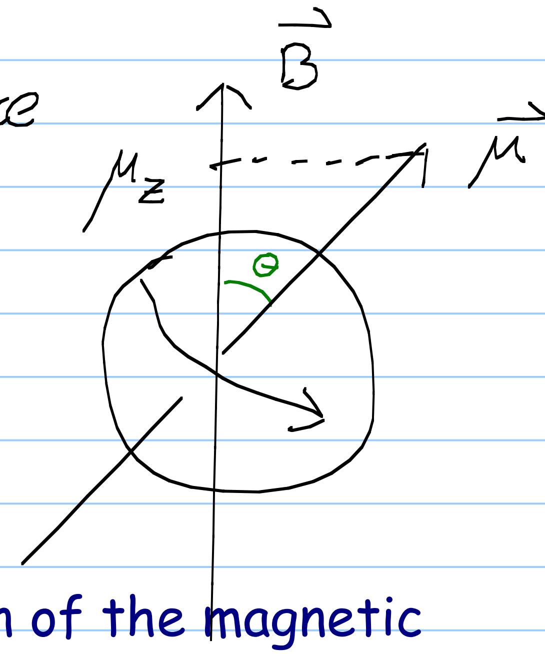
$$\rightarrow \mu = \pi r^2 \cdot \frac{qv}{2\pi r} = \boxed{\frac{qvr}{2}}$$

The orbital angular momentum: $L = mvr$

$$\Rightarrow \mu = \frac{q}{2m} L$$, with $\hbar$ natural unit of L,

we can write: $\mu_B = \frac{e\hbar}{2m_e}$ $\rightarrow$ μ_B is "Bohr magneton"

If $\vec{B}$ is uniform, the magnetic dipole will experience no net force



If $\vec{B}$ is not uniform, net force will be given by:

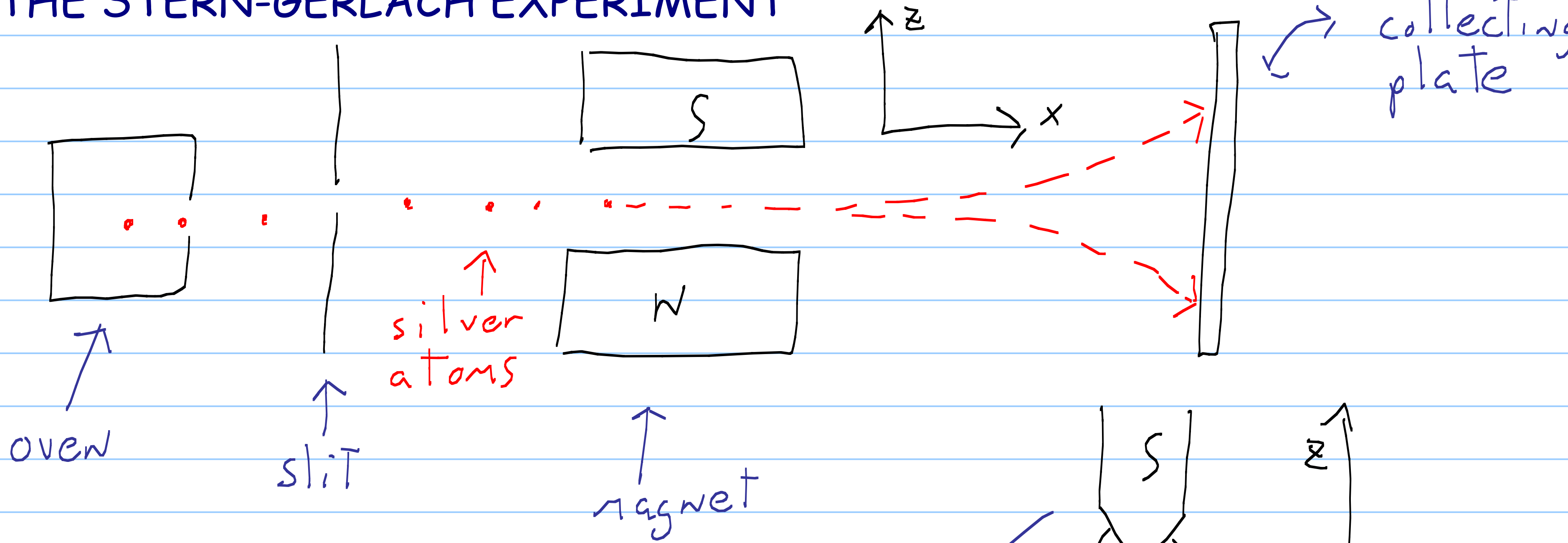
$$F_z = \mu_z \frac{\partial B_z}{\partial z}$$

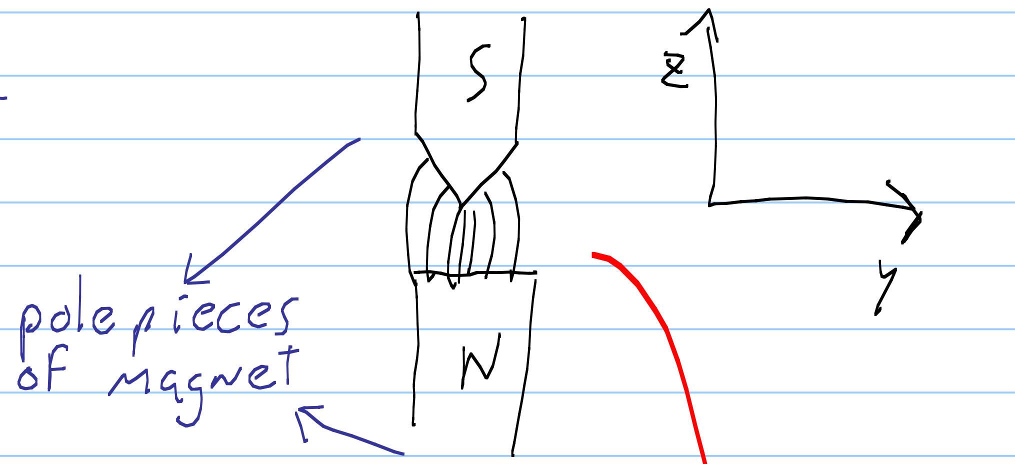
With classical physics, all values of the z projection of the magnetic dipole moment should be allowed.

From what we have learned from quantum physics, we know that the angular momentum will be quantised. BUT, there are no constaints (yet) on what are the allowed values of the projection of the angular momentum

With the Stern-Gerlach experiment we'll see that Lz is quantised.
This is called "space quantisation"

THE STERN-GERLACH EXPERIMENT





non-uniform B field!

- Oven emits silver atoms

- Atoms go through magnet with non-uniform B field along z direction

- Trajectory of atoms is bent is proportion to μ_z

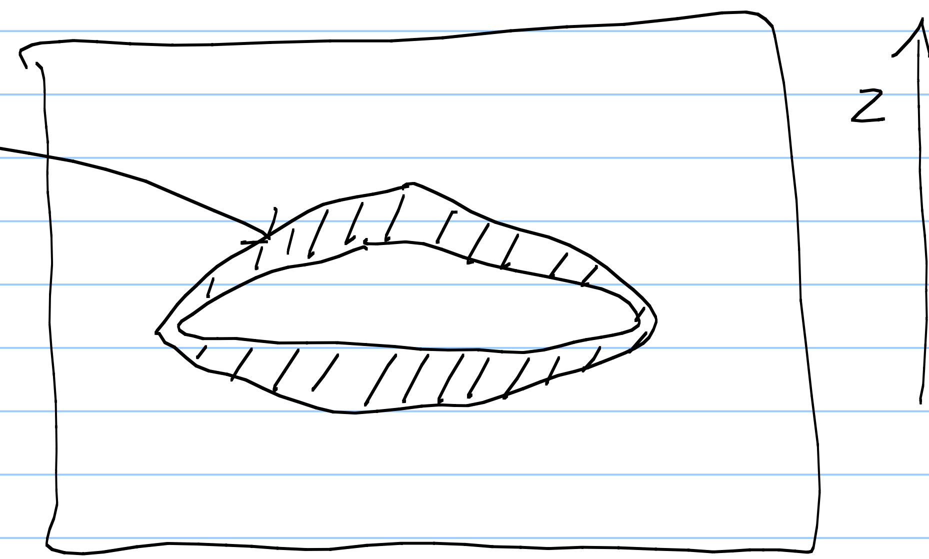
- Observe pattern of silver atoms on collecting plate

Observed pattern of silver atoms:

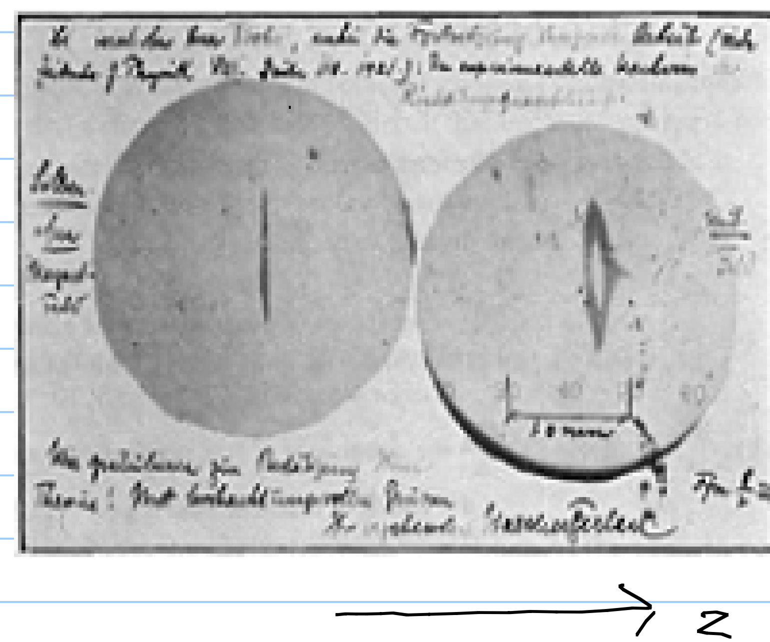
silver deposit

Observe: region where no atoms strike

$\Rightarrow$ quantisation of μ_z



A postcard from Gerlach to Bohr:
"...the experimental proof of directional quantisation. We congratulate you on the confirmation of your theory!"

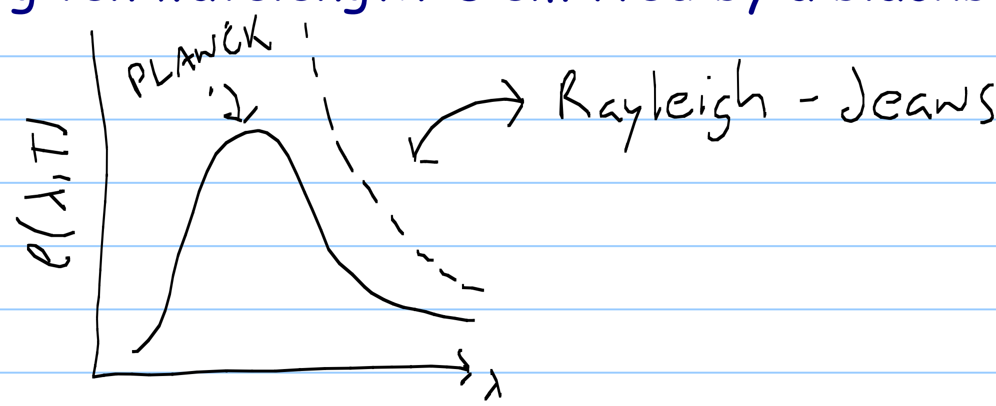


Space quantisation experiment in PHY225: Zeeman effect

RECAP of CHAPTER 1:

Blackbody Radiation:

How much radiation at a given wavelength is emitted by a blackbody at temperature T?



Problem: Classical physics predicts infinite amount of energy emitted! short wavelengths are responsible -> "Ultraviolet Catastrophe"

Planck's solution: The amount of energy emitted at a given wavelength is quantised

$$\boxed{E = nh\nu}$$

Perhaps for some yet to be understood reason atoms in cavity wall can only emit em radiation of a given amplitude?

RECAP of CHAPTER 1:

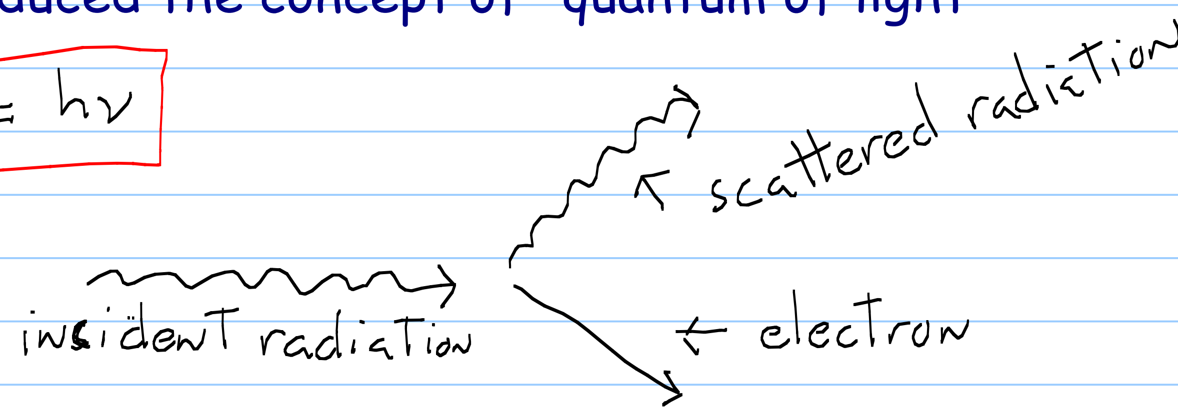
Photoelectric effect: electrons are emitted when light shines on a polished surface

- The problem: classical physics cannot explain the observations e.g.:
  - threshold frequency for electron emission
  - kinetic energy of electrons depends on frequency of light but not on intensity

- Einstein's solution: introduced the concept of "quantum of light" with energy:

$$\boxed{E_\gamma = h\nu}$$

Compton Scattering:



The idea of light as a particle took more time to mature.

Compton's scattering experiment gave clear evidence that em radiation behaved as particles with momentum.

RECAP of CHAPTER 1::

Rutherford Atom: a "planetary-type" system with the electron orbiting a heavy nucleus

problems with the model:

- -lifetime of such a system should be < 1 ns!
- -does not explain line spectra

Bohr's postulates:

- -Only certain electron orbits are allowed i.e. atom has a discreet set of stationary states -> implies that angular momentum is quantised
- -em radiation emitted or absorbed when electron goes through a transition between states

These postulates explain the line spectra

Franck-Hertz experiment confirms that internal energy states of atoms are quantised.

RECAP of CHAPTER 1::

The concept of light as electromagnetic waves was established prior to the onset of the quantum revolution

Various experiments and observations forced people to accept that light also has a particle nature

Louis de Broglie's hypothesis extended this matter-wave duality to ordinary particles: $\lambda = \frac{h}{mv}$

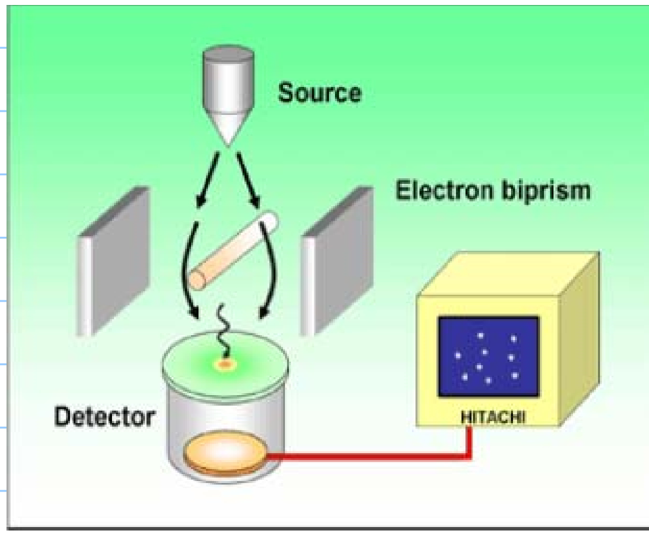
This hypothesis was confirmed with the Davisson-Germer and other experiments.

We know how to deal with particles and we know how to deal with waves (and you are learning about it in the waves course).

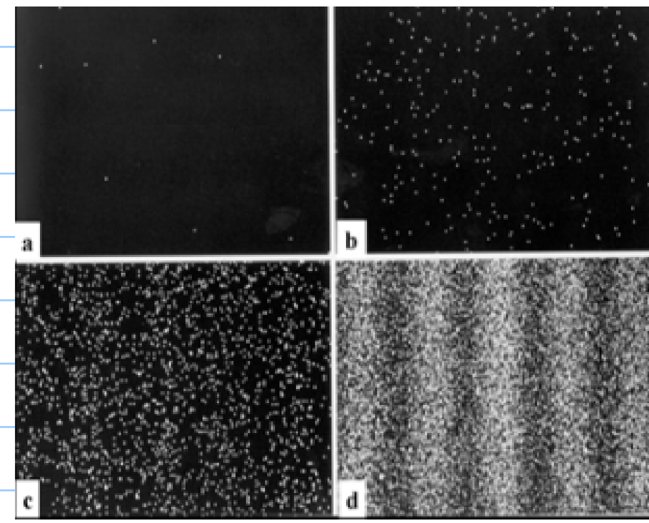
How do we deal with these particle-wave hybrids from a mathematical and physical standpoint?

Recall the Electron Biprism:

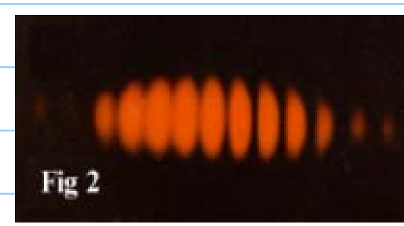
The experiment:



The results:



What one can observe is Young's double-slit experiment with light:



If the electron is a wave, it's a wave of what? How do we interpret it?

In Young's double slit experiment, the interference pattern represents the intensity of light on the screen (equal to the sum of the amplitudes squared)

For electrons, the pattern we see represents the probablility distribution for a single electron to be detected at a given point on the screen.

this would imply that the probablility of finding the electron is equal to the square of some wave function

this wave function must represent the probablility amplitude for finding an electron at a given point in space

$$P(x,y,z,t) \propto |\psi(x,y,z,t)|^2$$

↳ Probability of finding a particle at (x,y,z,t)

We'll call $\psi(x,y,z,t)$ the "wave function"

For a double-slit experiment where:

slit A is open : $P_A \propto |\psi_A|^2$

slit B is open : $P_B \propto |\psi_B|^2$

both slits are open : $P \propto |\psi_A + \psi_B|^2$

So, $P(r,t)\,dr = |\psi(r,t)|^2\,dr$ represents probability of finding particle within volume dr at time t.

$P(r,t)$ is the probability density

$$\Rightarrow \int |\psi(r,t)|^2\,dr = 1$$

$$\rightarrow \quad |\psi(r,t)|^2 = \psi^*(r,t)\,\psi(r,t)$$

We saw that the interference effect was obtained by adding two waves together. With the right setup, one could add an arbitrary number of waves:

$$\psi = c_n \psi_n$$

In the case of the double-slit experiment:

$$\psi = c_1 \psi_1 + c_2 \psi_2 \quad , \quad \text{in complex notation:}$$

$$\psi_1 = |\psi_1| e^{i\alpha_1} \qquad \psi_2 = |\psi_2| e^{i\alpha_2}$$

$$|\psi|^2 = |c_1 \psi_1|^2 + |c_2 \psi_2|^2 + 2\,\mathrm{Re}\{c_1 c_2^* |\psi_1||\psi_2| \exp[i(\alpha_1 - \alpha_2)]\}$$

Example: double-slit experiment with mono-energetic electrons. Detectors are placed on a vertical screen along y axis.

- When only slit 1 is open, the probability amplitude on the screen is given by:

$$\psi_1(y,t) = \frac{A_1 e^{-i(ky-\omega t)}}{\sqrt{1+y^2}}$$

- When only slit 2 is open, we have:

$$\frac{A_2 e^{-i(ky+\pi y-\omega t)}}{\sqrt{1+y^2}} \qquad (A_1, A_2 \text{ are real})$$

What is the probability that the electron will hit the screen as a function of y when:

A) both slits are open and a light source is used to determine which slit the electron went through

B) both slits are open and there is no light source

Example (cont.)

A) $$\text{Probability} = |\psi_1(\cancel{x},t)|^2 + |\psi_2(\cancel{x},t)|^2$$

$$= \psi_1^*\psi_1 + \psi_2^*\psi_2$$

$$= A_1^* \frac{e^{i(ky-\omega t)}}{\sqrt{1+y^2}} \frac{A_1 e^{-i(ky-\omega t)}}{\sqrt{1+y^2}} + \frac{A_2^* e^{i(ky+\pi y-\omega t)}}{1+y^2} \frac{A_2 e^{-i(ky+\pi y-\omega t)}}{\sqrt{1+y^2}}$$

$$= \frac{A_1^2}{1+y^2} + \frac{A_2^2}{1+y^2} = \frac{A_1^2 + A_2^2}{1+y^2}$$

B) $$\text{Probability} = |\psi_1(y,t) + \psi_2(y,t)|^2$$

$$= \psi_1^*\psi_1 + \psi^{2*}\psi_2 + \psi_1^*\psi_2 + \psi_2^*\psi_1$$

① ②

First Two Terms were calculated above

① $$\frac{1}{1+y^2} \cdot A_1A_2\, e^{i(ky-\omega t)}\, e^{-i(ky+\pi y-\omega t)} = \frac{A_1A_2}{1+y^2} e^{-i\pi y}$$

② $$\frac{1}{1+y^2} A_2A_1\, e^{i(ky+\pi y-\omega T)}\, e^{-i(ky-\omega t)} = \frac{A_2A_1}{1+y^2} e^{i\pi y}$$

Example cont.

$$① + ② = \frac{A_1 A_2}{1+y^2}\left(e^{i\pi y} + e^{-i\pi y}\right) = 2\,\frac{A_1 A_2}{1+y^2}\cos \pi y$$

$$\boxed{\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}}$$