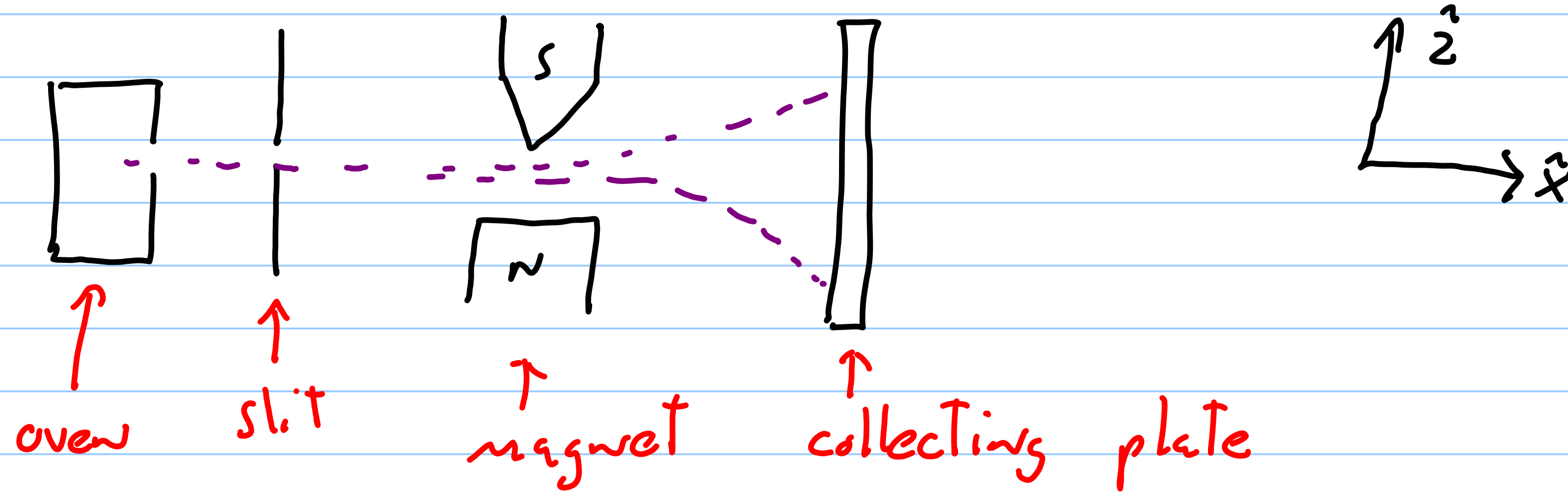
PRACTICE EXAM SOLUTIONS.

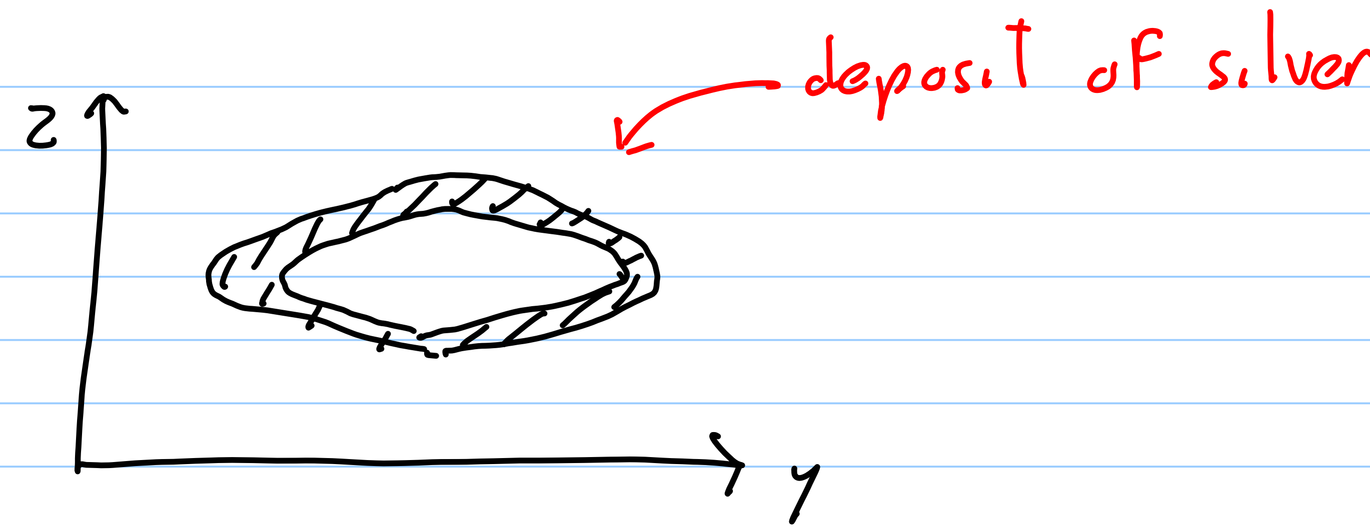
1 -

a) see page 35 of Textbook



An oven emits atoms of silver that are collimated using slits so that the atoms pass between the poles of a magnet as a narrow, parralel beam. The magnet poles are shaped to produce an inhomogeneous field. The atoms are deflected in the field and detected on a collecting plate

1 b)



1 c) → quantisation of angular momentum

→ "space quantisation"

→ intrinsic spin or spin $1/2$

d) Example problem page 16 of lecture 31

e) using result from d) : $P = \cos^2 \theta/2$

we have: $0°$, $P = 100\%$

$90°$, $P = 50\%$

$60°$, $P = 75\%$

2) a) $\langle \hat{H} \rangle = \sum_n |A_n|^2 E_n$

b) The probability of observing the system with an energy of E_n

c) and d) : example problem on page 11 of lecture 27.

3) This problem is equivalent to 6.13 in Textbook except we ask here for the Time-independent expression. See prob. set 5 solutions.

4) a) no. There will an uncertainty on the particle's momentum since $\Delta x \neq \infty$. This means that there will be a non-zero probability of finding the particle in the other half of the well. Also, $P(x)$ will evolve since a square wavefunction is the sum of many eigenfunctions.

b) : See lecture 22, page 19.

5)

a) $\vec{L} = \vec{r} \times \vec{p}$

b) $\hat{L} = \hat{r} \times \hat{p} = -i\hbar \, (\hat{r} \times \nabla)$

$= \hat{r} \times -i\hbar \frac{\partial}{\partial r}$

c) $\left(\sqrt{\frac{1}{7}}\right)^2 + A^2 + \left(\sqrt{\frac{2}{7}}\right)^2 = 1$

$A^2 = 1 - 3/7 \Rightarrow A^2 = 4/7 \Rightarrow A = \frac{2}{\sqrt{7}}$

d) $+\hbar$ corresponds to $m = +1$ which corresponds to state $|1,1\rangle$

$\hookrightarrow |\ell, m\rangle$

the coef. in front of $|1,1\rangle$ is $\sqrt{2/7}$.

Probability $= \left| \sqrt{2/7} \right|^2 = 2/7$

e) the state $|\psi\rangle$ can be written as

$$|\psi\rangle = \begin{pmatrix} \sqrt{2/7} \\ 2/\sqrt{7} \\ 1/\sqrt{7} \end{pmatrix}, \quad \langle \hat{L}_z \rangle = \langle \psi | \hat{L}_z | \psi \rangle$$

$$= \left(\sqrt{2/7},\ 2/\sqrt{2},\ 1/\sqrt{7} \right) \begin{pmatrix} \hbar & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hbar \end{pmatrix} \begin{pmatrix} \sqrt{2/7} \\ 2/\sqrt{7} \\ 1/\sqrt{7} \end{pmatrix}$$

$$= \hbar \left(\sqrt{2/7},\ 0,\ -1/\sqrt{7} \right) \begin{pmatrix} \sqrt{2/7} \\ 2/\sqrt{7} \\ 1/\sqrt{7} \end{pmatrix} = \hbar \cdot \left(2/7 - 1/7 \right)$$

$$= \hbar/7$$

$$\langle \hat{L}_x \rangle = \langle \psi | \hat{L}_x | \psi \rangle$$

$$= \left(\sqrt{2/7},\ 2/\sqrt{2},\ 1/\sqrt{7}\right) \begin{pmatrix} 0 & \frac{\hbar}{\sqrt{2}} & 0 \\ \frac{\hbar}{\sqrt{2}} & 0 & \frac{\hbar}{\sqrt{2}} \\ 0 & \frac{\hbar}{\sqrt{2}} & 0 \end{pmatrix} \begin{pmatrix} \sqrt{2/7} \\ 2/\sqrt{7} \\ 1/\sqrt{7} \end{pmatrix} \quad \ldots$$

$\langle L^2 \rangle$: system is in state $\ell = 1$

$$L^2 = \ell(\ell+1)\hbar^2 = 2\hbar^2$$

or

$$\left(\sqrt{2/7},\ 2/\sqrt{2},\ 1/\sqrt{7}\right) \begin{pmatrix} 2\hbar^2 & 0 & 0 \\ 0 & 2\hbar^2 & 0 \\ 0 & 0 & 2\hbar^2. \end{pmatrix} \begin{pmatrix} \sqrt{2/7} \\ 2/\sqrt{7} \\ 1/\sqrt{7} \end{pmatrix} = 2\hbar^2$$