UNIVERSITY OF TORONTO

Faculty of Arts and Science

DECEMBER 2006 PRACTICE EXAM

PHY256H1F

Exam Questions:

1. (25 points) The Stern-Gerlach Experiment

   - a) (4 points) Describe the setup of the Stern-Gerlach Experiment.
   - b) (3 points) Plot the result of this experiment (include axis labels).
   - c) (4 points) Give two important features of quantum physics that emerge from the result of this experiment.
   - d) (7 points) Find the eigenvalues and eigenstates of the spin operator $\hat{S}$ of an electron in the direction of a unit vector $\hat{n}$, where $\hat{n}$ lies in the "xz" plane.
   - e) (7 points) I use a Stern-Gerlach apparatus to select a "spin up" electron with respect to the z direction i.e. the eigenvalue of $\hat{S}_z$ that I obtain is $+\frac{\hbar}{2}$. If I measure the spin of the electron in the direction of the unit vector $\hat{n}$ given above, what values can I obtain and with what probabilities if the angle θ of $\hat{n}$ with respect to the z axis is: i) $\theta = 0$ degrees, ii) $\theta = 90$ degrees, iii) $\theta = 60$ degrees

2. (25 points) Postulates of Quantum Physics

   - for a particle in an infinite potential well, the expansion postulate states that the particle's wave function can be expanded in terms of eigenfunctions:

     $$\psi(x) = \sum_{n=1}^{\infty} A_n \psi_n(x)$$

     Here, the Schrodinger equation can be expressed as $\hat{H}\psi_n(x) = E_n\psi_n(x)$.

     – a) (5 points) Give the expression for the expectation value of $\hat{H}$ in terms of E_n and A_n

     – b) (5 points) What does $|A_n|^2$ represent physically?

   - A quantum mechanics postulate states that the time evolution of the state vector is given by the time-dependent Schrodinger equation. The state vector at a time t can be expressed as: $|\psi(t)>= exp(\frac{-i}{\hbar}\hat{H}t)|\psi(0)>$. Suppose a physical system is in the state $|\psi(0)>$: $\begin{pmatrix} 3/5 \\ 0 \\ 4/5 \end{pmatrix}$ and the Hamiltonian can be expressed as: $\hat{H} = \begin{pmatrix} 3\epsilon & 0 & 0 \\ 0 & 0 & 5\epsilon \\ 0 & 5\epsilon & 0 \end{pmatrix}$.

     – c) (8 points) For such a system, find what are the energies that can be measured and their respective probabilities.

     – d) (7 points) Using the eigenvalues of $\hat{H}$, write the column vector that represents $|\psi(t)>$.

3. (15 points) A particle of mass m is fixed at one end of a rigid rod of negligible mass and length R. The other end of the rod is attached at the origin such that the particle can rotate in the xy plane.

   - Write down the system's total energy in terms of its angular momentum L.

   - Write down the system's time-independent Schrodinger equation.

- Give the energy levels of the system.

4. (20 points) One-Dimensional Problems: a particle is known to be localized in the left half of a one-dimensional infinite potential well with walls at $x = \pm\frac{a}{2}$. The wave function is given by $\psi(x) = \sqrt{\frac{2}{a}}$ for -$\frac{a}{2} < x < 0$ and $\psi(x) = 0$ for $0 < x < \frac{a}{2}$.

   - a) (8 points) Will the particle remain localized on the left-hand side of the well at later times? why?
   - b) (12 points) Calculate the probability that an energy measurement will yield the ground state energy (n=1). Remember that the eigenfunctions for a similar infinite well with sides at x=0 and x=a are given by $\psi_n(x) = \sqrt{\frac{2}{a}} sin(\frac{n\pi x}{a})$.

5. (30 points) Orbital Angular Momentum

   - a) (5 points) Write the expression for the angular momentum vector in classical mechanics in terms of the position vector and the momentum vector.
   - b) (5 points) In the position representation, write the expression for the angular momentum operator in quantum mechanics in terms of the position operator and the momentum operator.
   - Consider a system which is the state: $|\psi >= \frac{1}{\sqrt{7}}|1,-1> + A|1,0>$ $+\sqrt{\frac{2}{7}}|1,1>$, where A is a real positive constant. The kets on the right-hand side are the eigenvectors $|l,m>$.
     - c) (5 points) Calculate A such that $|\psi>$ is normalized.
     - d) (5 points) What is the probability associated with a measurement that gives "$\hbar$" for the z component of the angular momentum?
     - e) (5 points) Calculate the expectation values for $\hat{L_x}, \hat{L_z}, \hat{L}^2$ if the system is in the state $|\psi>$ given at the beginning of the question.
     - f) (5 points) Calculate $\Delta\hat{L_x}$ if the system is in the state $|\psi>$ given at the beginning of the question.

Formulas and Constants:

$\Delta\hat{A} = \sqrt{< \hat{A}^2 > - < \hat{A} >^2}$

Infinite well eigenfunctions: $\psi_n(x) = \sqrt{\frac{2}{a}} sin(\frac{n\pi x}{a})$

$e^{i\theta} = cos(\theta) + isin(\theta)$

$sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$

$cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}$

$$\hat{L_x} = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \hat{L_y} = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \hat{L_z} = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\hat{S}_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \hat{S}_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \hat{S}_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$