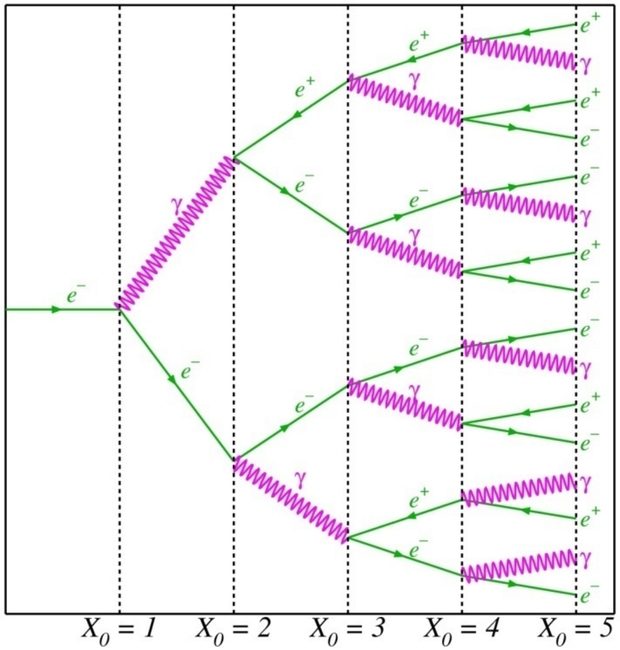


Electromagnetic Showers

incident γ E_0

χ_0 ↓

Converts to 2 electrons $E_e = \frac{E_0}{2}$

$\sim 2\chi_0$ ↓

Each electrons will have emitted a brems photon of energy $E_\gamma = \frac{E_0}{4}$

Now have 4 particles energy $\frac{E_0}{4}$

Number of particles after $t \cdot \chi_0$ $N = 2^t$

Average energy $E(t) = \frac{E_0}{2^t}$

At the critical energy $E_t(\max) = \frac{E_0}{2^{t\max}} = E_C$

assume this is max depth

$$N_{\max} \approx \frac{E_0}{E_c} \qquad t_{\max} = \frac{\ln(E_0/E_c)}{\ln 2}$$

$t_{\max} \sim \ln E_0$ shower grows as *ln E*

$N_{\max} \propto E_0$ linear energy measurement

$\sigma_E \sim \sqrt{N} \sim \sqrt{E}$ resolution improves with energy

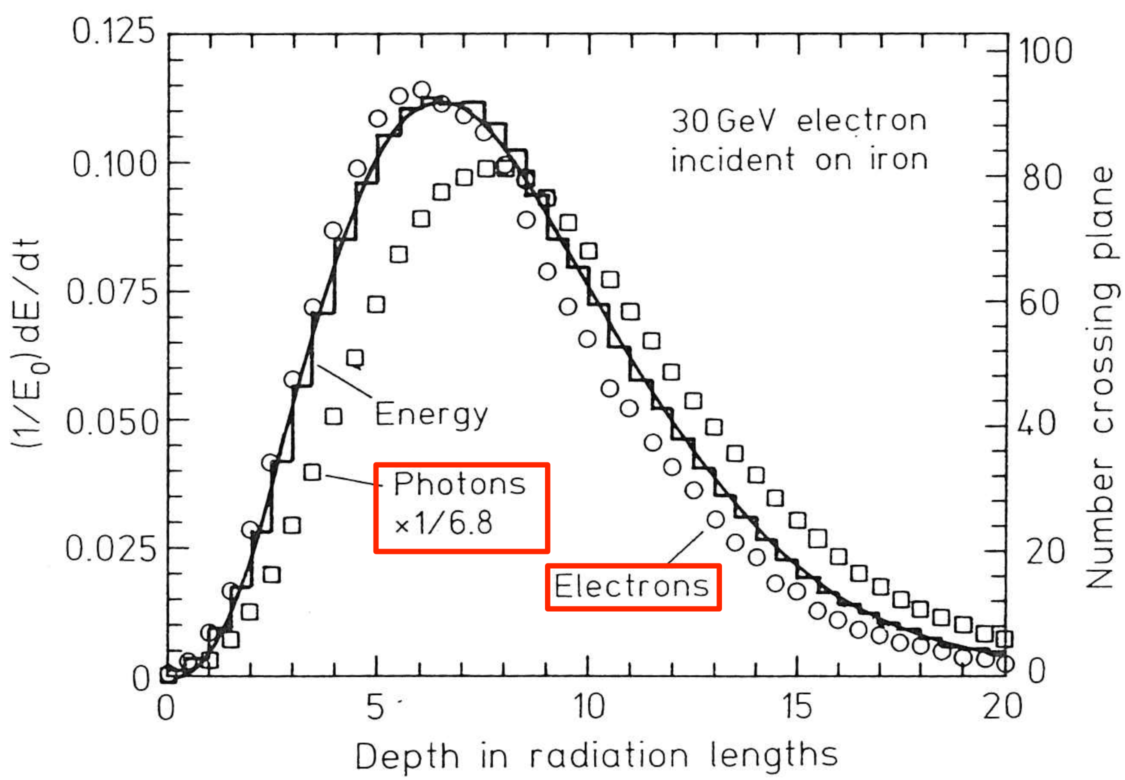
Monte Carlo Simulation of an EM Shower

$$\frac{dE}{dt} = E_0 \frac{b(bt)^{a-1} e^{-bt}}{\Gamma(a)} \Rightarrow t_{max} = \frac{a-1}{b} = 1.0(\ln y + C_i)$$

$t_{max} \sim \ln E_0$

$y = E/E_C$

$C_i = -0.5(e), +0.5(\gamma)$



Calc $t_{max} = (\ln y + C_i)$

↓

Put b=0.5

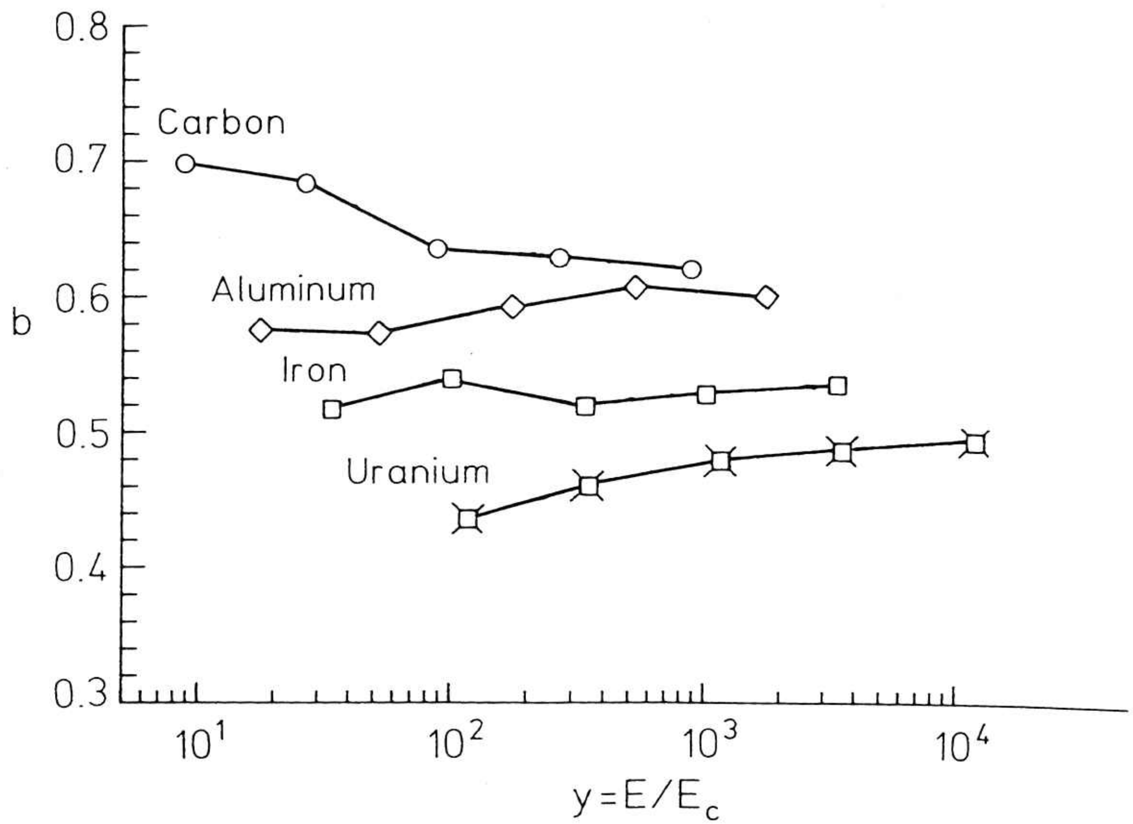
↓

Get a from $t_{max} = \frac{a-1}{b}$

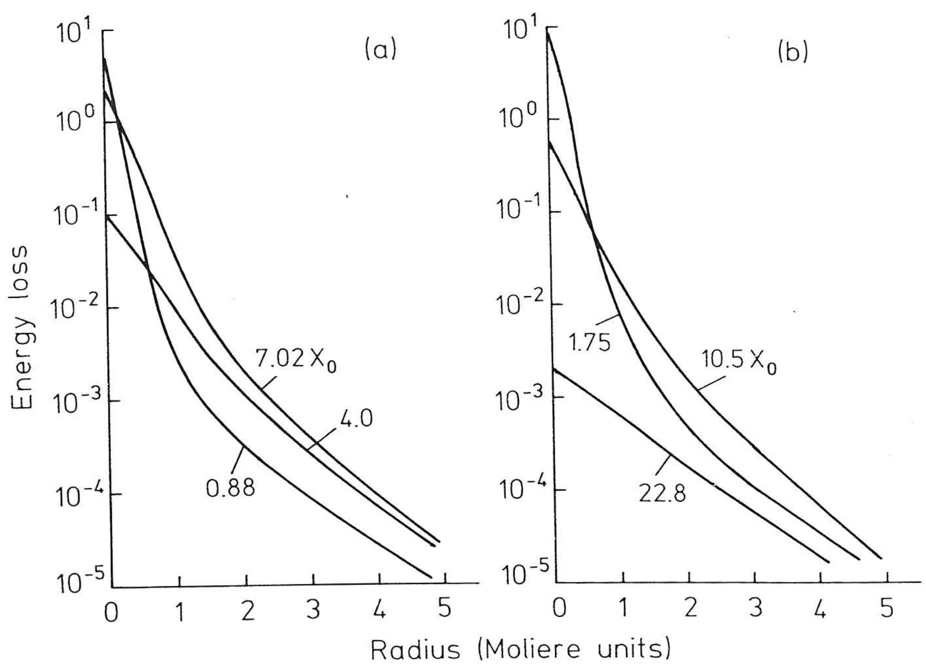
↓

$$\frac{dE}{dt}$$

“b” is sort of material independent



Transverse Shower Profile



- Shower Broadens as it develops
  - Pair
  - Brems
  - Compton
  - Multiple Coulomb

- Shower Broadens as it develops
  - dense central core
  - spreading with depth

- Moliére Radius

$$R_M = \chi_0 \frac{E_S}{E_C}$$

$$E_S = m_e c^2 \sqrt{\frac{4\pi}{\alpha}} = 21.2\,MeV$$

- Like radiation length, Moliére radius scales for different materials

- In terms of Moliére radius, shower width is roughly independent of material - 90% of energy in $2 \times R_M$

Comparison of Hadronic & Electromagnetic Showers