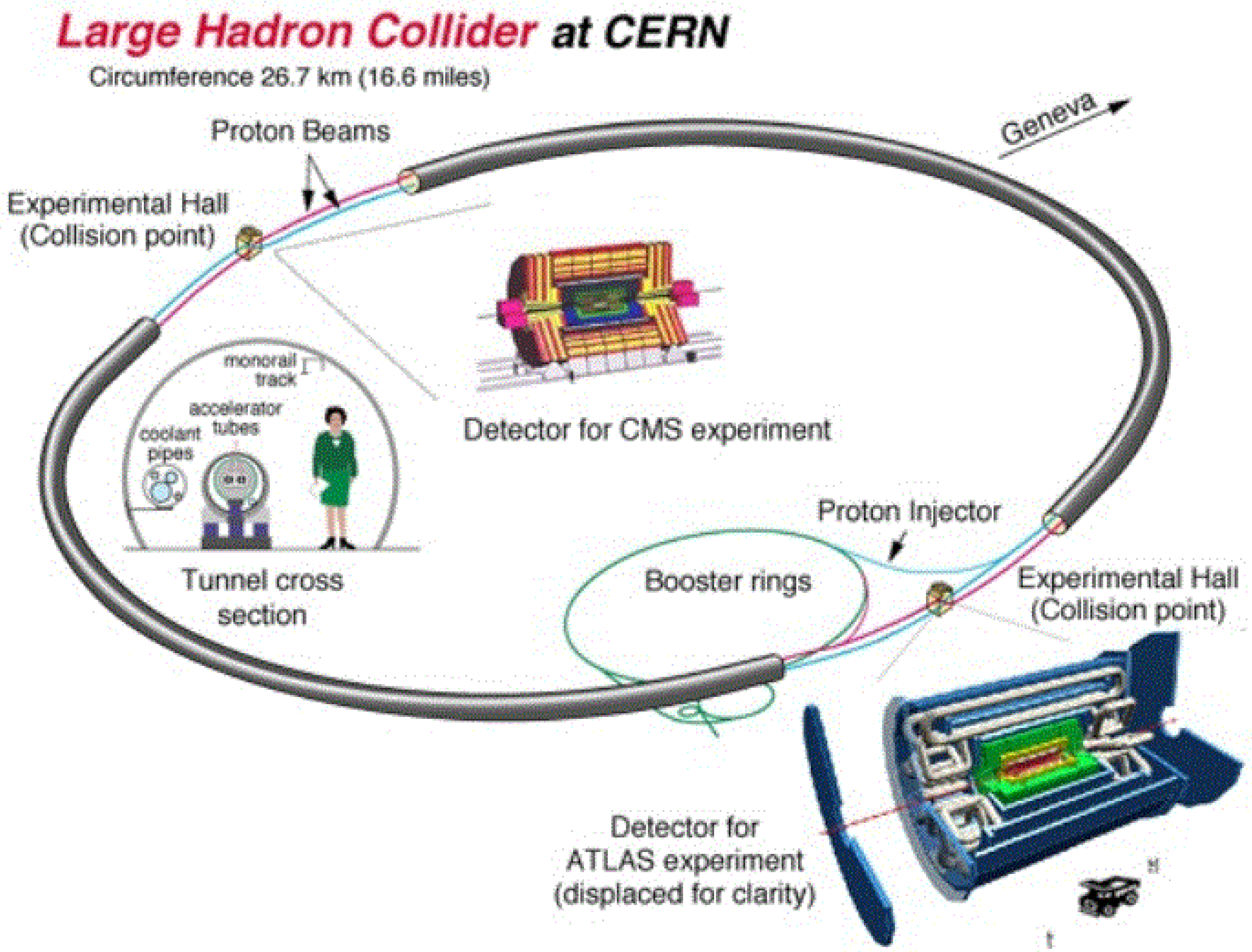
CARTOON OF MODERN ACCELERATOR COMPLEX



Applications of Particle Accelerators

Nuclear Physics
- Electron Proton Accelerators

High Energy Physics
- Fixed Target Accelerators
- Colliding Beam Storage RIngs
- Linear Colliders

Power Generation
- Inertial Fusion
- Reactor Fuel Breeding

Industry
- X-Ray Radiography
- Ion Implantation
- Isotope Production
- Materials Testing
- Food Sterilization
- X-Ray Lithography

Synchrotron Radiation (Electron Storage Rings)
- Atomic & Molecular Physics
- Condensed Matter Physics
- Earth Science
- Chemistry
- Molecular & Cell Biology
- Surface Physics

Coherent Radiation
- Free Electron Lasers

Medicine
- Radiotherapy
- Health Physics
- Imaging
- Microsurgery with tunable FEL

ACCELERATORS

LINEAR	CIRCULAR
ELECTROSTATIC	CYCLOTRON
RF LINAC	BETATRON
	SYNCHROCYCLOTRON
	SYNCHROTRON
	STRONG FOCUSING
	WEAK FOCUSING
	FFAG
	STORAGE RINGS

ELECTRONS, PROTONS, HEAVY IONS
POSITRONS, ANTIPROTON

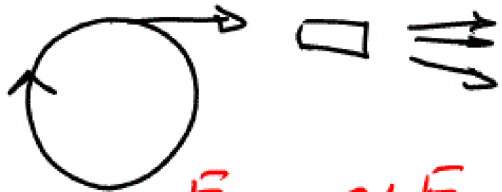
- DYNAMICS OF CHARGED PARTICLES IN EM FIELD
- SUPERCONDUCTING MAGNETS & RF CAVITIES
- VACUUM, RF, ETC ETC

⇒ COMPLEX SUBJECT

ACCELERATOR DESIGN

- FOR US → PRODUCE ENERGY / INTENSITY TO PROBE INTERESTING PHYSICS.

- FIXED TARGET



$K \to \mu \nu$

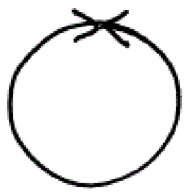
NEUTRINOS

SPS / KEK / TEV

JPARC / X_{FNAL}

$$E_{use} \sim E_{CM} \sim \sqrt{E_b}$$

- COLLIDER



$$E_{use} \sim E_{CM} = 2E_b$$

$E \sim 5\,GeV$ $\Upsilon \to B\bar{B}$ CESR, DORIS, B-FACTORIES, SUPER B

$E \sim 45\,GeV$ $Z^0 \to \mu^+\mu^-$ LEP

$E \sim 1\,TeV$ $p\bar{p} \to t\bar{t}$ TEVATRON

$E \sim 7\,TeV$ $pp \to H,\ SUSY$ LHC

INTENSITY / LUMINOSITY

- SUFFICIENT INTERACTION RATE TO OBSERVE PHYSICS IN PRACTICAL TIME SCALE

RATE → $R = \sigma \cdot \mathcal{L}$ ← LUMINOSITY

s^{-1} $\quad cm^2 \quad s^{-1}\,cm^{-2}$

$\sigma = 5\,nb$ $\quad e^+e^- \to \Upsilon \to B\bar{B}$

$\sigma = 30\,nb$ $\quad e^+e^- \to Z^0$

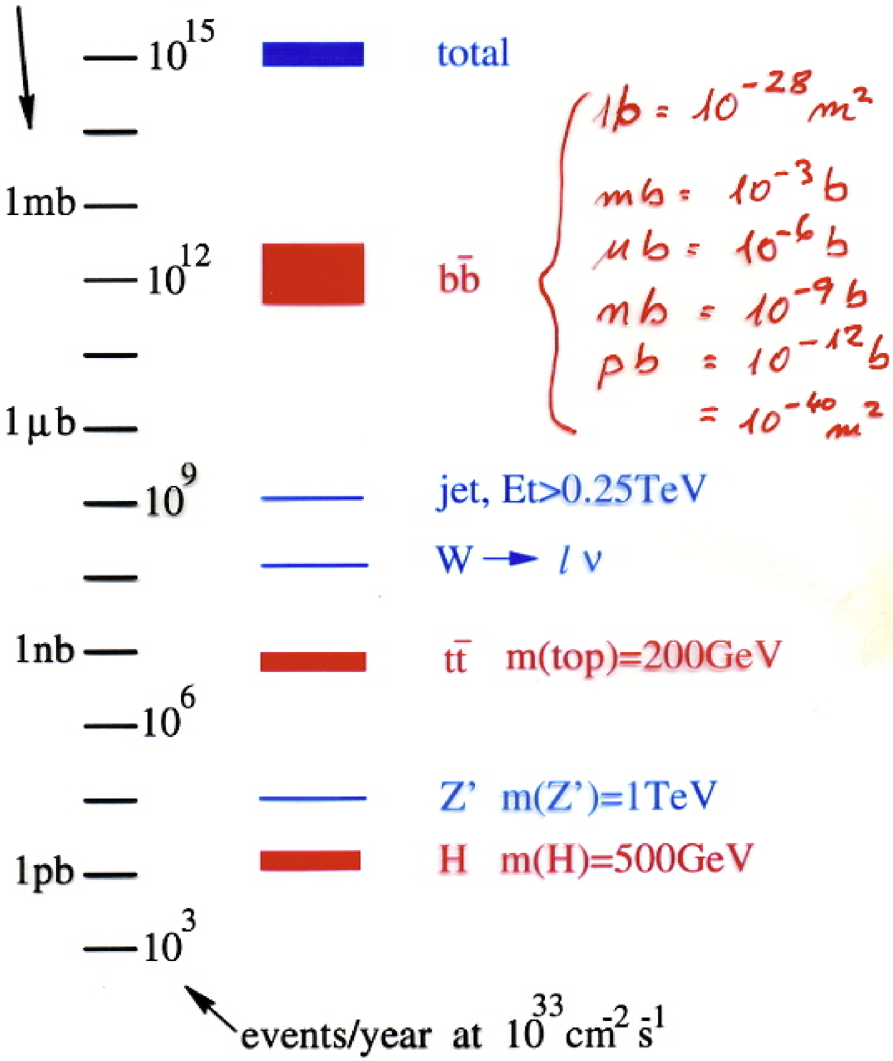
$\sigma = 50\,pb$ $\quad e^+e^- \to W^+W^-$

$\sigma = 1\,pb$ $\quad pp \to$ Higgs

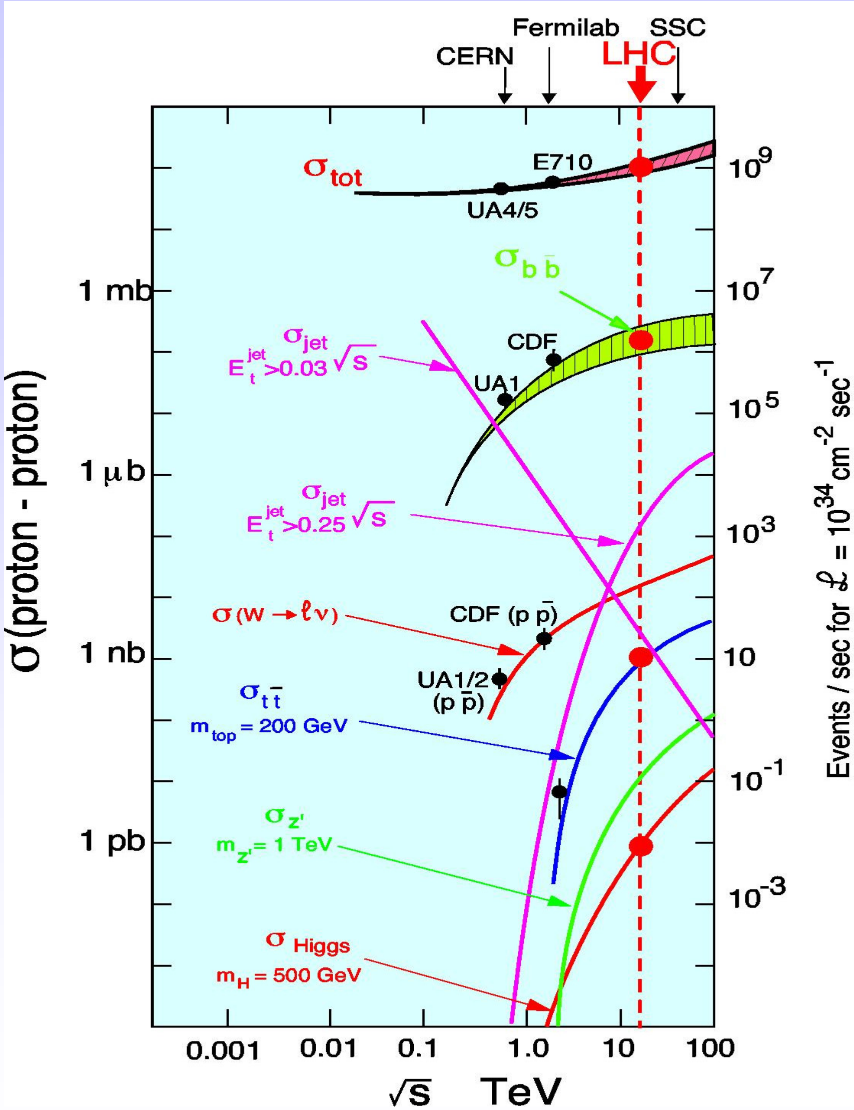
PP Cross Section

- High Luminosity needed
- Large background to overcome

14 TeV
pp cross section



Cross Sections and Production Rates



Rates for L = 10^{34} cm^{-2} s^{-1}: (LHC)

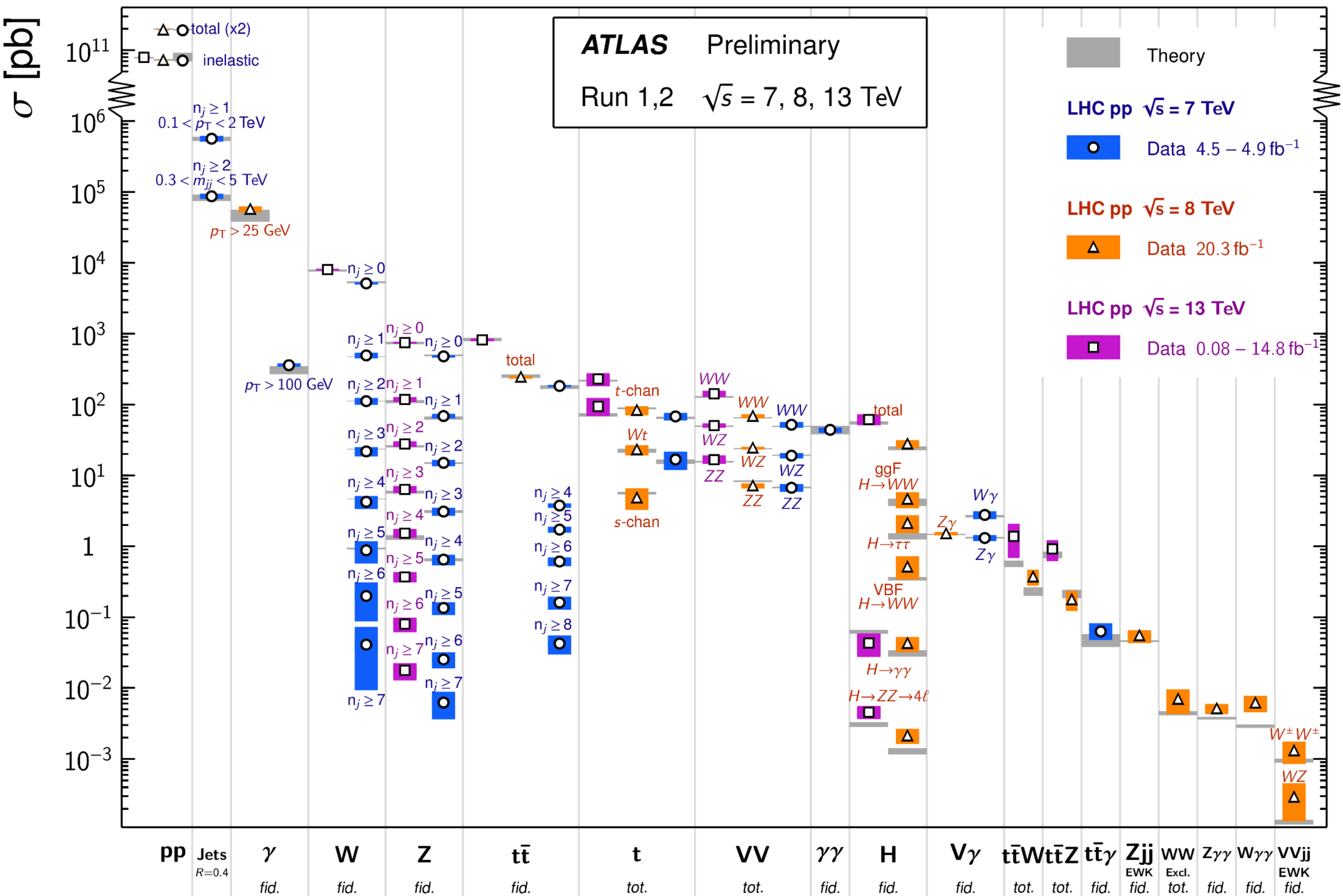
• Inelastic proton-proton reactions:	10^9 / s
• bb pairs • tt pairs	5 10^6 / s 8 / s
• W → e ν • Z → e e	150 / s 15 / s
• **Higgs (150 GeV)** • **Gluino, Squarks (1 TeV)**	**0.2 / s** **0.03 / s**

LHC is a factory for:
top-quarks, b-quarks, W, Z, ……. Higgs, ……

The only problem: you have to detect them !

Standard Model Production Cross Section Measurements

Status: August 2016



SYNCHROTRON RADIATION

WILLE P. 33

FOR A CHARGED PARTICLE IN A CIRCULAR ORBIT

POWER RADIATED $= P_s = \frac{e^2 c \gamma^2}{6\pi \varepsilon_0} \cdot \frac{1}{(mc^2)^2} \cdot \left(\frac{dp}{dt}\right)^2$ MOMENTUM

MOTION THRU ANGLE $d\alpha \rightarrow dp = p\, d\alpha$

$$\frac{dp}{dt} = p\,\omega = p\,\frac{v}{R} \rightarrow P_s = \frac{e^2}{6\pi\varepsilon_0} \frac{c}{(mc^2)^4} \cdot \frac{E^4}{R^2}$$

$\left\{ v = c,\; E = pc,\; \gamma = \frac{E}{mc^2} \right.$

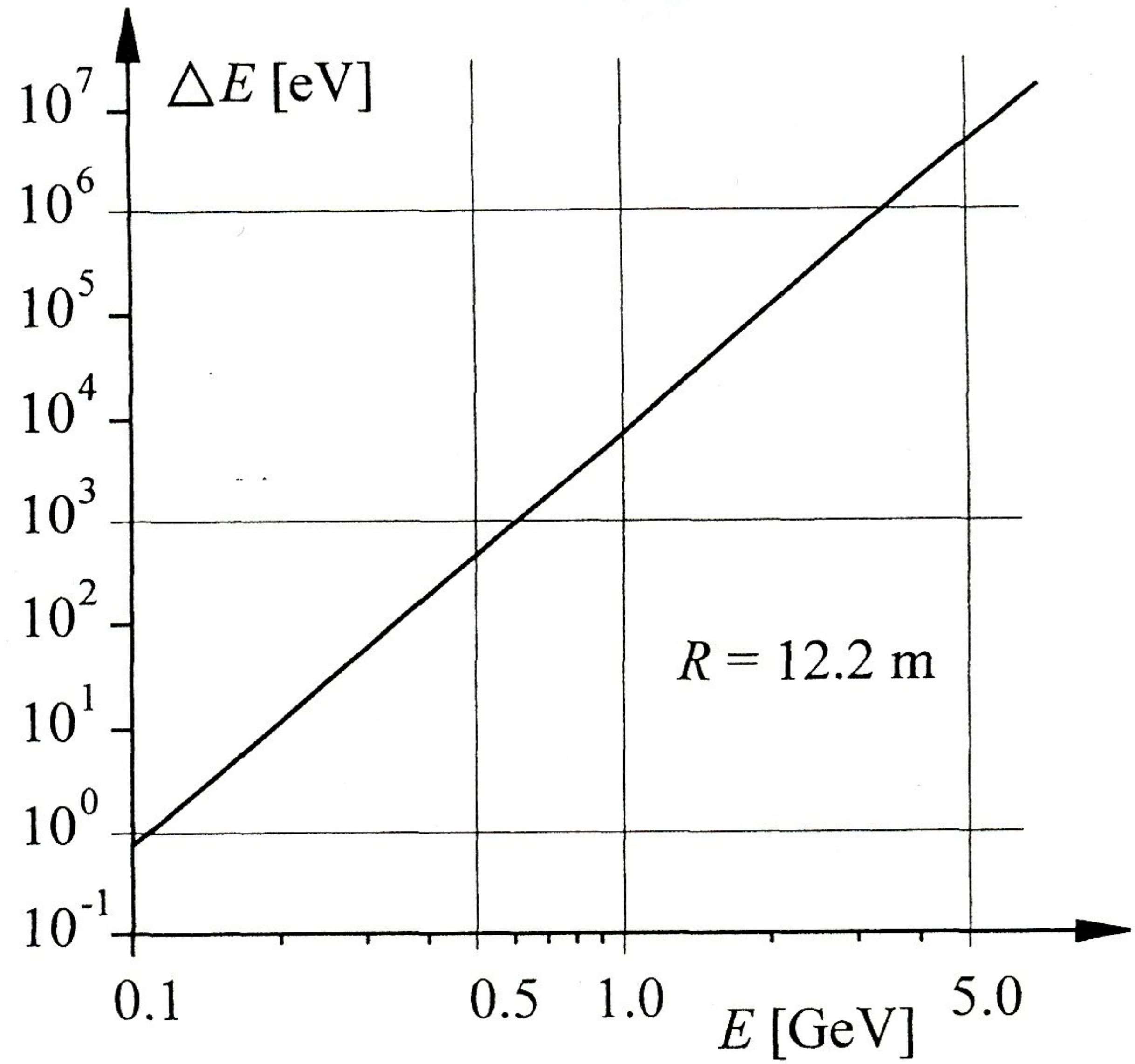
$$\Delta E = \oint P_s\, dt = P_s\, t_b = P_s \frac{2\pi R}{c}$$

$$\Delta E = \frac{e^2}{3\varepsilon_0 (mc^2)^4} \cdot \frac{E^4}{R} \rightarrow \frac{\Delta E_p}{\Delta E_e} = \left(\frac{m_e}{m_p}\right)^4 \sim 10^{-13}$$

$$\Delta E\,(\text{keV}) = 88.5\, \frac{E^4\,[\text{GeV}]^4}{R\,[\text{m}]}$$

THIS IS FOR ELECTRONS

SYNCHROTRON RADIATION IN A CIRCULAR MACHINE



SYNCHROTRON RADIATION IN VARIOUS ELECTRON MACHINES — LEP IS OBVIOUSLY LAST !

accelerator	L [m]	E [GeV]	R [m]	B [T]	ΔE. [keV]
BESSY I (Berlin)	62.4	0.80	1.78	1.50	20.3
DELTA (Dortmund)	115	1.50	3.34	1.50	134.1
DORIS II (Hamburg)	288	5.00	12.2	1.37	4.53×10^3
ESRF (Grenoble)	844	6.00	23.4	0.855	4.90×10^3
PETRA (Hamburg)	2304	23.50	195	0.40	1.38×10^5
LEP (Geneva)	27×10^3	70.00	3000	0.078	7.08×10^5

(~ 1 GeV PER TURN

LHC 6·24 keV / TURN

POWER RADIATED BY LEP

$$P_s = \frac{e^2}{\varepsilon_0} \cdot \frac{1}{6\pi} \cdot \frac{c}{(mc^2)^4} \cdot \frac{E^4}{R^2}$$

$$\hookrightarrow \frac{e^2}{\varepsilon_0} = 4\pi \hbar c \times 7.297 \times 10^{-3} \text{ MeV.fm}$$

$$= 18.1 \times 10^{-18} \text{ GeV.m}$$

$$P_s = 18.1 \times 10^{-18} [\text{GeV.m}] \cdot \frac{1}{6\pi} \cdot \frac{3\times 10^8 \text{ms}^{-1}}{[.511\times 10^{-3}]^4} \cdot \frac{[70]^4}{(3000)^2} \cdot \frac{[\text{GeV}]^4}{[\text{GeV}]^4} \cdot \frac{1}{[\text{m}]^2}$$

$$= 1.13 \times 10^4 \text{ GeV/s}$$

$1 \text{ GeV} = 1.6 \times 10^{-10} \text{ J}$

$$= 1.81 \times 10^{-6} \text{ W}$$

$\times 10^{11}$ PARTICLES / BUNCH

$\times 4$ BUNCHES

$$P_s = 7.2 \times 10^5 \text{ W}$$

ACCELERATOR BUZZ WORDS

LUMINOSITY
TRANSVERSE EMITTANCE
AMPLITUDE FUNCTION
BETATRON OSCILLATIONS
TUNE
SYNCHROTRON OSCILLATIONS
PHASE STABILITY
SYNCHRONOUS PHASE
LONGITUDINAL EMITTANCE
BEAM-BEAM TUNESHIFT
STRONG/WEAK FOCUSING
FODO STRUCTURES

DYNAMICS OF CHARGED PARTICLES IN ELECTRIC & MAGNETIC FIELDS

LORENTZ $\frac{dp}{dt} = e(\bar{E} + \bar{v} \times \bar{B})$

MAXWELL (MKS)

$\bar{\nabla} \cdot \bar{E} = \frac{1}{\varepsilon_0} \rho(\bar{r}, t)$ $\int \bar{E} \cdot d\bar{s} = \frac{1}{\varepsilon_0} \int \rho \, dV$

$\bar{\nabla} \cdot \bar{B} = 0$ $\int \bar{B} \cdot d\bar{s} = 0$

$\bar{\nabla} \times \bar{E} = -\partial \bar{B}/\partial t$ $\oint \bar{E} \cdot d\bar{\ell} = -\int \bar{B} \cdot d\bar{s}$

$\bar{\nabla} \times \bar{B} = \mu_0 \bar{j}(\bar{r}, t) + \frac{1}{c^2} \frac{\partial \bar{E}}{\partial t}$ $\oint \bar{B} \cdot d\bar{\ell} = \mu_0 \int \bar{j} \, d\bar{s} + \frac{1}{c^2} \int \bar{E} \cdot d\bar{s}$

MAGNETIC MATERIAL

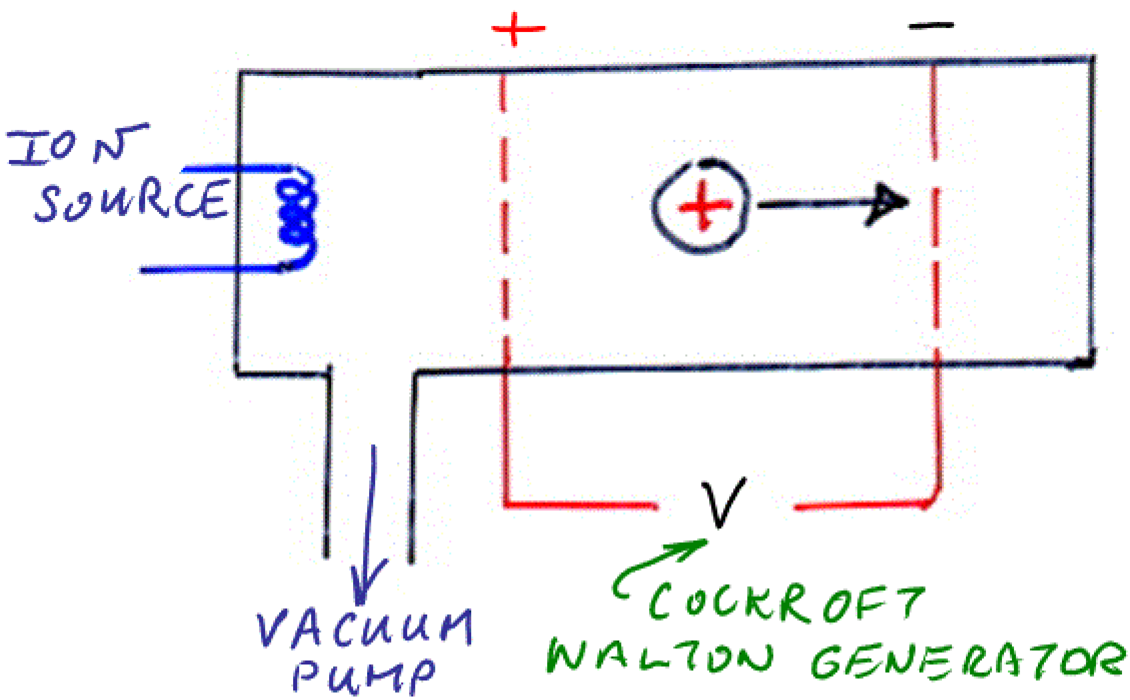
$\bar{\nabla} \times \bar{H} = \bar{j}(\bar{r}, t) + \varepsilon_0 \, \partial E/\partial t$

$\bar{B} = \mu^0 (\bar{H} + M)$

↑ MAGNETIZATION

SIMPLE ELECTROSTATIC ACCELERATOR

USED BY COCKROFT & WALTON — ARTIFICIAL RADIOACTIVITY



ELECTRIC FIELD

$$\bar{F} = q\,\bar{E}$$

CHARGE ON PARTICLE

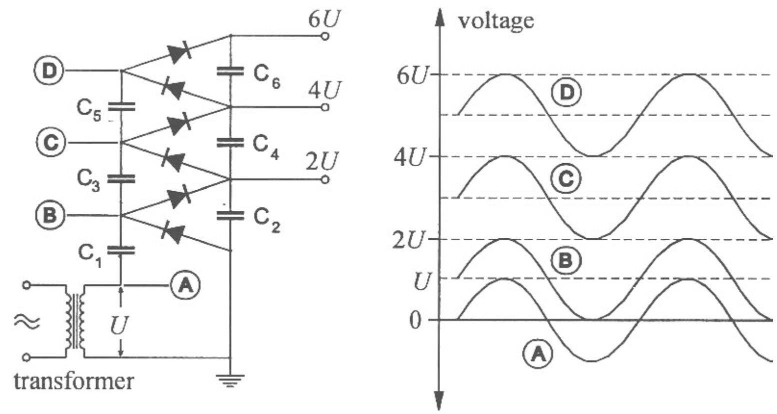
$$|\bar{E}| = \frac{V}{d}$$

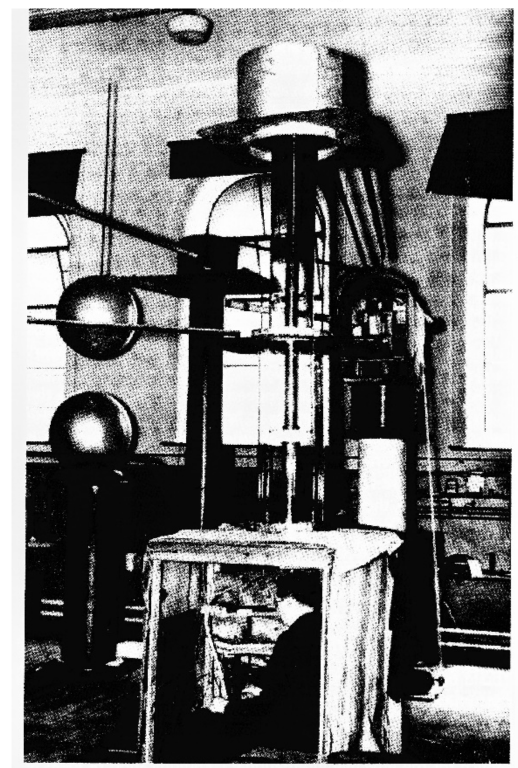
ENERGY GAINED BY CHARGED PARTICLE

$$E_{ACC} = Fd = qV$$

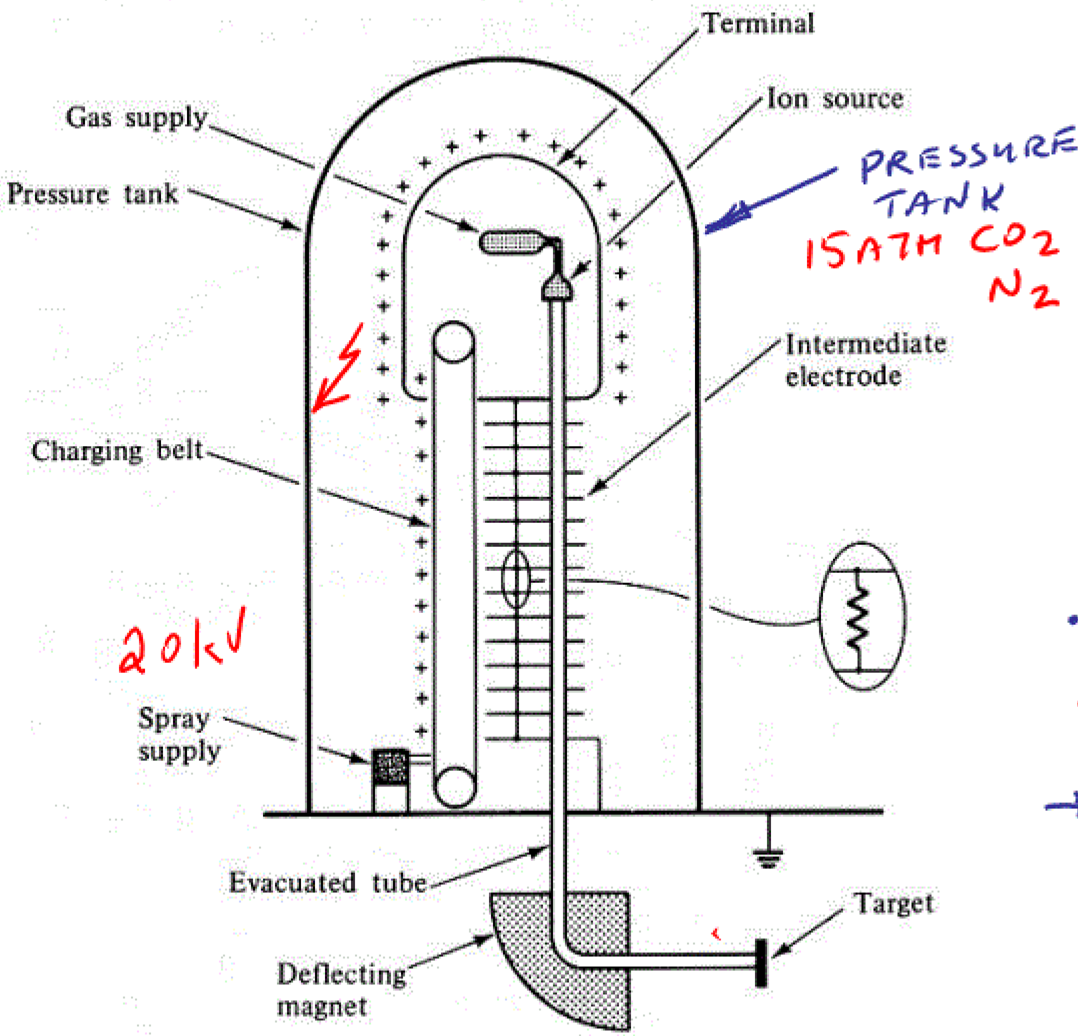
- TWO SHORTCOMINGS:
  - GENERATING HIGH VOLTAGE
  - INSULATING BEYOND ~ 100 kV (100 keV)

Cockcroft-Walton Generator





VAN DE GRAAFF



- TRANSPORT CHARGE

Q

TO TERMINAL OF CAPACITANCE

C

$$V = \frac{Q}{C}$$

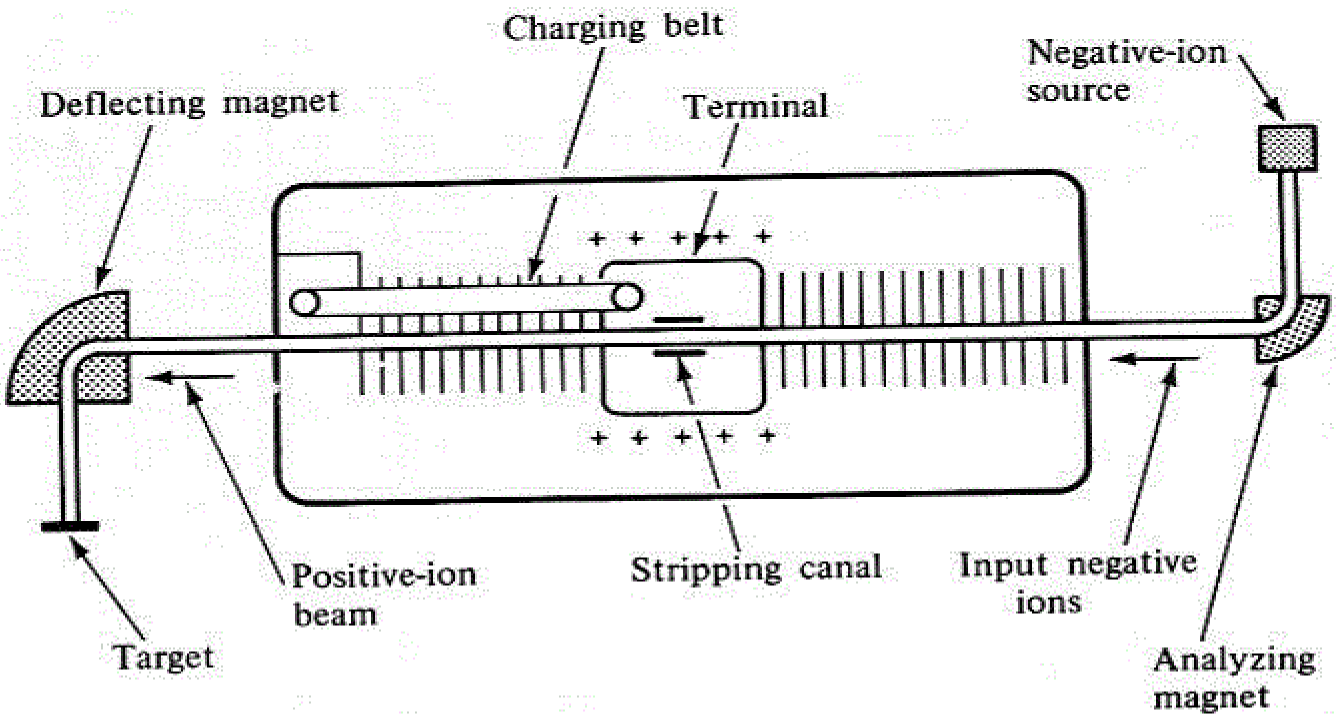
- LIMITATION ~ 12MV

→ VOLTAGE BREAKDOWN

→ NOT ENOUGH TO RESOLVE PROTONS IN THE NUCLEUS

~ 12 MeV

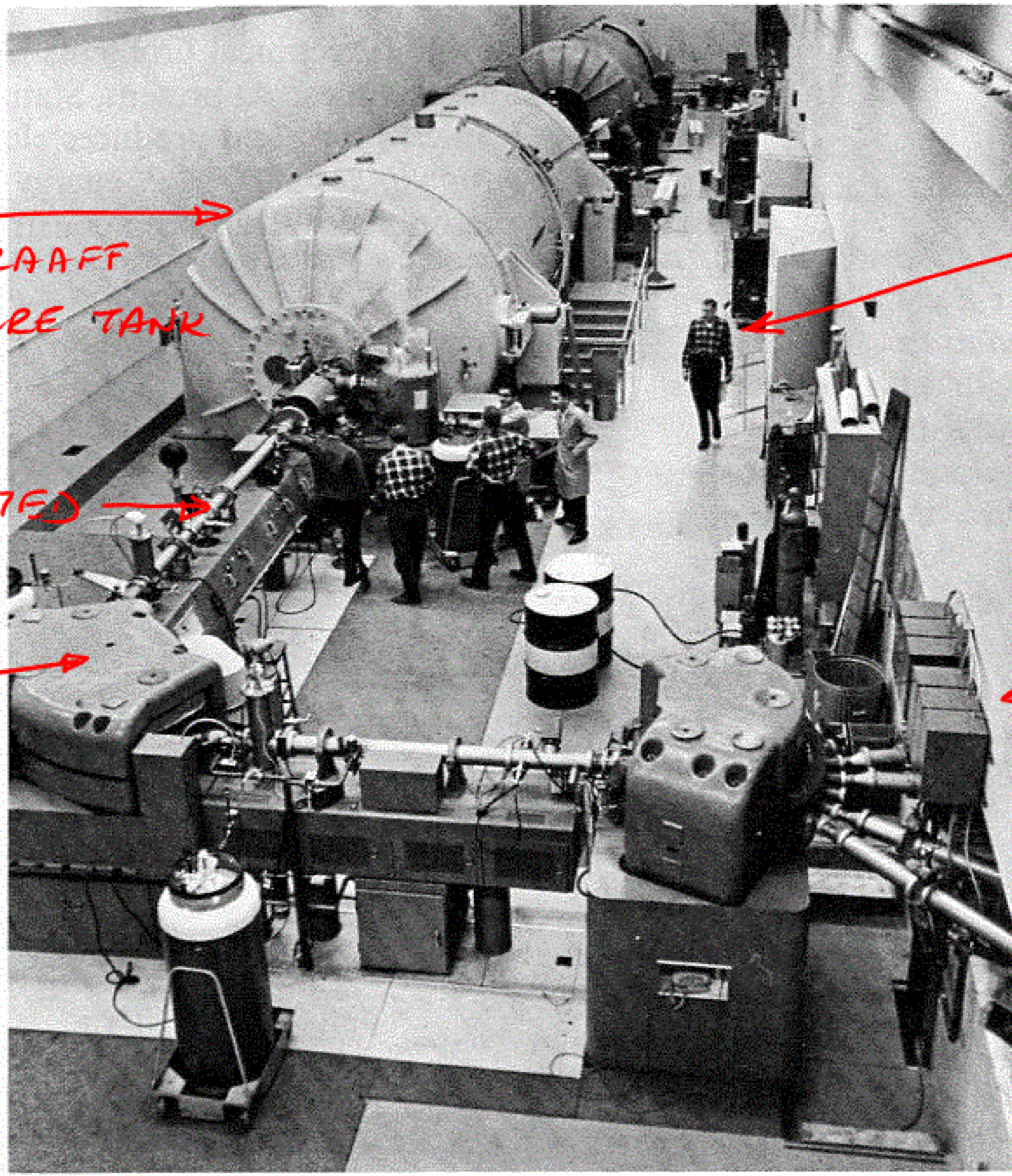
TANDEM VAN DE GRAAFF



- USE VOLTAGE ON TERMIAL TWICE
- ACCELERATE −VE IONS UP TO TERMINAL
- STRIP OFF TWO ELECTRONS INSIDE TERMINAL

  — ACCELERATE AWAY

- 40 MeV   CHALK RIVER HAD LARGE TANDEM


VAN DE GRAAFF
IN PRESSURE TANK
ACCELERATED
BEAM
BENDING
MAGNET
1960's
PHYSICIST
BEAMS TO
EXPERIMENTS

DC HIGH-VOLTAGE ACCELERATORS – TANDEM VAN DE GRAAFFS

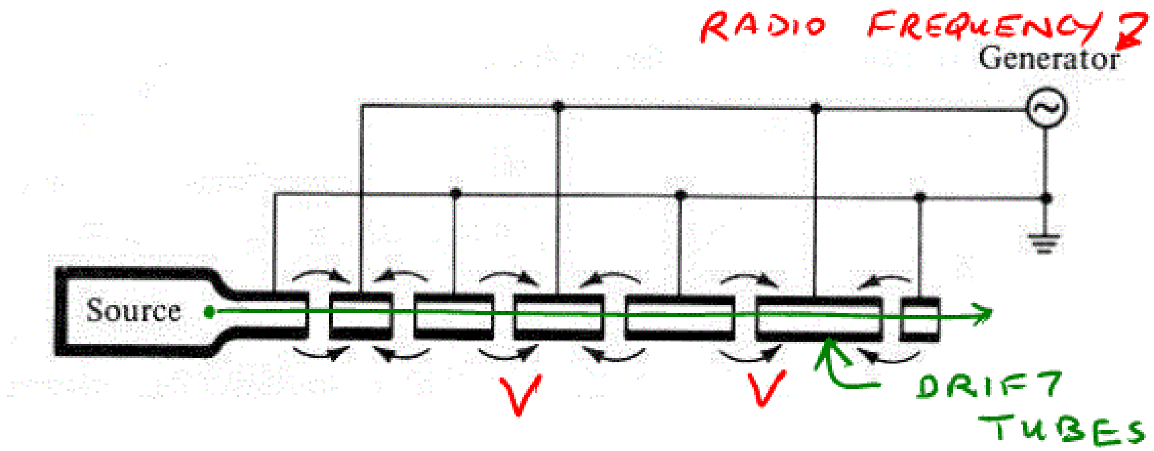
Yale 22-MV tandem.

Daresbury folded tandem
(20 MV in a 230-ft tower).

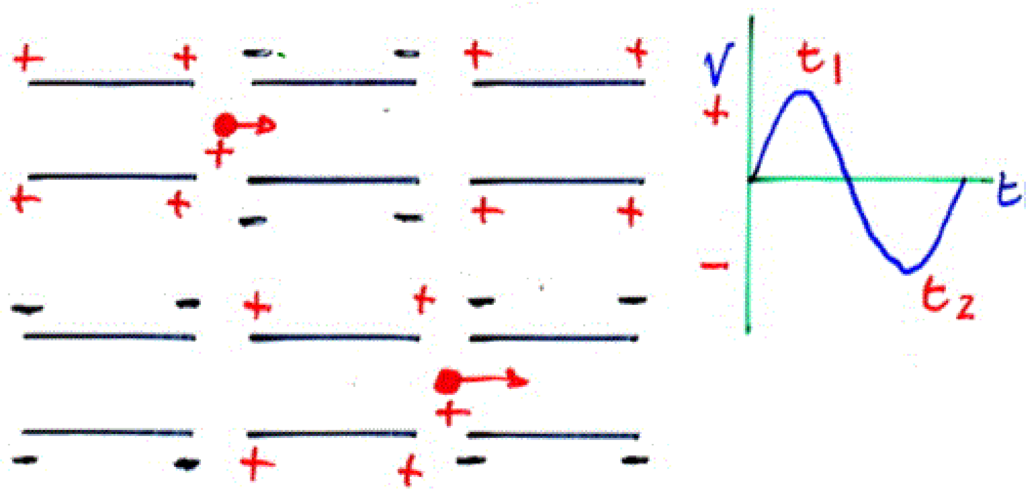
LINEAR ACCELERATOR (LINAC)

TORONTO USED TO HAVE 40 MeV LINAC

- INVENTED BY WIDEROE



- USE SAME RELATIVELY SMALL VOLTAGE IN MANY STEPS — REACH EQUIVALENT HIGH VOLTAGE



- FIELD ZERO INSIDE DRIFT TUBES
- PARTICLE MOVES ONE GAP → NEXT, IN TIME E-FIELD REVERSES
- PARTICLES ACCELERATING → LENGTH OF DRIF TUBES INCREASES

→ NON RELATIVISTIC

- PARTICLE ENTERING DRIFT TUBE n, ENERGY $n \cdot eV$

  (n: # GAPS TRAVERSED; V: VOLTAGE ACROSS GAP)

- NON-RELATIVISTIC

  KINETIC ENERGY $T = \frac{1}{2} m v^2$

$$v = \left(\frac{2 \cdot n e V}{m} \right)^{\frac{1}{2}} \quad \left(\frac{2T}{m} \right)^{1/2}$$

- THIS VELOCITY TAKES PARTICLE THRU DRIFT TUBE OF LEGTH L_n IN TIME FIELD TAKES TO REVERSE

$$t_n = L_n / v$$

- FREQUENCY OF RADIO FREQNECY OSCILLATOR f (Hz) HAS REVERSAL TIME $\frac{1}{2f}$

$$L_n = \frac{1}{2f} \left(\frac{2neV}{m} \right)^{\frac{1}{2}} \longrightarrow L_n \propto \sqrt{n}$$

NUMERICAL VALUES

$$L_n = \frac{1}{2f} \cdot v_n$$

TYPICALLY $v_n = 0.5c$; $f = 7\,MHz \rightarrow L_n = 10.7\,m$

- LOW RADIO FREQUENCY LEADS TO VERY LONG STRUCTURES
- PRACTICALLY NEED HIGH RADIO FREQUENCIES

KLYSTRONS $\rightarrow$ 100 MHz $\rightarrow$ 10 GHz

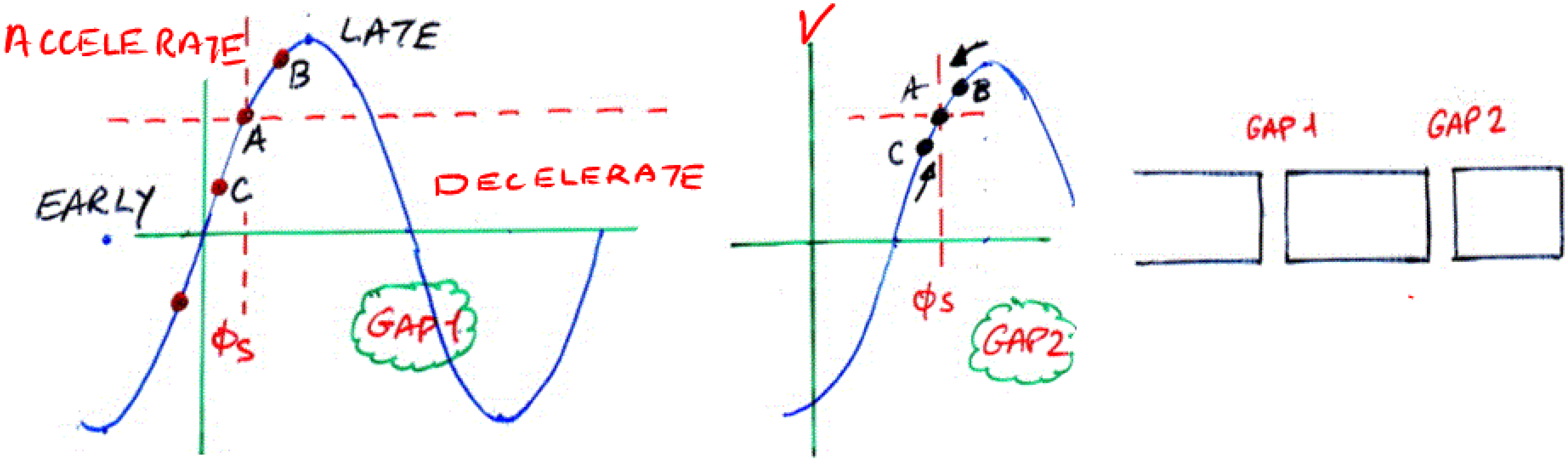
- THIS WIDEROE STRUCTURE IS OBSOLETE

  $\rightarrow$ VERY INEFFICIENT

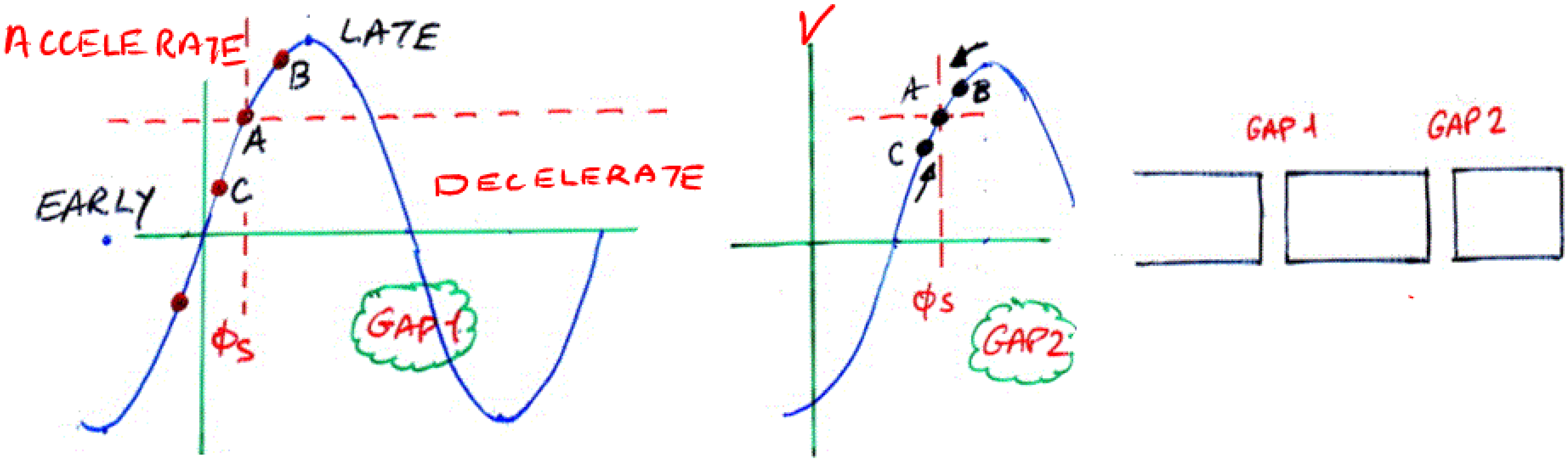
  $\rightarrow$ RADIATION LOSS

PHASE STABILITY IN LINAC

- TO MAINTAIN PRECISE SYNCHRONISM BETWEEN PARTICLE MOTION & RF OSCILLATOR SEEMS DIFFICULT → NOT SO

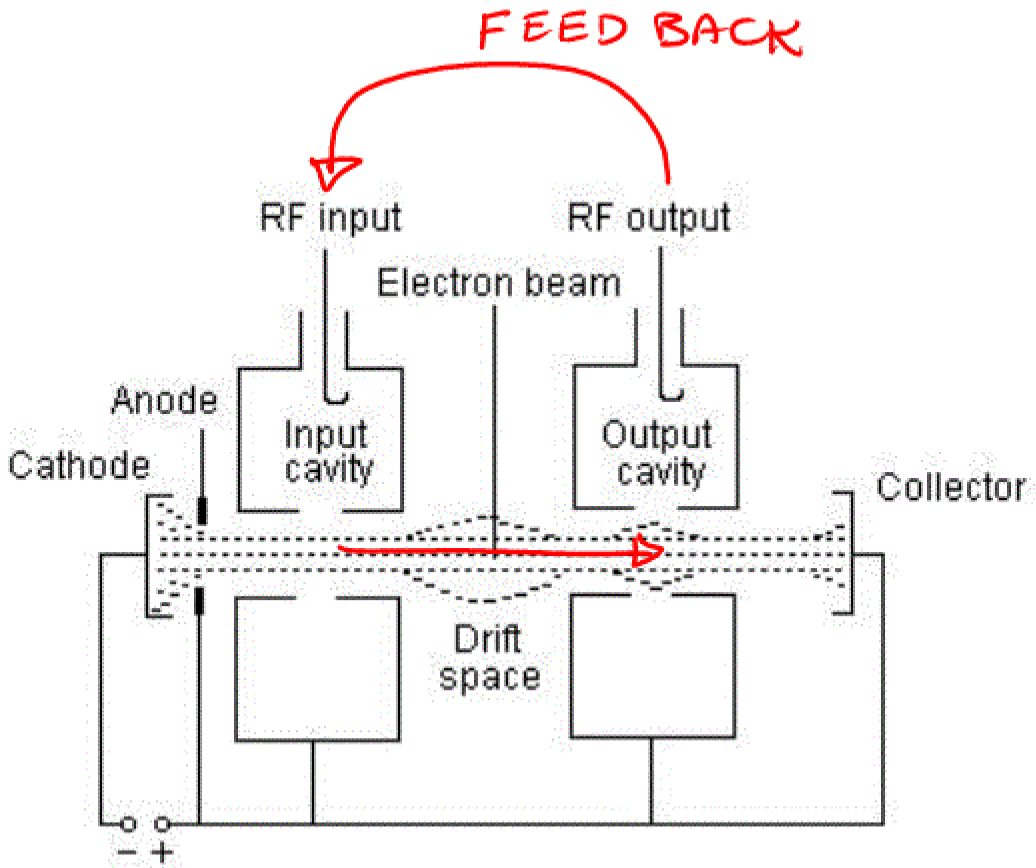


- PARTICLE A CROSSES GAP1 PHASE ϕ IN STEP WITH VOLTAGE
- GAP 2 — SAME VOLTAGE PHASE — AGAIN ACCELERATED
- PARTICLE B ARRIVE LATE, VOLTAGE HIGHER
  ACCELERATED MORE ARRIVES AT GAP2 EARLIER



- PARTICLE C ARRIVES EARLIER AT GAP 1
  - VOLTAGE LOWER, ACCELERATED LESS
  - ARRIVES LATER IN PHASE AT GAP 2
- B AND C CONVERGE IN PHASE WITH A
- NO NEED TO START WITH PARTICLES ALL IN PHASE WITH RADIO FREQUENCY OSCILLATOR

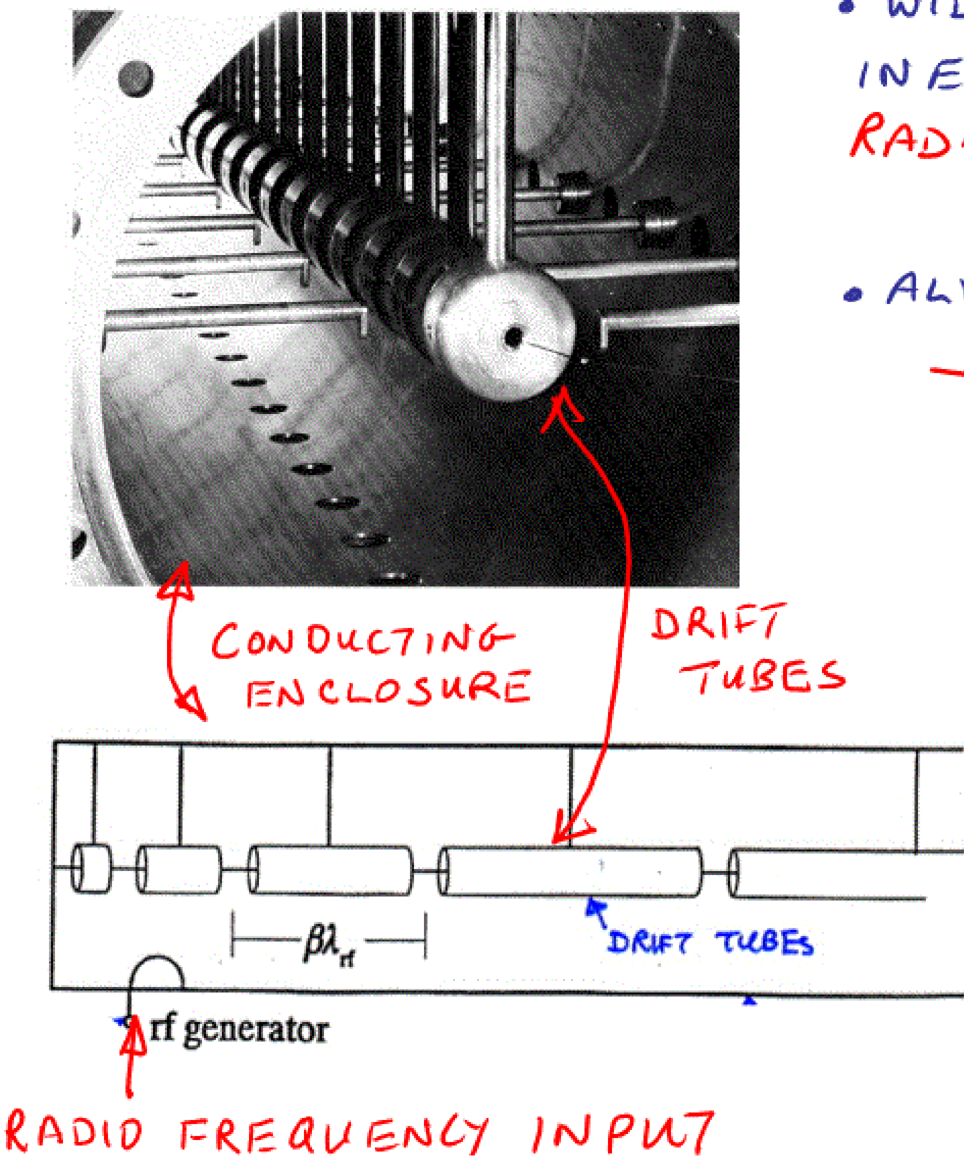
RADIO FREQUENCY POWER GENERATION



Two cavity klystron amplifier

2 CAVITY KLYSTRON OSCILLATOR

ALVAREZ LINAC STRUCTURE



- WIDEROE STRUCTURE VERY INEFFICIENT — RADIO FREQUENCY RADIATION LOSS
- ALVAREZ STRUCTURE
  — RESONANT CAVITY LIKE KLYSTRON
  - USED FOR PROTON SYNCHROTRON INJECTOR

    100 MeV → 100 MHz
- HIGH ENERGY ELECTRON ACCELERATORS

  40 GeV - 500 GeV   GHz RF

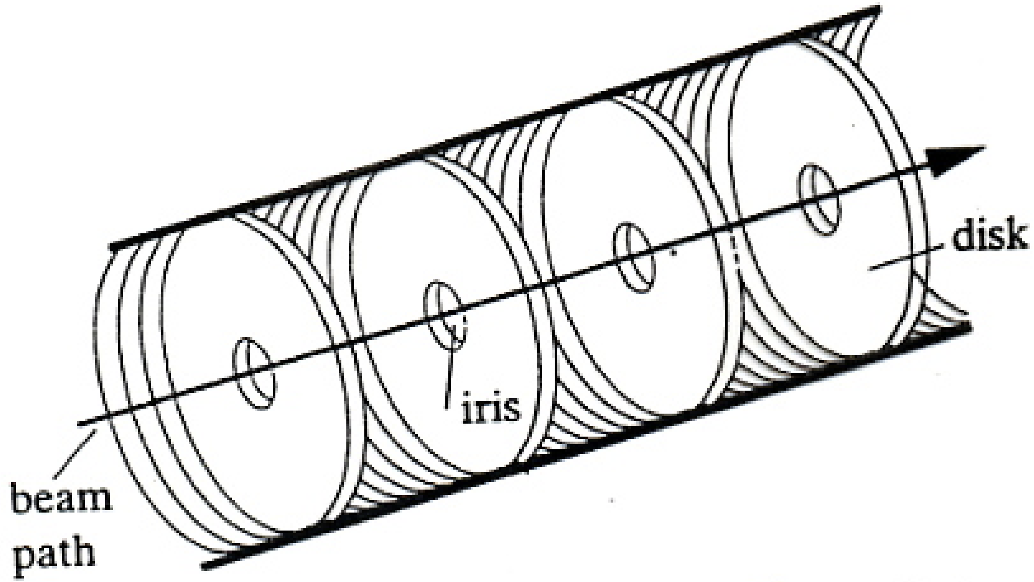


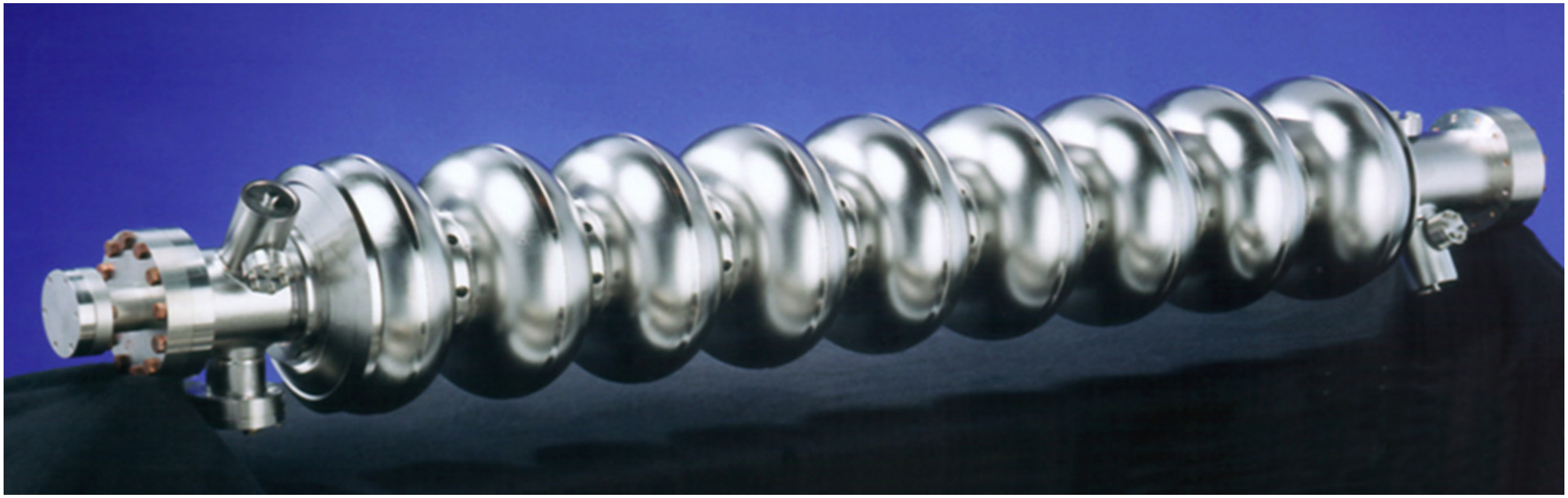
Fig. 2.8. Disk loaded accelerating structure for an electron linear accelerator (

- TRAVELING WAVE LINAC (SLAC)
- IRISES REDUCE v_{gap} OF EM WAVES

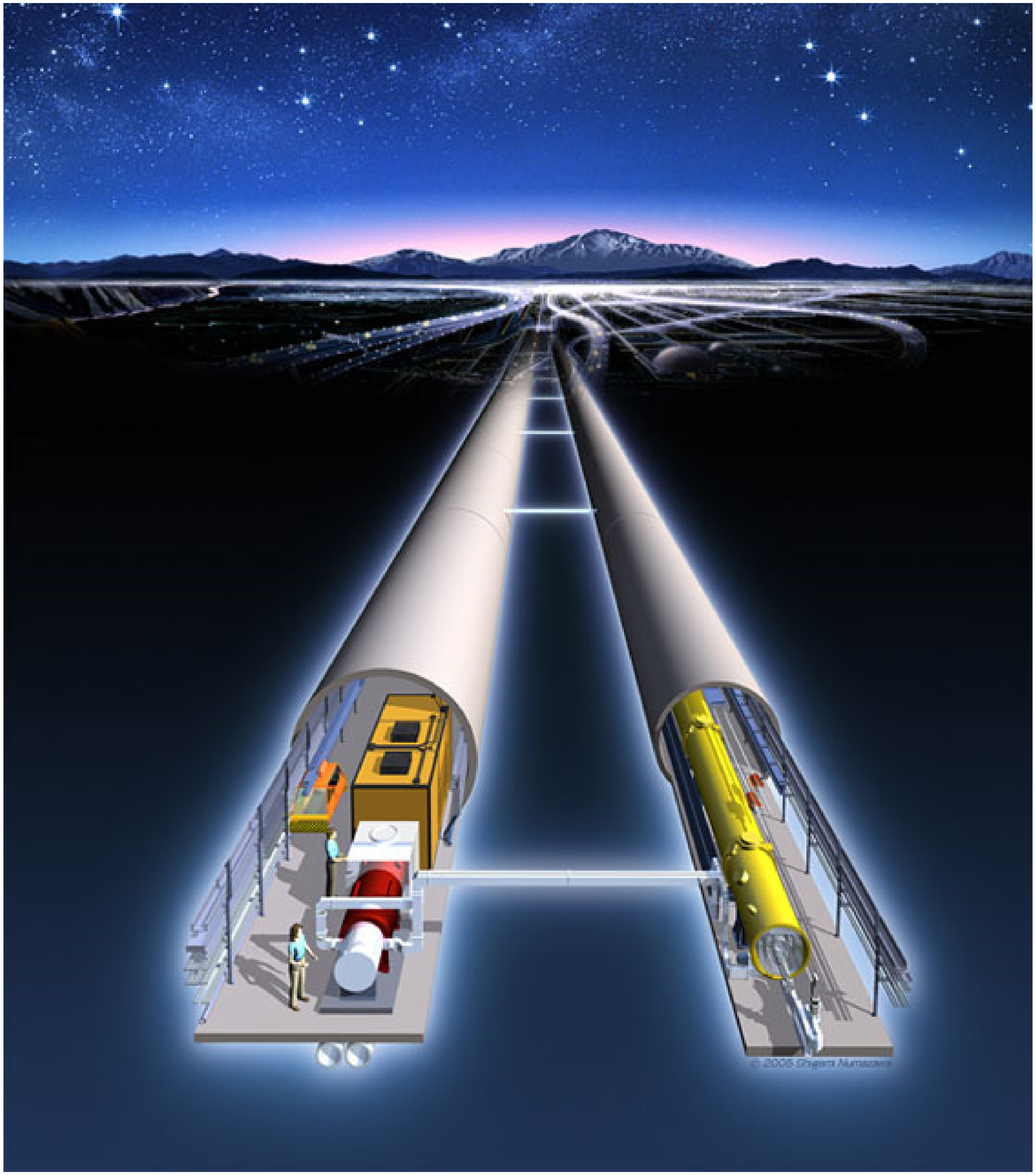
↓

- MATCH VELOCITY OF ELECTRONS

SLAC – 50 GeV Electron LINAC



ILC RF CAVITY STRUCTURE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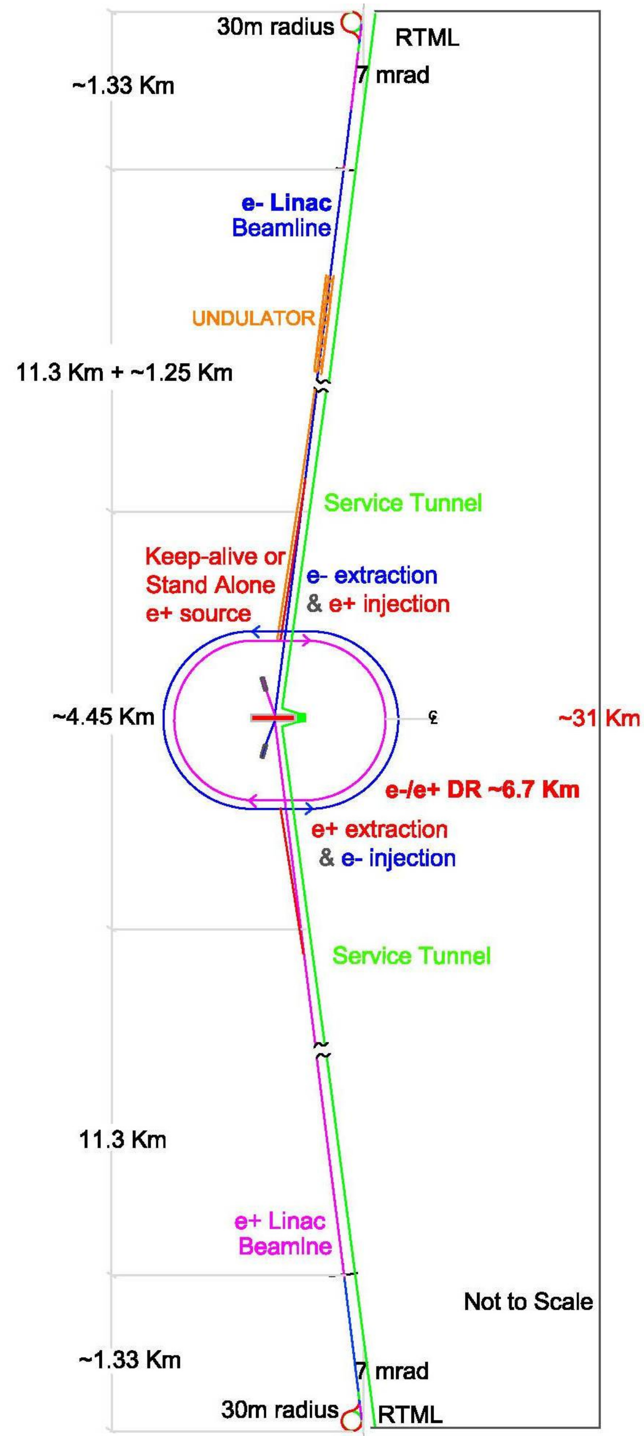
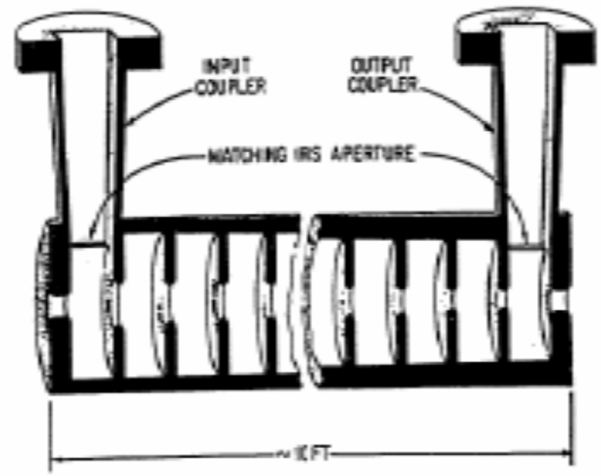

30m radius
RTML
7 mrad
~1.33 Km
e- Linac
Beamline
UNDULATOR
11.3 Km + ~1.25 Km
Service Tunnel
Keep-alive or
Stand Alone
e+ source
e- extraction
& e+ injection
~4.45 Km
~31 Km
e-/e+ DR ~6.7 Km
e+ extraction
& e- injection
Service Tunnel
11.3 Km
e+ Linac
Beamline
Not to Scale
~1.33 Km
7 mrad
30m radius
RTML

The ISAC 150-keV/u RFQ linac

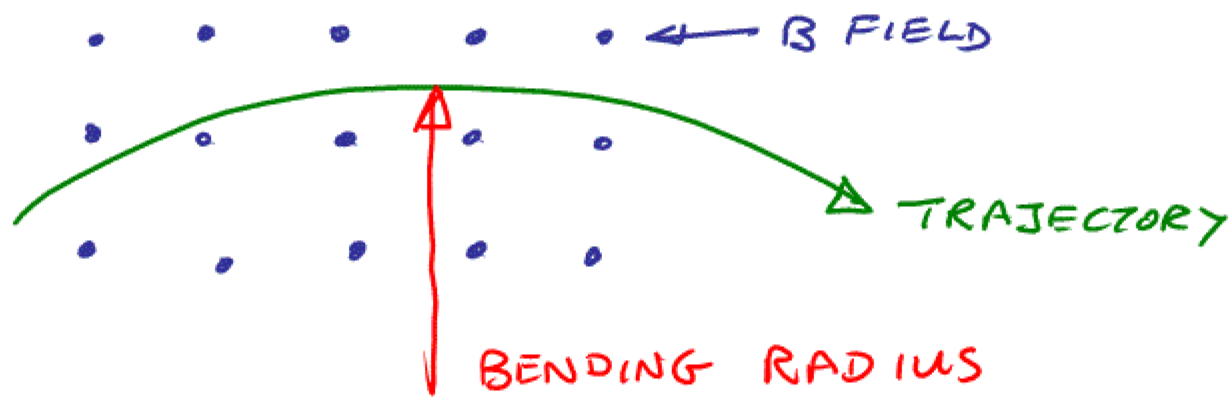


500 keV electron LINAC for Cancer Therapy

PARTICLE BENDING IN MAGNETIC FIELD

$$\bar{F} = q\left(\bar{E} + \frac{1}{c}\bar{v} \times \bar{B}\right)$$ LORENTZ

- FORCE FROM MAGNETIC FIELD NORMAL TO PARTICLE TRAJECTORY



- FOR NO ELECTRIC FIELD & B FIELD NORMAL TO PAGE

$$F = q\frac{v}{c}B\sin\theta \quad \leftarrow 90° = 1 \rightarrow \quad F = q\,\frac{vB}{c}$$

- FOR A PARTICLE MOVING IN A CIRCLE OF RADIUS ρ

CENTRIPETAL FORCE = LORENTZ FORCE

CIRCULAR ACCELERATORS

- AT PRESENT PARTICLE PHYSICS STILL DOMINATED BY CIRCULAR ACCELERATOR

CESR
PEP II
KEK
JPARC
LEP
SUPER-B

SPS
TEVATRON
AGS / RHIC
LHC

- CIRCULAR ACCELERATORS ARE MOST EFFICIENT & COMPACT WAY TO REACH HIGH ENERGY → UNTIL SYNCHROTRON RADIATION DOMINATES

SIZE OF ACCELERTOR OF GIVEN MOMENTUM

IN SI UNITS

CENTRIFUGAL FORCE = LORENTZ FORCE

$$m v \,[\mathrm{kg.m.s^{-1}}] = e\,[\mathrm{C}]\; B\,[\mathrm{T}]\; R\,[\mathrm{m}]$$

$$1 \frac{\mathrm{eV}}{c^2} = 1.78 \times 10^{-36}\ \mathrm{kg}$$

$$1 \frac{\mathrm{eV}}{c} = 1.78 \times 10^{-36}\ \mathrm{kg} \times 3 \times 10^{8}\ \mathrm{m\,s^{-1}}$$

$$1 \frac{\mathrm{eV}}{c} = 5.34 \times 10^{-28}\ \mathrm{kg.m.s^{-1}}$$

$$1 \frac{\mathrm{GeV}}{c} = 5.34 \times 10^{-19}\ \mathrm{kg.m.s^{-1}}$$

$$1\ GeV/c = 5.34 \times 10^{-19}\ kg \cdot m \cdot s^{-1}$$

FOR A 1 GeV/c PARTICLE

$$P\left[1\ \frac{GeV}{c}\right] \cdot 5.34 \times 10^{-19} = 1.6 \times 10^{-19}\ B[T]\ R[m]$$

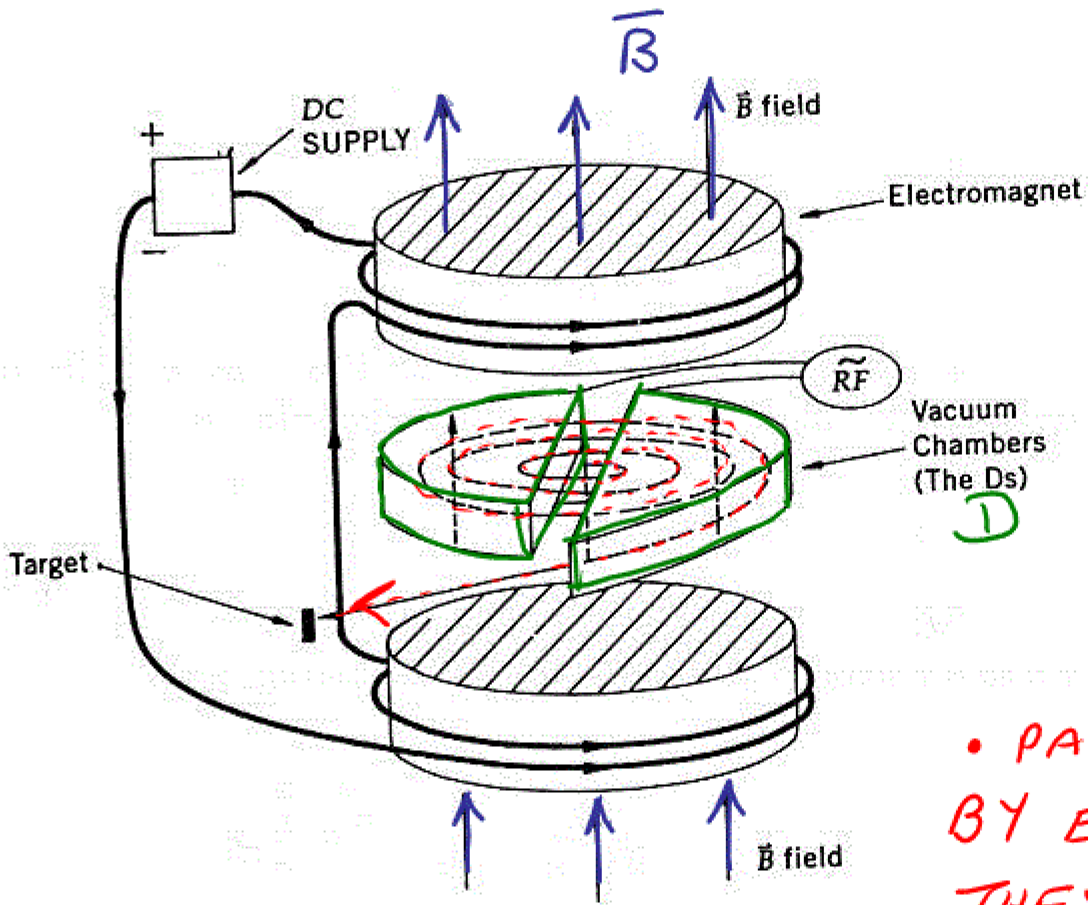
$$P\left[\frac{GeV}{c}\right] = 0.3\ B[T]\ R[m]$$

DORIS

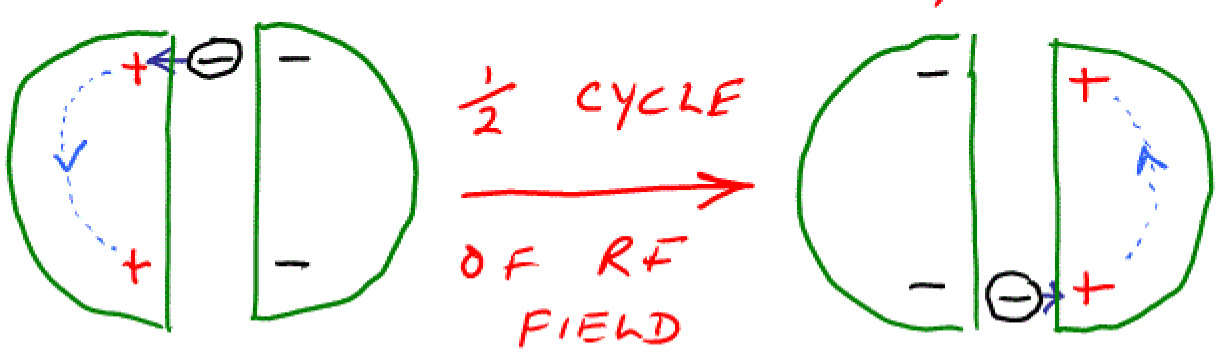
$$P = 0.3 \times 1.37\ [T] \times 12.2\ [m]$$

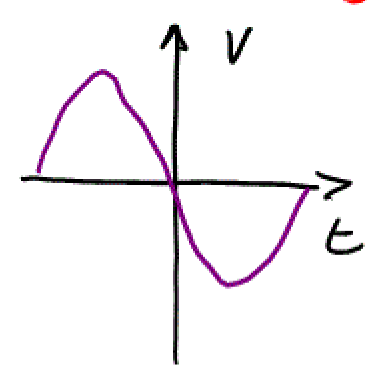
$$= 5\ GeV/c \quad \checkmark$$

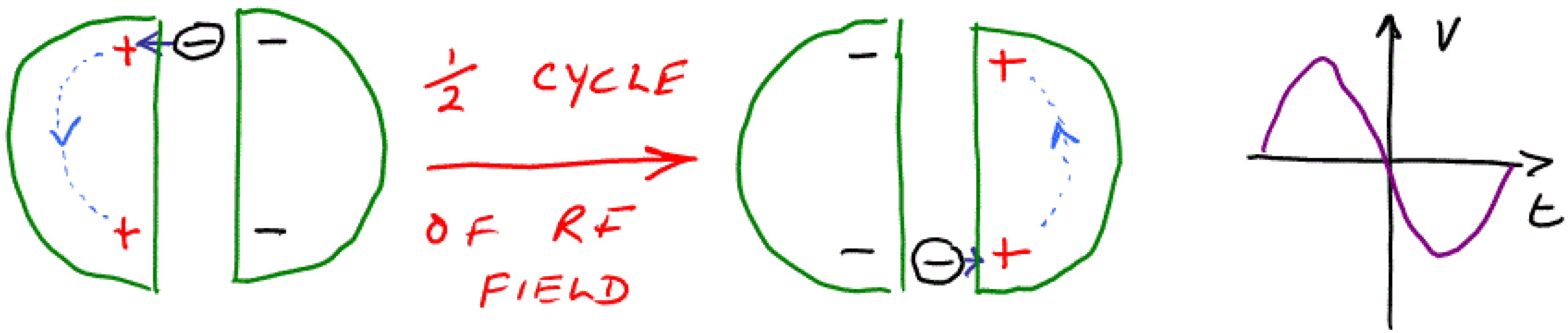
THE CYCLOTRON



- TWO HOLLOW D-SHAPED ELECTRODES, EXCITED BY HV RF OSCILLATOR
- NO ELECTRIC FIELD INSIDE "Ds"
- MAGNET FIELD INSIDE Ds
- PARTICLES ACCELERATED BY ELECTRIC FIELD AS THEY CROSS GAP BETWEEN D_s







CENTRIPETAL FORCE = LORENTZ FORCE
FOR AN ORBIT OF RADIUS r

NON-RELATIVISTIC

$$\frac{m v^2}{r} = q \frac{v \cdot B}{c}$$

$$\frac{v}{r} = \frac{q B}{m c} = \text{CONSTANT}$$

TIME FOR ORBIT = $2\pi r / v$

ORBITAL FREQUENCY = $v / 2\pi r$

IF RADIO FREQUENCY f = ORBITAL FREQUENCY

CONTINUOUS ACCELERATION

CONTINUOUS ACCELERATION

RADIO FREQUENCY = ORBITAL FREQUENCY

$$f = \frac{v}{2\pi r} = \frac{1}{2\pi}\frac{q}{m}\frac{B}{c} = \text{CONSTANT}$$

CYCLOTRON FREQUENCY

→ DOES NOT DEPEND ON RADIUS OF ORBIT

- PARTICLE STARTS AT SOURCE CLOSE TO CENTRE OF MACHINE
- SPIRALS OUT CONTINUOUSLY GAINING ENERGY FROM RESONANT RF.

THINK AGAIN ABOUT WHY A CYCLOTRON WORKS

$$F_c = F_L$$

$$\frac{mv}{r} = \frac{q \cdot B}{mc} = k$$

$$\frac{v}{r} = \text{CONSTANT} = \text{FREQUENCY}$$

AS r INCREASES, v INCREASES $\rightarrow$ $\frac{v}{r}$ = CONSTANT

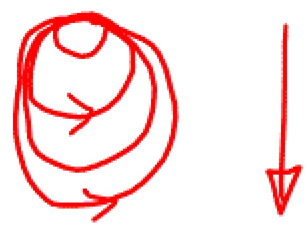
FOR A RELATIVISTIC PARTICLE $v = c =$ CONSTANT

$$\therefore \frac{v}{r} = \frac{c}{r} \neq \text{CONSTANT}$$

ELECTRON CYCLOTRON

"MICROTRON"

ELECTRON IS RELATIVISTIC FOR $E \sim 500$ keV



ORBITS INCREASE IN RADIUS DURING ACCELERATION

ANOTHER RELATIVISTIC EFFECT

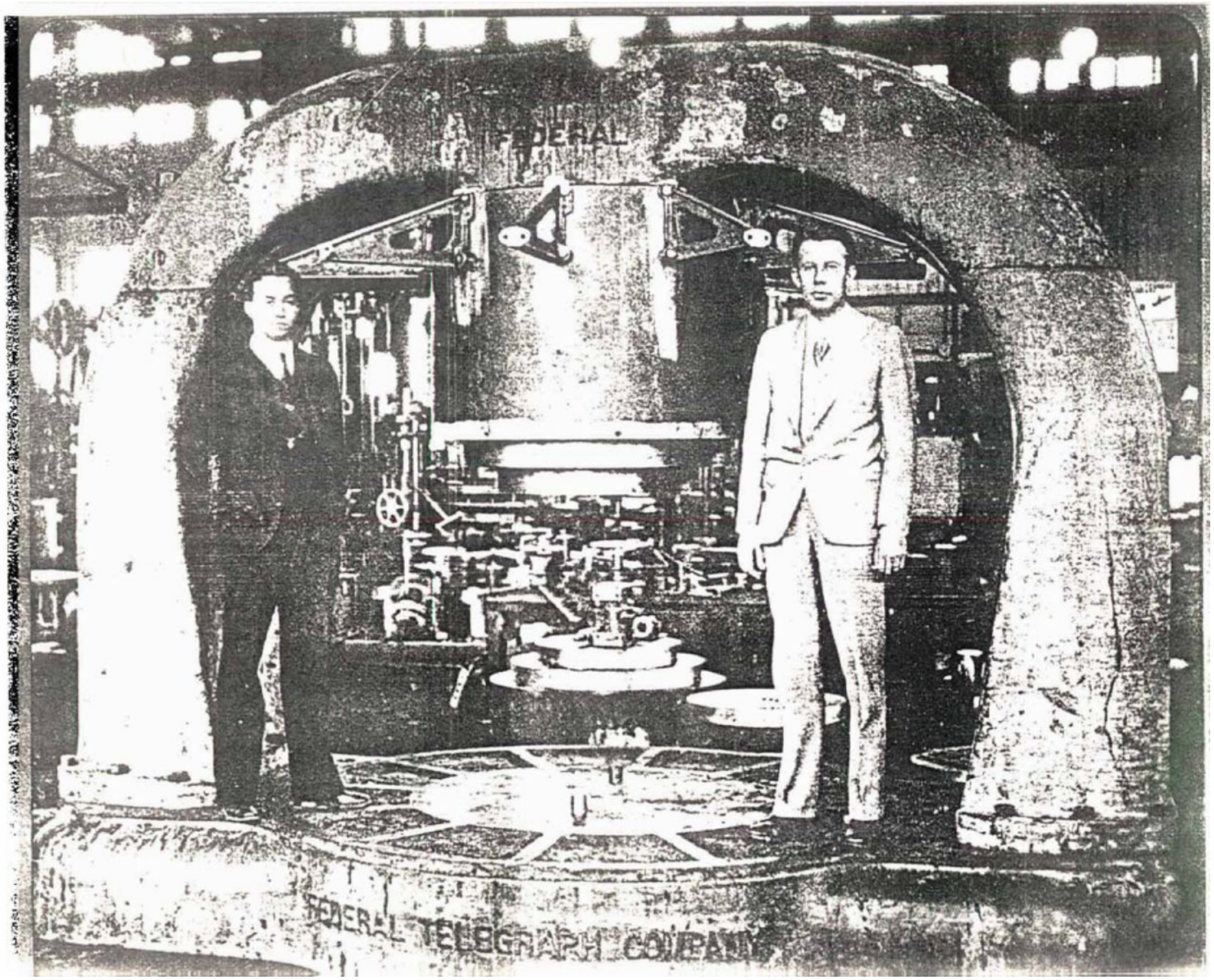
$$f = \frac{1}{2\pi}\frac{q}{m}\frac{B}{c}$$ = RF FREQUENCY = ORBITAL FREQUENCY

- AS PARTICLES ACCELERATE, TOTAL RELATIVISTIC ENERGY BECOMES ≈ MASS ENERGY
- IN THIS SITUATION $m \rightarrow m\gamma$ ← LORENTZ BOOST

$$f = \frac{1}{2\pi}\frac{q}{\gamma m}\frac{B}{c}$$

DURING ACCELERATION γ INCREASES & RESONANCE CONDITION FAILS

- INCREASE B — SYNCHROTRON
- DECREASE RF FREQUENCY — SYNCHROCYCLOTRON

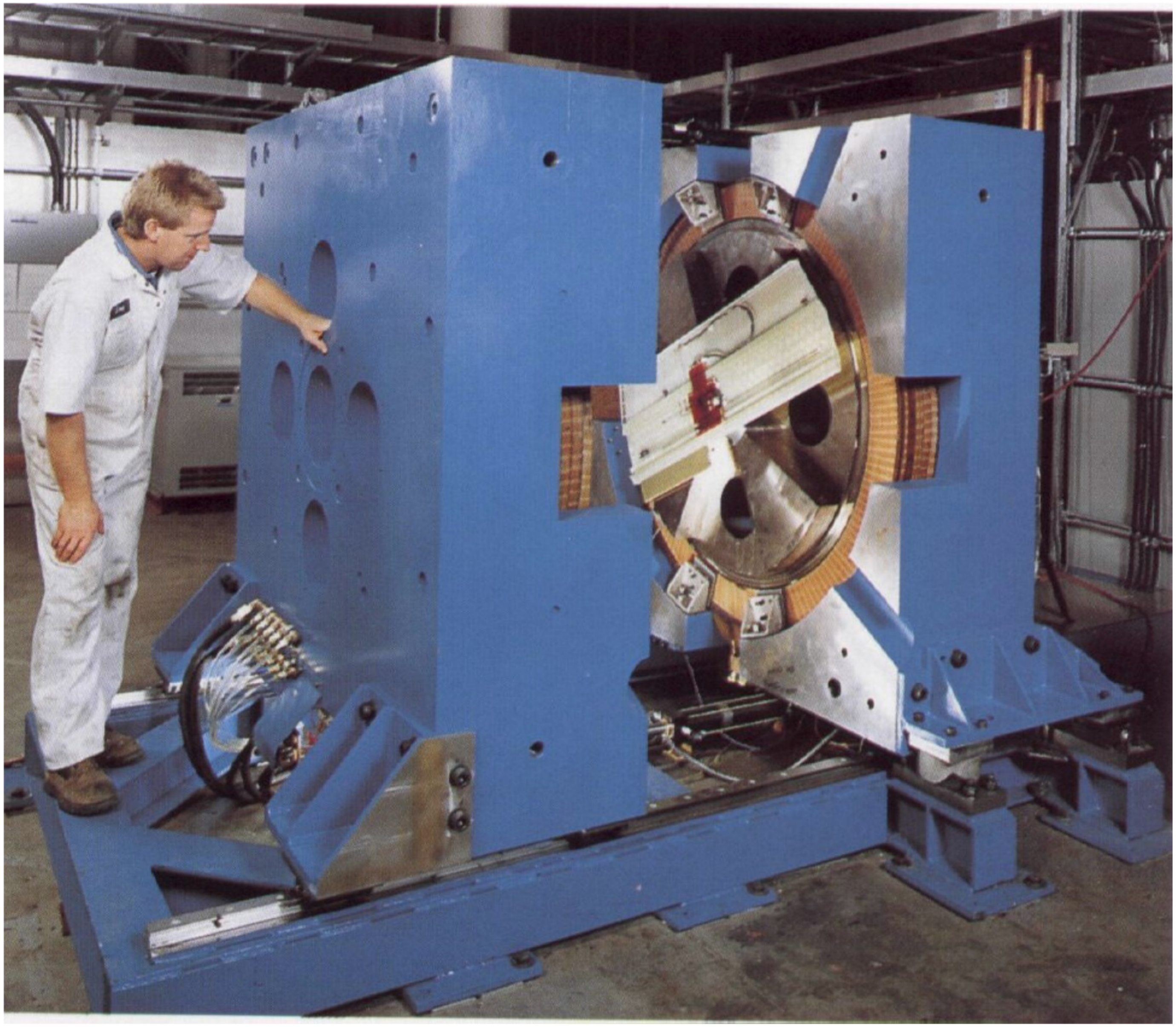


Livingston and Lawrence with the magnet of the “27-inch” (later “37-inch”) cyclotron on which most of Berkeley’s 1930s nuclear physics was performed. Lab wear was different then!

THE 184-INCH SYNCHROCYCLOTRON

The Berkeley 184” was begun in 1939 as a classical cyclotron, to be operated with Vrf = 1 MV, but WWII interrupted rf installation and it was used to test mass spectrographic separation of uranium isotopes. FM rf was installed in 1946, yielding 190 MeV d+ (700 MeV p in 1959).

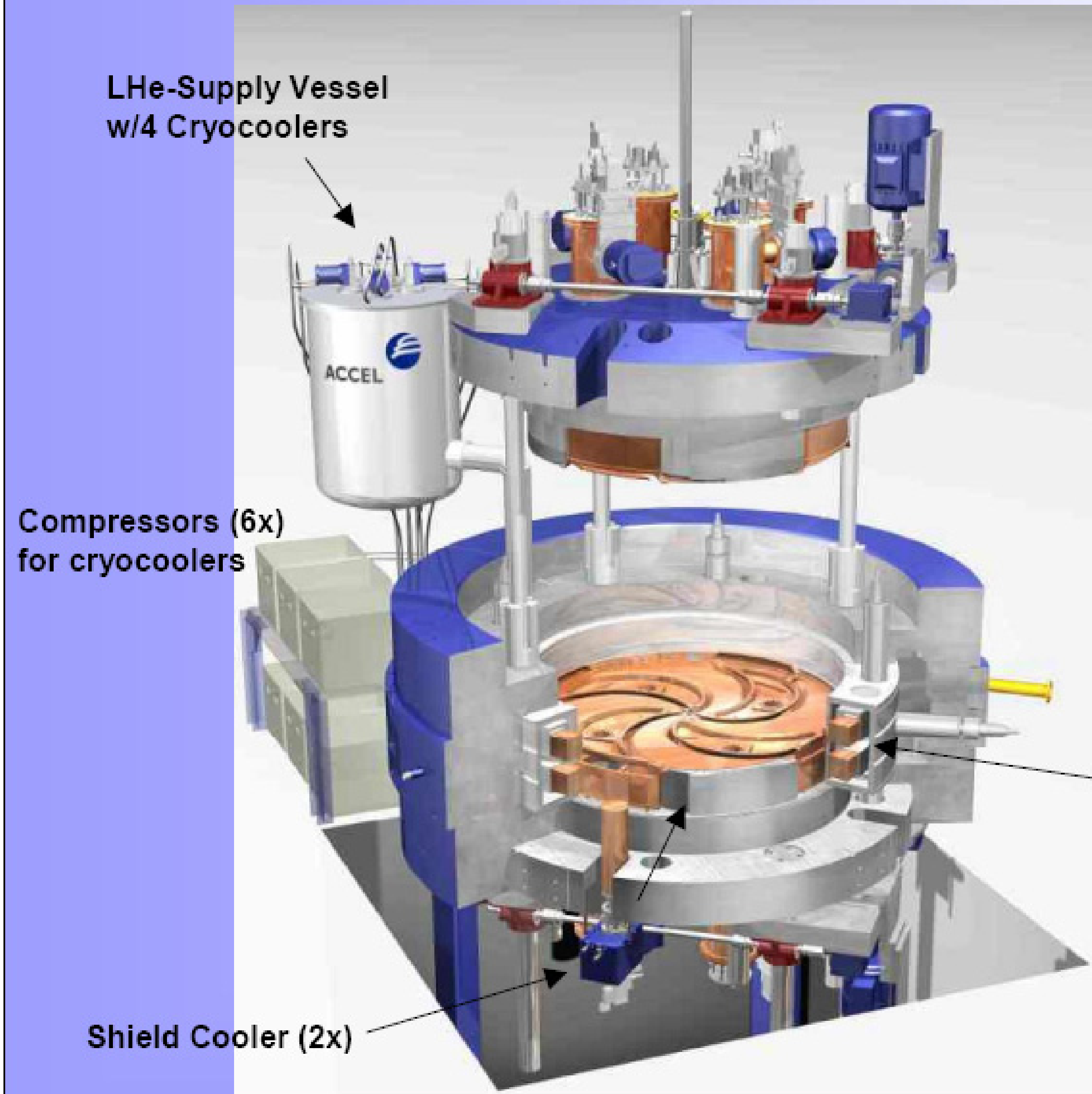
PET Medical Cyclotron

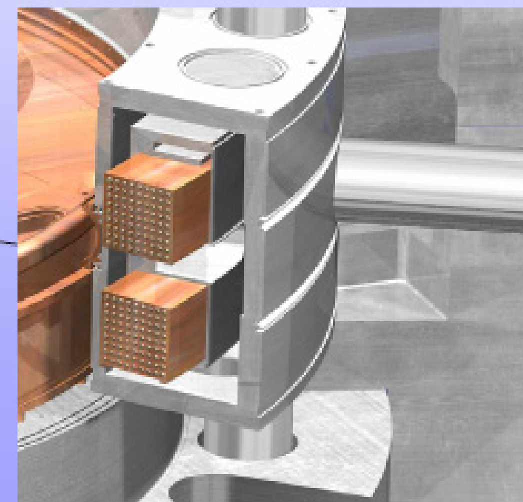


TRIUMF (Vancouver) 500 MeV Cyclotron

ACCEL

250 MeV Superconducting Proton Cyclotron



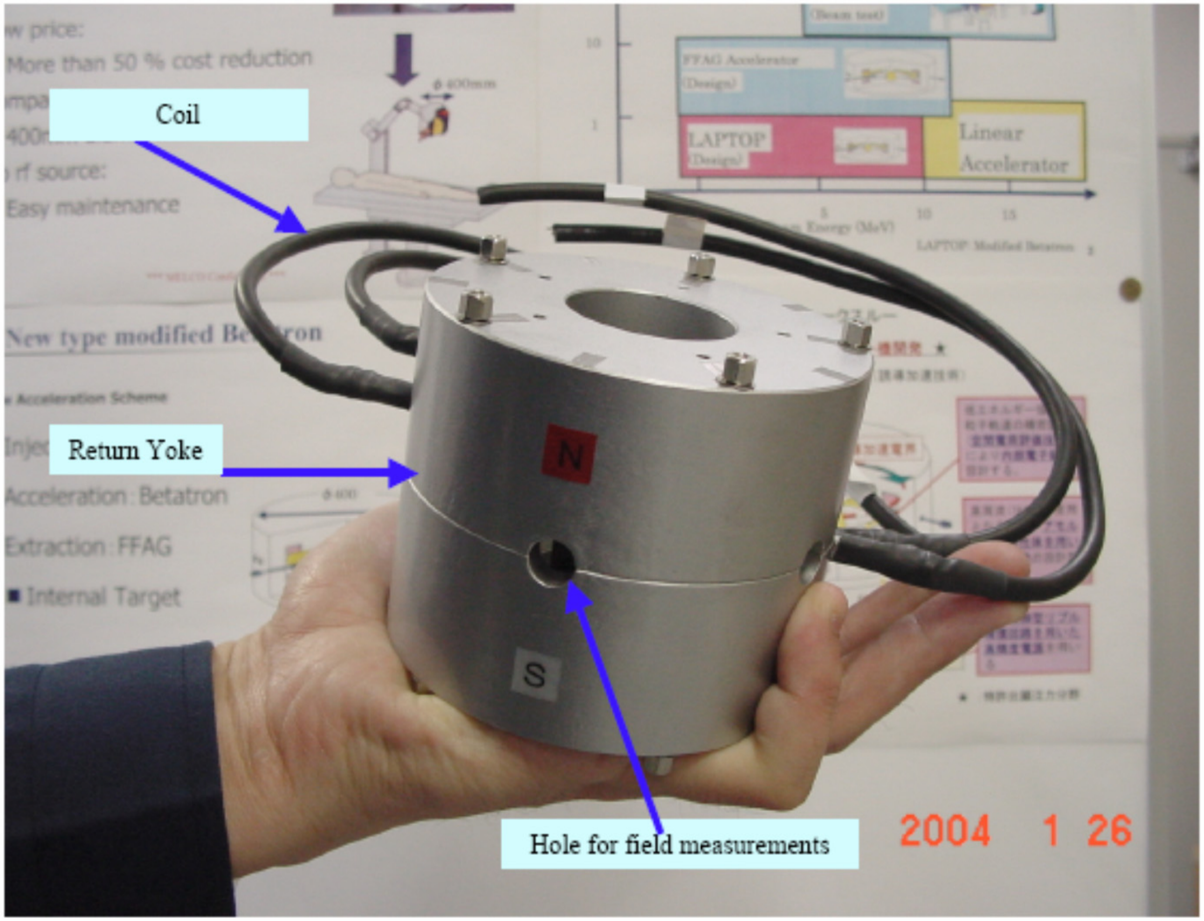


Superconducting Coil

Spiral Magnet

MITSUBISHI ELECTRIC

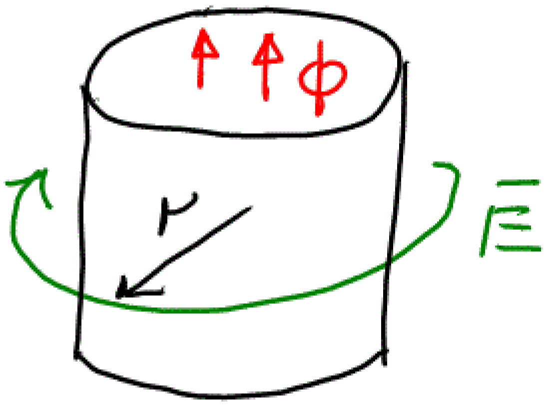
Changes for the Better



The present study is partially supported by the REIMEI Research Resources of Japan Atomic Energy Research Institute.

You can have your own cyclotron – from Mitsubishi

BETATRON → WIDEROE FIXED RADIUS ORBIT



- TIME VARYING MAGNETIC FIELD PRODUCES ELECTRIC FIELD TO ACCELERATE PARTICLES
  - → TRANSFORMER
  - → AT SAME TIME PROVIDES MAGNETIC GUIDE FIELD

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$ ← MAXWELL

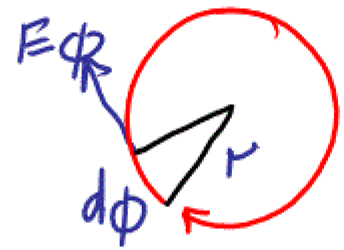
WANT TO INTEGRATE

STOKES $$\int_{LINE} \vec{v}\, d\vec{s} = \int_{AREA} \vec{\nabla} \times \vec{v}\, d\vec{A}$$

$$\int \vec{E}\, ds = -\int \frac{d\vec{B}}{dt} \cdot d\vec{A} = -\frac{d\Phi}{dt}$$

FLUX ENCLOSED BY ORBIT

FROM RIGHT HAND RULE



$\phi \rightarrow$ ANGLE

① $\int \bar{E}\, d\bar{s} = -\int E_\phi\, r\, d\phi = -2\pi r\, E_\phi \rightarrow E_\phi = \frac{1}{2\pi r} \frac{d\Phi}{dt}$

ACCELERATING ELECTRIC FIELD

SINCE PARTICLE IS MOVING IN A CIRCLE

$$\frac{\gamma m v^2}{r} - e(\bar{E} + \bar{v} \times \bar{B})$$

$\bar{E}$ RADIAL $= 0$

$$\frac{\gamma m v^2}{r} = e v B_\perp \rightarrow \frac{p v}{r} = e v B$$

$$\frac{1}{r} = \frac{eB}{p} \quad ②$$

PRACTICAL BETATRON HAS CONSTANT RADIUS ORBIT

$r =$ CONSTANT HOW?

USE $\frac{1}{r} = \frac{eB}{p}$ & CONDITION FOR r CONSTANT

$$\frac{d}{dt}\left(\frac{1}{r}\right) = \frac{d}{dt}\left(\frac{eB}{p}\right) = 0 \rightarrow -\frac{1}{r^2}\frac{dr}{dt} = e\left(\frac{\dot{B}}{p} - \frac{B\dot{p}}{p^2}\right) = 0$$

ACCELERATING ELECTRIC FIELD GIVES $\dot{p}$

$$\dot{p} = eE_\phi = \frac{e}{2\pi r}\dot{\Phi} \rightarrow \frac{\dot{B}}{p} = \frac{B}{p^2}\frac{e}{2\pi r}\dot{\Phi}$$

FROM TOP OF PAGE $B = p/er$

$$\frac{\dot{B}}{p} = \frac{p}{erp^2}\cdot\frac{e}{2\pi r}\cdot\dot{\Phi} \rightarrow \dot{\Phi} = 2\pi r^2 \dot{B}$$

REWRITE

$$\frac{d\bar{\Phi}}{dt} = 2\pi r^2 \frac{dB(r)}{dt}$$

ORBIT RADIUS

COMPLETE FLUX ENCLOSED BY ORBIT $\bar{\Phi} = \pi r^2 B_A$

AVERAGE FIELD ENCLOSED BY ORBIT

SO

$$\frac{d\bar{\Phi}}{dt} = \pi r^2 \frac{dB_A}{dt}$$

AT ORBIT

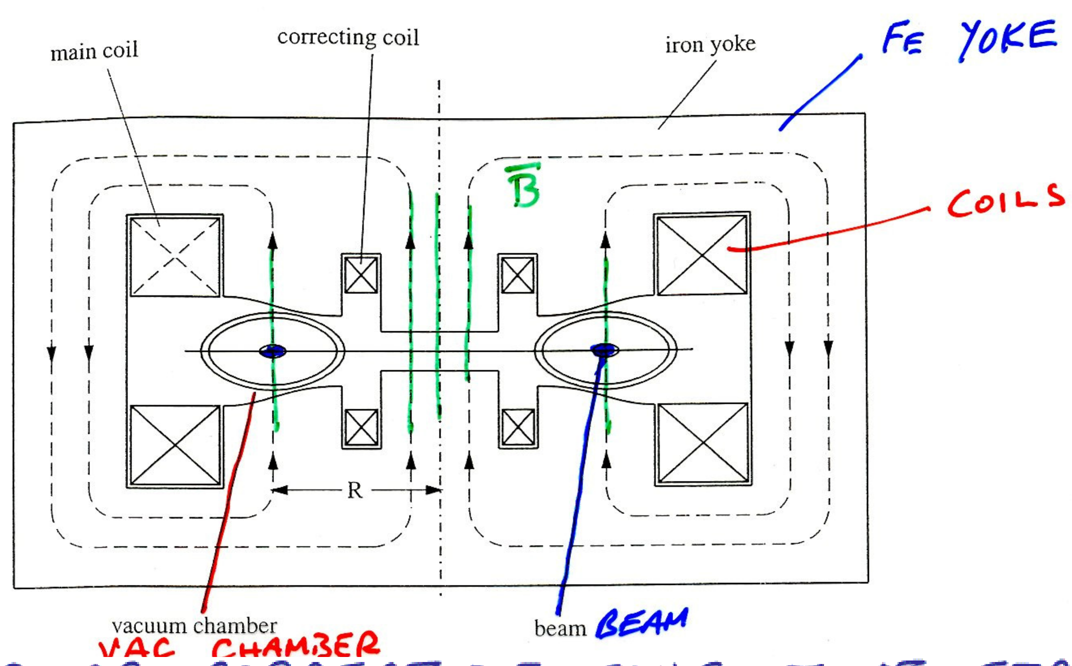
AVERAGE ENCLOSED

$$\frac{dB(r)}{dt} = \frac{1}{2}\frac{dB_A}{dt} \qquad \int dt \rightarrow B(r) = \frac{1}{2}B_A + B_0$$

WIDEROE $\frac{1}{2}$ CONDITION

FOR STABLE ORBIT

FIELD AT ORBIT = $\frac{1}{2}$ (AVERAGE FLUX DENSITY THRU ORBIT)



- GAP OR CORRECTION COILS TUNE STABILITY
- NOT PRACTICAL FOR HIGH ENERGY

MEDICAL
X-RAY FOR INDUSTRY

- LARGEST BETATRON — KERST, U. of ILLINOIS

$r = 1.23$ m

$B = 8.1$ kG

MASS = 350 t

$p_{MAX} = 300$ MeV/c

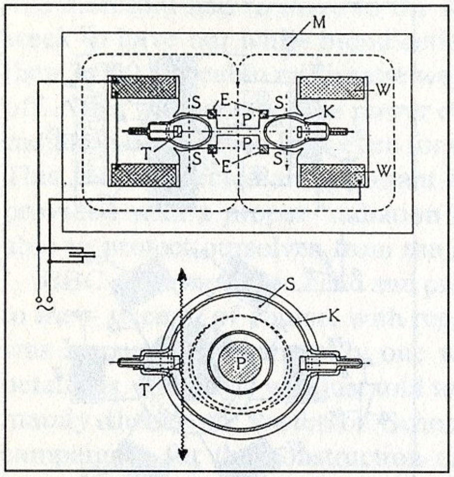


Fig. 10.3: Diagram of BBC's double-beam betatron.
M = Magnet yoke
P = Central magnet poles
S = Steering poles
W = Exciting coils
E = Expansion coils
K = Ring tube
T = Anticathode (target) [Wi62].

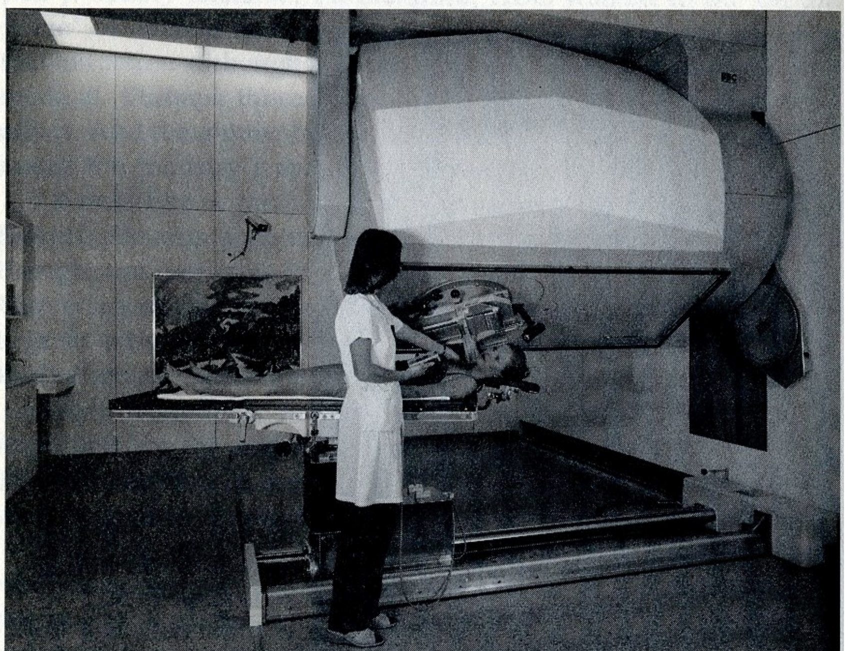


Fig. 10.4: Betatron radiation therapy, Inselspital Berne (phot.: BBC).

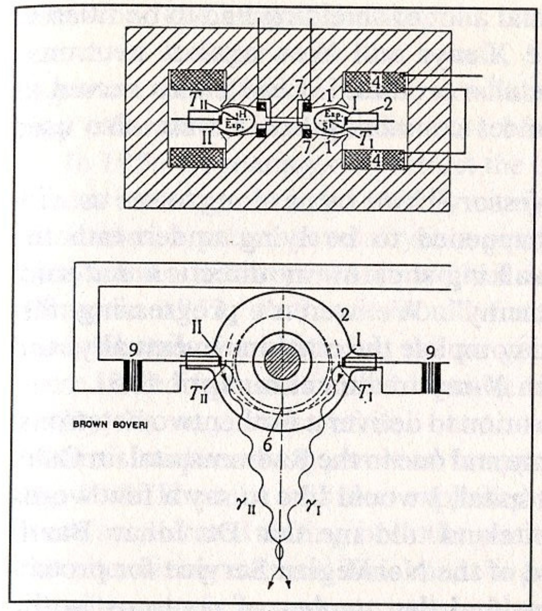


Fig. 10.5: BBC stereo two-beam betatron for materials testing.
1 = Magnet pole
2 = Ring tube
4 = Coil
6 = Orbit
7 = Expansion coil
9 = Impulse transformer
I+II = Electron sources
$T_I + T_{II}$ = Targets
$Y_I + Y_{II}$ = X-rays
[Se58].

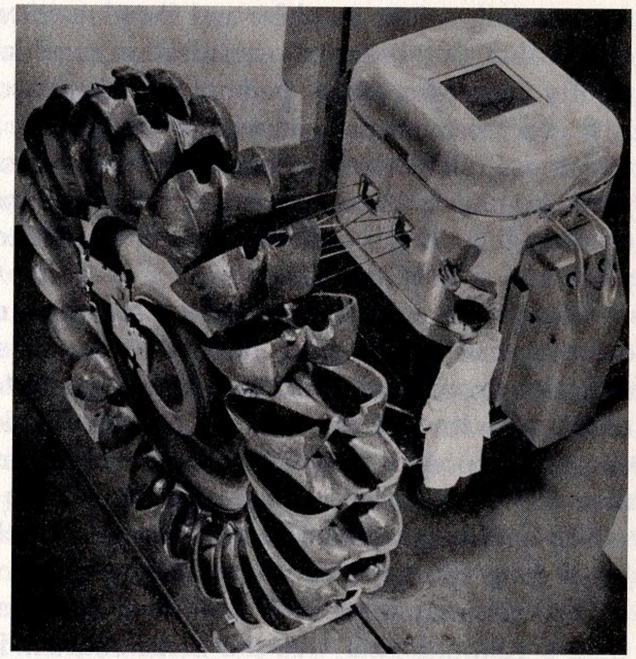
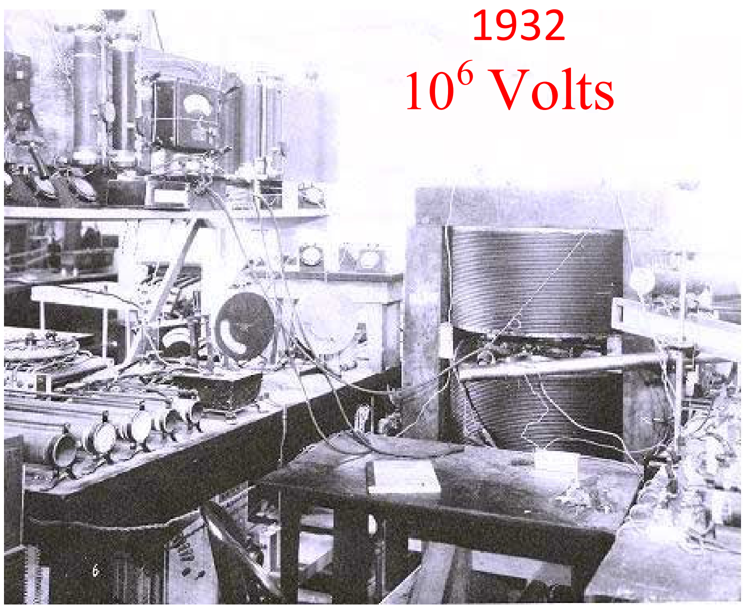
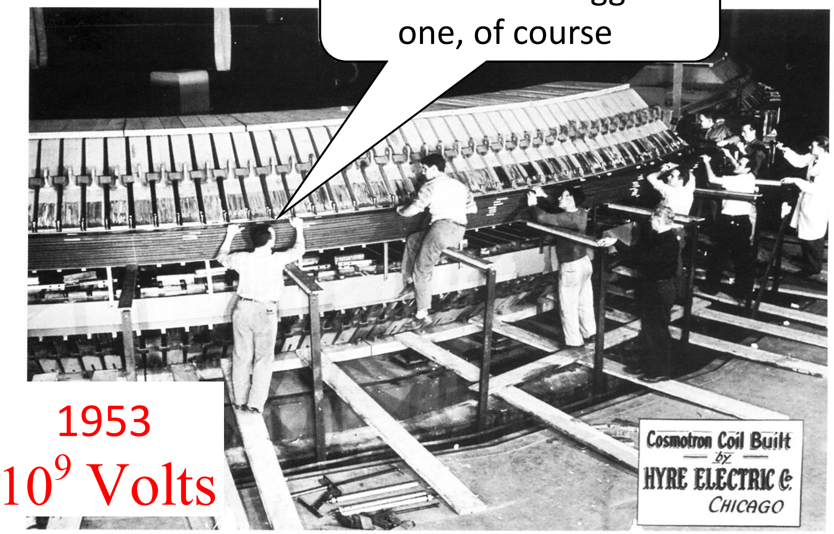


Fig. 10.6: A betatron being used to test a Pelton-wheel at Georg Fischer AG, Schaffhausen (photograph: BBC).

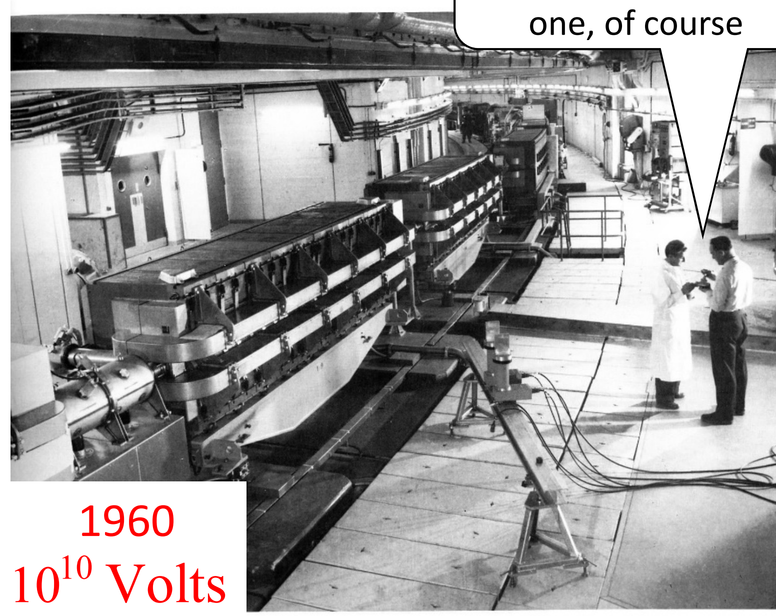
1931
10^4 Volts



1932
10^6 Volts

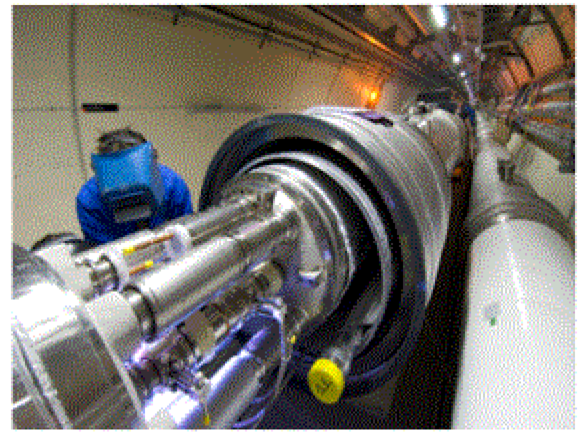


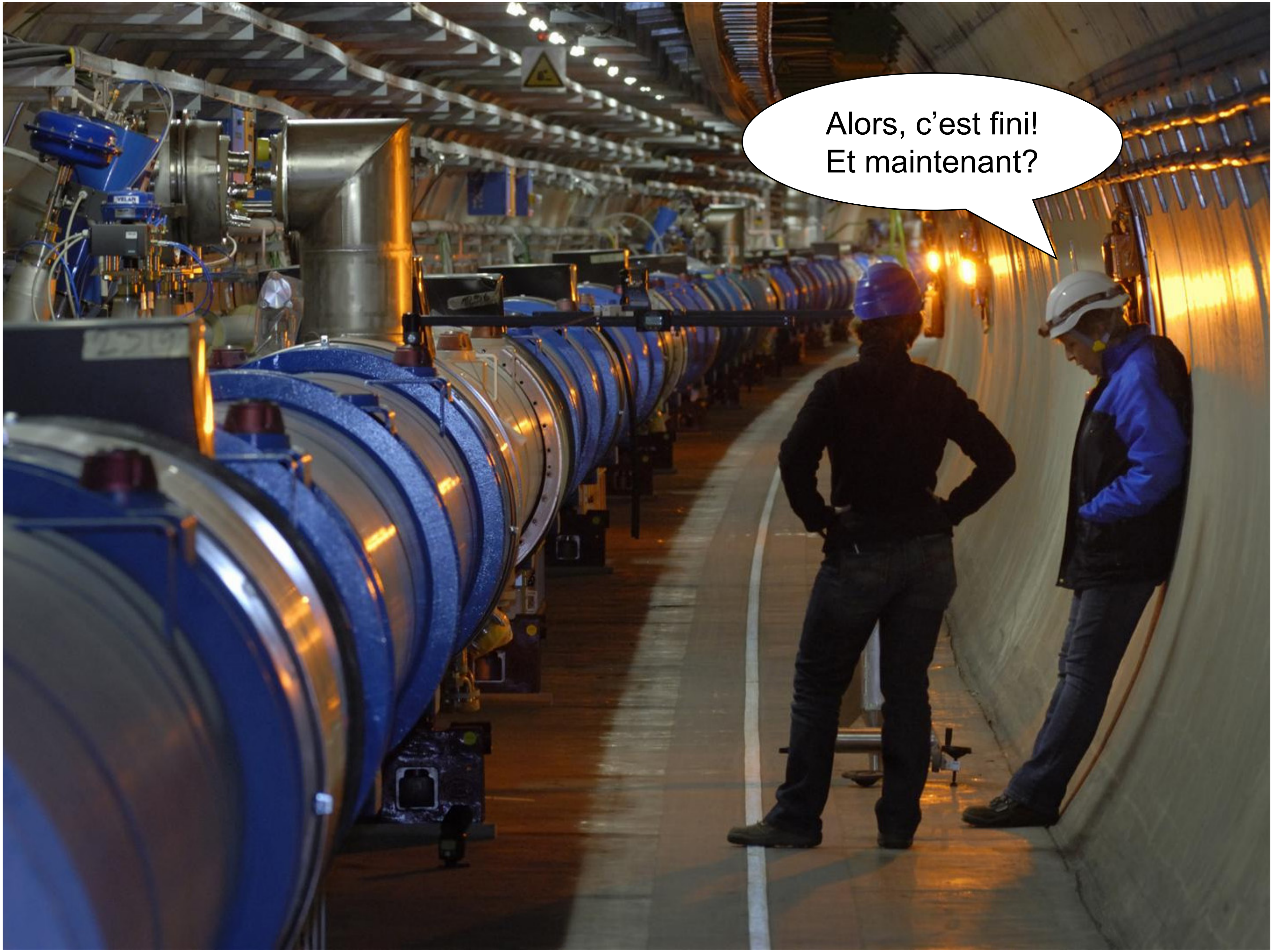
1953
10^9 Volts



1960
10^{10} Volts

BUILDING THE LHC

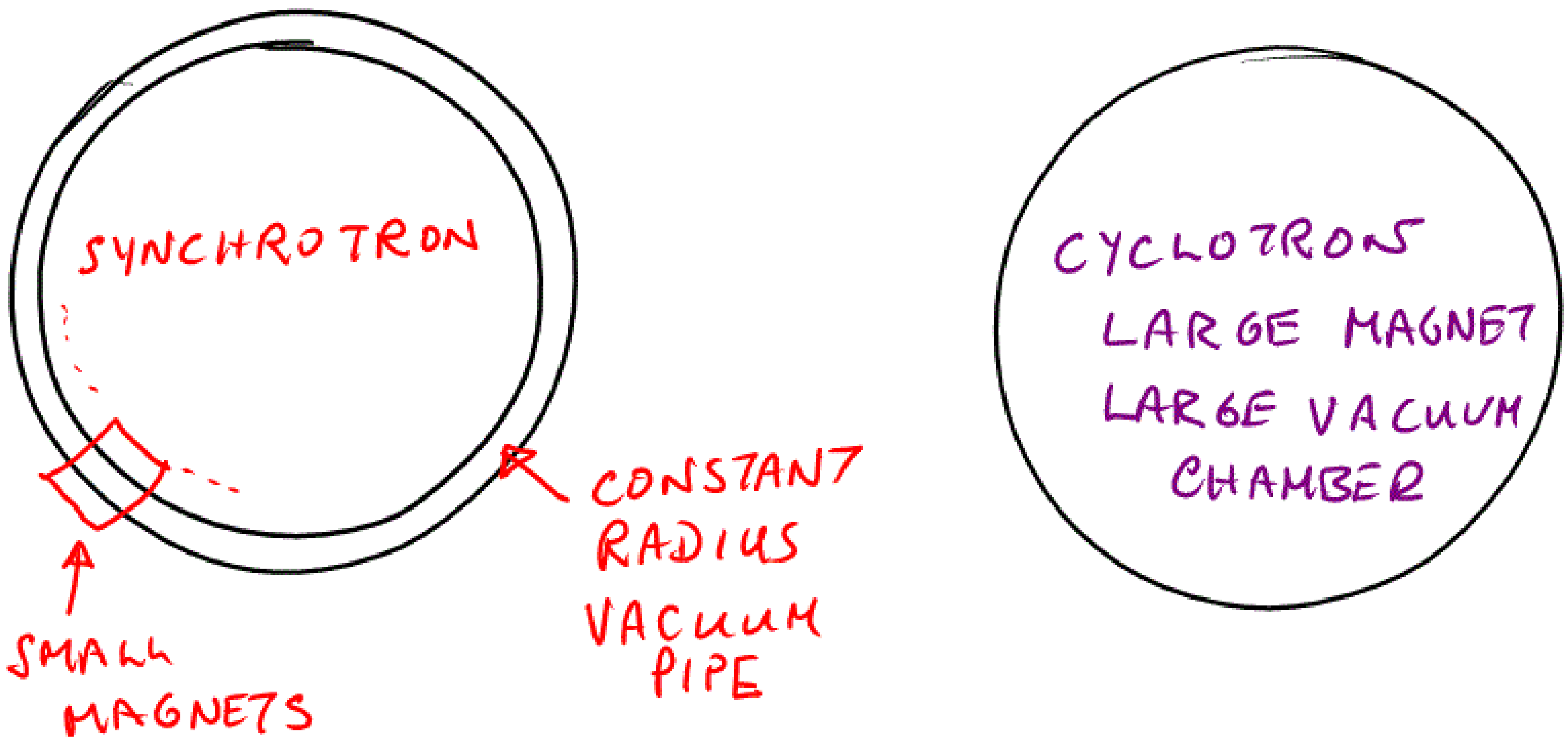



Alors, c'est fini!
Et maintenant?

FOR HIGH ENERGY, ONLY MACHINES WITH CONSTANT ORBIT RADIUS ARE PRACTICAL

$$\frac{1}{r} = \frac{eB}{p} \rightarrow p = eB \cdot r \rightarrow r \propto p$$

$p \rightarrow$ HIGH $\quad r \rightarrow$ HIGH



Synchrotron Ring Schematic