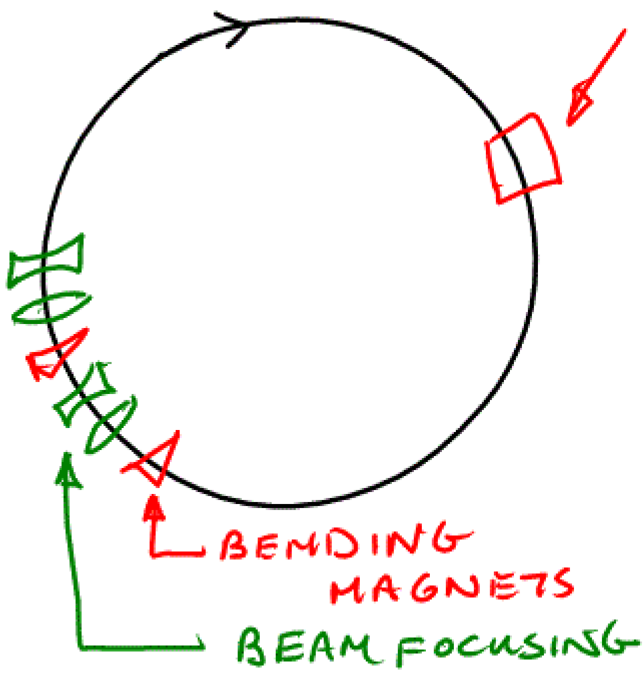
# RADIO FREQUENCY CAVITIES

- IN THE ELECTROSTATIC ACCELERATOR THE ACCELERATING VOLTAGE IS CONSTANT
- IN THE CYCLOTRON WE HAVE A RADIO FREQUENCY ACCELERATING VOLTAGE.
- WHAT ABOUT SYNCHROTRON?
- USE A RADIO FREQUENCY CAVITY WITH A RESONANT FREQUENCY EQUAL TO ORBITAL FREQUENCY (OR SOME MULTIPLE)
- IF Q-VALUE OF CAVITY IS LARGE, AND ENERGY TRANSFERRED TO BEAM SMALL

$\Rightarrow$ EFFICIENT USE OF RADIO FREQUENCY POWER




# ACCELERATING STRUCTURE

- TYPICALLY ENERGY TRANSFERRED FROM ACCELERATING FIELD TO BEAM IS SMALL
- SAY A PARTICLES RECEIVES 1MeV ON TRANSIT THROUGH CAVITY – $10^{10}$ PARTICLES / BUNCH
  - ENERGY EXTRACTED ~ m JOULE
- ENERGY STORED IN OSCILLATING FIELDS IN CAVITY MANY ORDERS OF MAGNITUDE GREATER

→ USE HIGH Q RESONANT CAVITIES

– LET'S HAVE A LOOK AT RF CAVITIES

# SIMPLE RF CAVITY SYSTEM

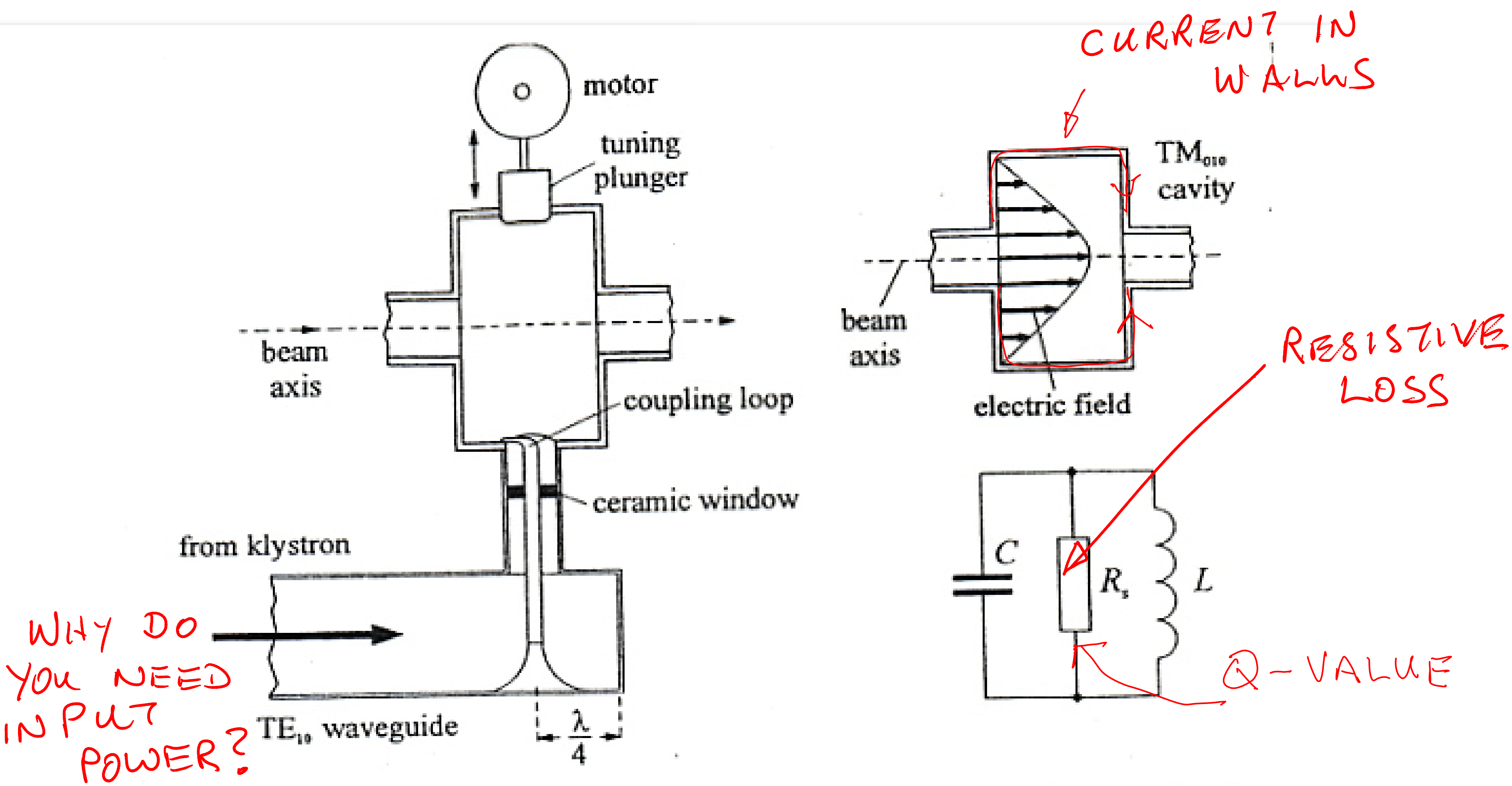



**Fig. 5.4** Design of a single-cell accelerating structure using the TM$_{010}$ mode. The exact resonant frequency is adjusted using a tuning plunger. The resonator is excited by an inductive coupling loop.

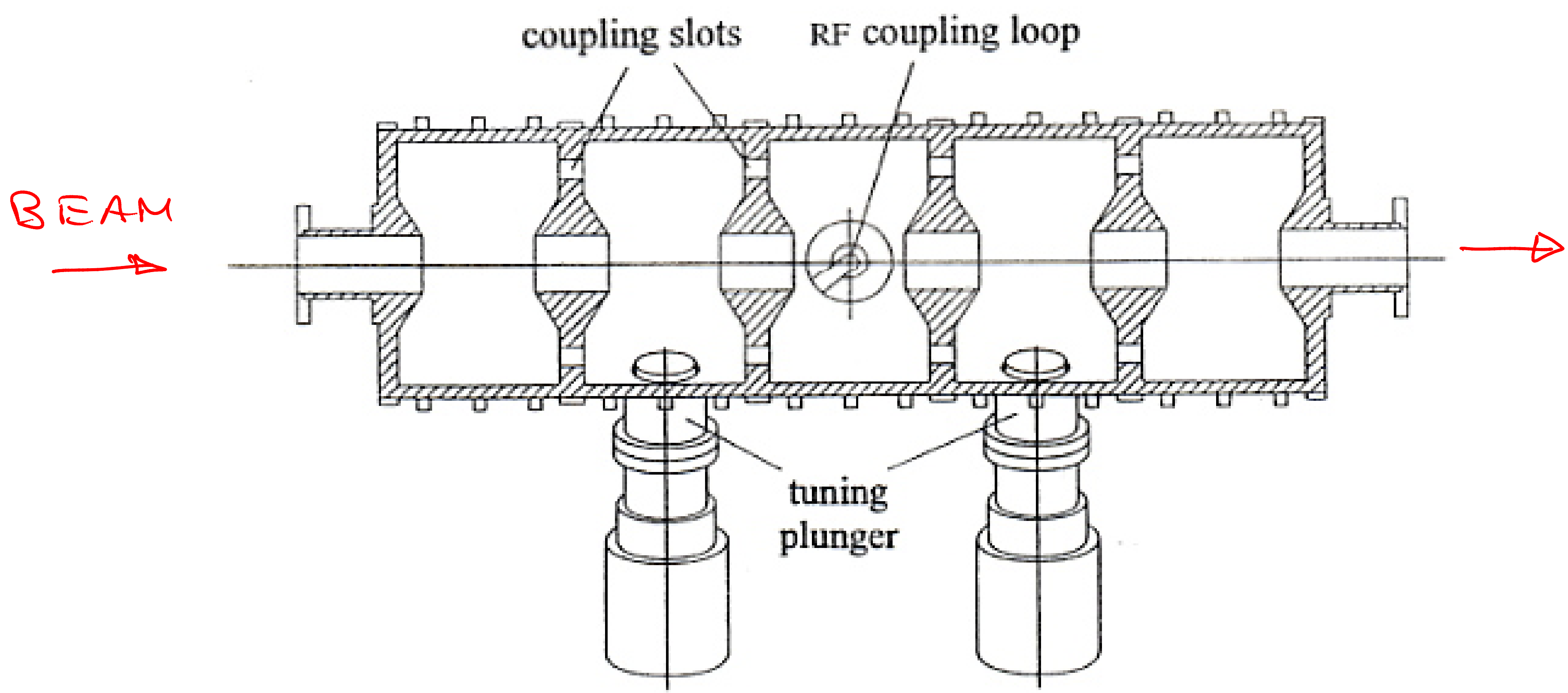
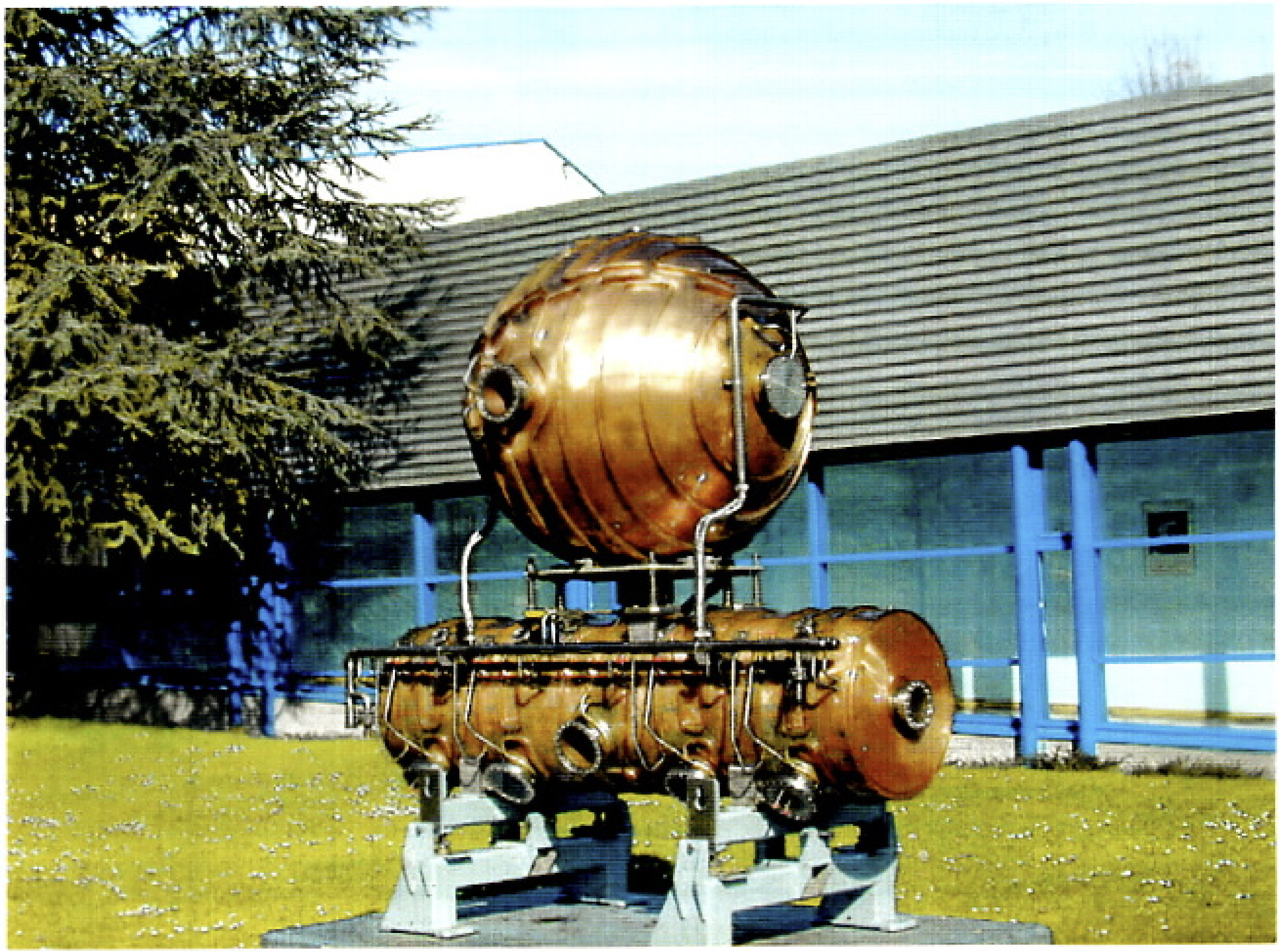


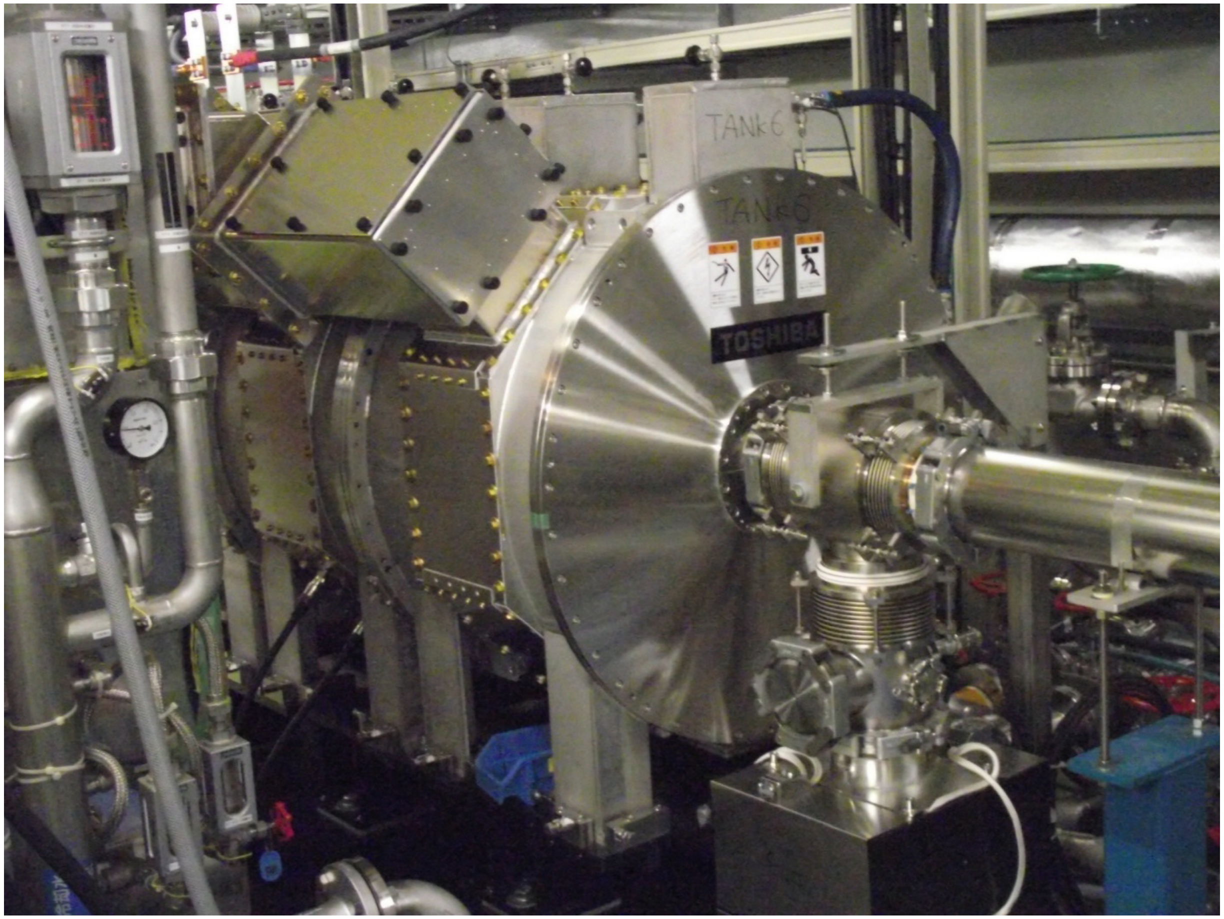

**Fig. 5.5** Layout of a five-cell accelerating structure. The power feed is coupled to the middle cell and two tuning plungers are sufficient for the entire structure.

$$\boxed{f_{\mathrm{RF}} = 500\ \mathrm{MHz}}$$
$$R_s = 3.0 \times 10^6\ \Omega$$
$$Z = 80\ \Omega$$
$$\boxed{Q = 38\,000}$$

$$\boxed{P_{\mathrm{RF}} = 50\ \mathrm{kW}}$$
$$\boxed{U_{\mathrm{cav}} = 548\ \mathrm{kV}}$$

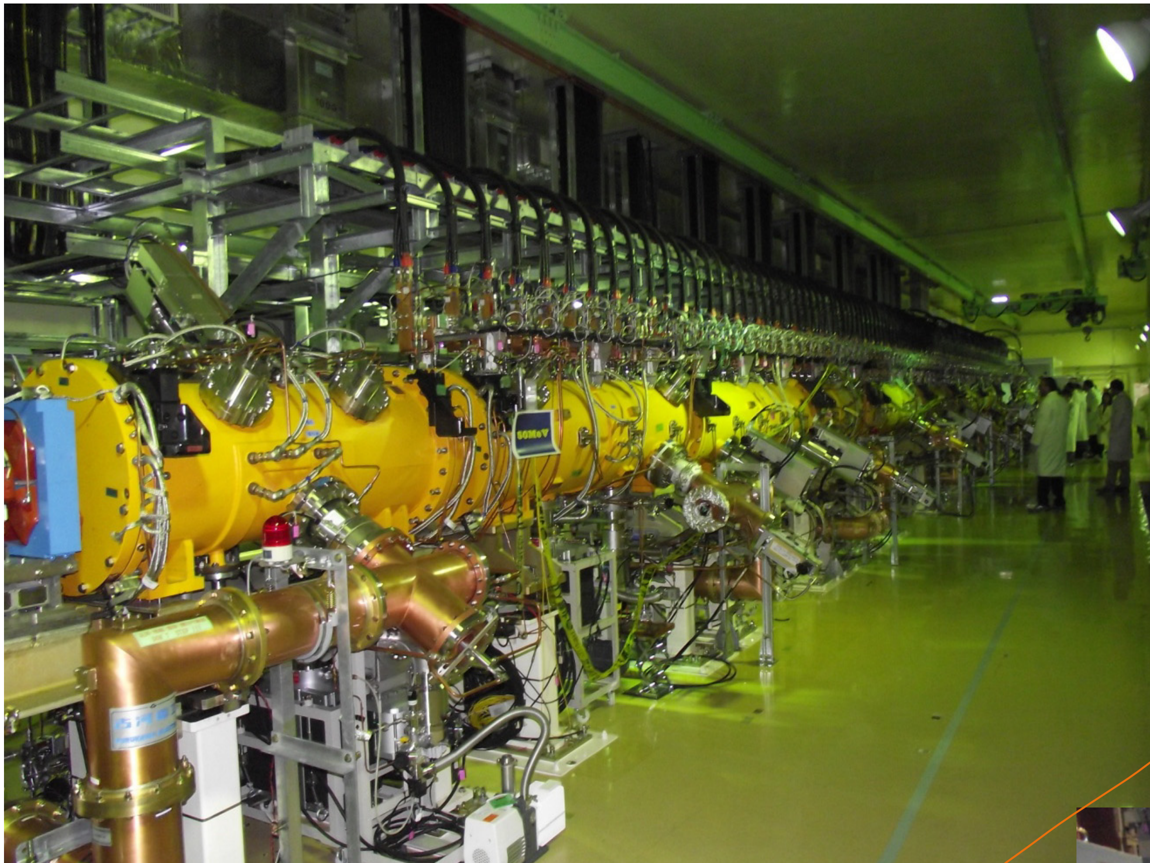
# Copper RF Cavity






RF CAVITY IN JPARC

KLYSTRONS

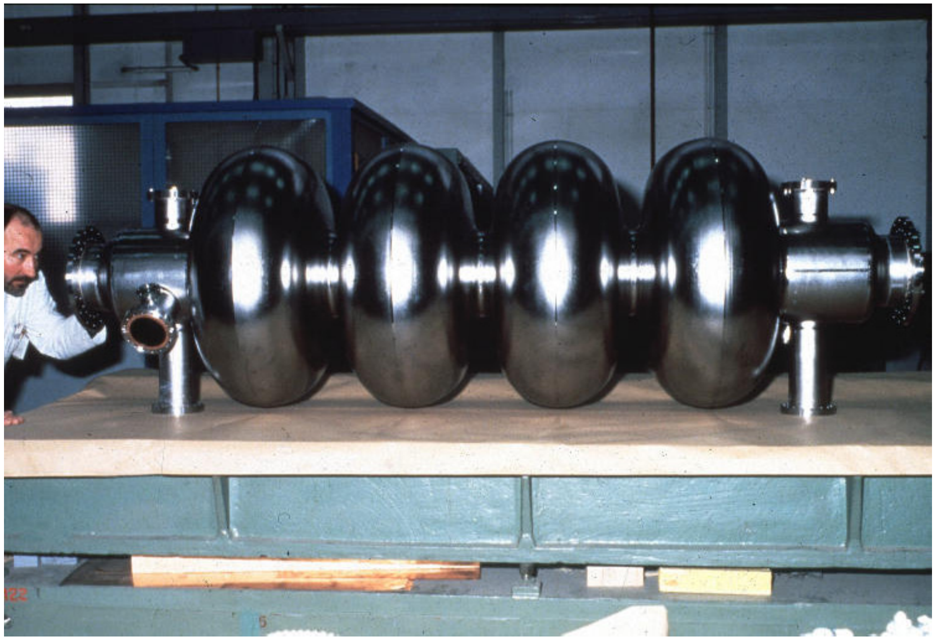


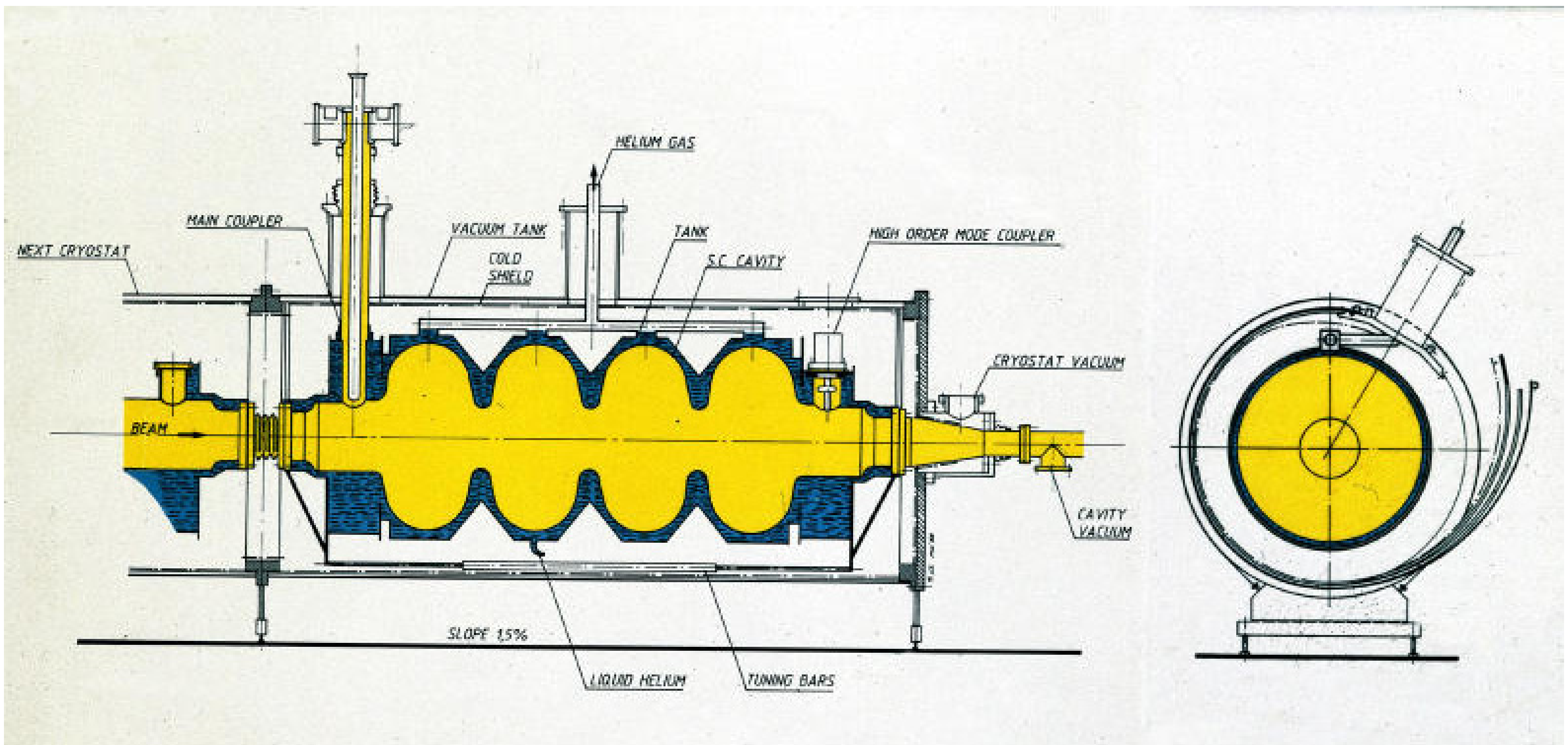
JPARC LINAC

KLYSTRON

KLYSTRON GALLERY

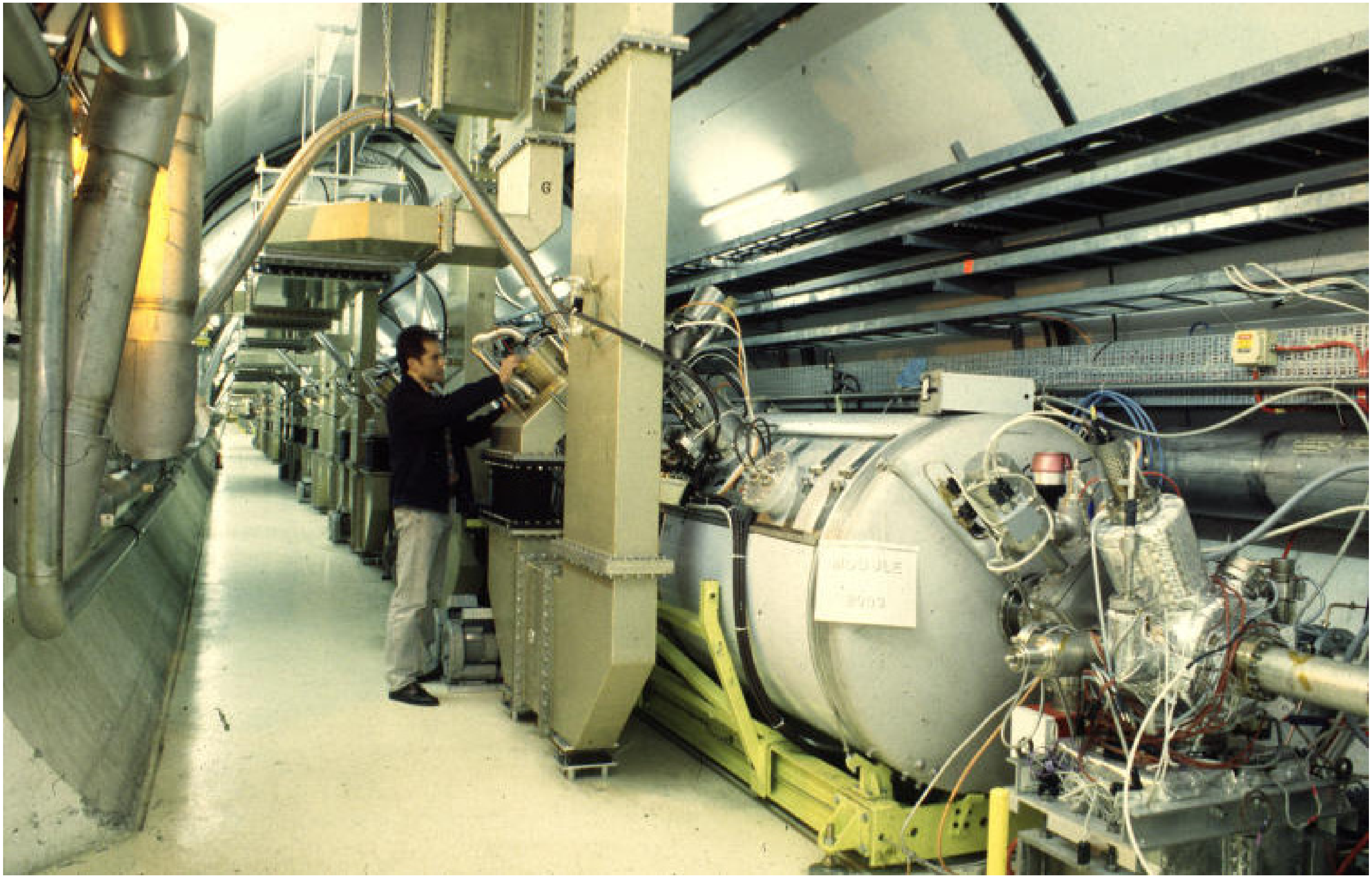
LEP
SUPER CONDUCTING
CAVITY



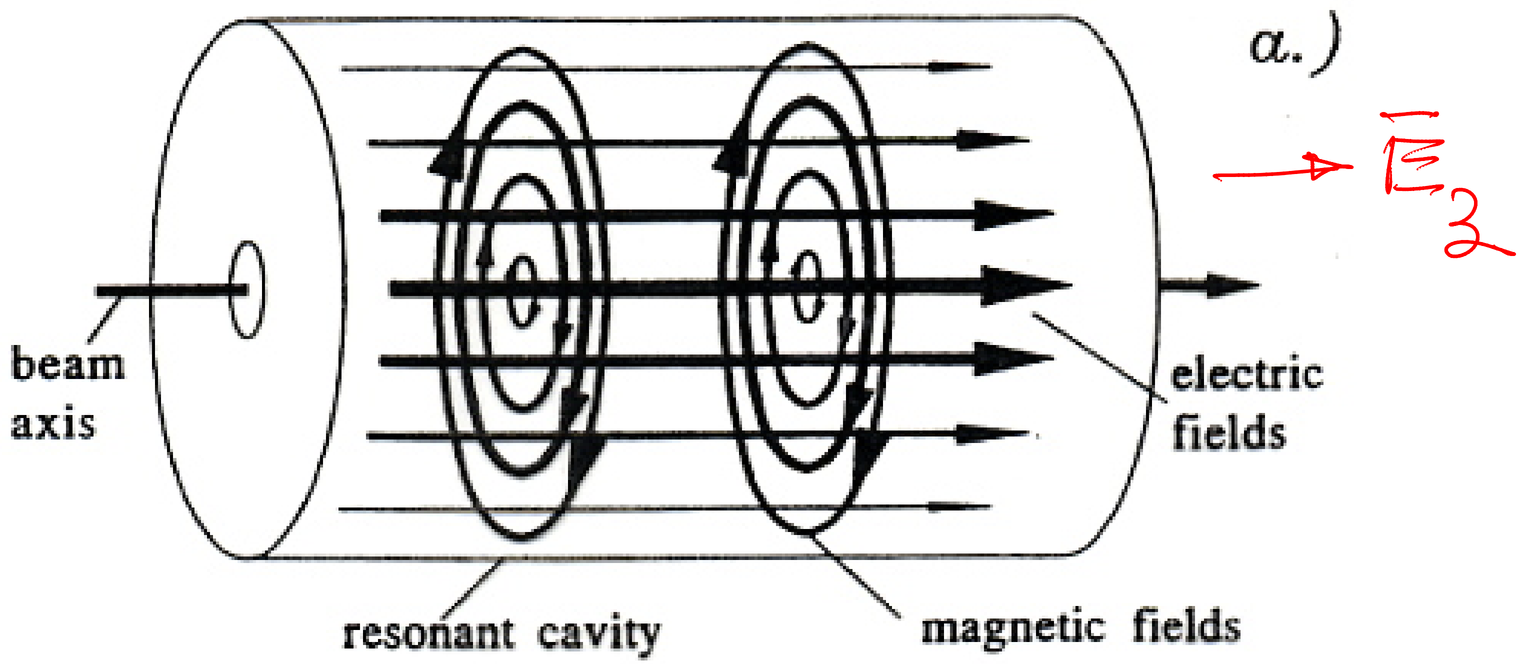



SUPERCONDUCTING CAVITY WITH ITS CRYOSTAT

# LEP SUPERCONDUCTING CAVITY

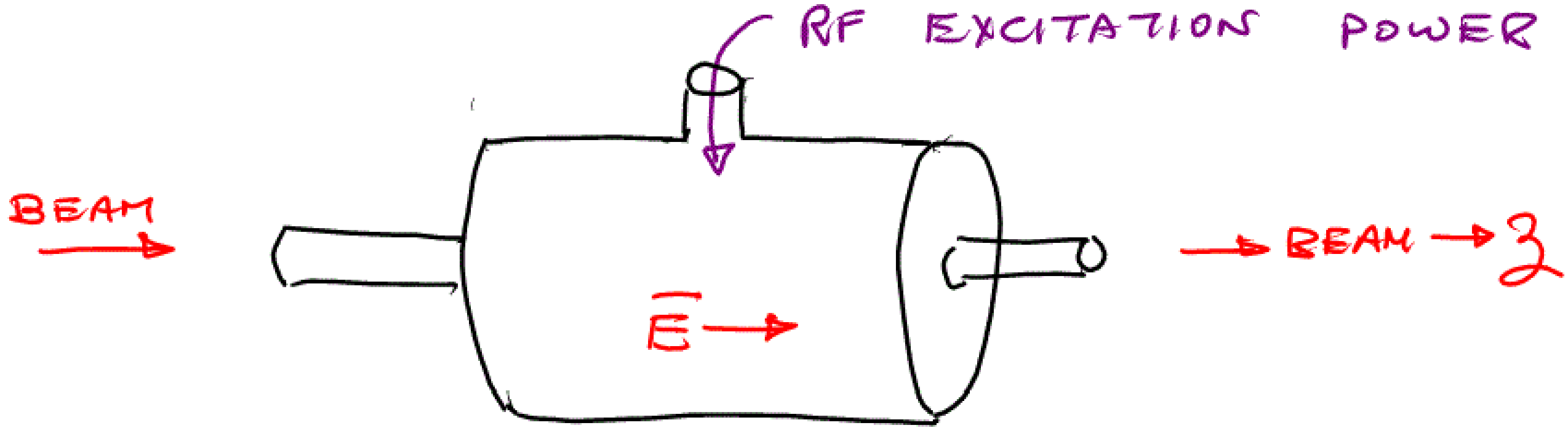


SOLVE LAPLACE EQUATION
WITH CYLINDRICAL BOUNDARY CONDITIONS

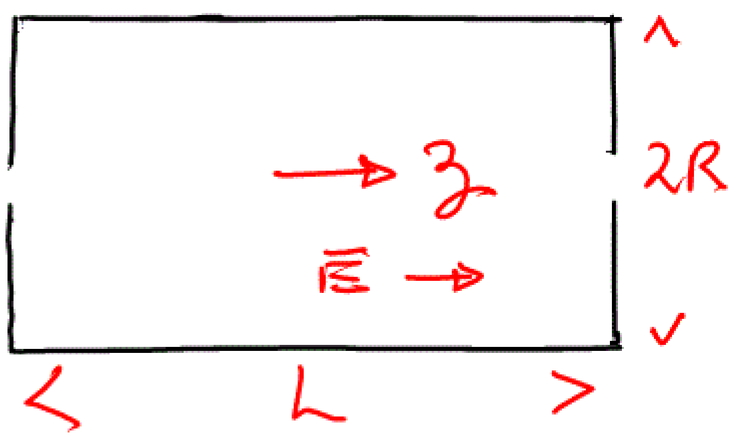



FREE ELECTROMAGNETIC
WAVES HAVE $E_z = 0$

# RADIO FREQUENCY ACCELERATING CAVITIES




- IDEALIZATION OF A CAVITY — PILL BOX
- BEAM ENTERS/EXITS THROUGH HOLES FOR BEAM PIPE.
- ANTENNA COUPLES RF POWER FROM KLYSTRON INTO CAVITY.




- WE WANT TO INVESTIGATE SPATIAL VA(R)IATION OF ACCELERATING $\bar{E}$
- NEED $E_z$ — FREE EM WAVE DOES NOT HAVE THIS
- BUT SOLVING LAPLACE EQUATION WITH BOUNDARY CONDITIONS
- THERE ARE VARIOUS MODES OF ELECTRIC AND MAGNETIC OSCILLATIONS

$TM_{010} \rightarrow$ ONLY $E_z$ AND $B_\theta$

MAXWELL

FOR $j \equiv 0$

$$\bar{\nabla} \times \bar{B} = \frac{1}{c^2}\frac{\partial \bar{E}}{\partial t} \rightarrow \frac{1}{r}\frac{\partial (r B_\theta)}{\partial r} = \frac{1}{c^2}\frac{\partial E_z}{\partial t}$$

$$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \rightarrow \frac{\partial E_z}{\partial r} = \frac{\partial B_\theta}{\partial t}$$

I'M USING CYLINDRICAL COORDINATES!

$$(\vec{\nabla}\times\vec{A})_r = \left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right)$$

$$(\vec{\nabla}\times\vec{A})_\theta = \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) \qquad \frac{\partial A_z}{\partial r} = \frac{\partial B_\theta}{\partial t}$$

$$(\vec{\nabla}\times\vec{A})_z = \frac{1}{r}\left(\frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right) \qquad \frac{1}{r}\frac{\partial (r A_\theta)}{\partial r} = \frac{1}{c^2}\frac{\partial E_z}{\partial t}$$

ONLY $B_\theta \neq 0$

AND $\frac{\partial B_\theta}{\partial z} = 0$

ONLY $E_z \neq 0$

$\frac{\partial E_z}{\partial \theta} = 0$

$$\frac{1}{r}\frac{\partial}{\partial r}(r B_\theta) = \frac{1}{c^2}\frac{\partial E_z}{\partial t}$$

$$\frac{1}{r}\frac{\partial r}{\partial r} B_\theta + \frac{r}{r}\frac{\partial B_\theta}{\partial r} = \frac{1}{c^2}\frac{\partial}{\partial t} E_z$$

DIFFERENTIATE W.R.T TIME

$$\frac{1}{r}\frac{\partial B_\theta}{\partial t} + \frac{\partial^2 B_\theta}{\partial t\,\partial r} = \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2}$$

$\frac{\partial E_z}{\partial r}$ $\qquad$ $\frac{\partial^2 E_z}{\partial r^2}$ $\leftarrow$ MAXWELL

$$\frac{1}{r}\frac{\partial E_z}{\partial r} + \frac{\partial^2 E_z}{\partial r^2} = \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2} \quad \text{--- } (1)$$

ONLY $E_z$ APPEARS — ONLY $E_z$ ACCELERATES PARTICLES

TIME VARIATION OF $E_z$ IN IDEAL CAVITY WILL JUST BE SINUSOIDAL.

$$E_z(r,t) = E(r)\, e^{i\omega t}$$

$$\frac{\partial}{\partial t} E_z = i\omega\, E(r)\, e^{i\omega t} = i\omega\, E_z$$

$$\frac{\partial^2}{\partial t^2} E_z = -\omega^2 E(r)\, e^{i\omega t} = -\omega^2 E_z$$

WRITE $\dfrac{\partial^2 E_z}{\partial r^2} = \dfrac{\partial^2 E(r)}{\partial r^2} e^{i\omega t}$

$$\frac{\partial E_z}{\partial r} = \frac{\partial E(r)}{\partial r} e^{i\omega t}$$

AND $\dfrac{\partial^2 E_z}{\partial t^2} = -\omega^2 E(r)\, e^{i\omega t}$

THEN FOR $\frac{1}{r}\frac{\partial E_z}{\partial r} + \frac{\partial^2 E_z}{\partial r^2} = \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2}$

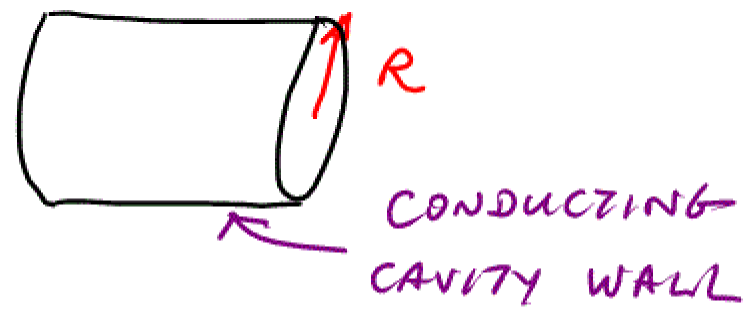
HAVE:

$$\frac{\partial^2 E(r)}{\partial r^2} + \frac{1}{r}\frac{\partial E(r)}{\partial r} + \frac{\omega^2}{c^2} E(r) = 0$$

THIS IS BESSEL'S EQUATION OF ORDER ZERO
IT DESCRIBES THE RADIAL VARION OF
$E_z$ WHICH HAS TIME VARIATION $e^{i\omega t}$
IT' SOLUTION IS:

$$E(r) = E_0 J_0\left(\frac{\omega}{c} \cdot r\right)$$

$$E(r) = E_0 J_0\left(\frac{\omega}{c} \cdot r\right)$$




AT $r = R$ – CAVITY WALL

ELECTRIC FIELD VANISHES

$$\frac{\omega}{c} \cdot R \rightarrow 1^{st} \text{ ZERO OF } J_0 \rightarrow \frac{\omega}{c} \cdot R = 2.4048$$

$$\frac{2\pi f}{c} \cdot R = 2.405;$$

RESONANT FREQUENCY

FOR $R \sim 30$ cm

$$f = \frac{c}{2\pi} \cdot \frac{2.4}{30} \sim 4 \times 10^8$$

TYPICAL CAVITY HAS RESONANT FREQUENCY ~ 500MHz

ILC 1.3 GHz

CLIC 30 GHz

Q VALUE $\Rightarrow$ LOSSES IN CAVITY MATERIAL ?

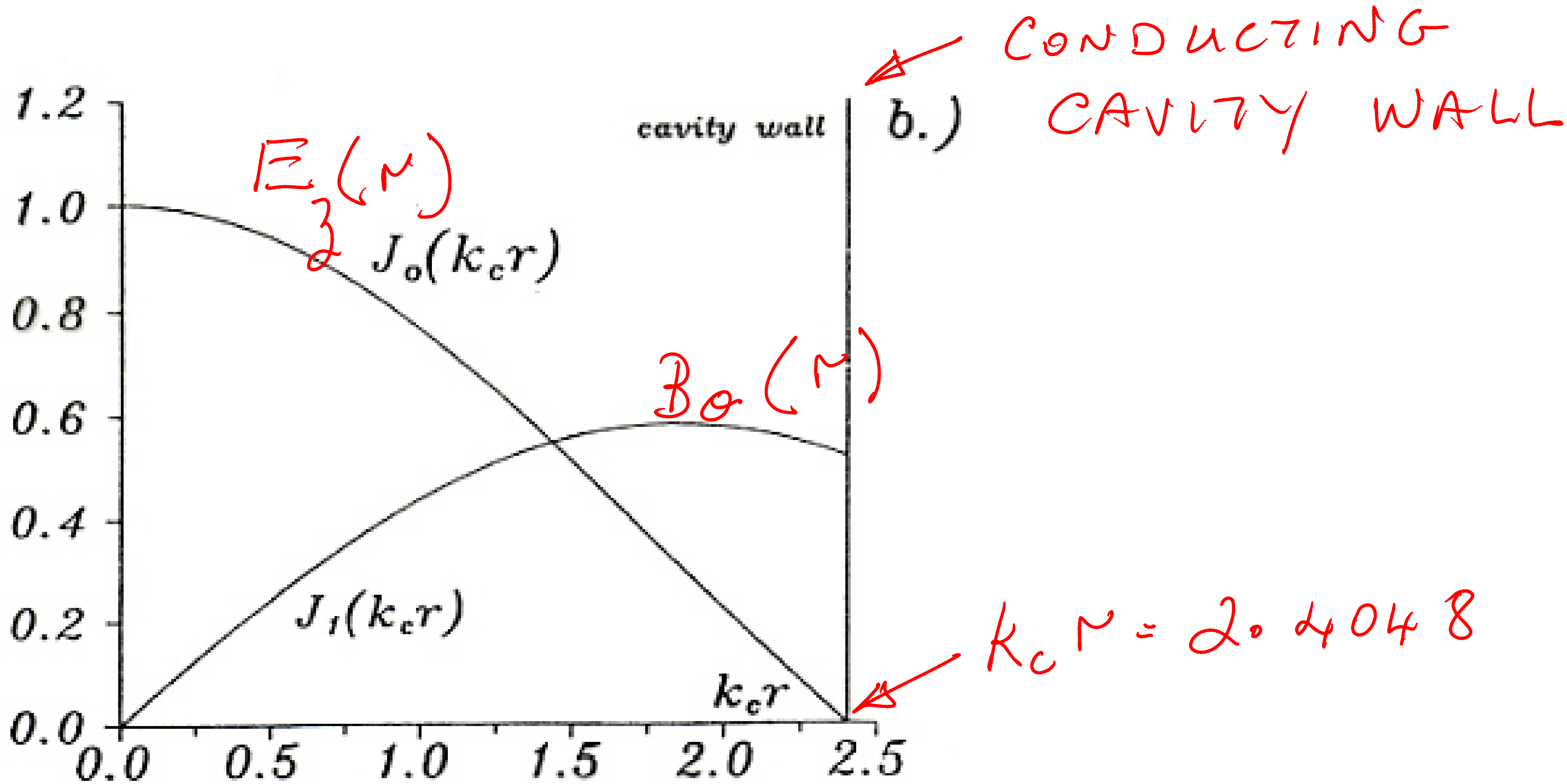
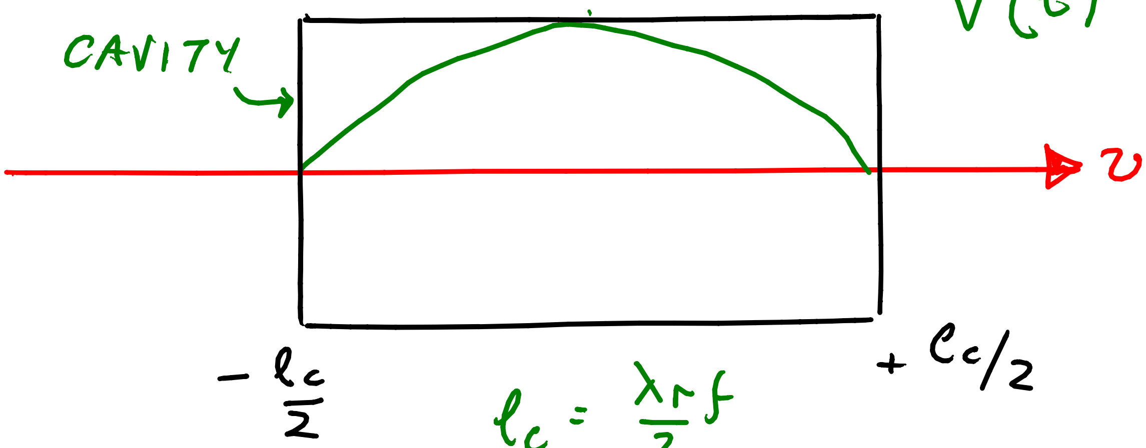



**Fig. 2.7.** Electromagnetic field pattern for a $TM_{010}$ mode in a circular waveguide a.) three dimensional field configuration, b.) radial dependence of fields

# BEAM ENERGY GAIN IN OSCILLATING $\vec{E}$ FIELD




$V(t)$ TIME VARIATION OF VOLTAGE DURING TRANSIT

CHARGED PARTICLE TRAVERSING CAVITY

$t = 0$, CENTRE OF CAVITY

$$l_c = \frac{\lambda_{rf}}{2}$$

$$e\,V(t) = e\,V_0 \cos \omega t = e\,V_0 \cos\left(\omega \frac{s}{v}\right)$$

$$\Delta E_{KIN} = F \cdot d = eEd$$

$$\Delta E_{KIN} = \int_{-\frac{l_c}{2}}^{+\frac{l_c}{2}} e\,E_0 \cos\left(\omega \frac{s}{2v}\right) ds$$

$$\Delta E_{KIN} = \int e \frac{V_0}{l_c} \cos\left(\omega \cdot \frac{s}{v}\right) ds = \frac{\sin u}{u}\, e V_0 \; ; \quad u = \frac{\omega l_c}{2v}$$

TRANSIT TIME FACTOR

PEAK VOLTAGE

# TRANSIT TIME FACTOR $\sin u/u = T$

$$\frac{\{\text{ENERGY GAINED BY PARTICLE PASSING THROUGH CENTRE OF CAVITY AT PEAK FIELD}\}}{\{\text{ENERGY GAIN FROM CONSTANT FIELD}\}}$$

WANT CLOSE TO 1

FOR $u = 0.8$ $\qquad \frac{\sin u}{u} = 0.9$

CAVITY LENGTH

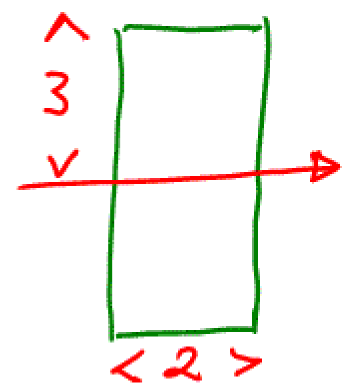
$$u = \frac{\omega l}{2v} = \frac{\omega l}{2c} \; ; \quad \frac{\omega l}{c} = 1.6$$

CAVITY RADIUS

BUT $\frac{\omega R}{c} = 2.405$

$$\frac{\omega/c \cdot R}{\omega/c \cdot l} = \frac{2.405}{1.60} \approx 3/2$$

TYPICAL CAVITY ASPECT RATIO



# CAVITY DIMENSIONS

- SAW THAT A CAVITY OF RADIUS ~30cm RESONATES AT ~ 500 MHz

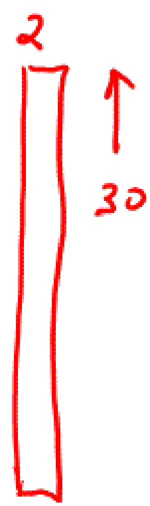
- FOR $\frac{\omega l}{2c} = 0 \cdot 8 \rightarrow L = \frac{1 \cdot 6 \times c}{2\pi\, 500 \times 10^6} = 15 \text{ cms}$

- WHY IS A PRACTICAL $T \sim 0 \cdot 8$ ?

FOR $T = 0 \cdot 998 \; (\simeq 1)$ $\quad T = \frac{\sin 0 \cdot 1}{0 \cdot 1}$

FROM ABOVE

$$L = \frac{0 \cdot 2 \cdot c}{2\pi\, 500 \times 10^6} = 2 \text{ cms}$$




- ELECTRICAL BREAK DOWN
- SMALL $\Delta E$ ON TRANSIT
- HARD TO MANUFACTURE

# CAVITY Q-VALUE

$$Q = \frac{\text{STORED ENERGY}}{\left(\begin{array}{c}\text{ENERGY LOST IN ONE}\\ \text{RADIAN OF OSCILLATION}\end{array}\right)}$$

WANT THIS TO BE LARGE !

ENERGY CONSTANTLY OSCILLATING $\bar{E} \leftrightarrow \bar{B}$

ONLY NEED TO CALC ENERGY IN $E_{MAX}$

$u = \varepsilon_0 \frac{E^2}{2}$ VOLUME

DIFFERENT $u$

$$U = \frac{\varepsilon_0}{2}\int E^2\, dV$$ ← VOLUME OF CAVITY

$E(r) = E_0 J_0\left(\frac{\omega}{c}\cdot r\right)$

$$= \frac{\varepsilon_0}{2}\int E_0^2 J_0^2\left(\frac{\omega}{c}\cdot r\right) dV \rightarrow E_0^2\frac{\varepsilon_0}{2}\int J_0^2\left(\frac{\omega}{c}\cdot r\right)\int_{\phi=0}^{2\pi} d\phi \int_{z=0}^{L} dz$$

$$= \frac{\varepsilon_0}{2} E_0^2 \underbrace{2\pi}_{\phi}\ \underbrace{L}_{z} \int_{r=0}^{R} J_0^2\left(\frac{\omega}{c}\cdot r\right) r\, dr$$

PUT $\rho = \frac{r}{R}$ $\Big\}$ $\begin{array}{ll} r & 0 \rightarrow R \\ \rho & 0 \rightarrow 1\end{array}$

$$= \varepsilon_0 E_0^2 \, \pi L \int_0^1 J_0^2\left(\frac{\omega}{c} \cdot \rho R\right) \rho R \cdot R \, d\rho$$

$$= \varepsilon_0 E_0^2 \, \pi L \, R^2 \int_0^1 J_0^2\left(\frac{\omega R}{c} \cdot \rho\right) \rho \, d\rho$$

$$\int_0^1 J_0^2(\alpha x)\, x \, dx = \frac{1}{2}\left[J_0'(\alpha)\right]^2 = \frac{1}{2}\left[J_1(\alpha)\right]^2 ; \quad J_0' = -J_1$$

$$= \varepsilon_0 E_0^2 V \cdot \frac{1}{2}\left[J_1(\alpha)\right]^2 ; \quad \alpha = \frac{\omega R}{c}$$

$$U = \frac{1}{2} \varepsilon_0 E_0^2 V \underbrace{J_1^2(2.405)}_{(0.52)^2}$$

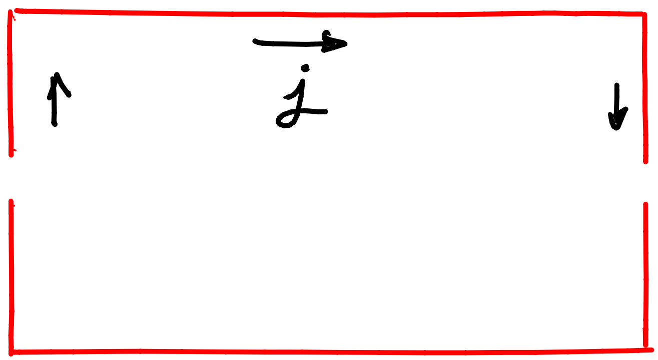
STORED ENERGY ~27% OF UNIFORM FIELD FILLING CAVITY

$\frac{1}{2} \varepsilon_0 E_0^2 V$

NOW NEED TO LOOK AT LOSSES TO GET Q

# OHMIC HEATING IN CAVITY WALL

OSCILLATING FIELD → INDUCED CURRENT → RESISTIVE LOSS




$j$ IS SURFACE CURRENT DENSITY

AMPERE → $\int B\,ds = \mu_0 i$

$$j = B_\theta / \mu_0$$

IN CAVITY $\quad B_\theta = B_\theta(r) \cdot e^{i\omega t}$

$$E_z = E_z(r)\, e^{i\omega t} = E_0 J_0\left(\frac{\omega}{c} \cdot r\right) e^{i\omega t}$$

WANT $B_\theta$ IN ORDER TO GET $j$

USE MAXWELL

$$\frac{\partial E_z}{\partial r} = \frac{\partial B_\theta}{\partial t}$$

$$\frac{\partial E_z}{\partial r} = E_0 \frac{\partial}{\partial r}\left[J_0\left(\frac{\omega}{c}\cdot r\right)\right] e^{i\omega t}$$

$$\frac{\partial B_\theta}{\partial t} = i\omega B_\theta(r)\, e^{i\omega t}$$

$$\frac{\partial E_z}{\partial r} = \frac{\partial B_\theta}{\partial t}$$

$$i\omega B_\theta(r) e^{i\omega t} = E_0 \underbrace{\frac{\partial}{\partial r}\left[J_0\left(\frac{\omega}{c}\cdot r\right)\right]}\, e^{i\omega t}$$

WHITTAKER & WATSON

"MODERN ANALYSIS"

1902

Page 359

$$\frac{d}{dz}\left\{z^{-n} J_n(z)\right\} = -z^{n} J_{n+1}(z)$$

$$n \to 0 \quad ; \quad z \to \frac{\omega}{c}\cdot r$$

$$\frac{c}{\omega}\frac{d}{dr}\left[J_0\left(\frac{\omega}{c}\cdot r\right)\right] = -J_1\left(\frac{\omega}{c}\cdot r\right)$$

SO $$i\omega B_\theta(r)e^{i\omega t} = -\frac{\omega}{c}E_0 J_1\left(\frac{\omega}{c}\cdot r\right)$$

$$|X|^2 = XX^*$$

$$|B_\theta(r)|^2 = \left|\frac{E_0}{c}J_1\right|^2$$

$$B_\theta(r) = \frac{E_0}{c}J_1\left(\frac{\omega}{c}\cdot r\right)$$

USE THIS TO CALCULATE THE CURRENT DENSITY IN THE CAVITY WALLS

$$B_\theta(r) = \frac{E_0}{c} J_1\left(\frac{\omega}{c}\cdot r\right)$$

- INSTANTANEOUS POWER LOSS INTO RESISTOR

$$P(t) = \mathrm{Re}\, I(t)\, \mathrm{Re}\, V(t)$$

- TIME AVERAGED POWER LOSS

$$\overline{P} = \int_{\text{CYCLE}} \mathrm{Re}\, I_0 e^{i\omega t}\, \mathrm{Re}\, V_0 e^{i\omega t} = \frac{1}{2} I_0 V_0$$

$$= \frac{1}{2} I_0^2 \rho$$

CAVITY POWER LOSS

CURRENT DENSITY

$$\int_{\text{CAVITY SURFACE}} \frac{1}{2} \rho_s J^2 = \frac{1}{2}\int \rho_s \left[\frac{B_\theta}{\mu_0}\right]^2 ds = \frac{1}{2} \rho_s \left(\frac{E_0}{\mu_0 c}\right)^2 \int J_1^2\left(\frac{\omega}{c}\cdot r\right) ds$$

SURFACE RESISTIVITY

$$P = \frac{1}{2}\rho_s\left(\frac{E_0}{\mu_0 c}\right)^2 \int J_1^{\,2}\left(\frac{\omega}{c}\cdot r\right) ds$$

$$= \frac{1}{2}\rho_s\left(\frac{E_0}{\mu_0 c}\right)^2 \left\{ \underbrace{2}_{\text{2 ENDS}} \times \underbrace{2\pi}_{\int \text{ OVER END}} \int_0^R J_1^{\,2}\left(\frac{\omega}{c}\cdot r\right) r\,dr + \underbrace{2\pi R L J_1^{\,2}}_{\substack{\int \text{ OVER SURFACE} \\ \text{OF CYLINDER} \\ \text{(RADIUS CONST)}}}\left(\frac{\omega}{c}\cdot R\right) \right\}$$

$$\int_0^{\alpha} J_1^{\,2}(u)\,u\,du = \frac{\alpha^2}{2}\left\{ J_1^{\,2}(\alpha) - J_0(\alpha)\,J_2(\alpha) \right\}$$

$$\int_0^{\frac{\omega}{c}\cdot R} J_1^2\left(\frac{\omega}{c}\cdot r\right)\left(\frac{\omega}{c}\right)^2 r\,dr = \left(\frac{\omega}{c}R\right)^2 \frac{1}{2}\left\{J_1^2\left(\frac{\omega}{c}\cdot R\right) - J_0\left(\frac{\omega}{c}\cdot R\right)J_2\left(\frac{\omega}{c}\cdot R\right)\right\}$$

ZERO ON CAVITY WALL

$$P = \frac{1}{2}\rho_s\left(\frac{E_0}{\mu_0 c}\right)^2\left\{4\pi\frac{R^2 J_1^2}{2} + 2\pi R L J_1^2\right\}$$

$$P = \frac{1}{2}\rho_s\left(\frac{E_0}{\mu_0 c}\right)^2 2\pi R L\left(1 + \frac{R}{L}\right)J_1^2$$

DEFINITION OF $Q = \dfrac{\omega U}{P}$

STORED ENERGY

$$= \frac{1}{2}\varepsilon_0 E_0^2 V J_1^2$$

$$= \frac{1}{2}\varepsilon_0 E_0^2 \overbrace{\pi R^2 L}^{} J_1^2$$

$\dfrac{P}{\omega} \rightarrow$ POWER LOSS PER RADIAN OF OSCILLATION

$$Q = \frac{\omega u}{P}$$ $\omega$ IS RESONANT FREQUENCY OF THE CAVITY

$$\omega = 2\pi f = \frac{2.405 \cdot c}{R}$$ $\frac{\omega}{c} \cdot R = 2.405$

SO $$Q = \frac{2.405\, c}{R} \frac{\frac{1}{2} \varepsilon_0 E_0^2 \pi R^2 L J_1^2}{\frac{1}{2} \rho_s \frac{E_0^2}{Z_0^2} 2\pi R L \left(1 + \frac{R}{L}\right) J_1^2}$$

Q VALUE OF CAVITY $$= \frac{2.405 \mu_0 c}{2 \rho_s \left(1 + \frac{R}{L}\right)}$$

WANT Q LARGE

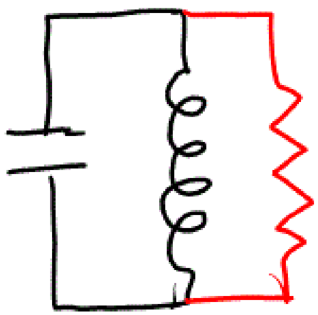
$\rightarrow \rho_s$ SMALL

$\rightarrow \frac{R}{L}$ SMALL $\rightarrow$ cf T

FOR Cu $Q \sim 10^4$ in 400 MHz REGION

FOR SUPERCONDUCTING CAVITIES $Q \sim 10^{11}$

# SHUNT IMPEDANCE




THE ENTIRE RF POWER IS CONVERTED TO HEAT IN CAVITY WALL → SHUNT IMPEDENCE

THIS IS WHAT GENERATES THE ACCELERATING VOLTAGE

$$P_{RF} = \frac{1}{2} V^2/R_s \rightarrow R_s = \frac{V^2}{2P_{RF}}$$

CAN ALSO THINK OF SHUNT IMPEDANCE AS

$$R_s = \frac{(\text{ENERGY GAIN/UNIT CHARGE})^2}{2P} \leftarrow \text{JUST CALCULATED}$$

$$\Delta E_{KIN} = eVT$$

$$= e E_0 \ell \cdot T \leftarrow \text{TRANSIT FACTOR}$$

CAVITY LENGTH

$$R_s = \frac{E_0^2 L^2 T^2}{2} \cdot \frac{2 \mu_0^2 c^2}{2\pi \rho_s E_0^2 \, R L \left(1+\frac{R}{L}\right) J_1^2(2.405)}$$

$$R_s = \frac{Z_0^2}{\pi \rho_s} \cdot \frac{L}{2R} \cdot \frac{T^2}{\left(1+\frac{R}{L} J_1^2[2.405]\right)}$$

DORIS – 5 GeV

$R_s = 3.6 \times 10^6 \, \Omega$

$P_{RF} = 50 \, kW$

$V_{CAV} = 548 \, kV$

PETRA – 30 GeV

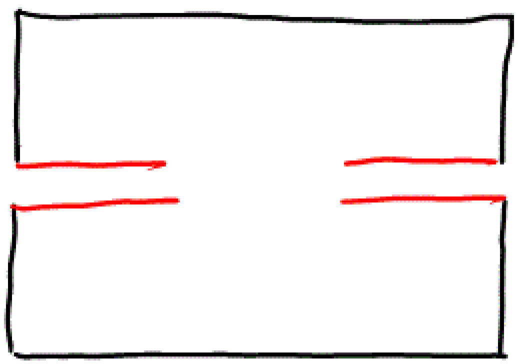
$R_s = 1.8 \times 10^6 \, \Omega$

$P_{RF} = 125 \, kW$

$V_{CAV} = 2.12 \, MV$

# CAVITY GEOMETRY

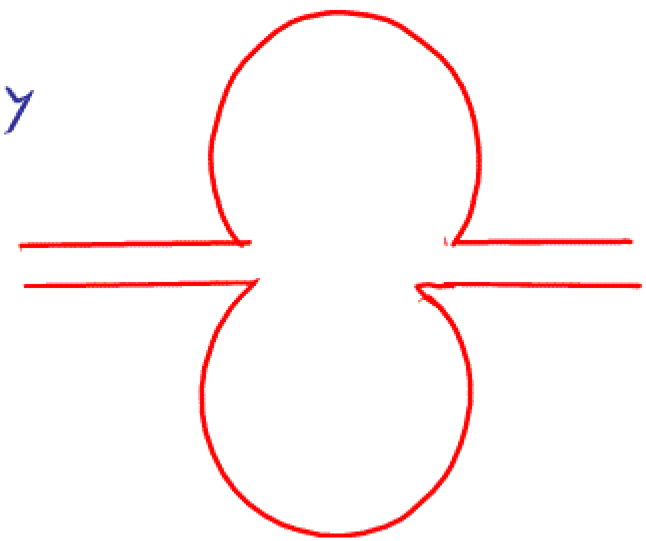
- CAN INCREASE ENERGY GAIN OF PILL BOX BY ADDING RE-ENTRANT SURFACES



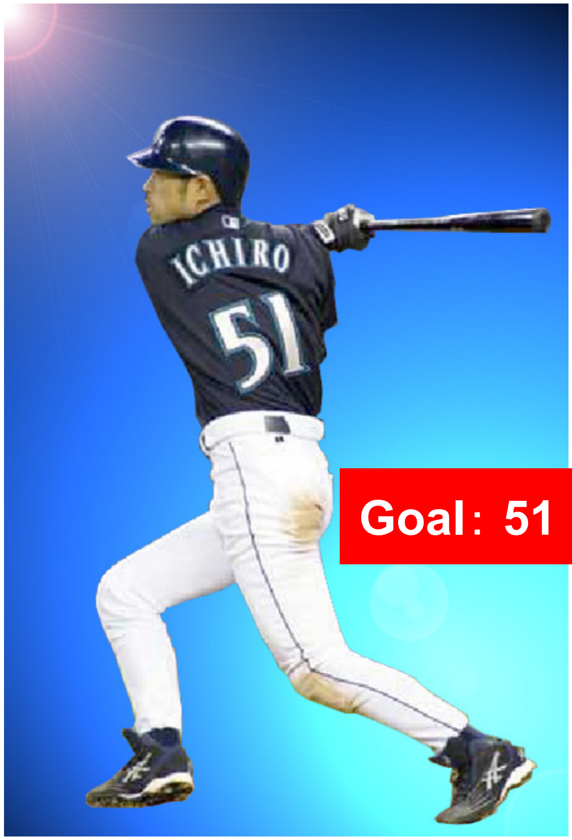
$E_{ACC} = V/d$ ← DECREASE

- RESISTIVE LOSSES MAINLY ON CYLINDRICAL WALLS

CAN RAISE Q BY →



# ICHIRO Cavity




Goal: 51 MV/m