

High Energy Physics experiments?

1. Collide Particles

Accelerators & Beams

$$E_{CM} \text{ and } \mathcal{L}$$

2. Detect Final State

Detectors

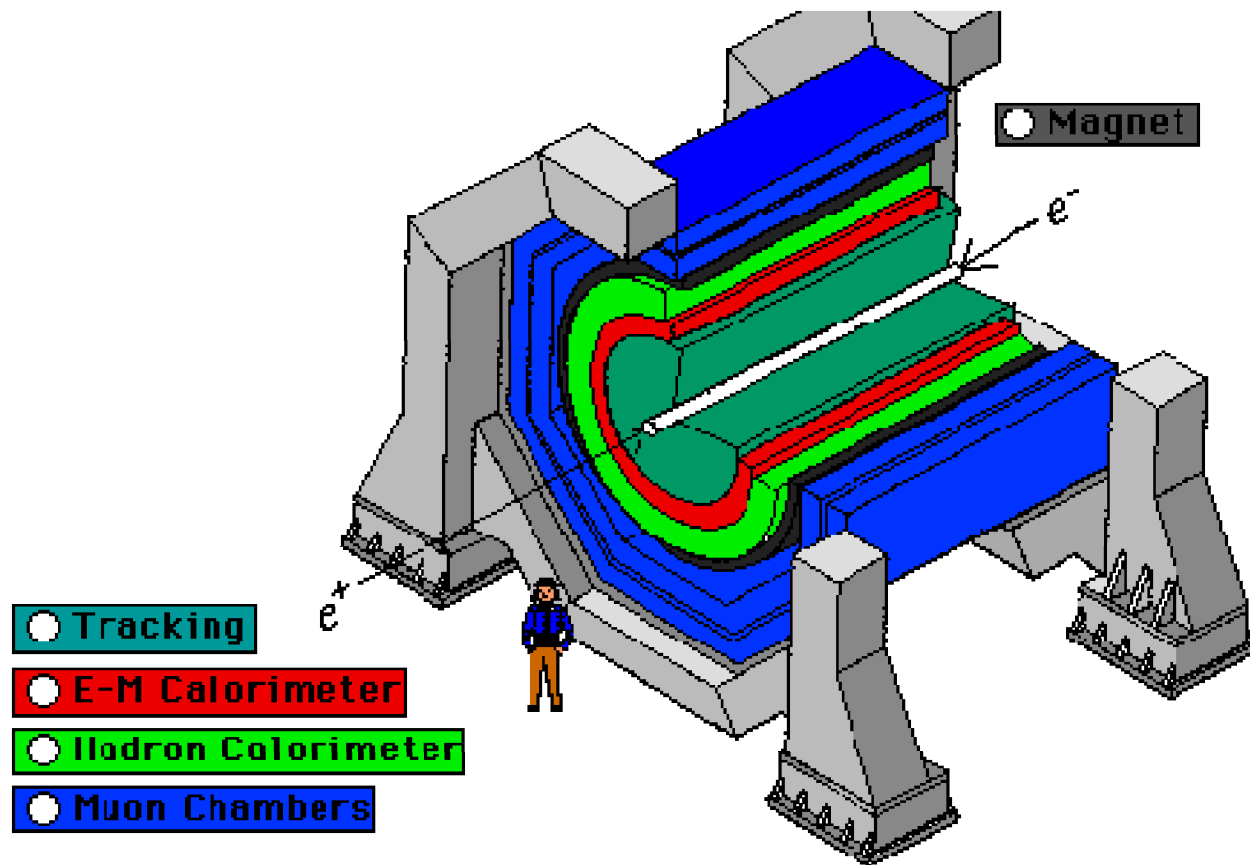
$$\frac{\sigma_{\mu}}{P_{\mu}}$$

3. Understand
Connection of 1 + 2

Analysis

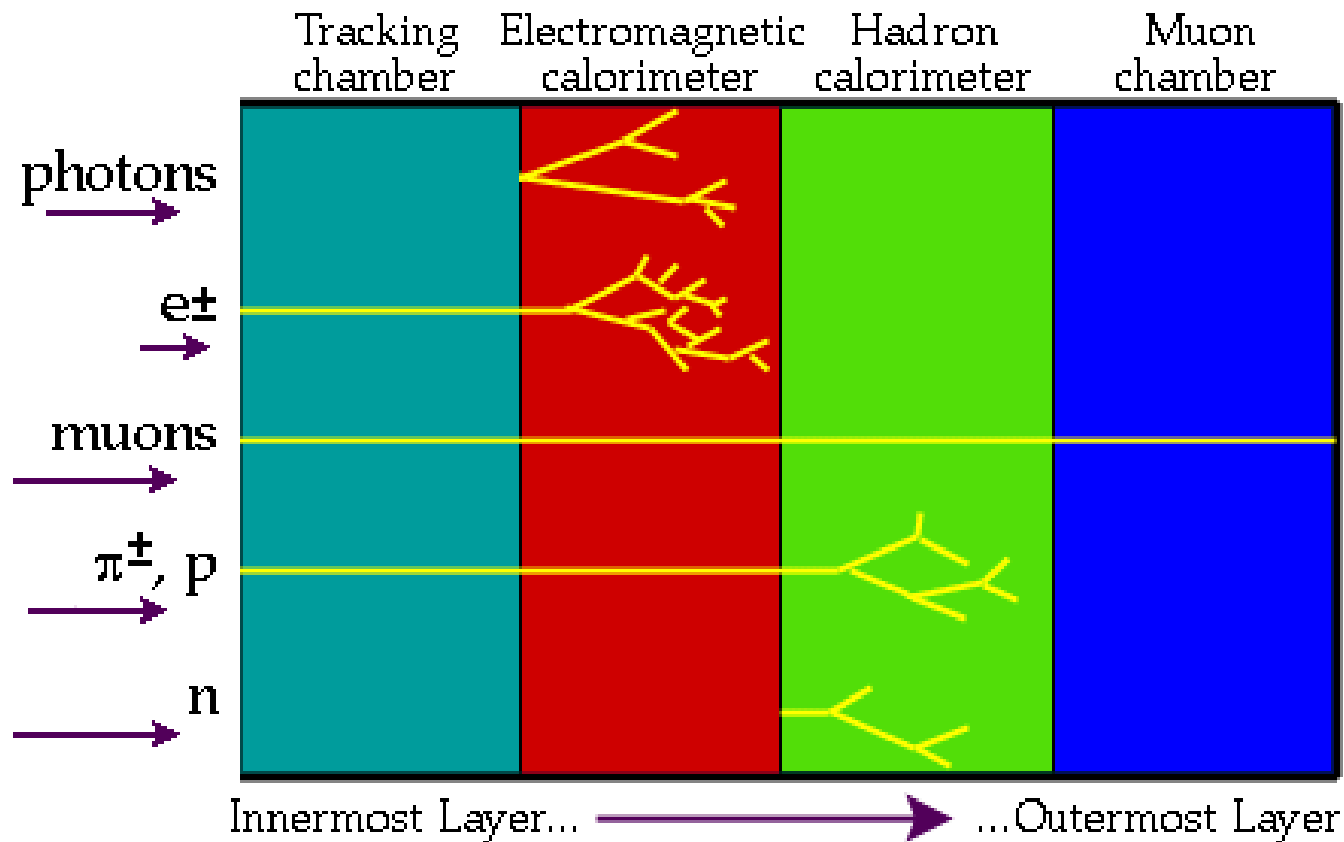
$$\frac{S}{B}$$

Generic Detector



- ▶ Layers of Detector Systems around Collision Point








Generic Detector

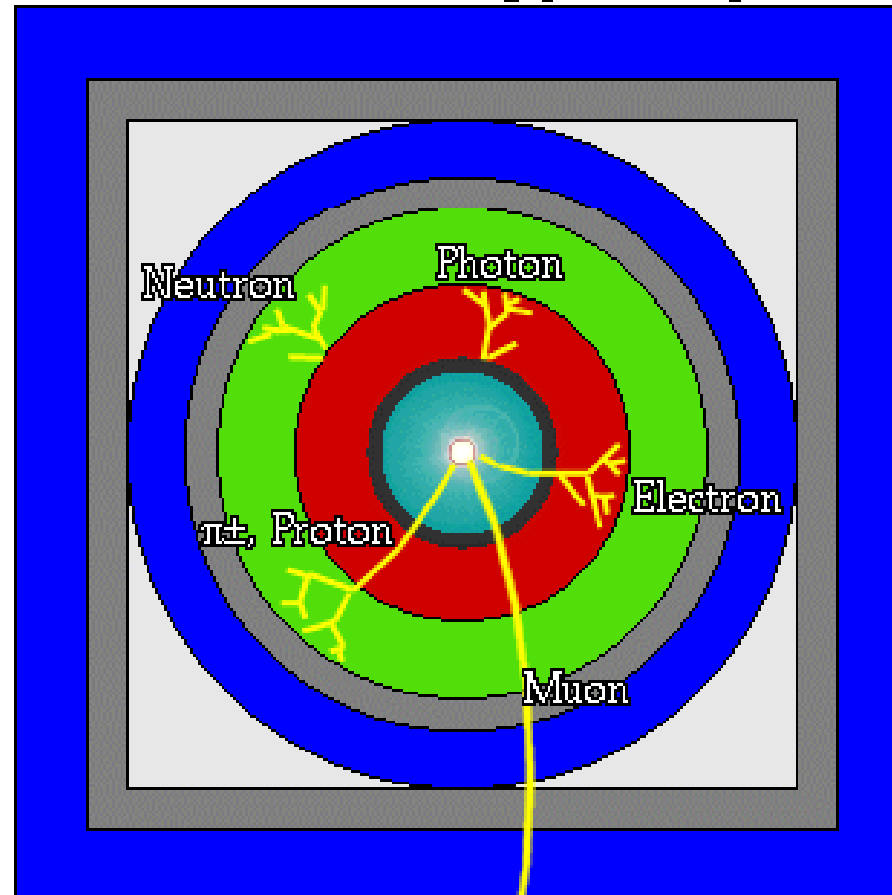


- ▶ Different Particles detected by different techniques.
 - ▶ Tracks of Ionization – Tracking Detectors
 - ▶ Showers of Secondary particles – Calorimeters

Generic Detector

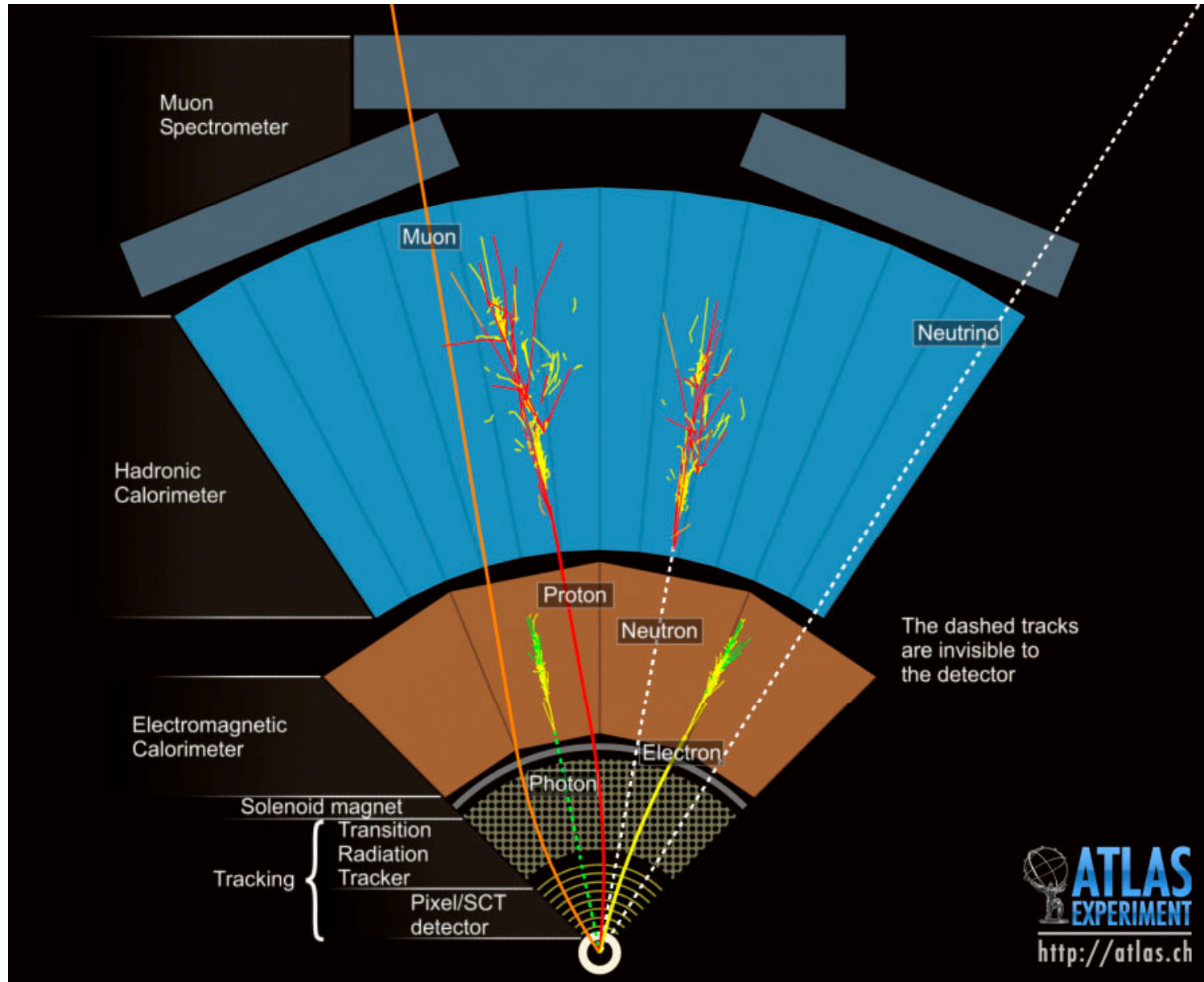
A detector cross-section, showing particle paths

-  Beam Pipe (center)
-  Tracking Chamber
-  Magnet Coil
-  E-M Calorimeter
-  Hadron Calorimeter
-  Magnetized Iron
-  Muon Chambers

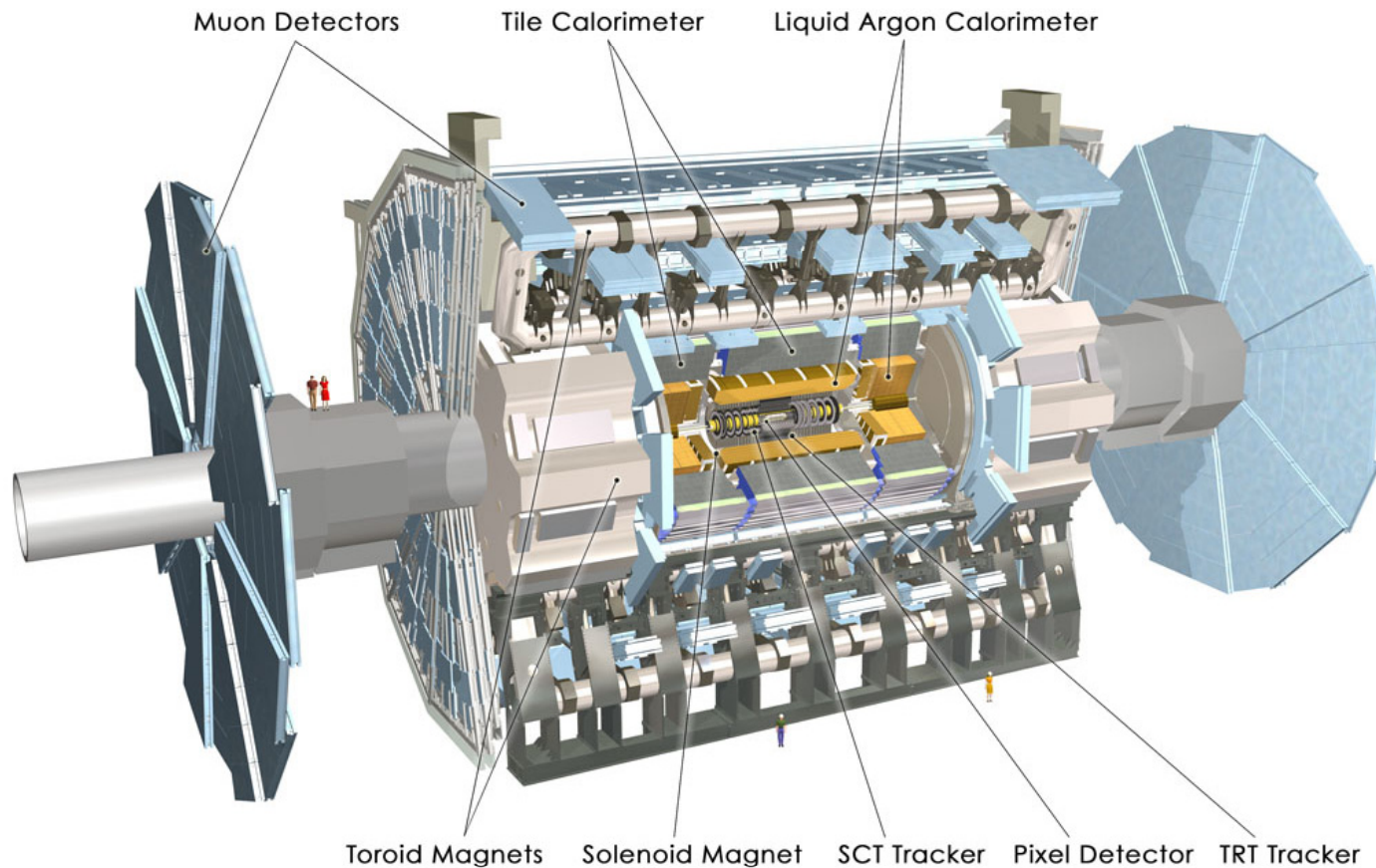


- ▶ Different Particles detected by different techniques.
 - ▶ Tracks of Ionization – Tracking Detectors
 - ▶ Showers of Secondary particles – Calorimeters

ATLAS Detector



ATLAS Detector

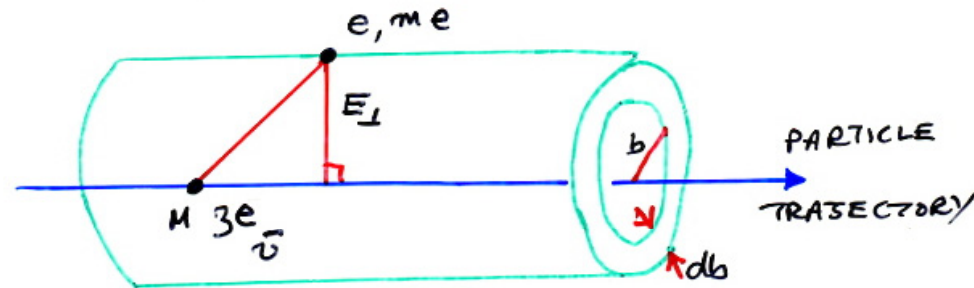


- ▶ Different Particles detected by different techniques.
 - ▶ Tracks of Ionization – Tracking Detectors
 - ▶ Showers of Secondary particles – Calorimeters

Interaction of Charged Particles with Matter

- All particle detectors ultimately use interaction of electric charge with matter
 - Track Chambers
 - Calorimeters
 - Even Neutral particle detectors $n \gamma \pi^0 \nu$
- Ionization
 - Average energy loss
 - Landau tail
- Multiple Scattering
- Cerenkov
- Transition Radiation
- Electron's small mass - radiation

Energy Loss to Ionization



- Heavy charged particle interacting with atomic electrons
- All electrons with shell at **impact parameter b**
- Energy loss $\Delta p = \Delta p_T$ - symmetry

$$\Delta p_T = \int_{-\infty}^{+\infty} F dt = e \int E_T dt = e \int E_T \frac{dt}{dx} dx = e \int E_T \frac{dx}{v}$$

- Gauss $\int \vec{E} \cdot \vec{n} dA = 4\pi Q_{ENCLOSED}$

$$\int E_T dA = 4\pi ze$$

$$\int E_T 2\pi b dx = 4\pi ze \quad \longrightarrow$$

$$\int E_T dx = \frac{2ze}{b}$$

$$\Delta p_T = \frac{2ze^2}{bv}$$

$$\Delta E = \frac{(\Delta p_T)^2}{2m_e}$$

$$\Delta E(b) = \frac{2z^2 e^4}{m_e v^2 b^2}$$

• Density of electrons

$$N_e \longrightarrow$$

$$-dE(b) = \Delta E(b) N_e dV$$

$$-dE(b) = \Delta E(b) N_e 2\pi b db dx$$

$$-dE(b) = \frac{4\pi z^2 e^4}{m_e v^2} N_e \frac{db}{b} dx$$


Physical limits of integration

$$\int_{b=0}^{\infty} \rightarrow \int_{b \min}^{b \max}$$

$$\frac{-dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln \left(\frac{b_{MAX}}{b_{MIN}} \right)$$

- Maximum $\Delta E \Rightarrow$ minimum b
- In a classical head-on collision $\Delta E_{MAX} = \frac{1}{2} m_e (2v)^2$
- Relativistically $\Delta E_{MAX} = \frac{1}{2} \gamma^2 m_e (2v)^2 = 2\gamma^2 m_e (v)^2$

$$\Delta E_{MAX} = \frac{2z^2 e^4}{m_e v^2 b_{MIN}^2}$$

$$b_{MIN}^2 = \frac{2z^2 e^4}{m_e v^2} \frac{1}{2\gamma^2 m_e (v)^2}$$


$$b_{MIN} = \frac{z e^2}{\gamma m_e v^2}$$

Physical limits of integration

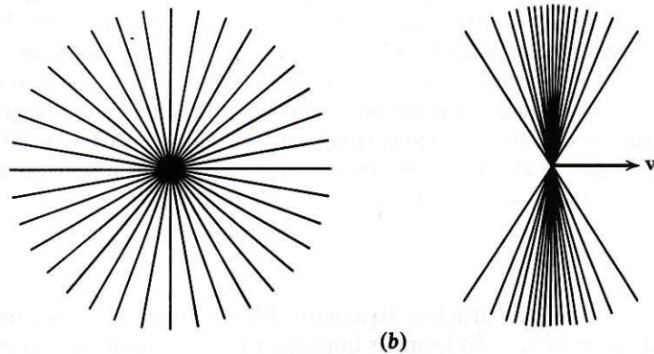
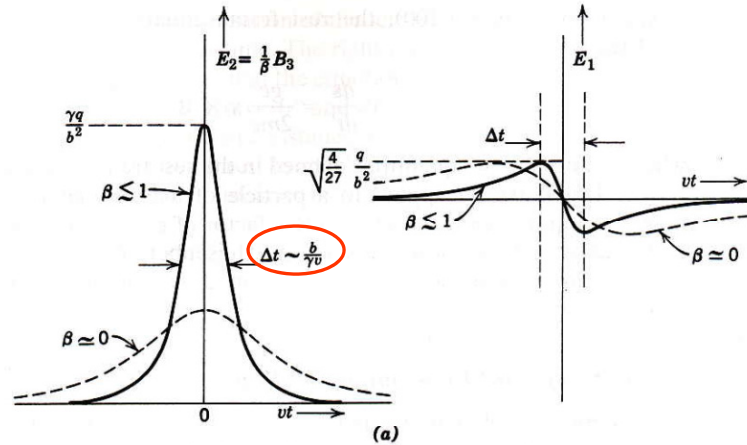


Figure 11.9 Fields of a uniformly moving charged particle. (a) Fields at the observation point P in Fig. 11.8 as a function of time. (b) Lines of electric force for a particle at rest and in motion ($\gamma = 3$). The field lines emanate from the *present* position

- Electrons bound in atoms
- Time of interaction must be small, compared to orbital period, **else energy transfer averages to zero**

- Orbital period $\tau \sim \frac{1}{\bar{\omega}}$
- Collision time $t \sim \frac{b}{\gamma v}$

$$\frac{b}{\gamma v} \leq \tau \sim \frac{1}{\bar{\omega}}$$

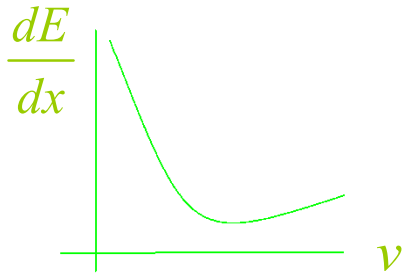
$$b_{MAX} = \frac{\gamma v}{\bar{\omega}}$$

- Put in integration limits

$$\frac{-dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln \left(\frac{\gamma^2 m_e v^3}{ze^2 \bar{\omega}} \right)$$

- Time for EM interaction $\Delta t \sim \frac{b}{\gamma v}$

Ionization Loss



$$\frac{-dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln \left(\frac{\gamma^2 m_e v^3}{ze^2 \bar{\omega}} \right)$$

$$-\frac{dE}{dx} \sim \frac{1}{v^2} \ln \left(\frac{\gamma^2 m v^3}{ze^2 \bar{\omega}} \right)$$

- This works for heavy particles like α
- Breaks down for $M \leq M_{PROTON}$
- Correct QED treatment gives **Bethe – Bloch** equation

Maximum energy transfer in single collision

$$-\frac{dE}{dx} = 2\pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2m_e \gamma^2 \beta^2 T_{MAX}}{I^2} \right) - 2\beta^2 - \delta - 2\frac{C}{Z} \right]$$

Mean excitation potential of material

Density correction

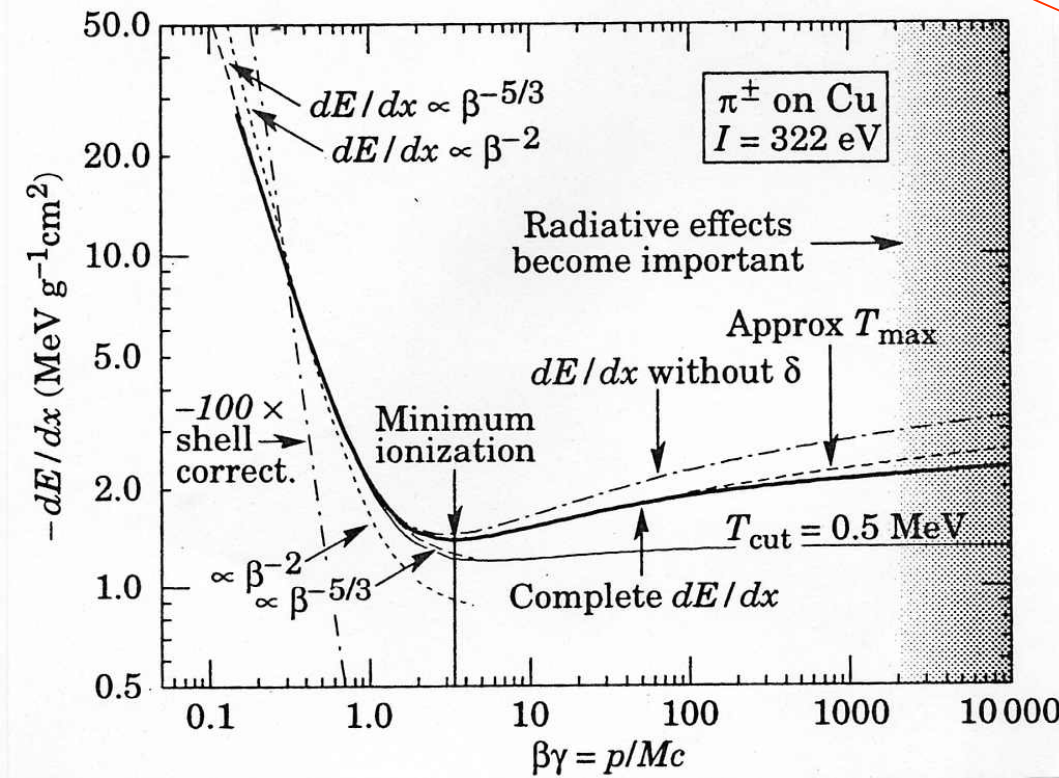
Shell correction

Bethe – Bloch Equation

$$-\frac{dE}{dx} = 2\pi N_A r_e^2 m_e c^2 \rho \frac{Z z^2}{A \beta^2} \left[\ln \left(\frac{2m_e \gamma^2 \beta^2 T_{MAX}}{I^2} \right) - 2\beta^2 - \delta - 2\frac{C}{Z} \right]$$

$$2\pi N_A r_e^2 m_e c^2 = 0.1535 \text{ MeV} \frac{\text{cm}^2}{\text{g}}$$

$$M \gg m_e \quad T_{MAX} \rightarrow 2m_e c^2 \beta^2 \gamma^2$$



- Mean excitation potential
This is main parameter in B – B
 $\sim \bar{\omega} h$

Hard to calculate

measure $\frac{dE}{dx}$ infer I

- Empirically

$$\frac{I}{Z} = 12 + \frac{7}{Z} \text{ eV} \quad Z < 13$$

$$\frac{I}{Z} = 9.76 + 58.8 \cdot Z^{-1.19} \text{ eV} \quad Z > 13$$

$$2\pi N_A r_e^2 m_e c^2 = 0.1535 \text{ MeV} \frac{\text{cm}^2}{\text{g}}$$

CM



$$P_R^* = m\gamma\beta^* \quad , \quad E_R = m\gamma \quad ,$$

$$M \gg m$$

$$\begin{aligned} E_R^{\text{LAB}} &= \gamma (E_R^* + \beta P_R^*) \\ &= \gamma (m\gamma + m\gamma\beta^2) \end{aligned}$$

$$\beta^* = \beta^{\text{LAB}}$$

ORIGINAL $E^{\text{LAB}} = m$

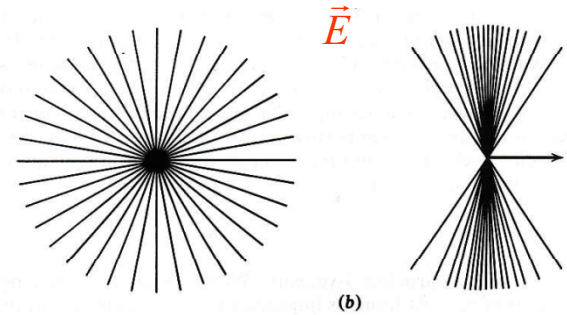
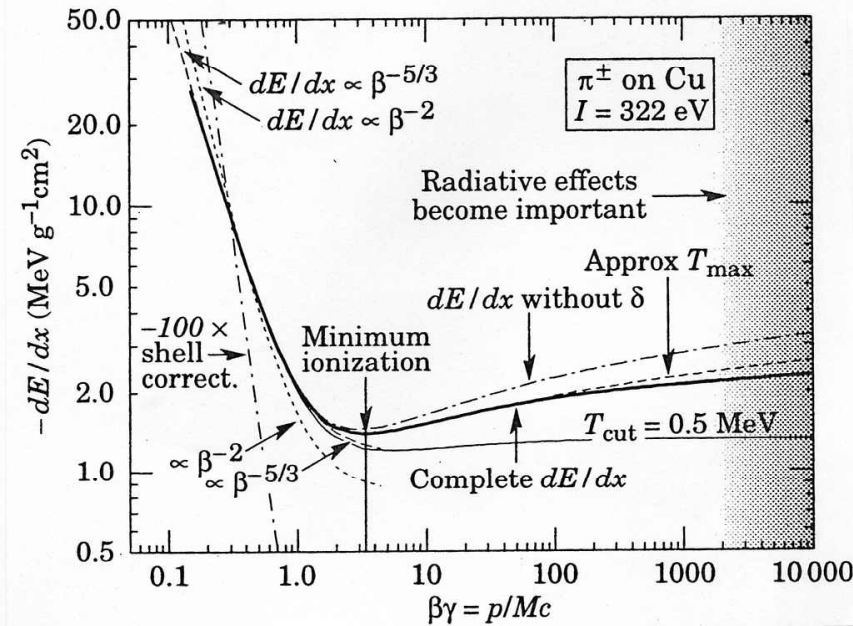
$$\begin{aligned} \Delta E &= m\gamma^2 + m\gamma^2\beta^2 - m \\ &= m(\gamma^2 - 1) + m\gamma^2\beta^2 \end{aligned}$$

$$\text{BUT } (\gamma^2 - 1) = \beta^2\gamma^2$$

$$\Delta E = 2m\gamma^2\beta^2$$

Relativistic rise & Density Correction

$$-\frac{dE}{dx} = 2\pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2m_e \gamma^2 \beta^2 T_{MAX}}{I^2} \right) - 2\beta^2 - \delta - 2\frac{C}{Z} \right]$$



$$b_{\max} = \frac{\gamma v}{\omega} \quad \text{Increases with energy}$$

- Electric field polarizes material along path
- Far off electrons shielded from field and contribute less

$$\frac{dE}{dx} \rightarrow \frac{dE}{dx} - \delta$$

- Polarization greater in condensed materials, hence density correction

Particle Identification

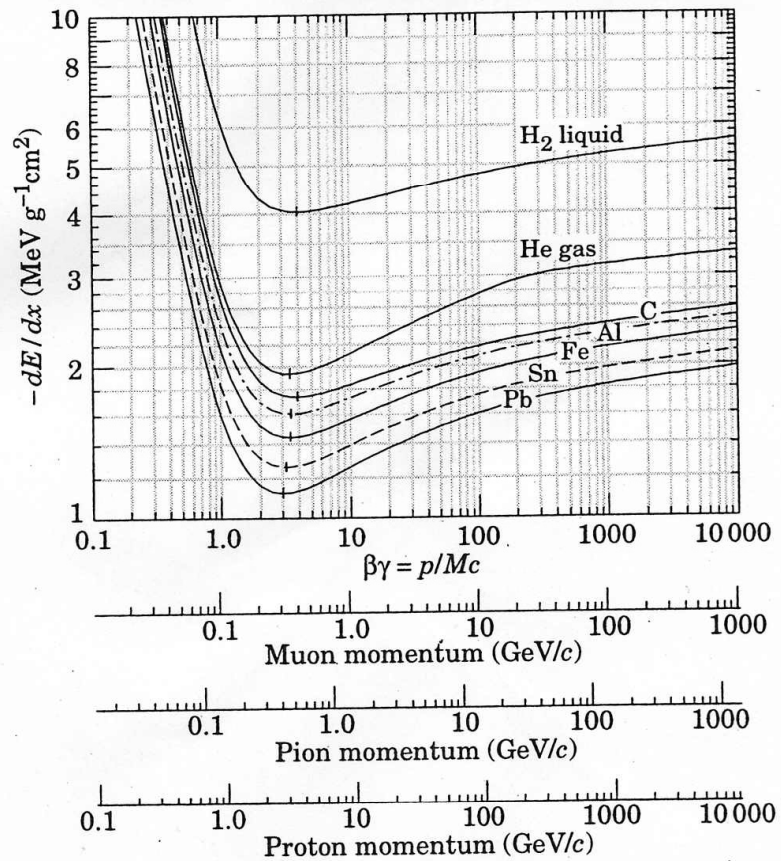
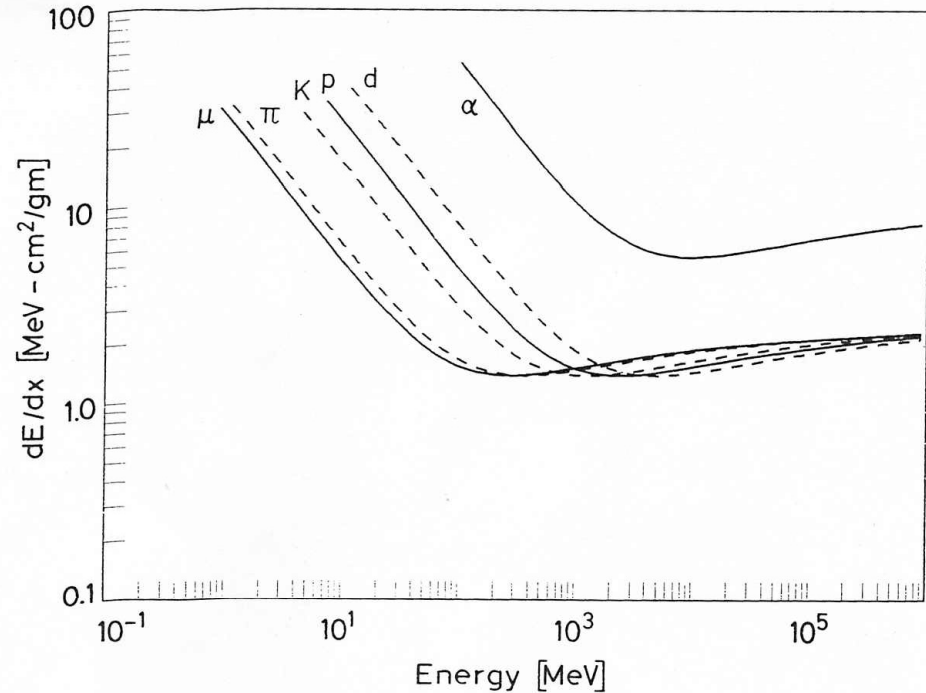


Figure 23.2: Energy loss rate in liquid (bubble chamber) hydrogen, gaseous helium, carbon, aluminum, tin, and lead.

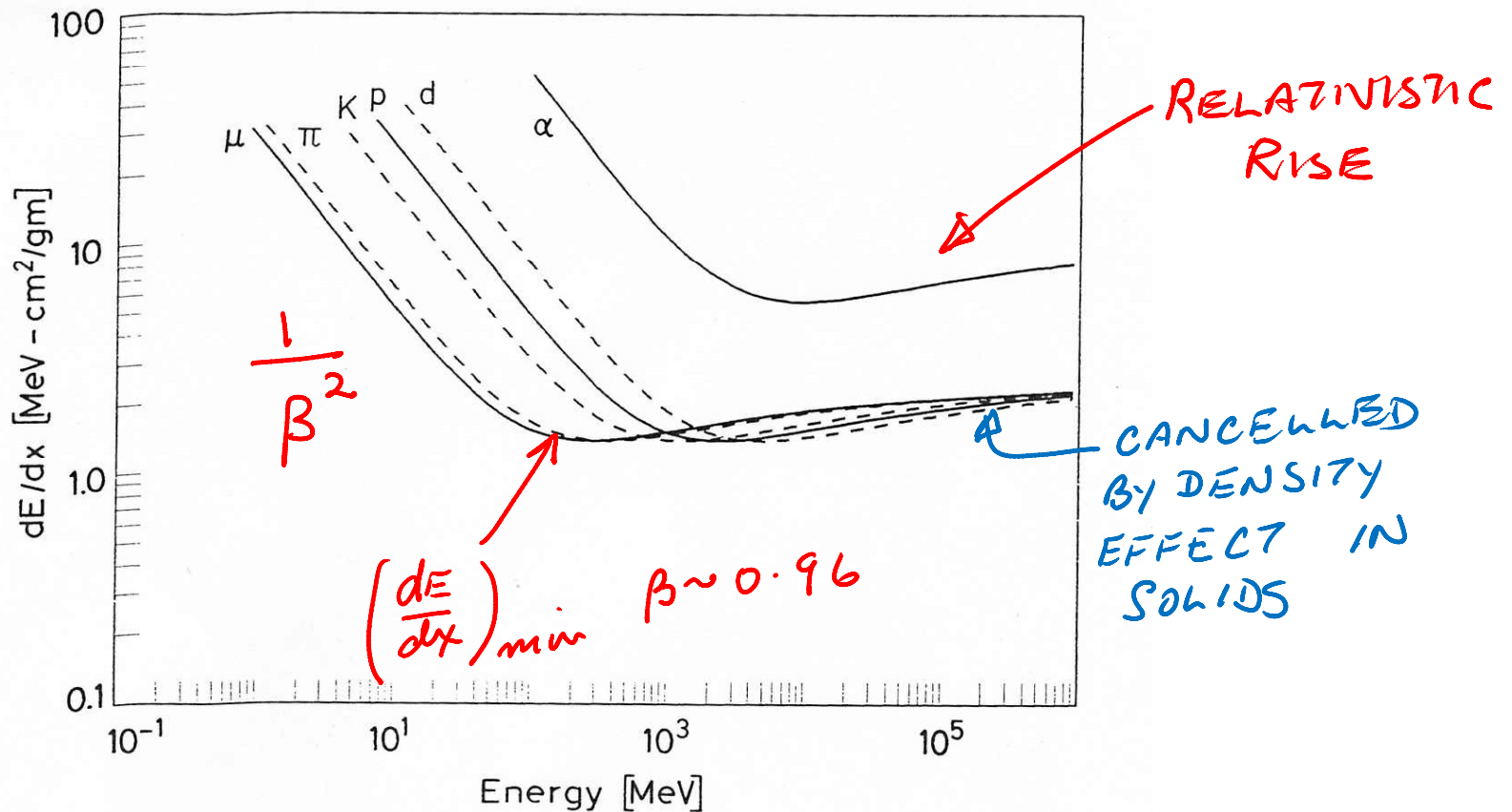


$\frac{dE}{dx}$ depends on velocity

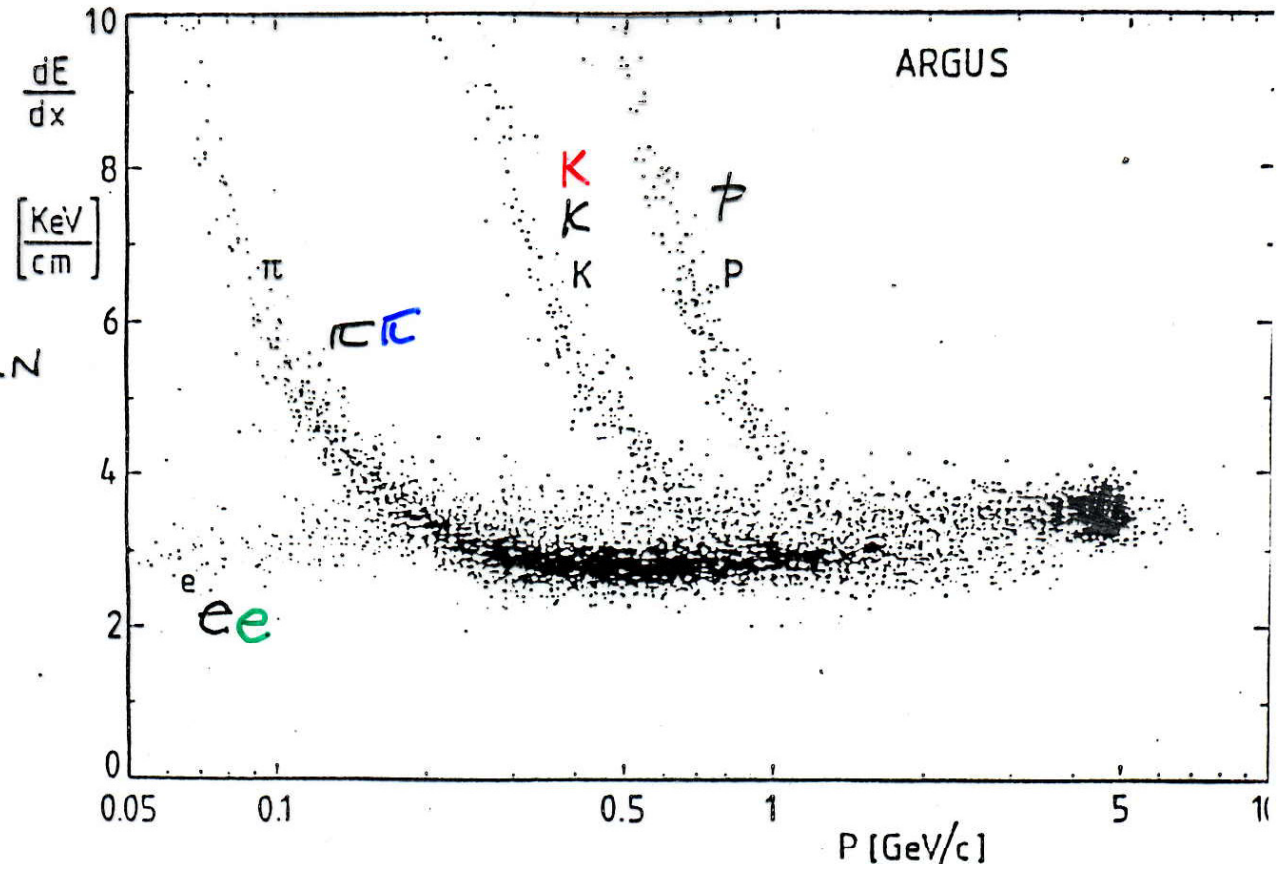
Usually measure $p = \gamma\beta M$

$\frac{dE}{dx}$ determines mass

Particle Identification



$\frac{dE}{dx}$ RESLN
 $\sim 5\%$



SCALING LAW FOR dE/dx

FOR PARTICLES IN THE SAME MEDIUM, BETHE-BLOCH HAS THE FORM

$$-\frac{dE}{dx} = z^2 f(\beta) \rightarrow \frac{dE}{dx} \equiv \frac{dE}{dx}(z^2, \beta)$$

← ONLY A FUNCTION OF VELOCITY

KINETIC ENERGY $T = (\gamma - 1)mc^2 \rightarrow \beta \equiv \beta(T/m)$

SO $-\frac{dE}{dx} = z^2 f(T/m)$

SCALING LAW

$$-\frac{dE_2}{dx}(T_2) \approx -\frac{z_2^2}{z_1^2} \frac{dE_1}{dx}\left(T_2 \frac{m_1}{m_2}\right)$$

Mass Stopping Power

$\frac{dE}{dx}$ expressed as (mass)x(thickness) is relatively constant over a wide range of materials

$$\varepsilon \frac{\left[\frac{Mass}{Area} \right]}{\left[\frac{Mass}{Area} \right]} - \frac{dE}{d\varepsilon} = - \frac{1}{\rho} \frac{dE}{dx} = z^2 \frac{Z}{A} f(\beta, I)$$

density → ρ
Roughly constant over periodic table → Z/A
In variation → $f(\beta, I)$

$\frac{dE}{d\varepsilon}$ → Independent of material

10 MeV proton loses same energy in

$$\frac{1gm}{cm^2} \text{ Cu or } \frac{1gm}{cm^2} \text{ Fe, Al,}$$

Mixtures of Materials

Bragg's Rule

$$\frac{1}{\rho} \frac{dE}{dx} = \frac{\omega_1}{\rho_1} \frac{dE}{dx_1} + \frac{\omega_2}{\rho_2} \frac{dE}{dx_2} + \dots$$

fraction by weight $\omega_i = \frac{a_i A_i}{A_{mixture}}$

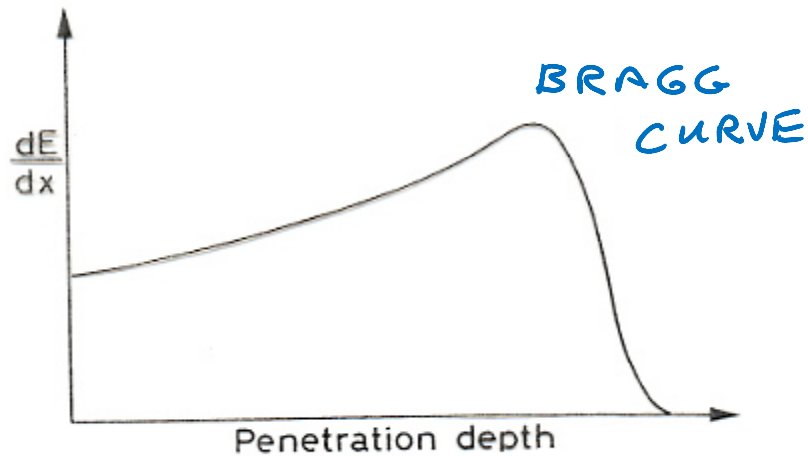
No of atoms of i element molecule → a_i

$$A_{mixture} = \sum a_i A_i$$

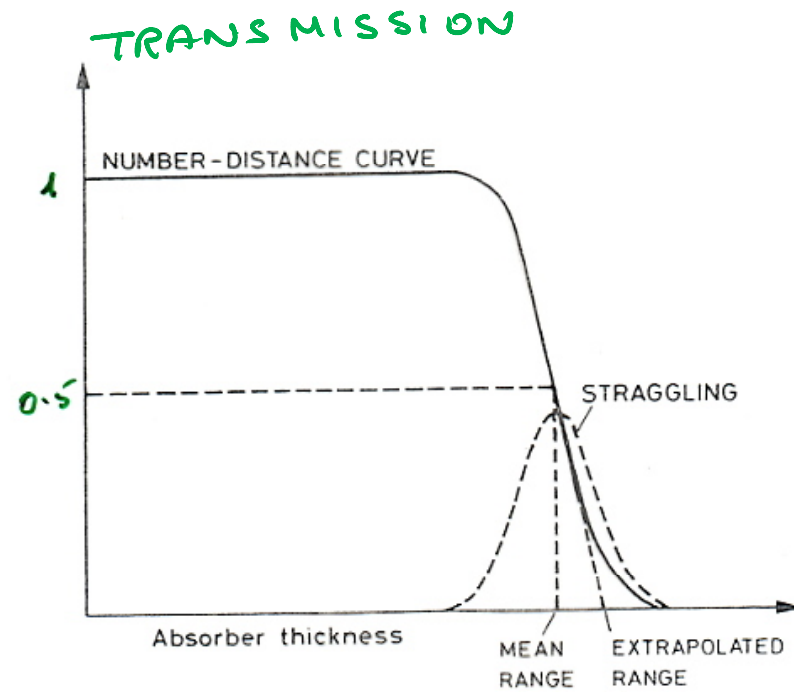
$$Z_{mixture} = \sum a_i Z_i$$

$$\ln I_{mixture} = \sum \frac{a_i Z_i}{Z_i} \ln I_i$$

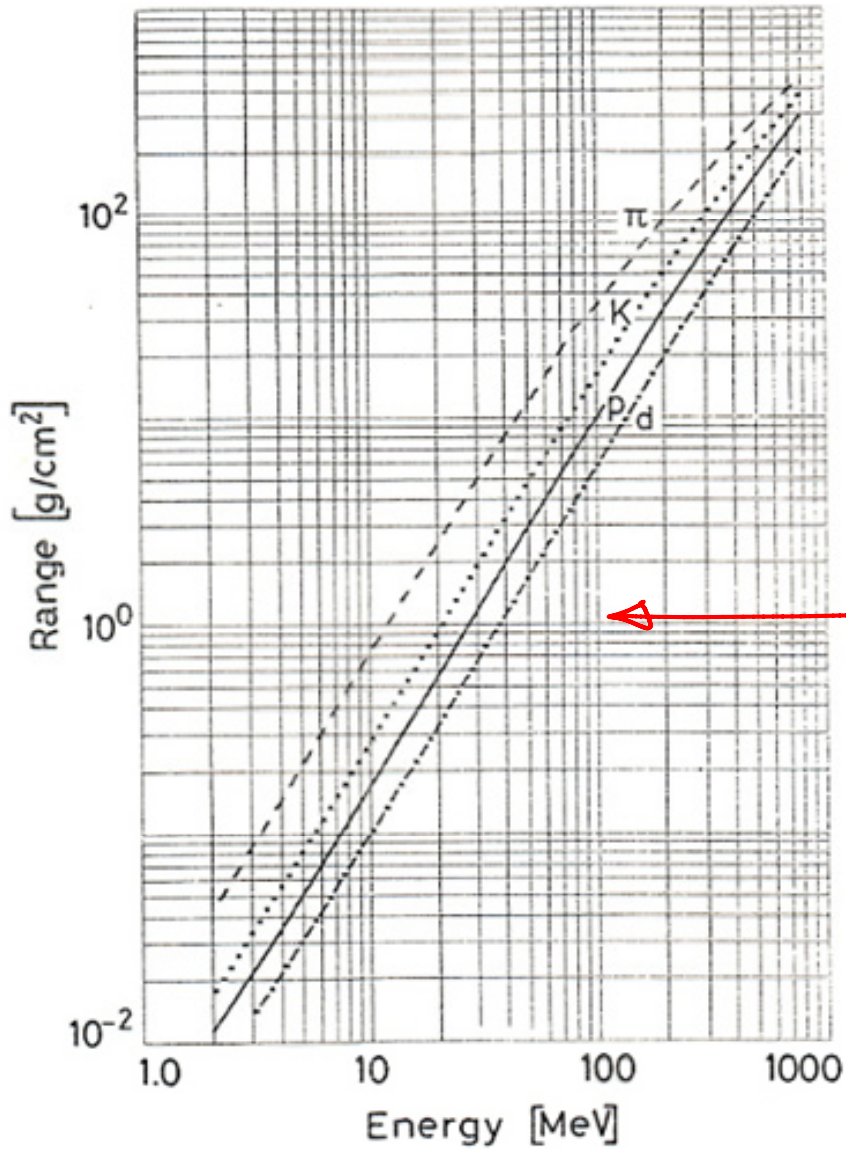
BETHE-BLOCH BREAKS DOWN
AS $\beta \rightarrow 0$



- HADRON THERAPY FOR TUMORS USING \bar{u}, p



RANGE STATISTICALLY
DISTRIBUTED FOR A BEAM
OF PARTICLES.



CALCULATED RANGE CURVES
IN ALUMINUM

$$R(T) = R_0(T_{min}) + \int_{T_{min}}^T \left(\frac{dE}{dx} \right)^{-1} dE$$

EMPIRICALLY
DETERMINED

MINIMUM
ENERGY AT
WHICH B-B
STILL VALID

NUMERICAL INTEGRATION OF B-B

Electron Energy Loss

- More complicated than heavy particles discussed so far
- Small mass \longrightarrow radiation (bremsstrahlung) dominates
- Above critical energy, radiation dominates
- Below critical energy, ionization dominates

$$\left(\frac{dE}{dx}\right)_{TOTAL} = \left(\frac{dE}{dx}\right)_{IONIZATION} + \left(\frac{dE}{dx}\right)_{RADIATION}$$

- What constitutes a heavy particle, depends on energy scale

Bethe Bloch for electrons

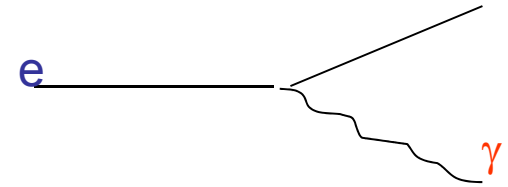
- Projectile deflected
- Projectile and atomic electrons have equal masses
- Also identical particles – statistics
- Equal masses $\longrightarrow T_{MAX} = \frac{T_E}{2}$

$$-\frac{dE}{dx} = 2\pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{\tau^2 (\tau + 2)}{2(I/m_e c^2)} \right) - F(\tau) - \delta - 2 \frac{C}{Z} \right]$$

KINETIC ENERGY IN UNITS OF $m_e c^2$

$F(\tau)^{Electron}$ \neq $F(\tau)^{Positron}$
identical *non-identical*

Bremsstrahlung



• Below ~ 100 GeV/c only important for electrons

• > 100 GeV/c becomes important for muons $\sigma \propto \left(\frac{e^2}{mc^2}\right)^2 \sim \frac{1}{m^2}$

• $E_B(\mu) \approx \frac{E_B(e)}{40,000}$ in the GeV range

$$-\left(\frac{dE}{dx}\right)_{RAD} = N \int_0^{v_0=E_0/h} h\nu \frac{d\sigma}{d\nu}(E_0, \nu) \sim \frac{1}{\nu} \quad N = \frac{\rho N_A}{A} \text{ atoms/cc}$$

$$-\left(\frac{dE}{dx}\right)_{RAD} = NE_0\Phi(Z^2) \quad \text{independent of } \nu \text{ function of material}$$

$$\Phi = 4Z^2 r_e^2 \alpha \left[\ln\left(183Z^{-1/3} - \frac{1}{18} - f(Z)\right) \right]$$

$\left(\frac{dE}{dx}\right)_{RAD} \propto E, Z^2$ can emit all energy in a few photons \rightarrow large fluctuations

$\left(\frac{dE}{dx}\right)_{ION} \propto \ln(E), Z$

Radiation Length

$$-\left(\frac{dE}{dx}\right)_{RAD} = NE_0\Phi(Z^2)$$

$$-\frac{dE}{E_0} = N\Phi(Z^2)$$

assume indep of E

$$E = E_0 \exp\left(-\frac{x}{\chi_0}\right)$$

$$\int \rightarrow \ln E - \ln E_0$$

$$\chi_0 = \frac{1}{N\Phi}$$

- χ_0 distance over which the electron energy is reduced by 1/e on average
- Radiation Length

$$\frac{1}{\chi_0} \approx \left[4Z(Z+1)\frac{\rho N_A}{A}\right] r_e^2 \alpha \left[\ln\left(\frac{183}{Z^{1/3}}\right) - f(z)\right]$$

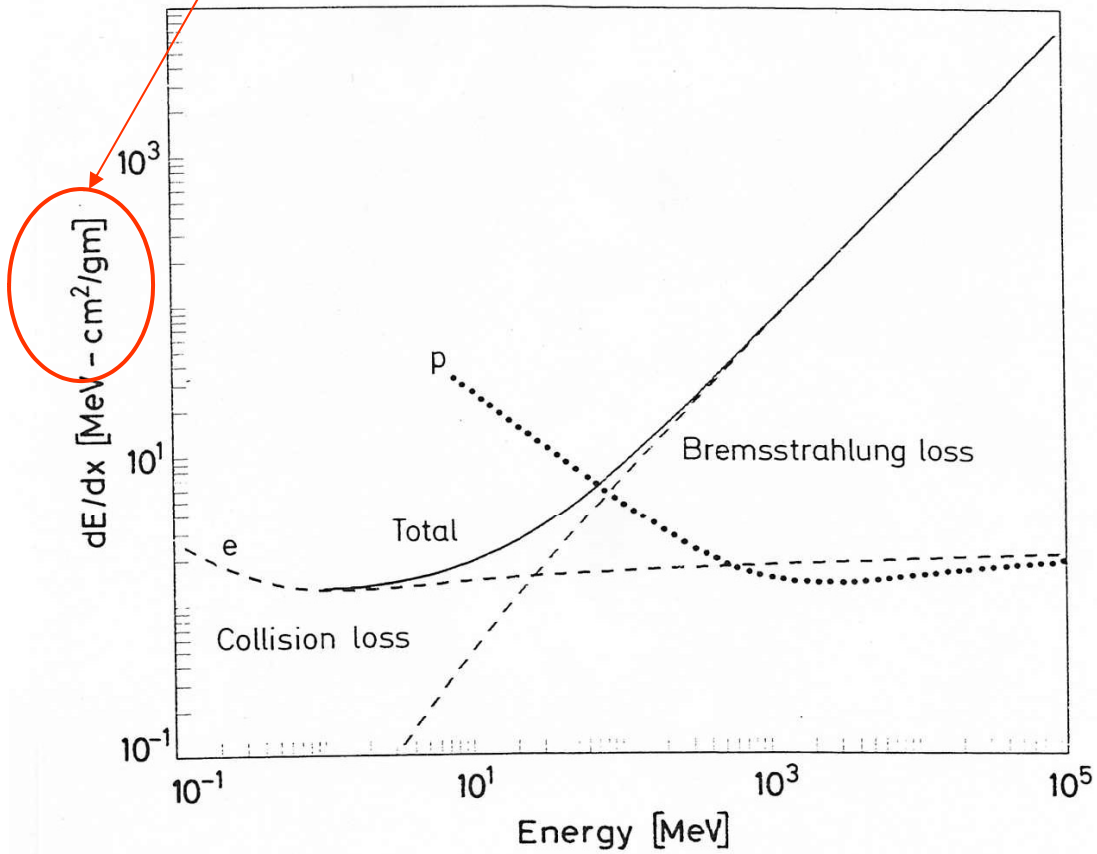
- for x expressed in units of χ_0

$$-\frac{dE}{dt} = E_0$$

ROUGHLY INDEPENDENT OF MATERIAL

Electron Energy Loss

approx valid for any material



electrons in Cu

Table 2.3. Radiation lengths for various absorbers

Material	[gm/cm ²]	[cm]
Air	36.20	30050
H ₂ O	36.08	36.1
NaI	9.49	2.59
Polystyrene	43.80	42.9
Pb	6.37	0.56
Cu	12.86	1.43
Al	24.01	8.9
Fe	13.84	1.76
BGO	7.98	1.12
BaF ₂	9.91	2.05
Scint.	43.8	42.4

$$X_0 = \frac{716.4 \text{ g cm}^{-2} A}{Z(Z + 1) \ln(287/\sqrt{Z})}$$

good approximation (3%) except for He

CRITICAL ENERGY FOR VARIOUS MATERIALS

	Ec (MeV)
Pb	9.51
Cu	24.8
Fe	27.4
Al	52
Water	92
Air	102

$$\left(\frac{dE}{dx}\right)_{RAD} = \left(\frac{dE}{dx}\right)_{ION}$$

High Energy Muons

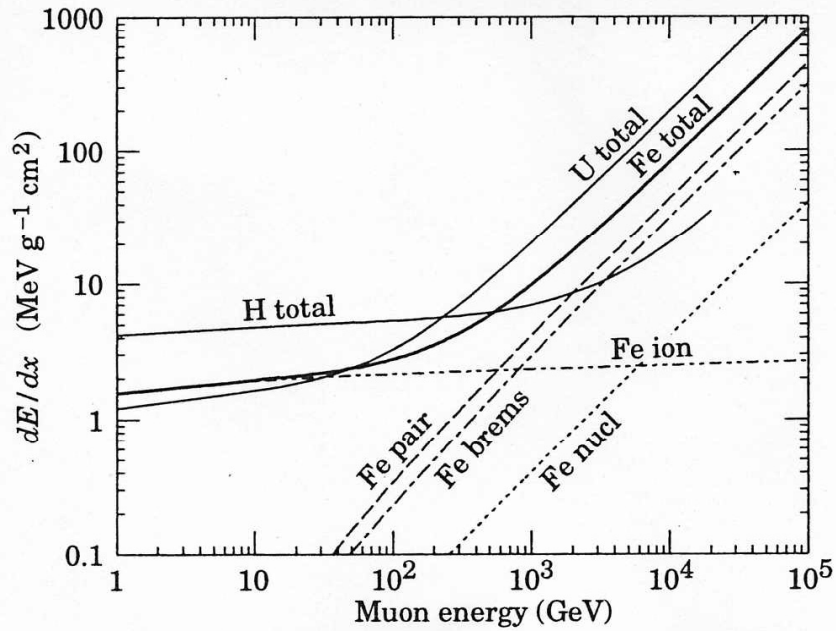
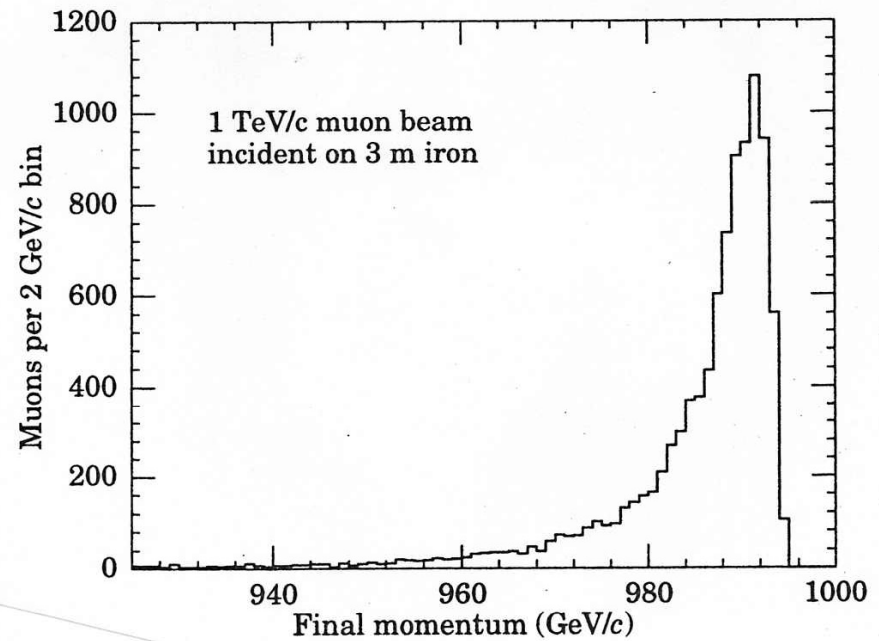
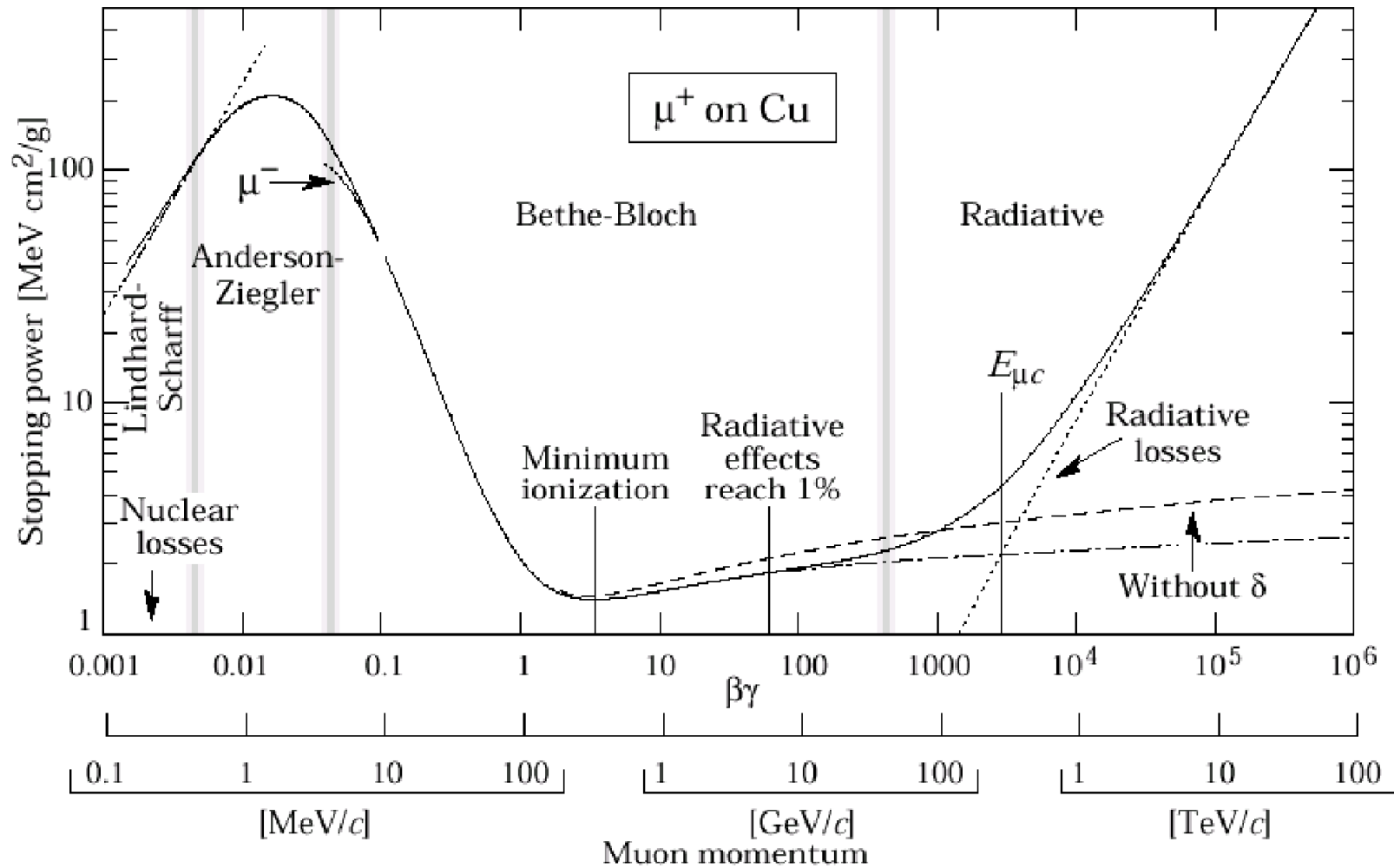


Figure 23.11: The average energy loss of a muon in hydrogen, iron, and uranium as a function of muon energy. Contributions to dE/dx in iron from ionization and the processes shown in Fig. 23.10 are also shown.



Muons in Cu



Energy Loss Distribution

- So far have discussed $\left(\frac{dE}{dx}\right)_{MEAN}$
- In general energy loss for a given particle $\Delta E \neq (\Delta E)_{MEAN}$
- For a mono-energetic beam
 - distribution of energy losses
- Thick Absorber – Gaussian Energy Loss
- Thin Absorber – Possibility of low probability, high fractional energy transfers

LANDAU DISTRIBUTION

THE IONIZATION LOSS dE/dx IN BETHE-BLOCH IS MEAN ENERGY LOSS

THERE IS ACTUALLY A LONG TAIL TO HIGH ENERGY LOSSES — THEORY DUE TO LANDAU

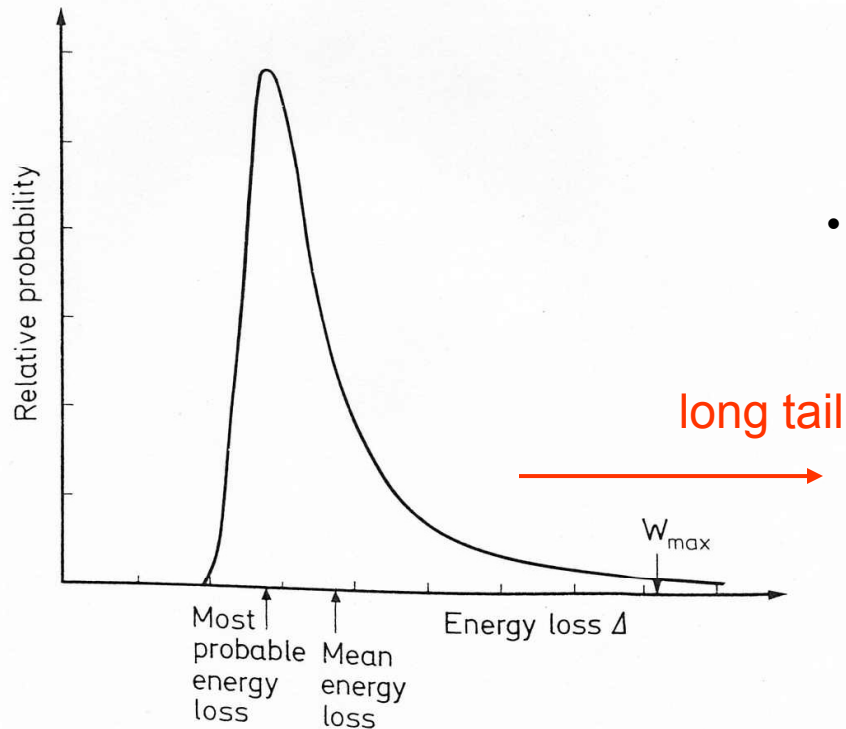
FOR MATERIAL DEPTH L , MOST PROBABLE ENERGY LOSS $\langle \Delta E \rangle \sim \langle dE/dx \rangle L$

LANDAU DEVIATIONS $\Delta E - \langle \Delta E \rangle$

PROBABILITY DISTRIBUTION ABOUT MOST PROBABLE ENERGY LOSS $\rightarrow P(\lambda) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}(\lambda + e^{-\lambda})\right\}$

$$\lambda = \frac{\Delta E - \langle \Delta E \rangle}{\langle \Delta E \rangle}$$

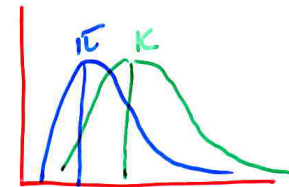
Typical Energy Loss in Thin Absorber



- Scintillator
- Wire Chamber Cell
- Si tracker wafer

- Practical Implications

- Use of dE/dx for particle ident
- Landau tails cause limitation in separation



- Various Calculations
- Landau – most commonly used
- Vavilov - “improved” Landau

- Position in tracking chamber
 - Landau tails smear resolution
- Separation of 1 from 2 particles in an ionization/scintillator counter
 - Landau tails smear ionization

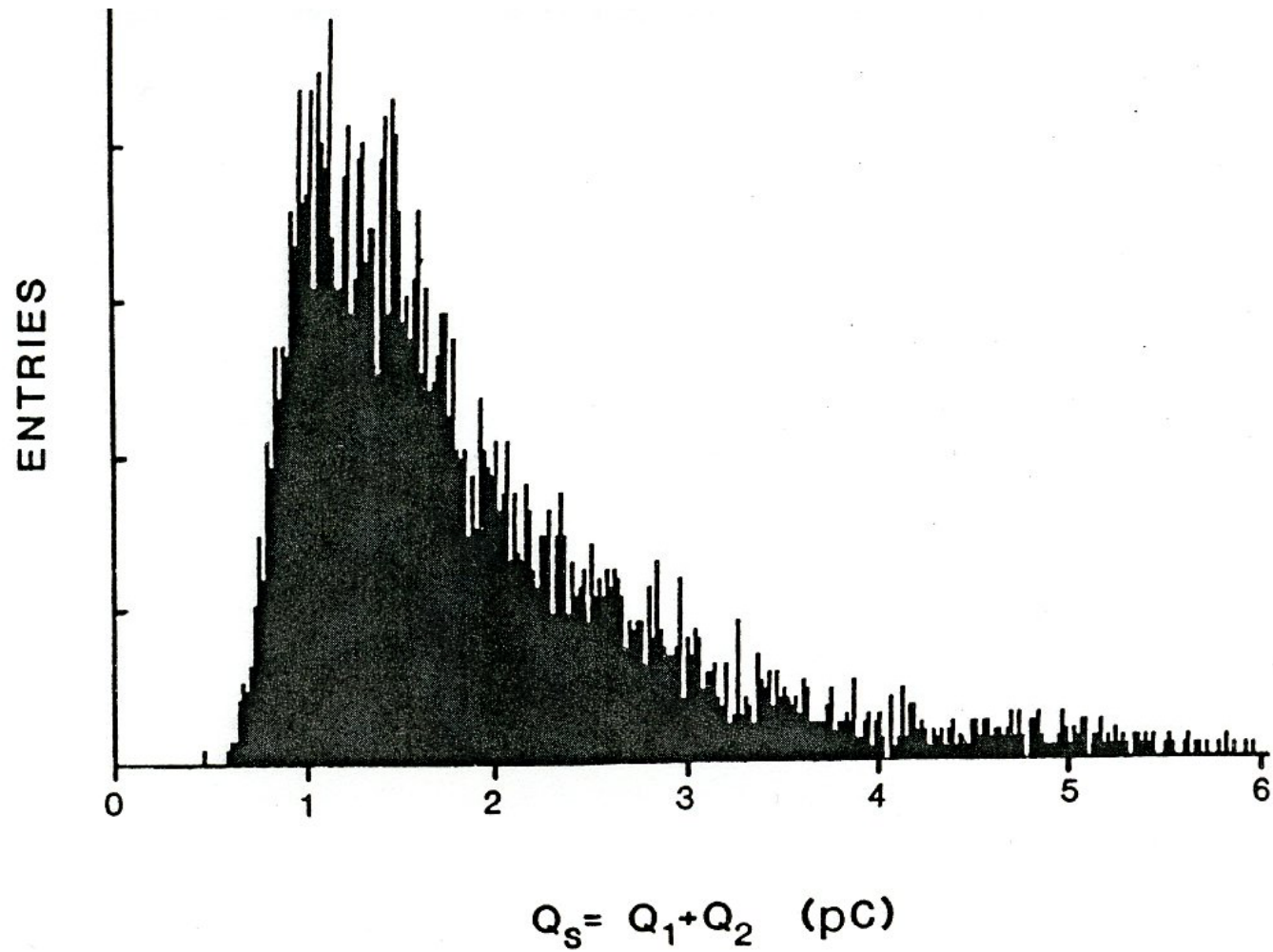
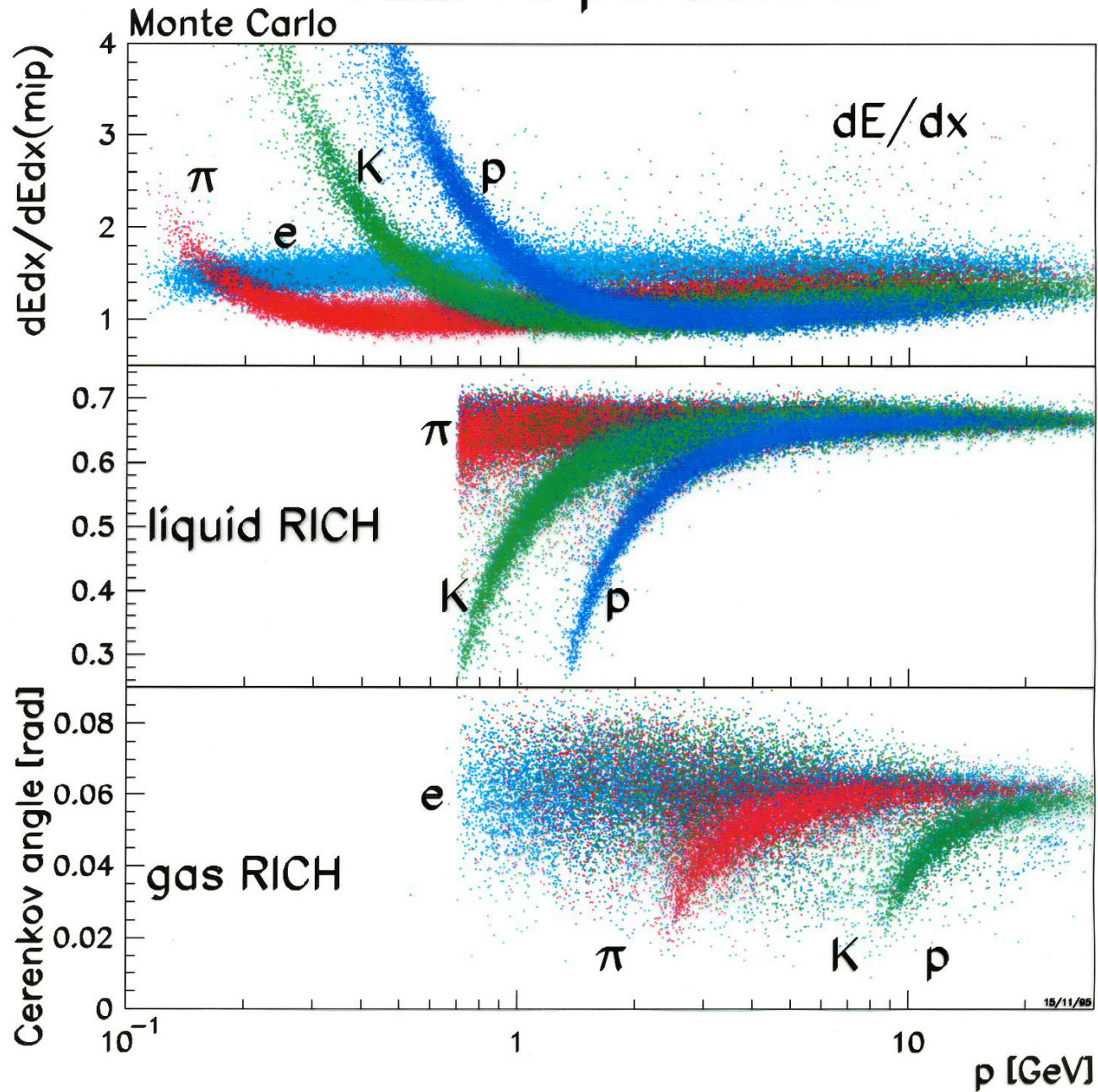


Fig. 6.9. Charge deposited on a wire chamber cathode by the passage of a MIP.

DELPHI particle ID



Cerenkov Radiation

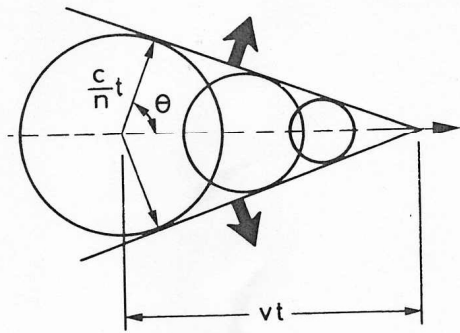
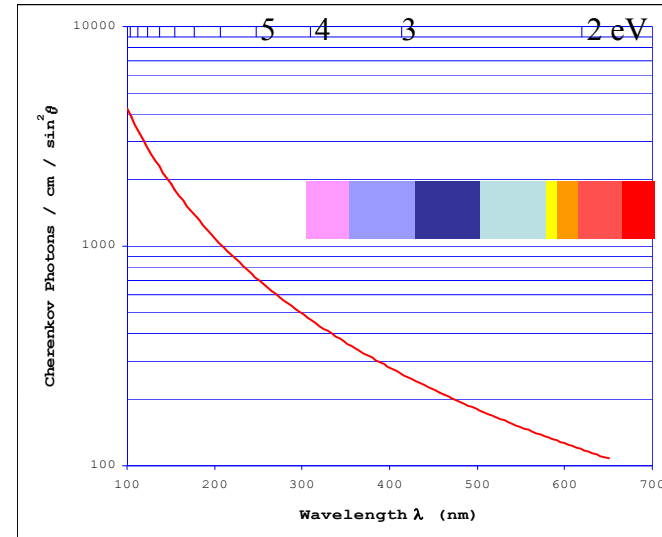


Fig. 2.9. Cherenkov radiation: an electromagnetic shock wave is formed when the particle travels faster than the speed of light in the same medium

$$\cos \theta = \frac{c/nt}{\beta ct} = \frac{1}{\beta n}$$

measure
known

medium	n	$\theta_{\max} (\beta=1)$	$N_{\text{ph}} (\text{eV}^{-1} \text{cm}^{-1})$
air	1.000283	1.36	0.208
isobutane	1.00127	2.89	0.941
water	1.33	41.2	160.8
quartz	1.46	46.7	196.4



$$\frac{dN}{dx} = 2\pi z^2 \alpha \sin^2 \theta \int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\lambda}$$

$$N[\lambda_1 \rightarrow \lambda_2] = 4.6 \cdot 10^6 \left[\frac{1}{\lambda_2(A)} - \frac{1}{\lambda_1(A)} \right] L(\text{cm}) \sin^2 \theta$$

$$475 z^2 \sin^2 \theta \text{ photons/cm}$$

$$350 \text{ nm to } 550 \text{ nm}$$

TRANSITION RADIATIONS

A CHARGED PARTICLE RADIATES WHEN IT CROSSES THE BOUNDARY BETWEEN MATERIALS WITH DIFFERENT DIELECTRIC CONSTANTS

DEPENDS ON PLASMA FREQUENCY $\omega_p = \sqrt{\frac{4\pi N_e e^2}{m}}$

$$\hbar \omega_p = \frac{m_0}{2} \sqrt{4\pi N_e} mc^2 \sim 30 \text{ MeV}, \text{ DENSITY} = 1$$

$$\text{TYPICAL ENERGY OF } \gamma = \gamma \hbar \omega_p / 4$$

$$\text{AVERAGE \# PHOTONS } E \gg \gamma \hbar \omega_p / 10$$

$$N_\gamma \approx 0.8 \alpha Z^2 \approx 0.59 \times 10^{-2} Z^2$$

TOTAL ENERGY EMITTED

$$E = \frac{\alpha Z^2 \gamma \hbar \omega_p}{3}$$

WEAK RADIATION
ONLY SEE FOR STACK
OF FOILS / BOUNDARIES

Multiple Coulomb Scattering

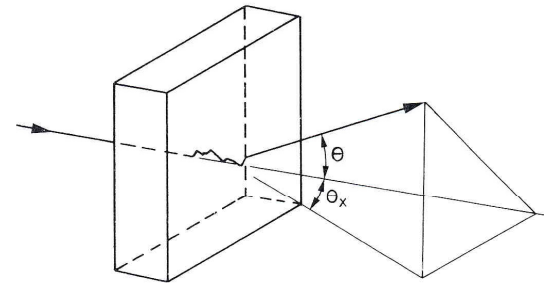
- Can be a very important limitation on detector angle/momentum resolution
- For Charged particles traversing a material (ignore radiation)
 - Inelastic collisions with electrons - ionization
 - elastic scattering from atomic nuclei

Rutherford scattering

$$\frac{d\sigma}{d\Omega} = z_1^2 z_2^2 r_e^2 \frac{(m_e c / p\beta)^2}{4 \sin^4 \frac{\theta}{2}}$$

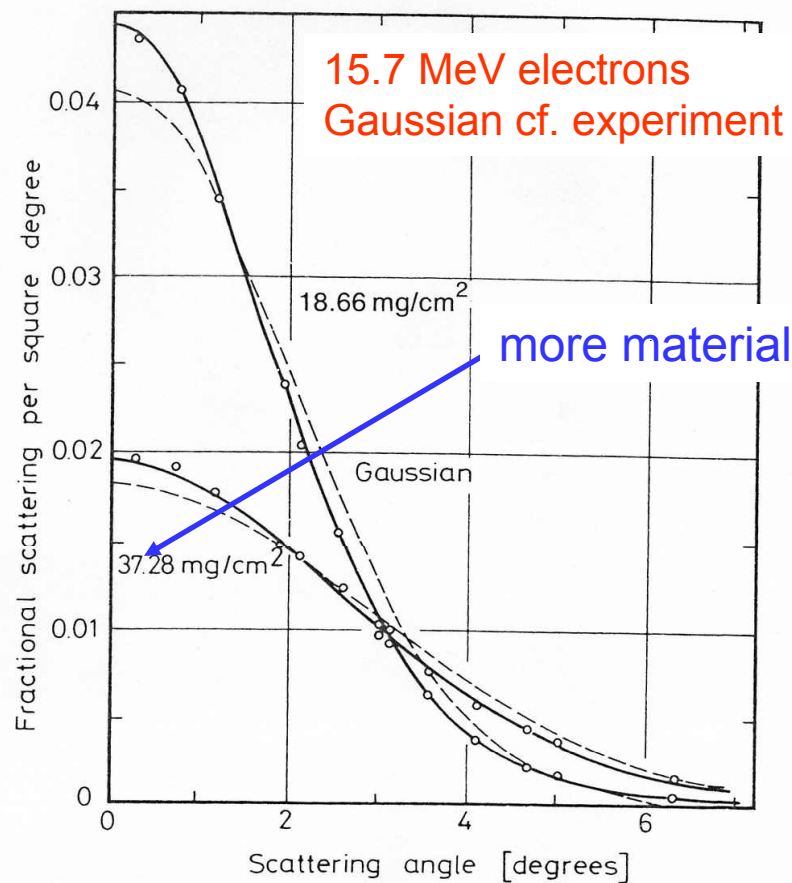
vast majority of scatters – small angle

MOUÏÈRE →
MORE SOPHISTICATED



- θ is polar angle
- number of scatters > 20
- negligible energy loss
- Gaussian statistical treatment is usually ok

Gaussian Multiple Scattering



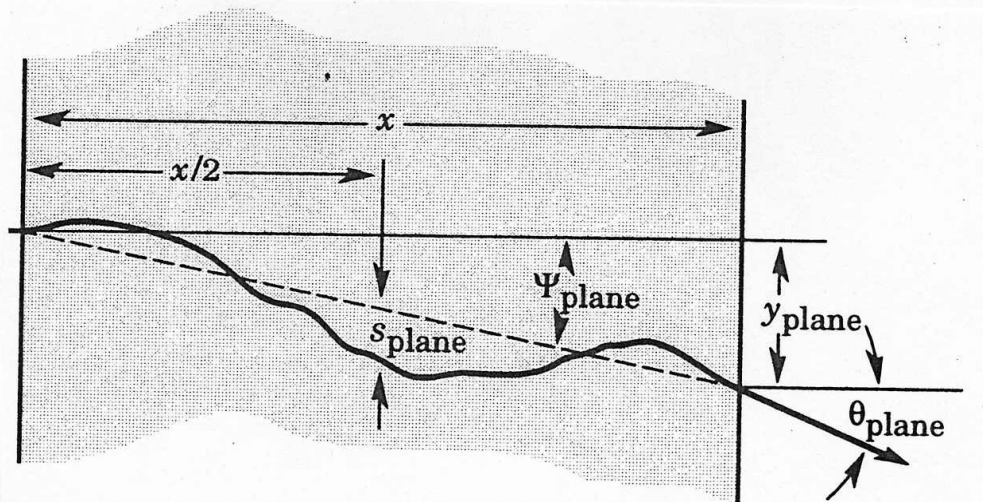
probability of scattering through θ

$$P(\theta) \approx \frac{2\theta}{\langle \theta^2 \rangle} \exp\left(-\frac{\theta^2}{\langle \theta^2 \rangle}\right) d\theta$$

$\sqrt{\langle \theta^2 \rangle}$ RMS scattering angle

Gaussian Multiple Scattering

For detectors usually interested in RMS scattering angle – projected on a plane
 most detectors measure in a plane



$$\theta_0 = \theta_{PLANE}^{RMS} = \frac{1}{\sqrt{2}} \theta_{SPACE}^{RMS}$$

$$\theta_0 = \frac{13.6 \text{ MeV}}{\beta c p} z \sqrt{\frac{x}{\chi_0}} \left[1 + 0.038 \ln \left(\frac{x}{\chi_0} \right) \right]$$

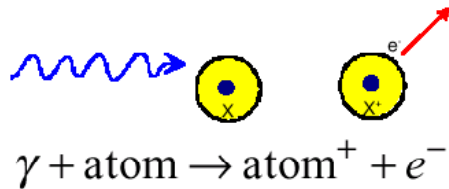
$$\Psi_{PLANE}^{RMS} = \frac{1}{\sqrt{3}} \theta_0$$

$$y_{PLANE}^{RMS} = \frac{x}{\sqrt{3}} \theta_0$$

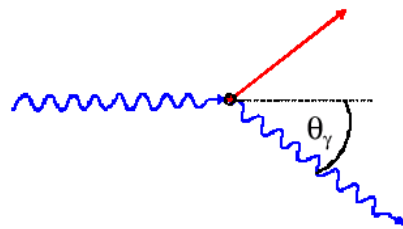
$$s_{PLANE}^{RMS} = \frac{x}{4\sqrt{3}} \theta_0$$

Energy Loss of Photons in Matter

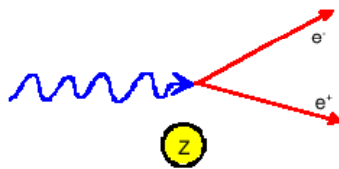
Important for Electromagnetic Showers



- Photoelectric Effect



- Compton Scattering

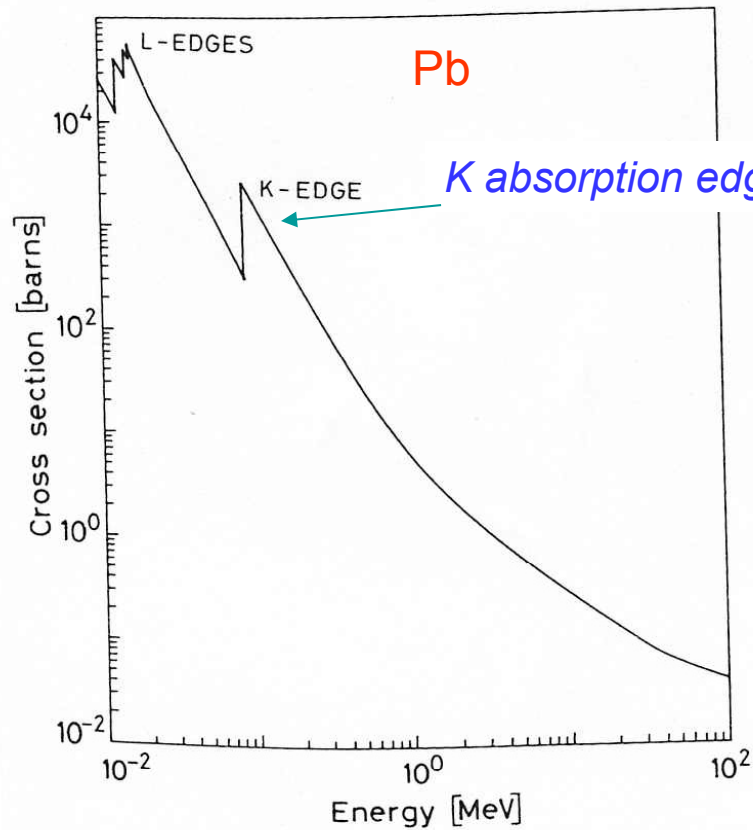


- Pair Production – completely dominant above a few MeV
- For a beam of γ or survival probability of a single γ

$$I(x) = I_0 e^{-\mu x}$$

absorption coefficient

Photoelectric Effect

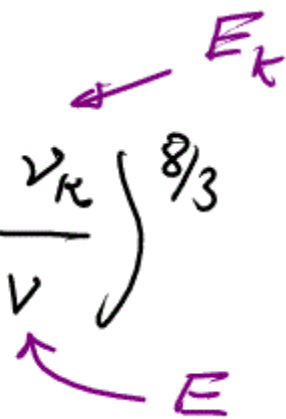


- Atomic electron absorbs photon and is ejected
- Cross section for absorption increases with decreasing energy until $E_e = h\nu - \text{Binding Energy}$
- Then drops because not enough energy to eject K-shell electrons
- Dependence on material $\sim Z^{4 \rightarrow 5}$

PHOTO ELECTRIC EFFECT IS HARD TO TREAT ACCURATELY

→ DEPENDS ON KNOWLEDGE OF ATOMIC ELECTRON WAVE FUNCTIONS

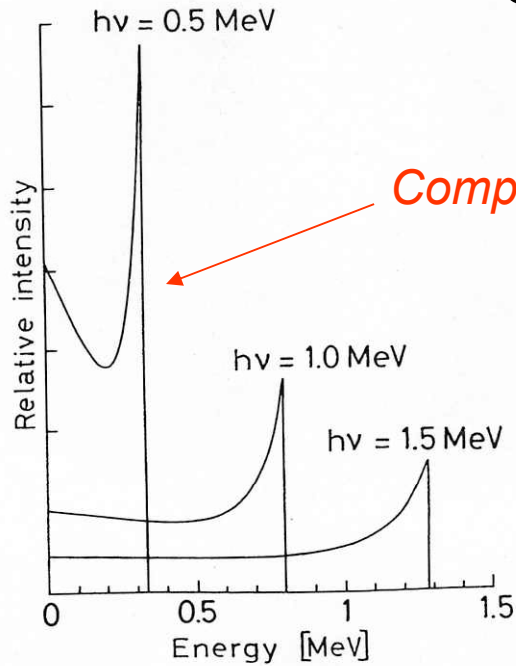
CLOSE TO K-EDGE

$$\text{CROSS SECTION} = \frac{6.3 \times 10^{-18}}{Z^2} \left(\frac{\nu_K}{\nu} \right)^{8/3}$$


DEPENDENCE ON Z

$$\sim Z^{4-5} \quad \text{AT HIGH } Z$$

Compton Scattering

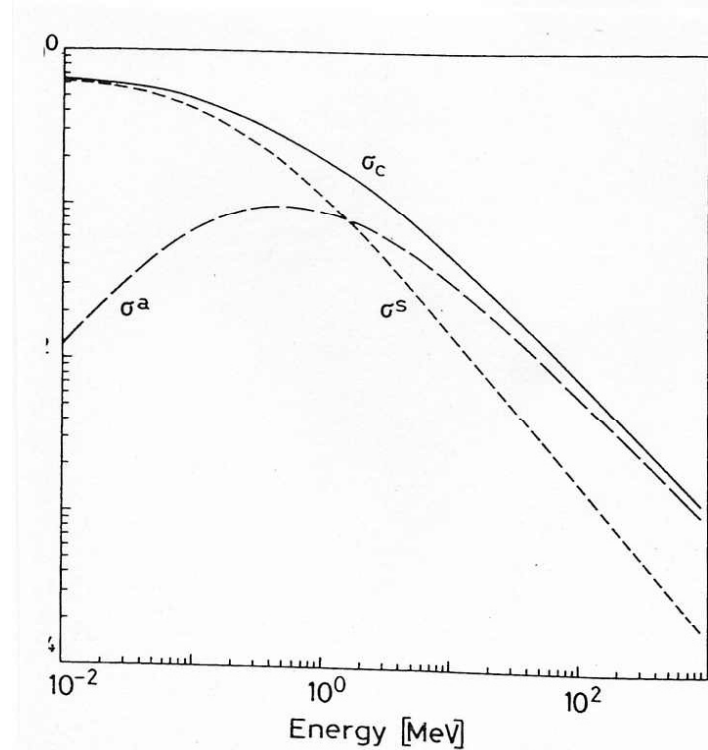


Compton Edge – Detector Calibration

$$T_{MAX} = h\nu \left(\frac{2\gamma}{1+2\gamma} \right)$$

$$\gamma = \frac{h\nu}{m_e c^2}$$

Fig. 2.24. Energy distribution of Compton recoil electrons. The sharp drop at the maximum recoil energy is known as the *Compton edge*



Pair Production

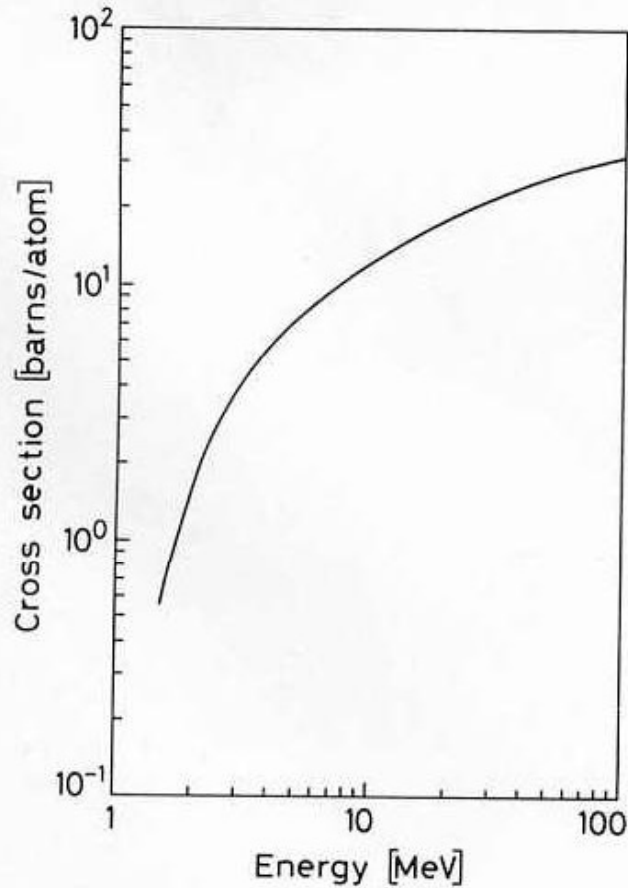


Fig. 2.25. Pair production cross section in lead

- Central to electromagnetic showers
- Can only occur in field of nucleus
- Rises with energy cf. Compton and PE
- Same Feynman diagram as Brems

Mean Free Path

$$\frac{1}{\lambda_{PAIR}} = \frac{7}{9} 4Z(Z+1) Nr_e^2 \alpha \left[\ln\left(\frac{183}{Z^{1/3}}\right) - f(z) \right]$$

$\sim Z^2$

$$\lambda_{PAIR} \approx \frac{9}{7} \chi_0 \quad \text{Closely related to Radiation Length}$$

Photon Absorption as Function of Energy

