UNITS IN SUBATOMIC PHYSICS

- PHYSICS & TECHNOLOGY → SI UNITS USED
- RELATIVISTIC PHYSICS CGS / GAUSSIAN

  NATURAL SYSTEM → NO "$4\pi\varepsilon_0$" IN EM

LENGTH	METER	m
TIME	SECOND	s
ENERGY	ELECTRON VOLT	eV
MASS	"	eV/c²
MOMENTUM	"	eV/c

- SCALE OF COLOR FORCE
- SCALE OF ELECTROWEAK FORCE

10^6 eV = MeV — BINDING ENERGY OF NUCLEI

10^9 eV = GeV — MASS ENERGY OF PROTON

10^{12} eV = TeV — MASS ENERGY OF HIGGS BOSON

- 10^{-15} m = FEMTOMETER
  = 1 FERMI
  = DIAMETER OF PROTON

- TIME FOR LIGHT SIGNAL TO CROSS PROTON

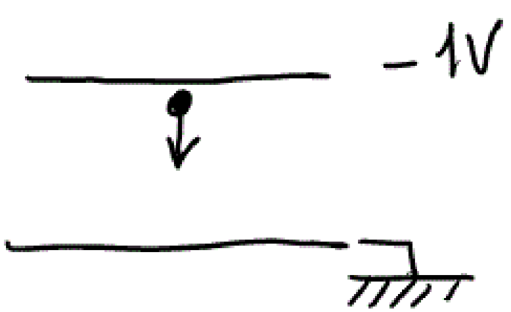
$$\approx \frac{10^{-15}\,[m]}{3\times 10^{8}\,[m]}\,[s] \approx 10^{-23}\ \text{sec}$$

- TIME SCALE OF STRONG INTERACTION

WHY USE ELECTRON VOLTS?

- CONVENIENT ENERGY UNIT
- SUBATOMIC PHYSICS EXPERIMENTS DONE BY ACCELERATING BEAMS OF PARTICLES IN ELECTRIC FIELDS, AND SCATTERING OFF TARGET

- ONE ELECTRON VOLT (eV) IS THE KINETIC ENERGY AN ELECTRON GAINS BY ACCELERATING THRU A POTENTIAL OF ONE VOLT



$$1\,\text{eV} = e \times 1\,\text{VOLT}$$

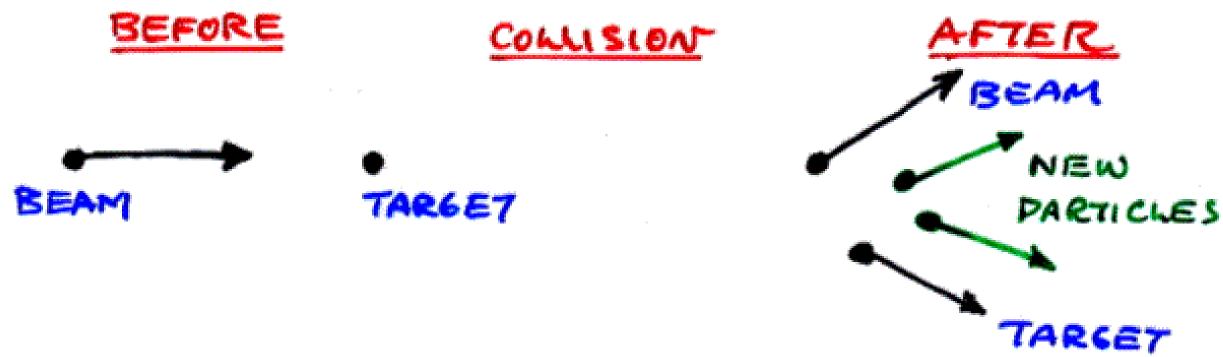
$$= 1.60 \times 10^{-19}\,(\text{COULOMB}) \times 1\,\text{VOLT}$$

$$= 1.60 \times 10^{-19}\ \text{JOULES}$$

$$= 1.60 \times 10^{-12}\ \text{ERGS}$$

WHY ELECTRON VOLTS FOR MASSES ?

WHEN A BEAM OF RELATIVISTIC PARTICLES SCATTERS FROM A TARGET, SOME OF THE KINETIC ENERGY CAN APPEAR AS MASS → NEW PARTICLES



- CONVENIENT TO MEASURE MASS IN SAME UNITS AS ENERGY

$$E^2 = p^2 c^2 + m^2 c^4$$

TOTAL RELATIVISTIC ENERGY

MOMENTUM

VELOCITY OF LIGHT

MASS
SOMETIMES CALLED "REST MASS"

$$E^2 = p^2 c^2 + m^2 c^4$$

- FOR A PARTICLE WITH NO MASS

PHOTON — γ
NEUTRINO — ν (?)

$$E = p \cdot c \longrightarrow p = \frac{E}{c} = \left.\frac{[eV]}{c}\right\}$$

UNIT OF MOMENTUM

MASSLESS PARTICLE $E = 1\,eV \rightarrow p = 1\,eV/c$

- FOR A MASSIVE PARTICLE AT REST — IN ITS REST FRAME

$$E = mc^2 \longrightarrow m = \frac{E}{c^2} = \left.\frac{[eV]}{c^2}\right\}$$

UNIT OF MASS

PARTICLE OF MASS $1\,eV/c^2$ AT REST

HAS A TOTAL ENERGY OF $1\,eV$

$$E^2 = p^2c^2 + m^2c^4 \quad \text{—— } ①$$

$$E^2 = p^2 + m^2 \quad [\text{WHERE } C = 1]$$

EACH TERM OF ① HAS DIMENSIONS $[E]^2$

EXAMPLE – PARTICLE MASS = $1000 \frac{MeV}{c^2}$, $p = 1000 \frac{MeV}{c}$

$$E^2 = 10^6 \frac{MeV^2}{c^2} \cdot c^2 + 10^6 \frac{MeV^2}{c^4} \cdot c^4$$

$$E^2 = 2 \times 10^6 \ MeV^2$$

THE PARTICLE HAS A TOTAL RELATIVISTIC ENERGY

$$E = \sqrt{2} \times 10^3 \ MeV$$

Table 1.1. *Units in high energy physics*

(a)

Quantity	High energy unit	Value in SI units
length	1 fm	10^{-15} m
energy	1 GeV = 10^9 eV	1.602×10^{-10} J
mass, E/c^2	1 GeV/c^2	1.78×10^{-27} kg
$\hbar = h/(2\pi)$	6.588×10^{-25} GeV s	1.055×10^{-34} J s
c	2.998×10^{23} fm s^{-1}	2.998×10^8 m s^{-1}
$\hbar c$	0.1975 GeV fm	3.162×10^{-26} J m

PROTON MASS

(b)

natural units, $\hbar = c = 1$	
mass, Mc^2/c^2	1 GeV
length, $\hbar c/(Mc^2)$	1 GeV^{-1} = 0.1975 fm
time, $\hbar c/(Mc^3)$	1 GeV^{-1} = 6.59×10^{-25} s
Heaviside–Lorentz units, $\epsilon_0 = \mu_0 = \hbar = c = 1$	
fine structure constant	$\alpha = e^2/(4\pi) = 1/137.06$

Relations between energy units

1 MeV = 10^6 eV   1 GeV = 10^3 MeV   1 TeV = 10^3 GeV

$\hbar c = 1 = 0.1975$ GeV$\cdot$fm

$\hbar = 1 = 6.59 \times 10^{-25}$ GeV$\cdot$s

CHOOSE $\hbar c = 1$ [NATURAL UNITS] $= 0.1975$ [$GeV \cdot fm$]

CAN CONVERT BETWEEN UNITS

LENGTH

[$GeV \cdot fm$]

$$\frac{\hbar c}{m c^2} = \frac{\hbar c}{GeV} [L] = 0.1975 \left[\frac{GeV \cdot fm}{GeV}\right]$$

$$\left[\frac{GeV}{c^2} \cdot c^2\right]$$

$$1\,GeV^{-1} = 0.1975\,fm$$

TIME

$$\frac{\hbar c}{m c^3} [T] = \frac{\hbar}{GeV} = 6.588 \times 10^{-25} \left[\frac{GeV \cdot s}{GeV}\right]$$

$$1\,GeV^{-1} = 6.588 \times 10^{-25}\,s$$

$$\hbar = c = 1$$

RANDOMNESS OF DECAYS

- QUANTUM MECHANICS → IN AN ENSEMBLE OF UNSTABLE PARTICLES, ANY ONE MAY DECAY AT RANDOM IN A SMALL TIME INTERVAL.
- EACH PARTICLE DECAYS AFTER A RANDOM TIME → ENSEMBLE CHARACTERIZED BY MEAN LIFETIME
- NUMBER DECAYING IN TIME INTERVAL dt:

$$dN = -\lambda N(t)\, dt$$

#DECAYING

PROBABILITY PER UNIT TIME FOR DECAY → DECAY CONSTANT TRANSITION RATE

#UNDECAYED AT START OF TIME INTERVAL

TIME INTERVAL

$$\frac{dN}{N(t)} = -\lambda\, dt$$

$$dN = -\lambda N(t)\,dt \rightarrow \frac{dN}{N} = -\lambda\,dt$$

$$\int_{N(0)}^{N(t)} dN/N = -\lambda \int_0^t dt \rightarrow \ln N(t) - \ln N(0) = -\lambda t$$

$$N(t) = N(0)\,e^{-\lambda t}$$

SURVIVAL EQUATION

INTENSITY OF RADIATION = ACTIVITY

$$I(t) = \frac{-dN(t)}{dt} = \underbrace{N(0)\,\lambda}\,e^{-\lambda t}$$

$$I(t) = I(0)\,e^{-\lambda t}$$

INTENSITY OF EMITTED RADIATION

INITIAL ACTIVITY OR INTENSITY

WHAT IS THE MEAN OF AN EXPONETIAL?

MEAN OF ANY FUNCTION

$$\bar{x} \equiv \int x f(x)\,dx \Big/ \int f(x)\,dx$$

$$\tau \equiv \frac{\int_0^\infty t\,dN(t)}{\int_0^\infty dN(t)} = \frac{\int_0^\infty t \exp(-\lambda t)\,dt}{\int_0^\infty \exp(-\lambda t)\,dt}$$

$$\int_0^\infty x^n e^{-ax}\,dx = n!/a^{n+1} \quad ; \quad \int e^{ax}\,dx = \frac{1}{a} e^{ax}$$

$$\tau = \left[(1/\lambda)^2 / (1/\lambda) \right]_0^\infty$$

$$= 1/\lambda$$

UNITS OF RADIOACTIVITY

- CURIE (Ci) AMOUNT OF RADIOACTIVE MATERIAL IN WHICH NUMBER OF DISINTEGRATIONS PER SECOND = 1g OF RADIUM

  $3.7 \times 10^{10}\ s^{-1}$

- BECQUEREL (Bq) ONE DISINTEGRATION PER SECOND

  STARTED WHOLE SUBJECT BY DISCOVERING RADIOACTIVITY IN 1896

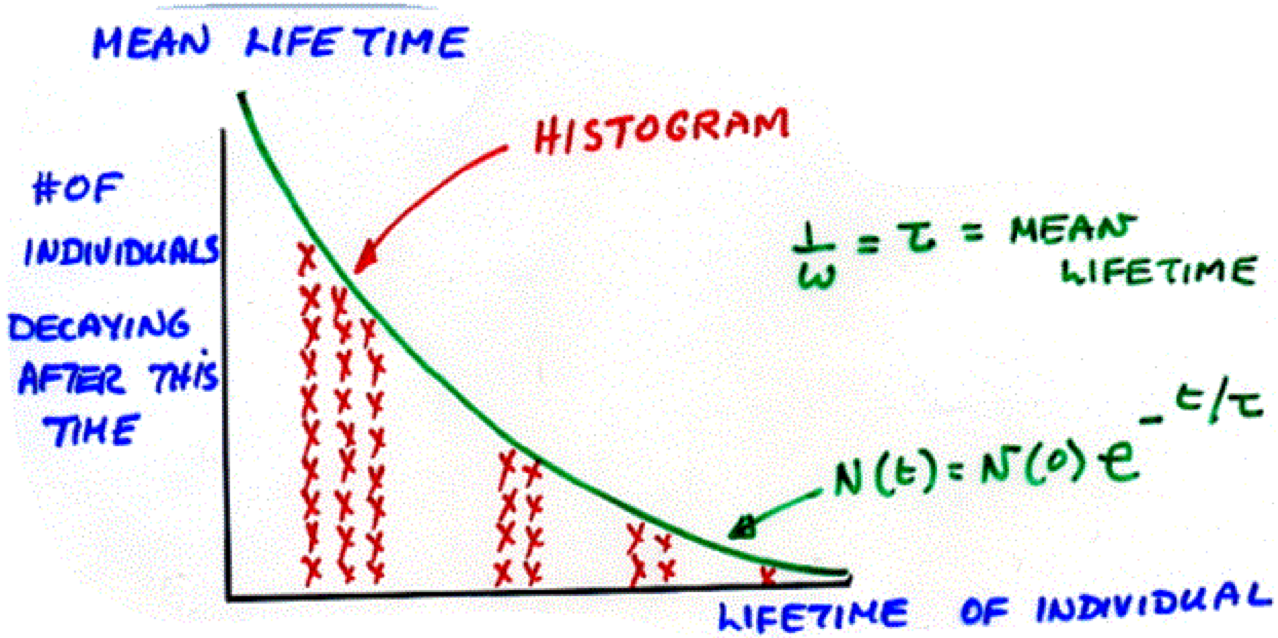
? HOW MUCH RADIOACTIVITY DID CHERNOBYL RELEASE

1000'S OF CURIES !

— WHAT ABOUT FUKUSHIMA → FIND OUT !

http://en.wikipedia.org/wiki/Comparison_of_Fukushima_and_Chernobyl_nuclear_accidents

MEAN LIFETIME



- INDIVIDUAL DECAY RANDOM
- POPULATION CHARACTERIZED BY MEAN LIFETIME

ENORMOUS RANGE OF LIFETIMES

PROTON DECAY	$> 10^{33}$ YEARS
$^{238}U \to {}^{234}Th + \alpha$	6.5×10^{9} YEARS
$^{215}Po \to {}^{211}Pb + \alpha$	2.0×10^{3} s
$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$	2.2×10^{-6} s
$\pi^0 \to \gamma\gamma$	8.3×10^{-17} s
$\Delta^{++} \to p\pi^+$	6×10^{-24} s

DECAY RATE GOVERNED BY FORCE (INTERACTION) CAUSING THE DECAY PROCESS

DECAY CHAINS

IMPORTANT FOR RADIOACTIVE NUCLEI

DECAY CHAIN $A \rightarrow B \rightarrow C$

DECAY CONSTANTS λ_A λ_B λ_C

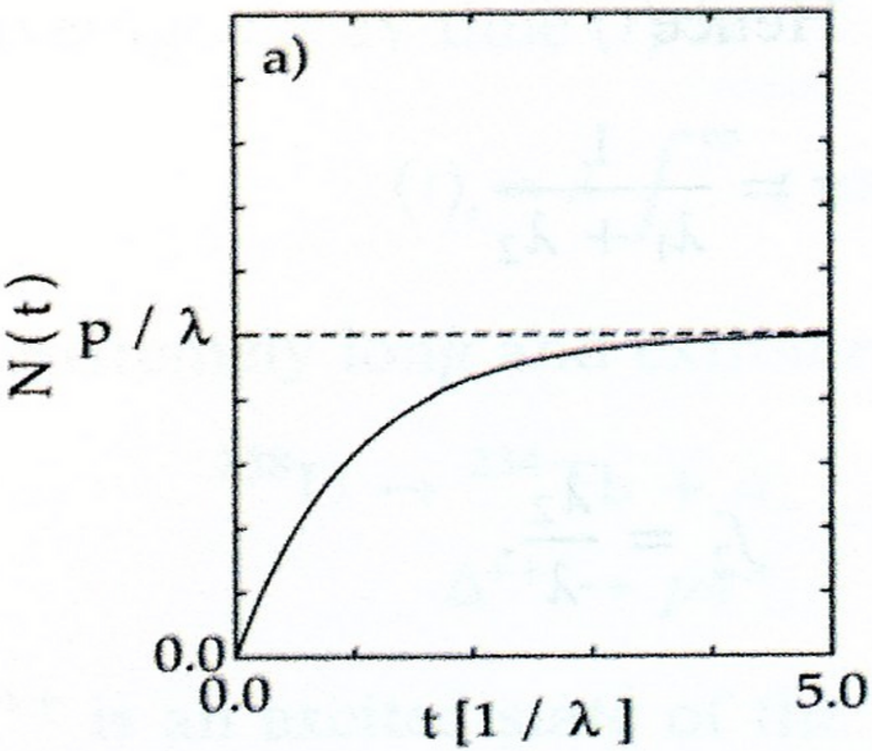
$$N_A(t) = N_A(0)\, e^{-\lambda_A t}$$

$$\frac{dN_B(t)}{dt} = -\lambda_B N_B + \lambda_A N_A$$

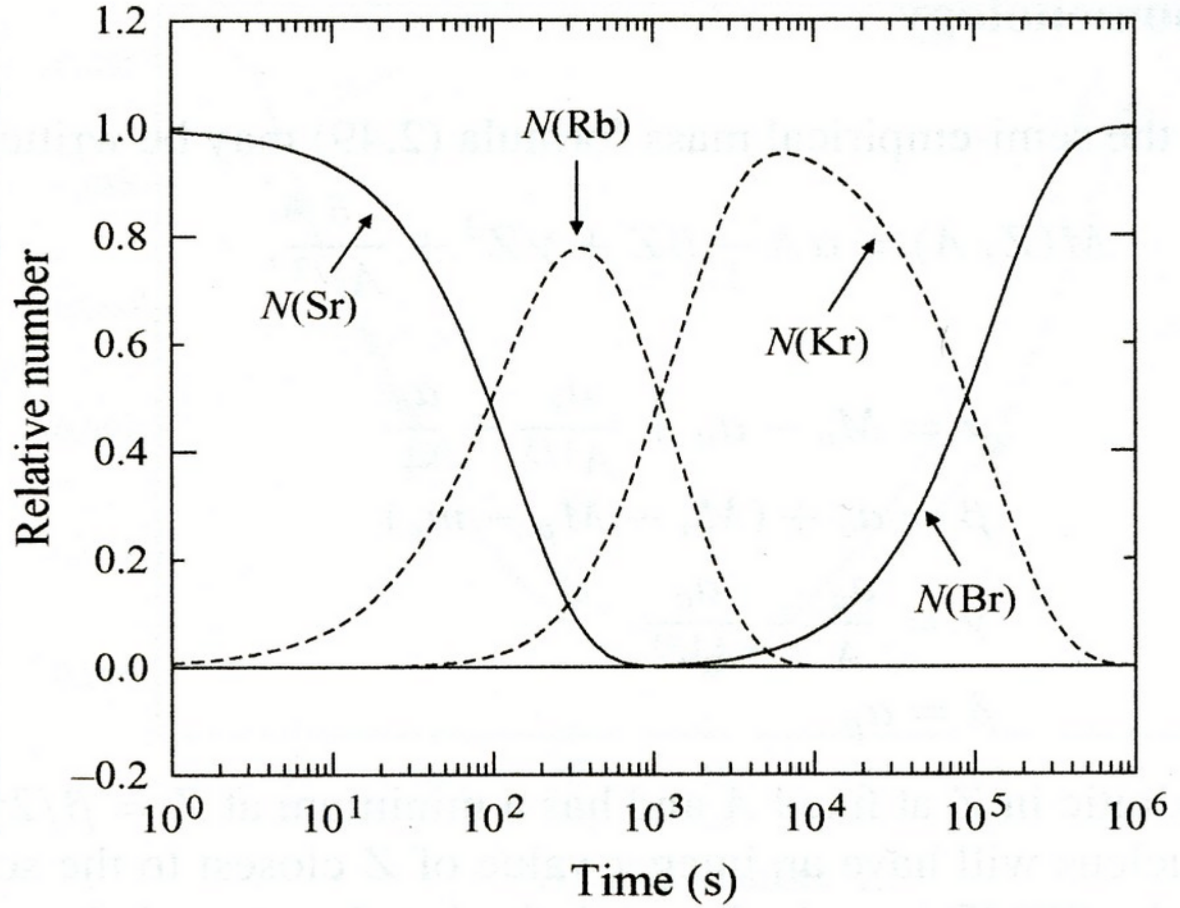
INTEGRATE $$N_B(t) = \frac{\lambda_A}{\lambda_B - \lambda_A} \cdot N_A(0) \left[e^{-\lambda_A t} - e^{-\lambda_B t} \right]$$

AGAIN

$$N_C(t) = \lambda_A \lambda_B N_A(0) \left[\frac{e^{-\lambda_A t}}{(\lambda_B - \lambda_A)(\lambda_C - \lambda_A)} + \frac{e^{-\lambda_B t}}{(\lambda_A - \lambda_B)(\lambda_C - \lambda_B)} + \frac{e^{-\lambda_C t}}{(\lambda_A - \lambda_C)(\lambda_B - \lambda_C)} \right]$$



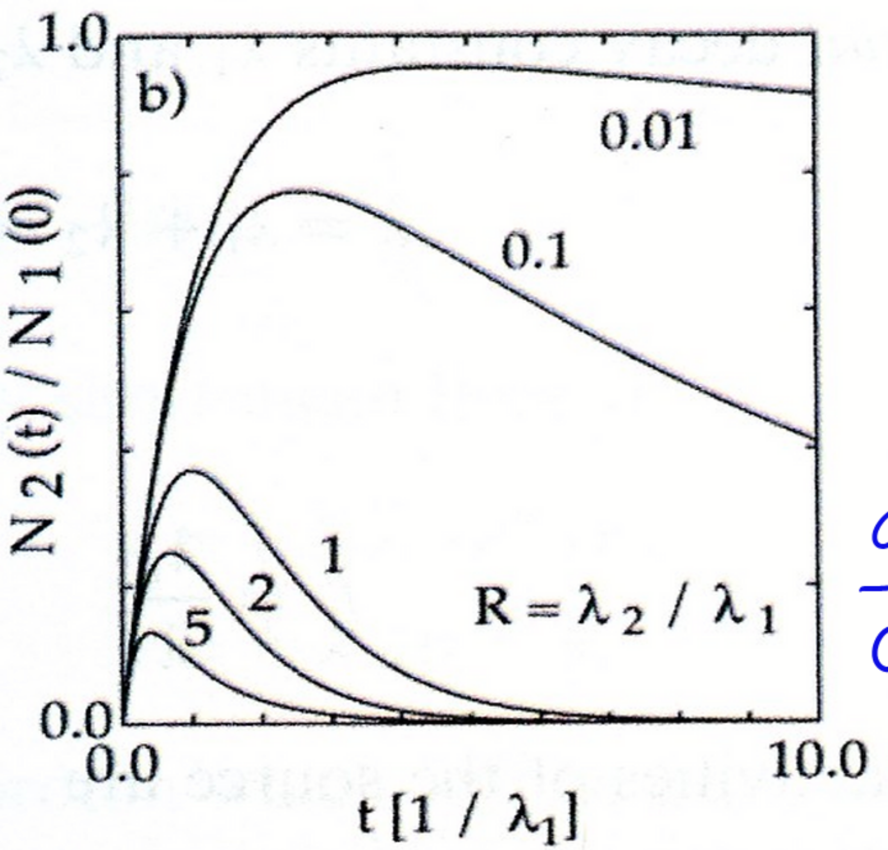
BUILD UP OF A SINGLE DECAY PRODUCT



$${}^{79}_{38}\mathrm{Sr} \rightarrow {}^{79}_{37}\mathrm{Rb} + e^+ + \nu_e \qquad (2.25\ \mathrm{min})$$
$$\hookrightarrow {}^{79}_{36}\mathrm{Kr} + e^+ + \nu_e \qquad (22.9\ \mathrm{min})$$
$$\hookrightarrow {}^{79}_{35}\mathrm{Br} + e^+ + \nu_e \qquad (35.04\ \mathrm{hr})$$

TIME VARIATION OF COMPONENTS OF A DECAY CHAINS

BALANCE BETWEEN PRODUCTION & DECAY



$$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(0) \left[e^{-\lambda_1 t} - e^{-\lambda_2 t} \right]$$

N_2 IS MAXIMUM WHEN $\frac{dN_2}{dt} = 0$

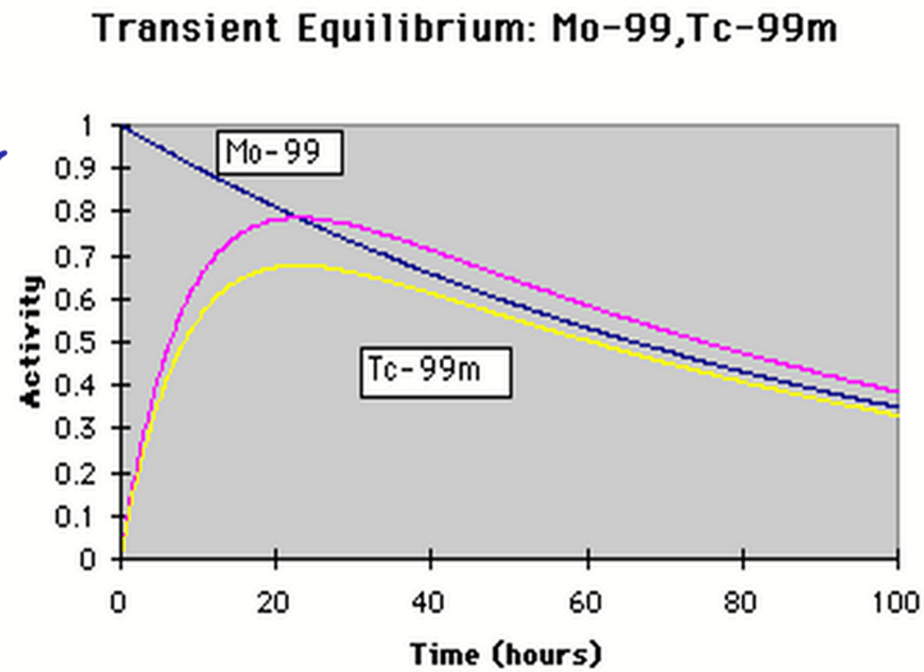
$$\frac{dN_2}{dt} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(0) \left[-\lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t} \right]$$

$$\frac{\lambda_2 \lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t_M} = \frac{\lambda_1^2 e^{-\lambda_1 t_M}}{\lambda_2 - \lambda_1} , \quad \lambda_1^2 = \lambda_2 \lambda_1 e^{(\lambda_1 - \lambda_2) t_M}$$

$$\frac{\lambda_1}{\lambda_2} = e^{(\lambda_1 - \lambda_2) t_M} \implies t_M = \frac{1}{\lambda_1 - \lambda_2} \ln \frac{\lambda_1}{\lambda_2}$$

TRANSIENT EQUILIBRIUM

$$N_d = \frac{\lambda_p}{\lambda_d - \lambda_p} N_p(0) \left\{ e^{-\lambda_p t} - e^{-\lambda_d t} \right\}$$



$\tau_p > \tau_d \rightarrow \lambda_p < \lambda_d$

$t \rightarrow$ SEVERAL $\times\, \tau_d$

$$N_d = \frac{\lambda_p}{\lambda_d} \underbrace{N_p(0)\, e^{-\lambda_p t}}$$

$$N_d \lambda_d = \lambda_p N_p$$

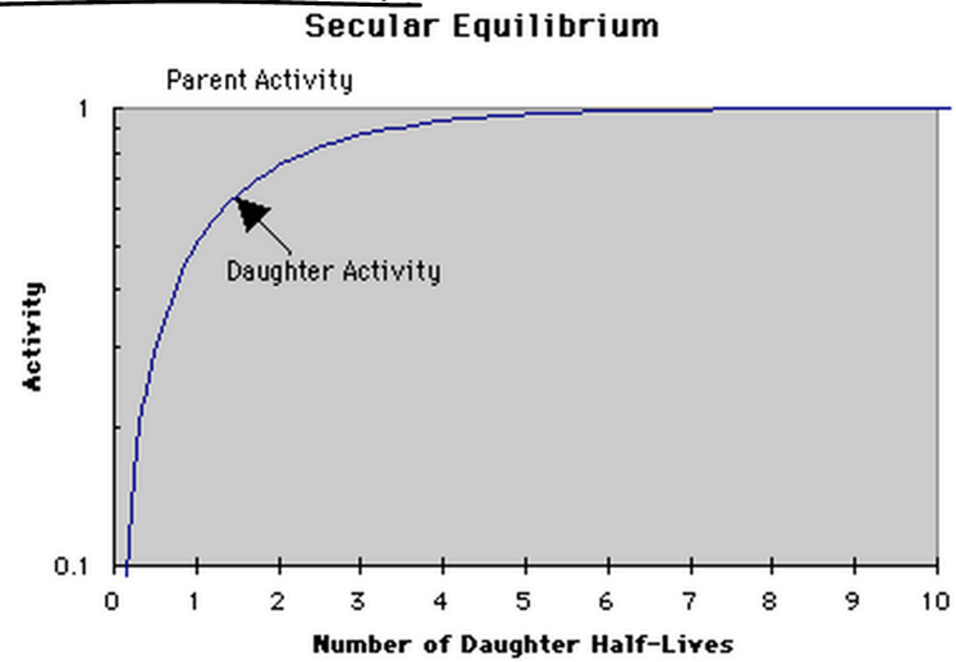
$$A_d = A_p$$

BOTH DECAY $e^{-\lambda_p}$

$A = N\lambda =$ ACTIVITY

SECULAR EQUILIBRIUM

$$N_d = \frac{\lambda_p}{\lambda_d - \lambda_p} N_p(0) \left\{ e^{-\lambda_p t} - e^{-\lambda_d t} \right\}$$



$$\tau_p >>> \tau_d$$

$$\lambda_p <<< \lambda_d$$

$$N_d = \frac{\lambda_p}{\lambda_d} N_p(0) \left\{ 1 - e^{-\lambda_d t} \right\}$$

$$N_d \lambda_d = \lambda_p N_p(0) \left\{ 1 - e^{-\lambda_d t} \right\}$$

$$A_d = A_p \left\{ 1 - e^{-\lambda_d t} \right\}$$

$$A_d = A_p = \text{CONSTANT}$$

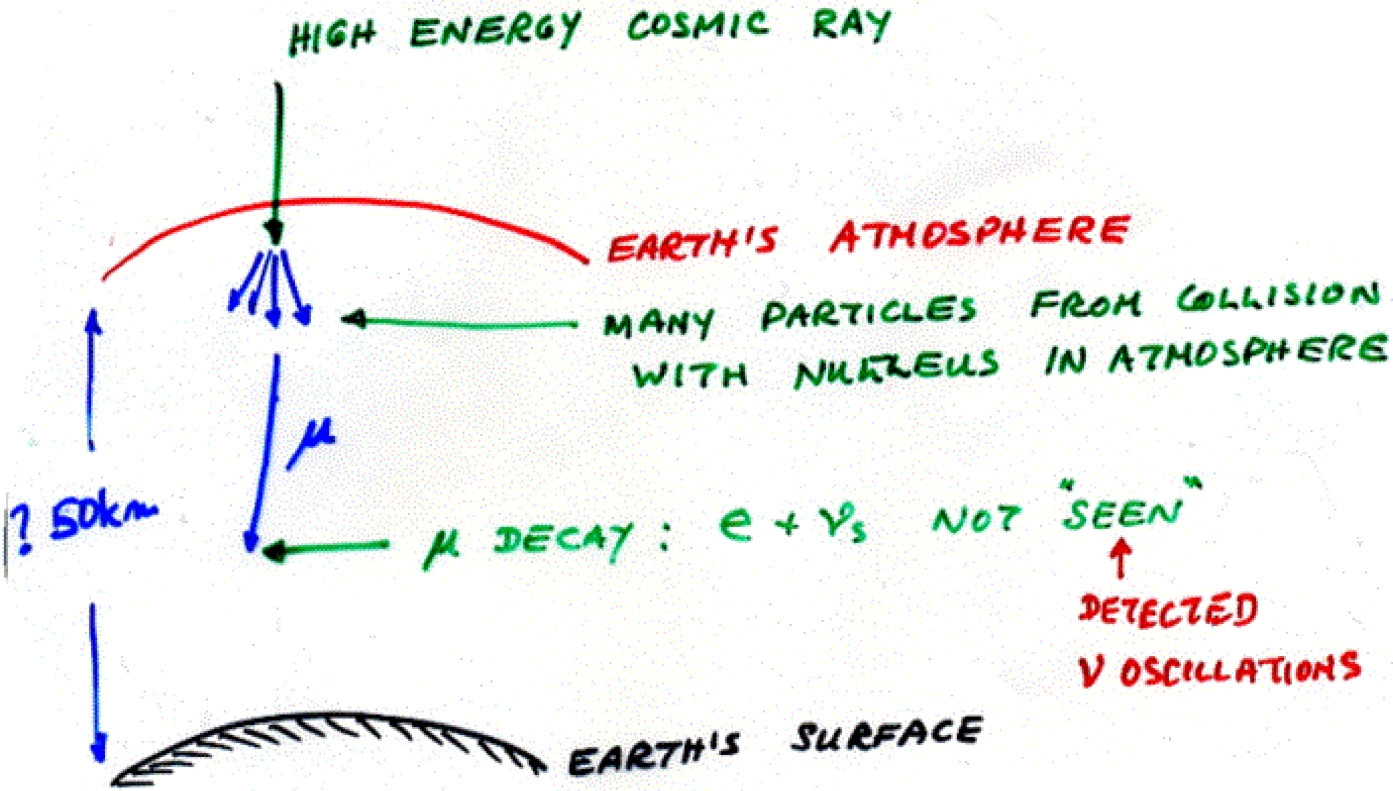
$$t > \tau_d$$

$$t << \tau_p$$

SPECIAL RELATIVITY & LIFETIME

LIFETIME ONLY MAKES SENSE IN SPECIAL FRAME

- REST FRAME OF PARTICLE



eg $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$

$\tau = 2{\cdot}2 \times 10^{-6}\,s$

- ASSUME μ TRAVELLING AT SPEED OF LIGHT

MEAN DISTANCE TO DECAY: $c\tau$

$$= 3\times10^8 \times 2 \times 10^{-6}\ \frac{[m]\,[s]}{[s]}$$

$$\approx 600\ m.$$

- TIME TO REACH EARTH'S SURFACE FROM 50km

$$t_{50} = \frac{50 \times 10^3}{3 \times 10^8} = \frac{50}{3} \times 10^{-5} \approx 2 \times 10^{-4}\,s$$

- HOW MANY UNDECAYED AFTER THIS TIME?

$$\text{FRACTION SURVIVING} = \frac{N(t_{50})}{N(0)} = e^{-t_{50}/\tau_{LAB}}$$

$$= \exp\left\{\frac{2 \times 10^{-4}}{2 \times 10^{-6}}\right\} \sim 10^{-44} \sim 0$$

- WHY DO WE SEE ANY MUONS AT EARTH'S SURFACE?

$\tau_{LAB} \neq \tau_\mu \Rightarrow$ $\tau^{REST} = t_2^{REST} - t_1^{REST}$ DEFINES τ IN REST FRAME

SO! $$\tau_{LAB} = \gamma t_2^{REST} - \gamma t_1^{REST}$$

$$\tau_{LAB} = \gamma \tau_\mu^{REST}$$

THAT'S MORE SENSIBLE!

ASSUME μ HAVE ENERGY OF 100 GeV

$$E = pc/\beta \quad \text{AND} \quad \gamma^2 = \frac{1}{1-\beta^2}$$

$$\gamma^2 = \left(1 - \frac{p^2c^2}{E^2}\right)^{-1} = E^2/(E^2 - p^2c^2)$$

$$\therefore \gamma = E/mc^2$$

MASS OF μ IS $m_\mu = 106$ MeV/c^2

$$\gamma = \frac{10^5\,[\text{MeV}]}{10^2\,\frac{[\text{MeV}]}{[c^2]}\cdot c^2} \approx 10^3 \rightarrow \tau_{LAB} = 10^3 \times 2.2 \times 10^{-6}$$
$$= 2.2 \times 10^{-3}$$

- FRACTION LEFT AT SEA LEVEL

$$= \exp - \left\{\frac{t_{50}}{\tau_{LAB}}\right\} = \exp - \left\{\frac{2 \times 10^{-4}}{2 \times 10^{-3}}\right\} = \exp - \left\{10^{-1}\right\} = .905$$

- FRACTION LEFT AT SEA LEVEL $\sim 90\%$