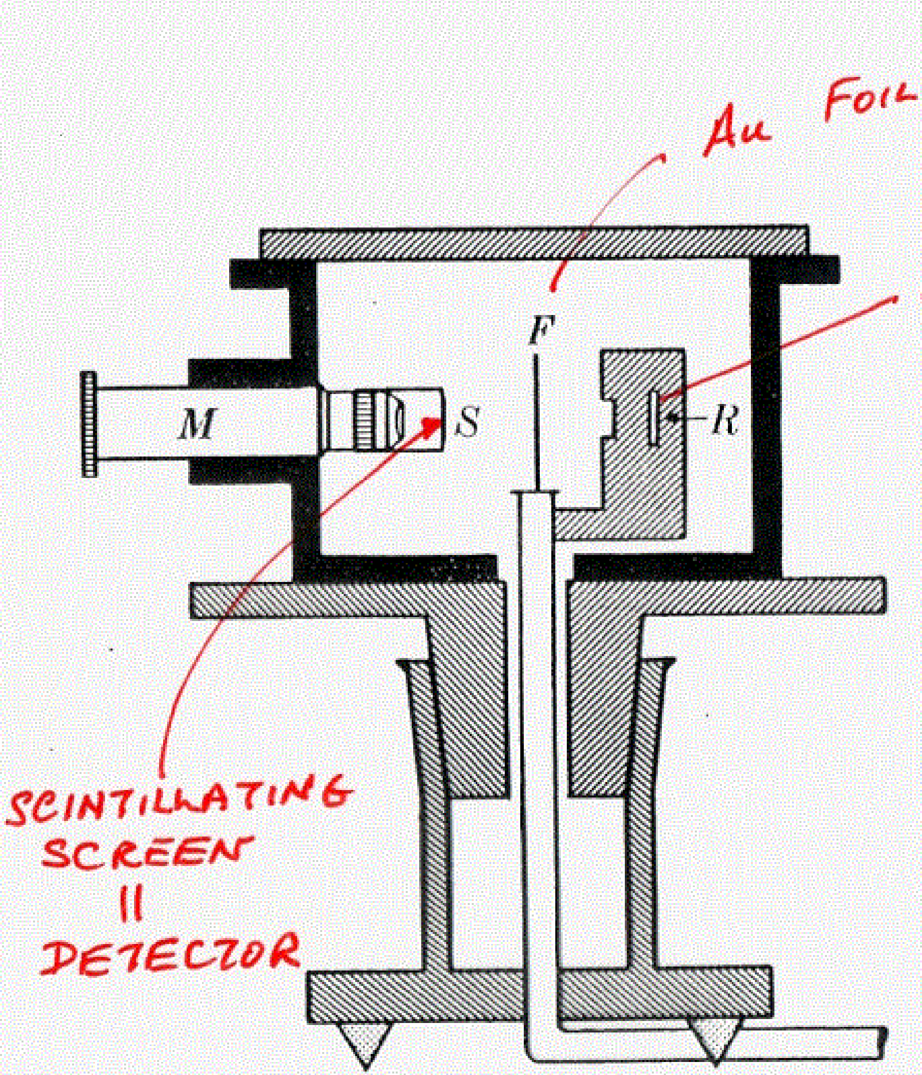
SCATTERING EXPERIMENTS & STRUCTURE OF MATTER

- METHODOLOGY FOR INVESTIGATING SMALLER & SMALLER DISTANCE SCALES ⟶ RUTHERFORD SCATTERING (GEIGER & MARSDEN)
- YOU MUST ALREADY HAVE SEEN THIS
- LETS LOOK AT IT IN SIMPLEST TERMS

  – CLASSICAL MECHANICS TAKES YOU A LONG WAY

  - ARCHETYPAL EXPERIMENT
  - PROBES STRUCTURE OF MATTER
  - REVEALS NATURE OF FORCE ACTING BETWEEN BEAM & TARGET



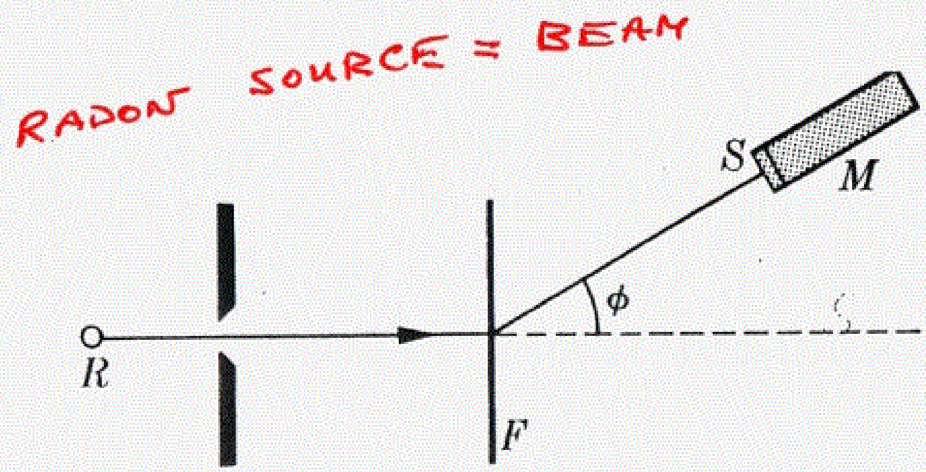
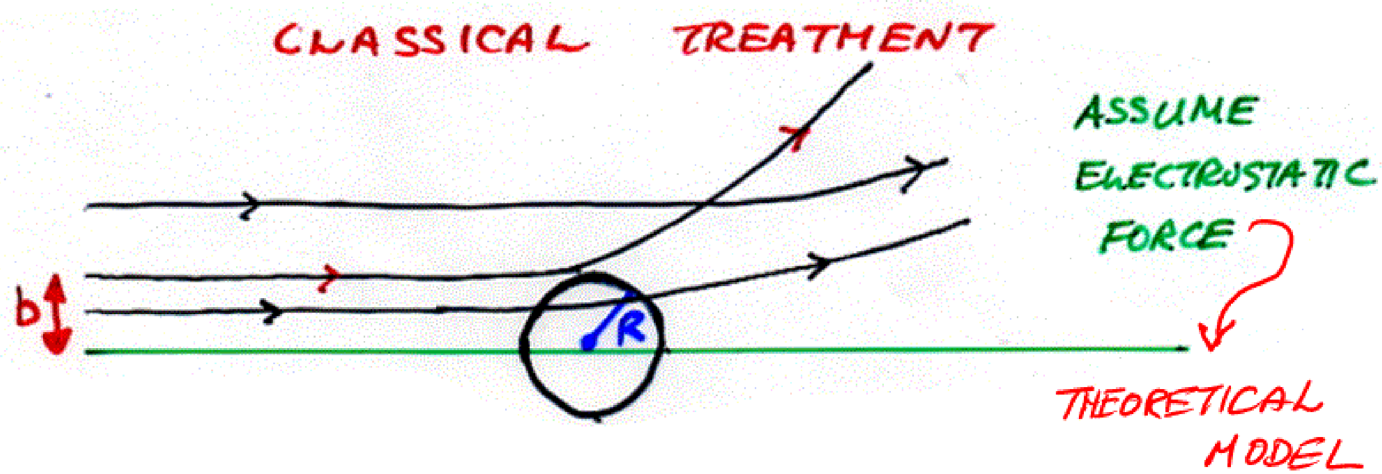


Fig. 3–2. Apparatus of Geiger and Marsden for testing the angular dependence of α-particle scattering. [*Phil. Mag.* **25**, 604 (1913).]

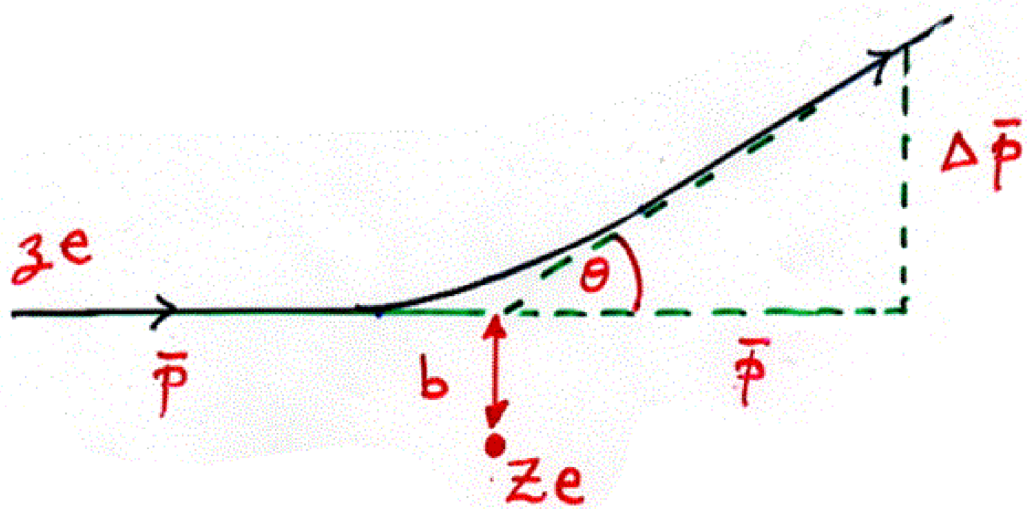
RUTHERFORD SCATTERING – SIMPLE PICTURE



- GEIGER & MARSDEN FIRED 7.7 MeV α AT THIN GOLD FOIL
- WANTED TO UNDERSTAND DISTRIBUTION OF +VE CHARGE WITHIN THE ATOM.
- GREATEST DEFLECTION – PROJECTILE GRAZES PERIPHERY OF TARGET
  - $b > R$ ELECTROSTATIC REPULSION FALLS OFF
  - $b < R$ PASSES THRU CHARGE SOME CHARGE REPELS α ONE WAY, SOME OTHER
- WHAT IS b CORRESPONDING TO MAXIMUM DEFLECTION ?

i.e. WHAT IS SIZE OF NUCLEUS ?

IDEALIZED MODEL FOR SIMPLE CALCULATION



NON RELATIVISTIC

$$M_\alpha \gg p_\alpha$$

$$\Downarrow$$

$$E^2 = p^2 + m^2$$

- ASSUME THAT θ IS SMALL
  - ALONG LINE OF FLIGHT α SLOWED BY COULOMB REPULSION ON APPROACH, AFTER SCATTER ACCELERATED BY SAME AMOUNT $\Rightarrow$ $M_{\text{NUCLEUS}} \gg M_\alpha$
- APROXIMATE — ALL DEFLECTION AN IMPULSE TRANSVERSE TO ORIGINAL DIRECTION, AS α PASSES NUCLEUS

$$\text{POTENTIAL} = \frac{Ze}{b}$$

$\rightarrow$ ASSUMES ONLY COULOMB FORCE ACTING

$\hookrightarrow$ "MODEL"

FORCE ACTING ON α, b AWAY FROM NUCLEUS

$$F = zZe^2 / b^2$$

• APROXIMATION → FORCE ACTS FOR DISTANCE b ON EITHER SIDE OF NUCLEUS, ALWAY NORMAL TO ORIGINAL DIRECTION OF α.

TIME FORCE ACTS FOR → $\Delta t = 2b/v$ ← VELOCITY

$$\Delta p / \Delta t = F \rightarrow \Delta p = F \cdot \Delta t$$

DEFINITION OF FORCE

$$\Delta p = \frac{Zze^2}{b^2} \cdot \frac{2b}{v}$$

APPROXIMATELY DEFLECTION (SCATTERING) ANGLE:

$$\sin\theta \approx \theta \approx \frac{\Delta p}{p}$$

$$\theta \approx \frac{Zze^2}{b^2} \cdot \frac{2b}{v} \cdot \frac{1}{mv}$$

NON RELATIVISTIC

$\theta = 0.1$

$\frac{2}{\theta} = 20$

$\cot\frac{\theta}{2} = 20$

YES $\frac{2}{\theta} \approx \cot\frac{\theta}{2}$

$$\theta \approx \frac{Zze^2}{b} \cdot \frac{2}{mv^2}$$

"CORRECT" TREATMENT

$$b \approx \frac{zZe^2}{mv^2\theta} \cdot 2$$

$$\frac{zZe^2}{2E} \cdot \cot\frac{\theta}{2}$$

- GEIGER & MARSDEN OBSERVED MAXIMUM DEFLECTIONS OF ~ 1 RADIAN (NOT SMALL)

- PUT $b = R$ — RADIUS OF NUCLEUS

$\theta = 1$ RADIAN

$$R = 2 \cdot zZe^2/mv^2$$

$$= 2 \cdot zZe^2/2E$$

$$R = \frac{zZe^2}{E}$$

$\theta = 1$ RAD

$\frac{2}{\theta} = 2$

$\cot\frac{\theta}{2} = 1.83$

NON RELATIVISTIC

$E = \frac{1}{2}mv^2$

$$R = zZe^2/E$$

FOR α ON GOLD FOIL $z=2$, $Z=79$

AND $E = 7.7$ MeV NON RELATIVISTIC

$$R = \frac{2 \times 79 \times e^2}{7.7\ \text{MeV}}$$

- FORTUNATELY WE DO NOT HAVE TO FOOL AROUND WITH e^2 IN COULOMBS, OR WHAT EVER

"FINE STRUCTURE CONSTANT" $\Longrightarrow$ $\frac{e^2}{\hbar c} = \frac{1}{137}$

$$e^2 = \frac{\hbar c}{137} \longleftarrow \hbar c = 197.3\ \text{MeV}\cdot\text{fm}$$

So $$e^2 = \frac{197.3}{137} \text{ MeV}\cdot\text{fm} = 1.44 \text{ MeV}\cdot\text{fm}$$

$$R = \frac{2\times 79}{7.7} \times 1.44 \frac{[\text{MeV}]}{[\text{MeV}]}\cdot[\text{m}] \times 10^{-15}$$

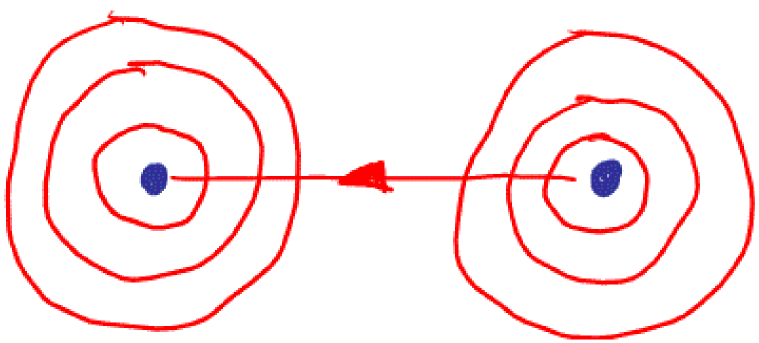
$$\boxed{R \sim 3\times 10^{-14}\,\text{m}}$$

LARGE ANGLE SCATTER $\Longrightarrow$ NUCLEAR RADIUS 10^4 TIMES SMALLER THAN BOHR RADIUS OF ATOM

MACROSCOPIC PHENOMENA REVEAL THE MICROCOSM

QUANTUM FORCES & FEYMAN DIAGRAMS

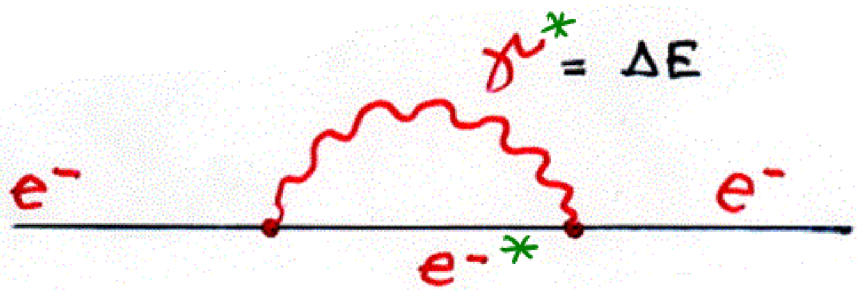
- IN SIMPLE MODEL OF RUTHERFORD SCATTERING → USED CONCEPT OF FORCE.
- WHAT "IS" A FORCE → ANYTHING WHICH CHANGES MOMENTUM OF A PARTICLE
- FORCE = MASS X ACCELERATION $= m \cdot \frac{\Delta v}{\Delta t} = \frac{\Delta p}{\Delta t}$
- PHYSICAL MEASUREMENT ⇒ MOMENTUM CHANGE
- CAN HAVE DIFFERENT ABSTRACT IDEAS OF WHAT "CAUSES" MOMENTUM CHANGE
- CLASSICALLY → FIELDS CAUSED BY POTENTIALS



← ACTION AT A DISTANCE

NON QUANTUM CONCEPT.

- IN RELATIVISTIC QUANTUM MECHANICS
  - FORCES CAUSED BY EXCHANE OF VIRTUAL PARTICLES (GAUGE BOSONS)
- HOW CAN THIS EXCHANGE BE A FORCE? — ANSWER ON 2 LEVELS
- SIMPLE — EXCHANGE CAUSES MOMENTUM CHANGE — BY DEFINITION A FORCE.



$*$ → VIRTUAL PARTICLE

$$E^2 = p^2 + m^{*2}$$

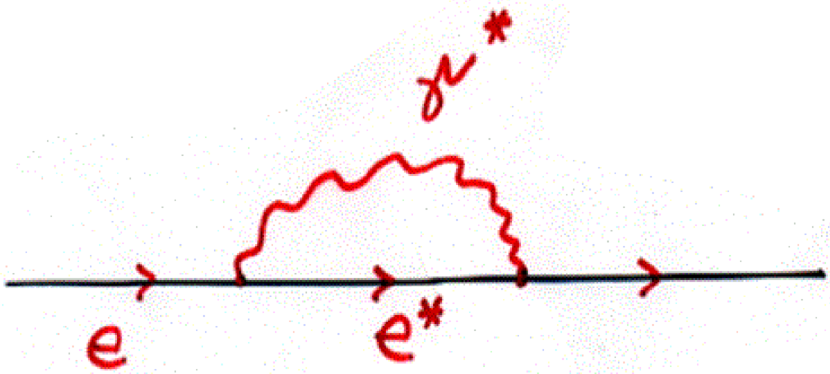
NOT PHYSICAL MASS

- ACCORDING TO HEISENBERG UNCERTAINTY
  - FREELY PROPAGATING ELECTRON CAN EMIT AND RE-ABSORB A PHOTON
- REABSORBTION HAS TO OCCUR WITHIN TIME

TIME VIRTUAL PARTICLE CAN EXIST

$$\Delta t \lesssim \hbar / \Delta E$$

HEISENBERG



BETTER WAY TO THINK OF IT

γ CARRIES 4-MOMENTUM AWAY FROM ELECTRON

- IF $E_e^2 = p_e^2 c^2 + m_e^2 c^4 \rightarrow m_e^2 c^4 = E_e^2 - p_e^2 c^2$ BEFORE THE EMISSION OF THE γ, THEN AFTER EMISSION $m_e^2 c^4 \neq E_e'^2 - p_e'^2 c^2 = m_e^{*2} c^4$

- REAL γ $\quad E_\gamma^2 - p_\gamma^2 c^2 = 0 \longleftarrow$ MASSLESS
- VIRTUAL γ $\quad E_\gamma'^2 - p_\gamma'^2 = m_\gamma^{*2} c^4 \longleftarrow$ γ & e WILL HAVE NON PHYSICAL MASSES FOR $\Delta E \rightarrow$ "OFF MASS SHELL"

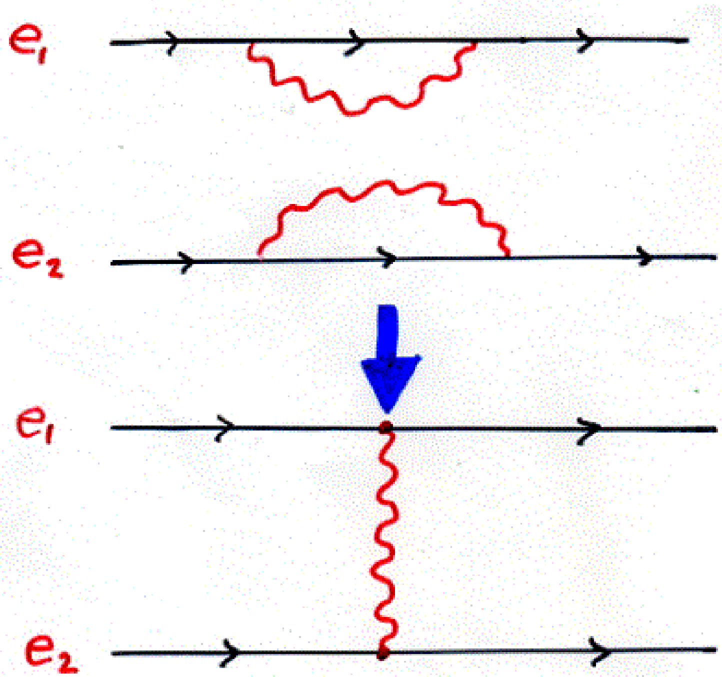
VIRTUAL PARTICLES

$\Downarrow$

TRANSMIT FORCES

} TRANSFER MOMENTUM

INTERACTION → FORCE BETWEEN PARTICLES



VIRTUAL PARTICLE EMISSION

4 MOMENTUM TRANSFER BETWEEN e_1 & e_2

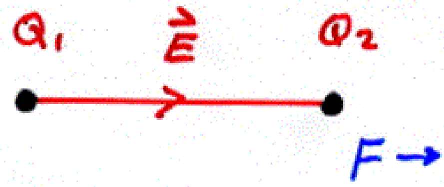
- 4 MOMENTUM TRANSFER — BOTH ENERGY & MOMENTUM
- TWO ELECTRONS IN INITIAL & FINAL STATE ENSURE THAT ENERGY & MOMENTUM CONSERVED

PARTICLES IN INITIAL & FINAL STATE ARE "ON MASS SHELL"

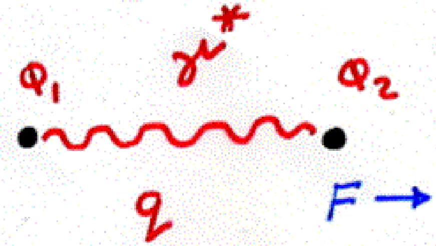
$$E^2 = p^2 + m^2$$

$$E^2 = p^2c^2 + m^2c^4$$

CLASSICAL ⟷ QUANTUM ANALOGY



CLASSICAL FIELD



EXCHANGE OF VIRTUAL QUANTUM.

- $\Delta E \Delta t \lesssim \hbar$ PROPAGATION TIME OF VIRTUAL PARTICLE

$$t \sim \Delta t \lesssim \frac{\hbar}{mc^2}$$

$\sim \Delta E$

MASS OF VIRTUAL PARTICLE

- ASSUME VIRTUAL PARTICLE PROPAGATES AT VELOCITY OF LIGHT

RANGE OF FORCE $R = c \cdot \Delta t \lesssim \dfrac{\hbar}{mc^2}$

FOR $m \rightarrow 0$; $R \rightarrow \infty$ → COULOMB $m_\gamma = 0$

FOR M LARGE ; R SMALL

MASS OF EXCHANGED VIRTUAL PARTICLE DETERMINES RANGE OF FORCE

QUANTUM FIELD

- VIRTUAL PARTICLE
- ALL INTERACTIONS OCCUR AT POINTS IN SPACE-TIME
- UNFAMILIAR
  → BUT MAKES SENSE !

CLASSICAL FIELD

- SPOOKY ACTION AT A DISTANCE
- FAMILIAR ?

WHAT IS AN AMPLITUDE?

PROBABILITY $\propto$ $|$AMPLITUDE$|^2$

IN NON-RELATIVISTIC QUANTUM MECHANICS

$$M(\bar{q}) = \int d^3\bar{r}\ V(\bar{r}) \exp\left(i\bar{q}\cdot\bar{r}/\hbar\right)$$

$\bar{q} = \bar{q}_f - \bar{q}_i$ (3-MOMENTUM)

$V(\bar{r})$ – SCATTERING POTENTIAL

BORN APPROXIMATION

$\longrightarrow$ LOWEST ORDER IN PERTURBATION THEORY

$$M(\bar{q}) = \int d^3\bar{r}\, V(\bar{r}) \exp\left(\frac{i\bar{q}\cdot\bar{r}}{\hbar}\right)$$

PUT IN YUKAWA POTENTIAL $V(r) = \frac{-g^2}{4\pi} \frac{e^{-r/R}}{r}$

g — "COUPLING CONSTANT"

R — "RANGE OF FORCE" $\hbar \to 1$

$$M(\bar{q}) = \int d^3\bar{r} \left(\frac{-g^2}{4\pi}\right) e^{-\frac{r}{R}} \cdot e^{-\bar{q}\cdot\bar{r}}$$

FOR A SPHERICALLY SYMMETRICAL POTENTIAL

$$d^3\bar{r} = r^2 dr\, d(\cos\theta)\, d\phi$$

AND $\bar{q}\cdot\bar{r} = q r \cos\theta$

$$M(\vec{q}) = \frac{-g^2}{4\pi} \int_0^{2\pi} d\phi \int_0^{\infty} dr \, \frac{r^2}{r} e^{-r/R} \int_{-1}^{+1} d\cos\theta \, e^{iqr\cos\theta}$$

NOT TOO HARD – USE $\int x e^{ax} = \frac{e^{ax}}{a^2}(ax-1)$

$$= \frac{-g^2}{2iq} \cdot \frac{iqR}{1-(iqR)^2}$$

$R \equiv \frac{\hbar}{mc}$

MASS OF VIRTUAL PARTICLE

$$= -g^2 \frac{1}{\frac{1}{R^2} + q^2}$$

$$= -g^2 \frac{1}{m^2+q^2}$$

$$M(q) = \frac{-g^2}{m^2+q^2}$$

COUPLING CONSTANT

MASS OF EXCHANGE

3-MOMENTUM TRANSFER

RELATIVISTIC TREATMENT

$$M(q^2) = \frac{g^2}{q^2-m^2}$$

COUPLING CONSTANT

4 MOMENTUM TRANSFER

MASS OF EXCHANGE PARTICLE

FEYMAN DIAGRAMS

- FEYNMAN DIAGRAM → GRAPHICAL REPRESENTATIONS OF QUANTUM MECHANICAL AMPLITUDES

- PROBABILITY OF A PROCESS $\propto$ $\left(\text{AMPLITUDE} \right)^2$

- IF A PROCESS CAN OCCUR IN 2 DIFFERENT WAYS

$$\text{PROB} \propto \left(A_1 + A_2 \right)\left(A_1 + A_2 \right)$$

A_1, A_2 → COMPLEX NUMBERS

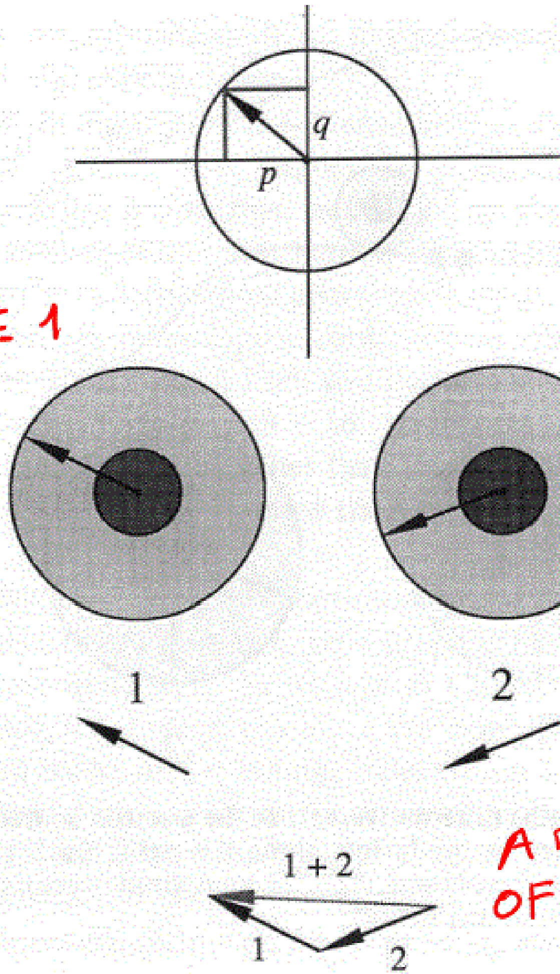
∴ INTERFERENCE cf → 2 SLIT EXPERIMENT WITH ELECTRONS

Q. M. AMPLITUDE ⟺ SPACE – TIME DIAGRAM

PARTICLE MOVING THRU SPACE - TIME NEEDS TO BE LABELED BY PHASE & AMPLITUDE

AMPLITUDE 1

PHASE 1



AMPLITUDE 2

PHASE 2

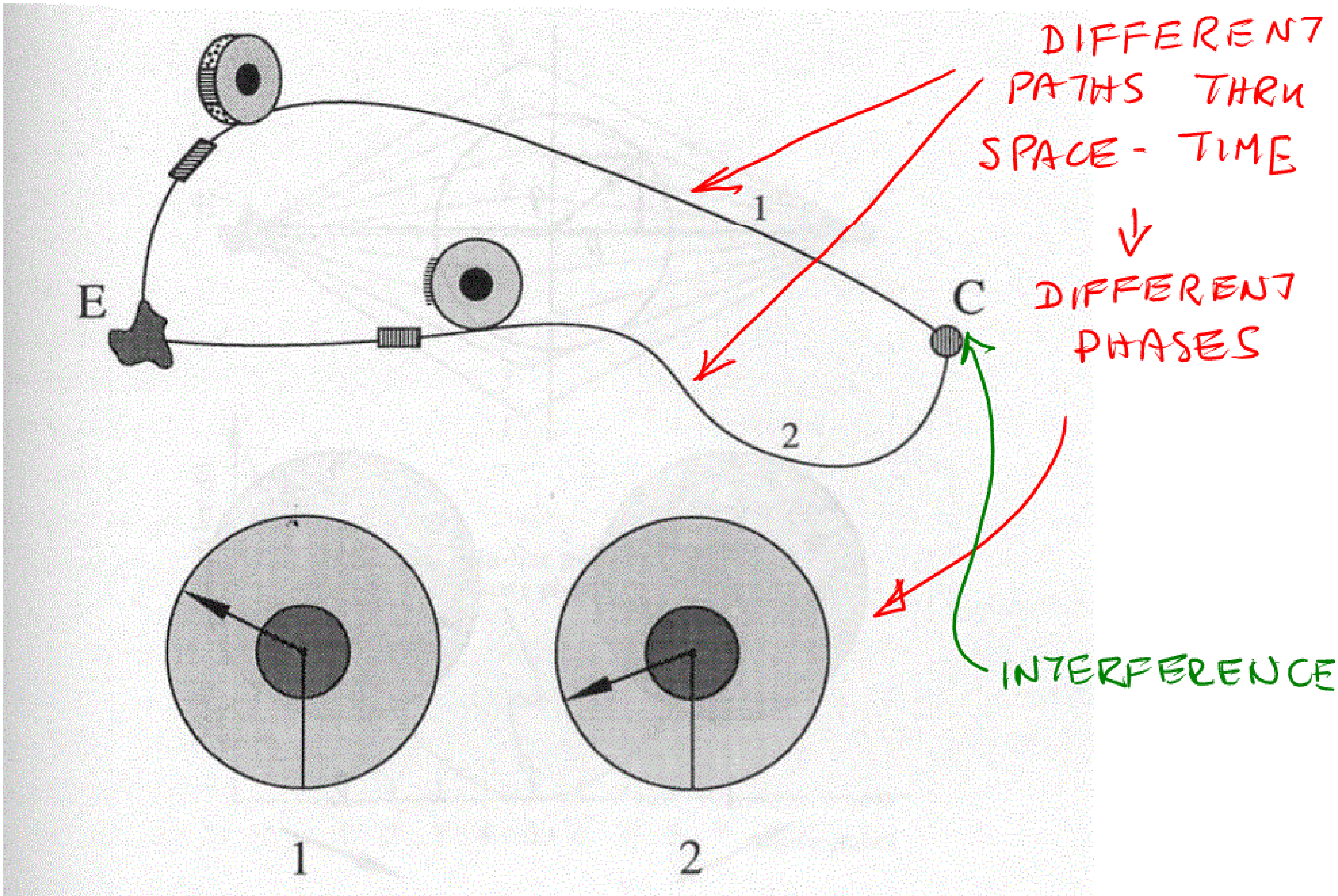
ADDITION OF AMPLITUDES

FREE PARTICLE

$$\psi = A e^{i(Et - px)\hbar}$$

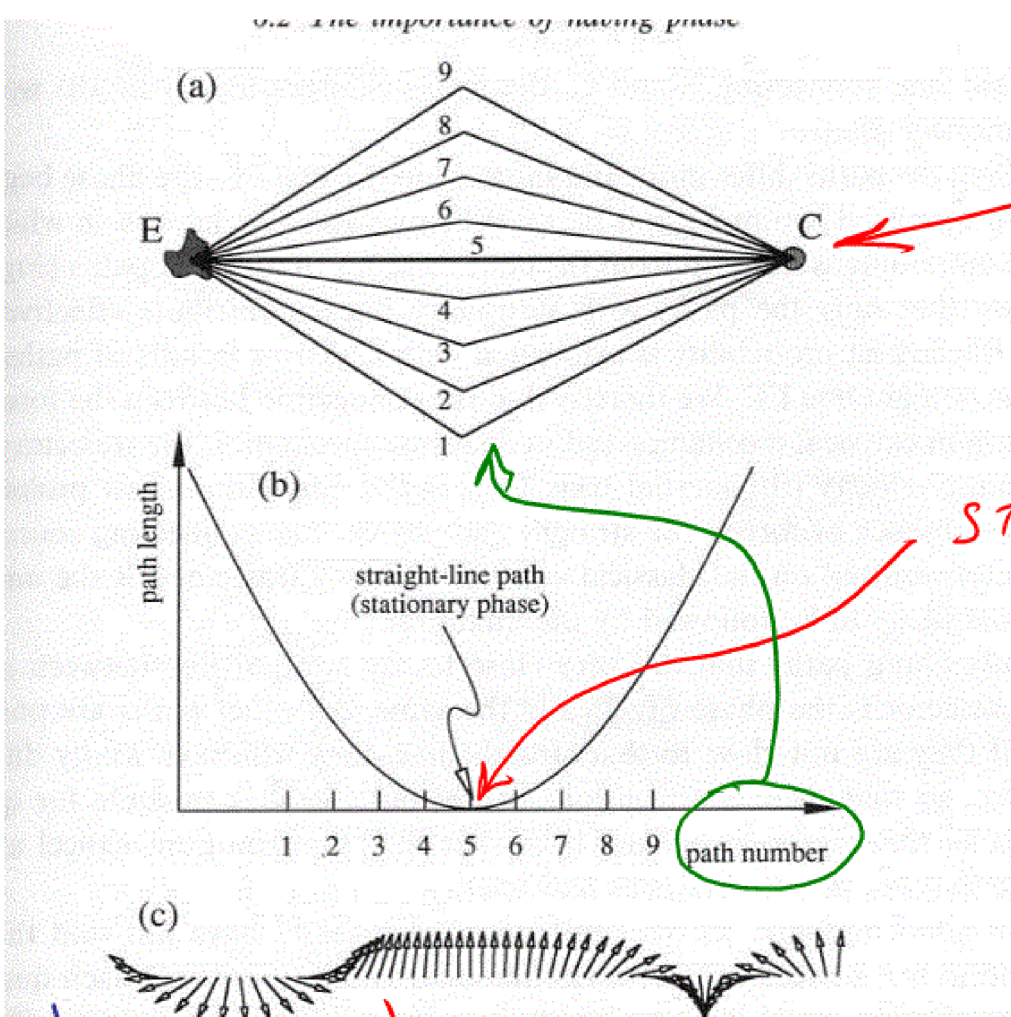
PHASE VARIES ALONG PATH

PLANE WAVE STATE


E
C
1
2
1
2
DIFFERENT
PATHS THRU
SPACE - TIME
DIFFERENT
PHASES
INTERFERENCE

FREE PARTICLE PROPAGATING $E \to C$

POSITIONS IN SPACE

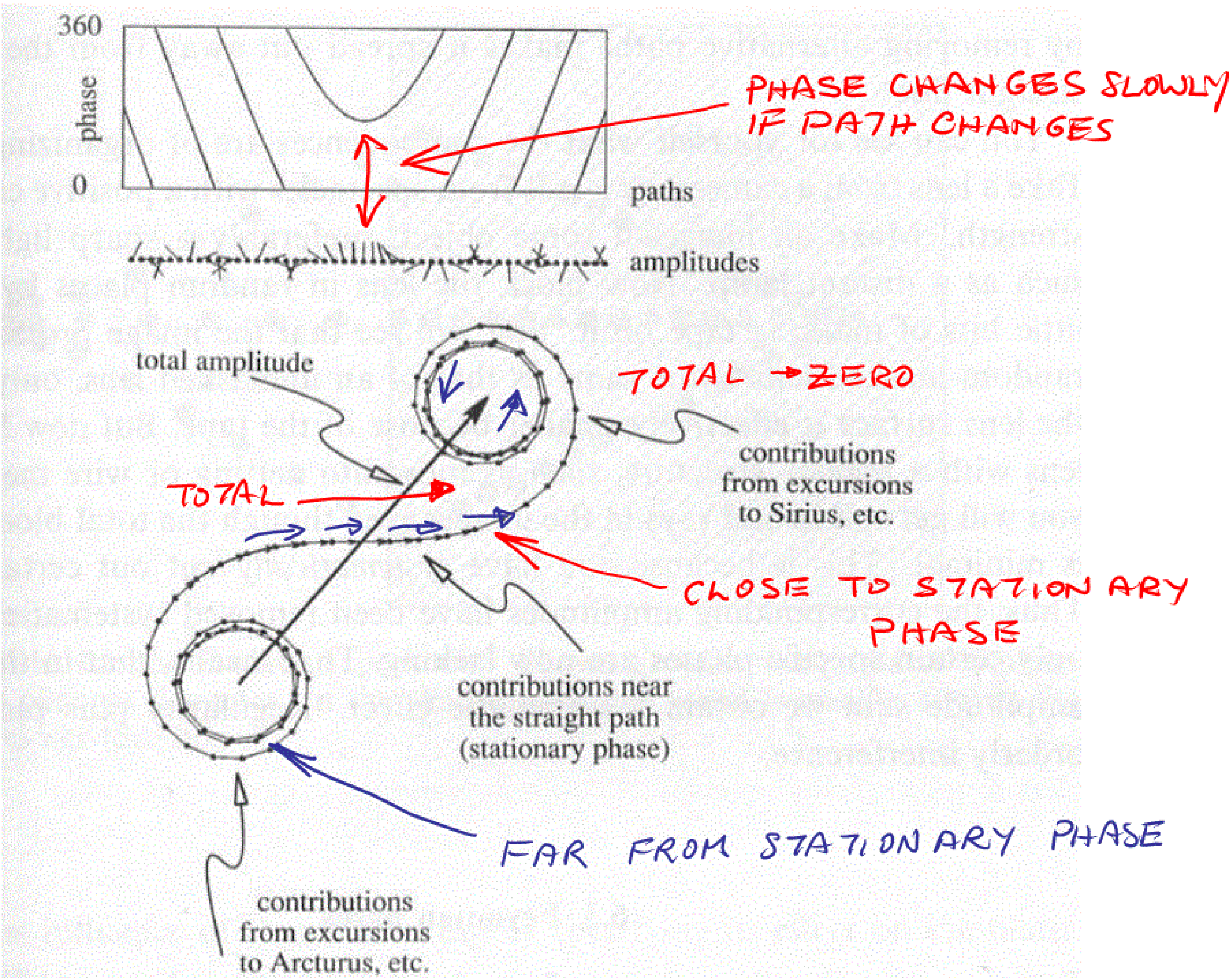


STATIONARY PHASE

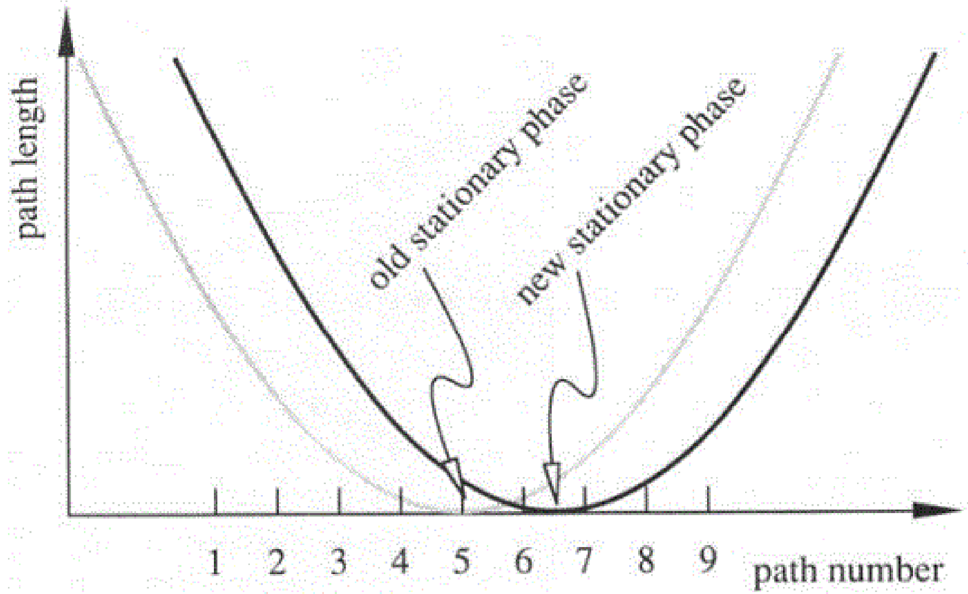
PHASES FAR FROM STRAIGHT LINE INTERFERE DESTRUCTIVELY

PHASES CLOSE TO STRAIGHT LINE INTERFERE CONSTRUCTIVELY

ADD AMPLITUDES FROM ALL POSSIBLE PATHS



PARTICLE FOLLOWS PATH OF LEAST ACTION



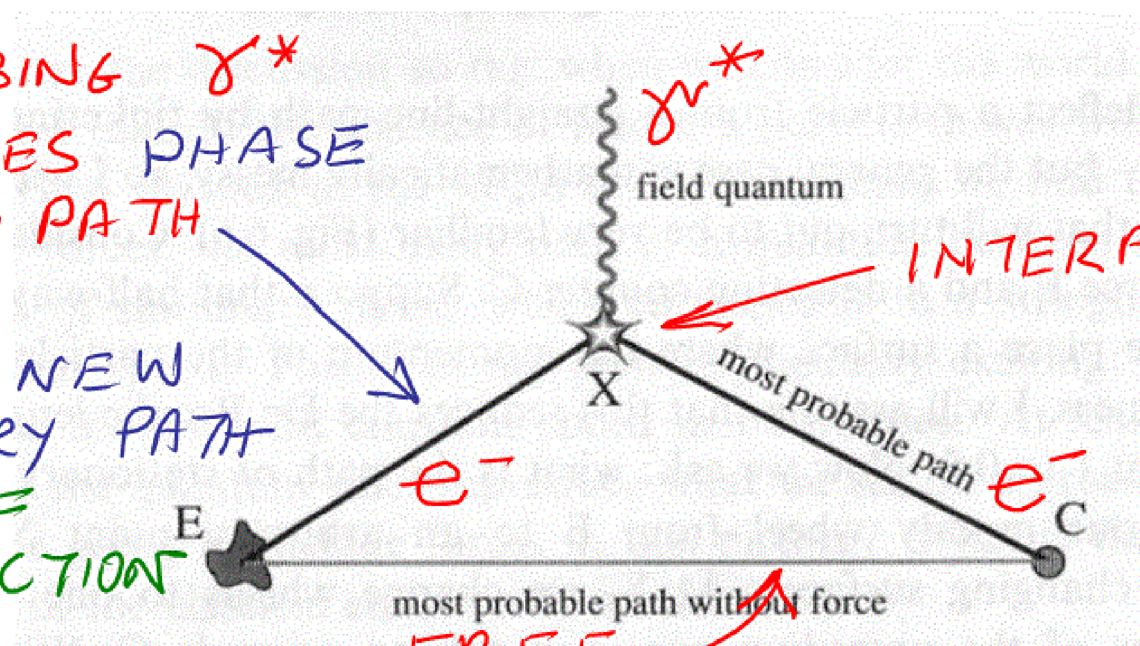
PHASE = $\frac{\text{ACTION}}{\hbar}$

ABSORBING γ^* CHANGES PHASE ALONG PATH

↓

BECOMES NEW STATIONARY PATH

PATH OF LEAST ACTION



γ^*

INTERACTION

e^-

e^-

FREE

• IN QUANTUM MECHANICS

$$\delta(\text{PHASE}) = \frac{\delta(\text{ACTION ALONG PATH})}{\hbar}$$

$\hbar$ IS QUANTUM OF ACTION

$$\delta\phi \propto \delta S$$

$$\delta\phi = F\,\delta S$$

PHASE CHANGE ALONG PATH

FIELD DENSITY

CHANGE IN ACTION ALONG PATH

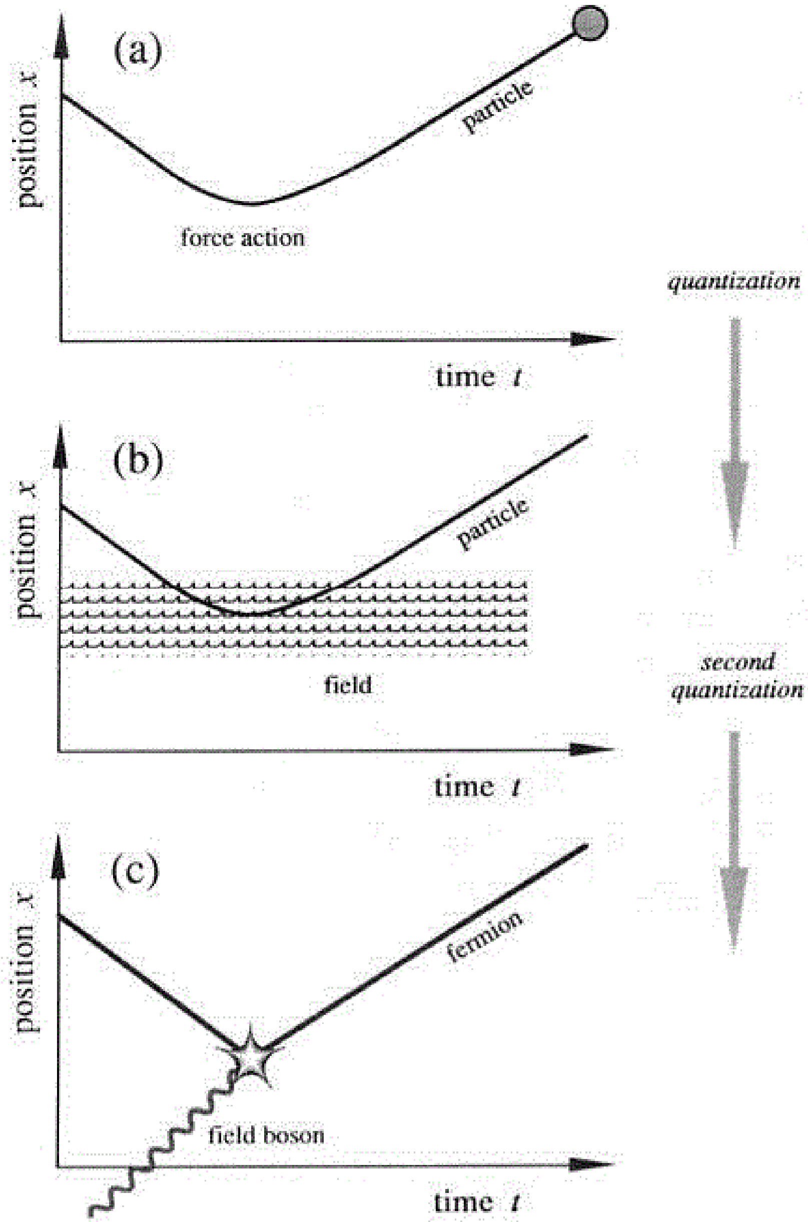
$$P_{E\to C} = \Big[\sum_{\text{PATH}} \delta\phi\Big]^2$$

TOTAL PROBABILITY

SUM OVER ALL POSSIBLE PATHS

SPACE - TIME DIAGRAM

CLASSICAL



NON-RELATIVISTIC QUANTUM MECHANICS

QUANTUM FIELD THEORY

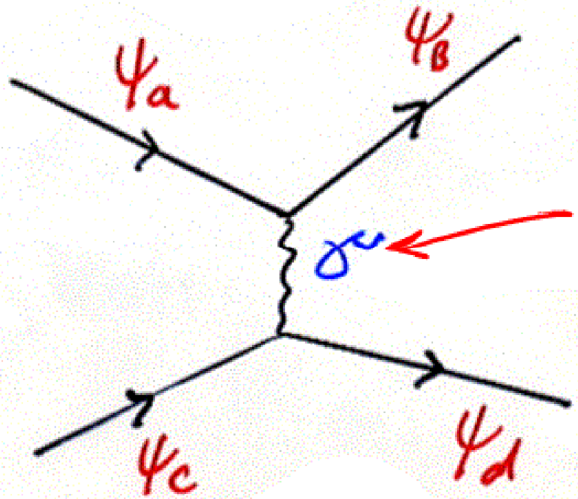
QUANTUM AMPLITUDE

SPACE - TIME DIAGRAM = QUANTUM AMPLITUDE

QUANTUM FIELD THEORY

PHASE CHANGE = MOMENTUM CHANGE = SCATTERING OF PARTICLES

$a\,b \rightarrow c\,d$



CARRIES A PHASE

ELECTRIC CHARGE

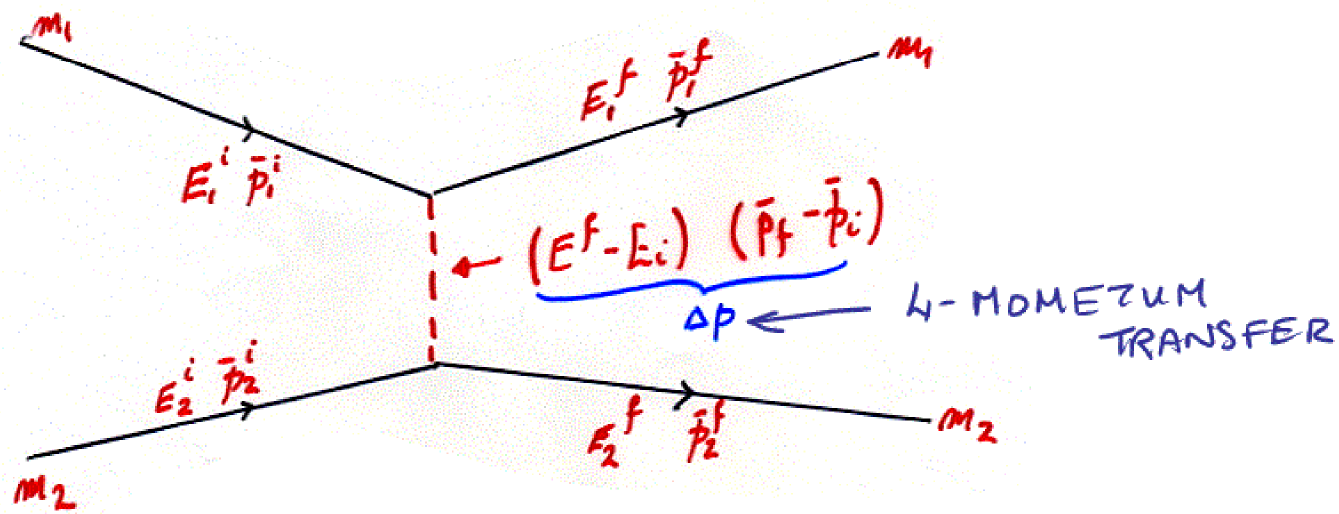
AMPLITUDE $\sim \bar{\psi}_b \bar{\psi}_d \dfrac{e^2}{q^2} \psi_a \psi_c$

ANNIHILATE a, b

CREATE b, d WAVE FUNCTIONS

MOMENTUM TRANSFERED BY γ^μ (PROPAGATOR)

SCATTERING ANGLE & MOMENTUM TRANSFER



USE 4-VECTOR, PUT $c=1$

$$(\Delta p)^2 = (p_1^i - p_1^f)^2 = p_1^{i\,2} + p_1^{f\,2} - 2\, p_1^i \cdot p_1^f$$ ← 4-VECTORS

$$p^2 = (E, \vec p)^2$$
$$= E^2 - \vec p^{\,2}$$
$$= m^2$$

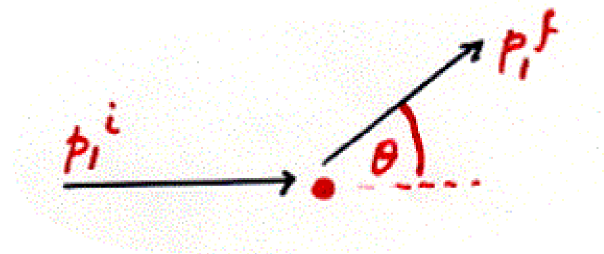
$$(E_1^i, p_1^i)\,(E_1^f, p_1^f)$$
$$= 2E_1^f E_1^i - 2\,\vec p_1^{\,i} \cdot \vec p_1^{\,f}$$
$$= 2E_1^f E_1^i - 2\,|\vec p_1^{\,i}|\,|\vec p_1^{\,f}| \cos\theta$$

SCALAR

3VECTOR SCALAR PRODUCT

$$(\Delta p)^2 = m_1^2 + m_1^2 - 2E_1^i E_1^f + 2|\bar{p}_1^i|\,|\bar{p}_1^f| \cos\theta$$

IN LAB FRAME



IF $E_1 \gg m_1$

$$|\bar{p}_1|^2 = E_1^2 - m_1^2 \approx E_1^2$$

$$(\Delta p)^2 = 2E_1^f E_1^i (\cos\theta - 1)$$

TRUE IN ANY FRAME

4 - MOMENTUM SQUARED

$q^2 \sim \cos\theta$

$\Delta p = q$

4 - MOMENTUM TRANSFER GIVEN BY LABORATORY KINEMATICS

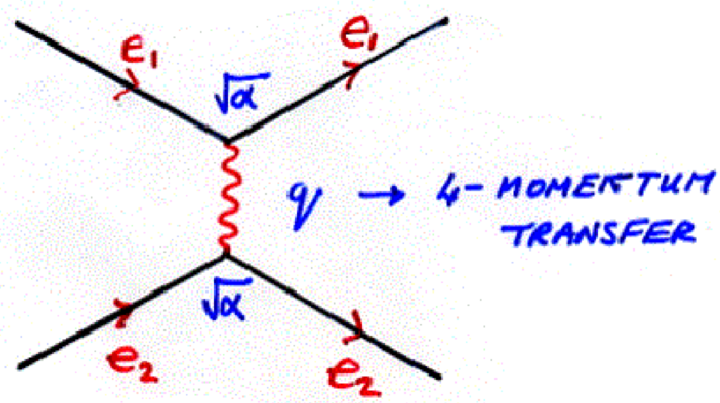
E^i INCOMING ENERGY

E^f OUTGOING ENERGY

θ SCATTERING ANGLE

SOME FEYNMAN DIAGRAMS

REMEMBER → DIAGRAM REPRESENT Q. M. AMPLITUDE



ELECTROMAGNETIC INTERACTION

PROBABILITY OF SCATTERING THROUGH $\theta \rightarrow P(\theta)$

$$P(\theta) = P(q^2) = |A(q^2)|^2$$

↑ QUANTUM MECHANICAL SCATTERING AMPLITUDE

SIMPLE "FEYMAN RULE"

$$A_{ee \to ee} \sim \frac{\sqrt{\alpha}\sqrt{\alpha}}{q^2} \sim \frac{e^2}{q^2}$$

$$\alpha = e^2/\hbar c$$

$$P_{ee \to ee} \sim e^4/q^4$$

PREDICTION →

SIMPLE FEYMAN RULE

$$P \propto |A|^2 \sim e^4/q^4$$

- SCATTERING DEPENDS ON $\sqrt{\alpha} \sim e$
- HARD TO SCATTER THROUGH LARGE ANGLES
  - → HAVE TO TRANSFER A LOT OF 4-MOMENTUM
  - → NEED "VERY VIRTUAL" PHOTON

- IN OUR SIMPLE MODEL OF RUTHERFORD SCATTERING

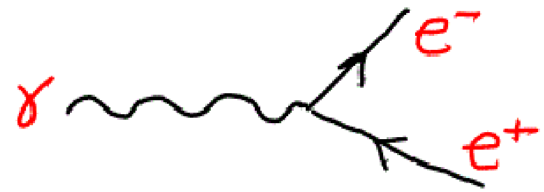
$$\Delta p^2 \sim q^2 \sim p^2 \theta^2$$

SO WE WOULD PREDICT THAT THE PROPABILITY FOR SCATTERING THROUCH θ

$$P(\theta) \sim \frac{e^4}{p^4 \theta^4}$$

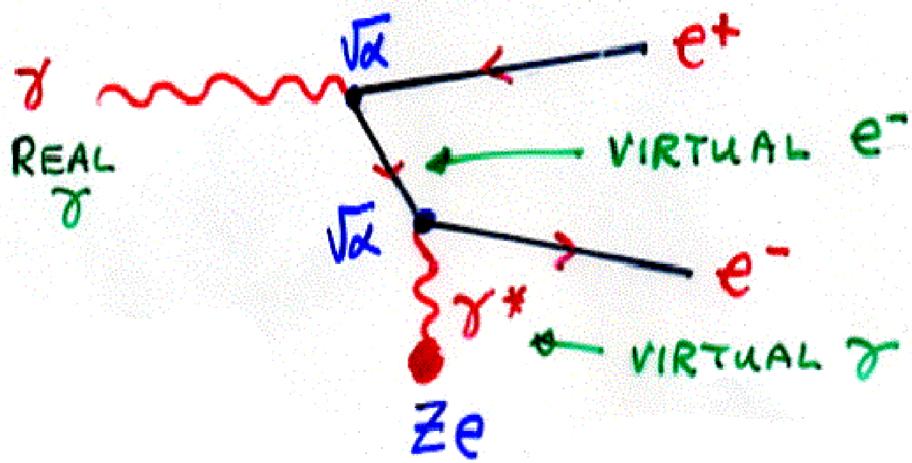
APPLY FEYMAN RULE TO OTHER SIMPLE E.M. PROCESSES

IN FREE SPACE CANNOT HAVE



DOES NOT CONSERVE ENERGY & MOMENTUM

- HOWEVER, A RECOILING NUCLEUS CAN PROVIDE MOMENTUM RECOIL TO



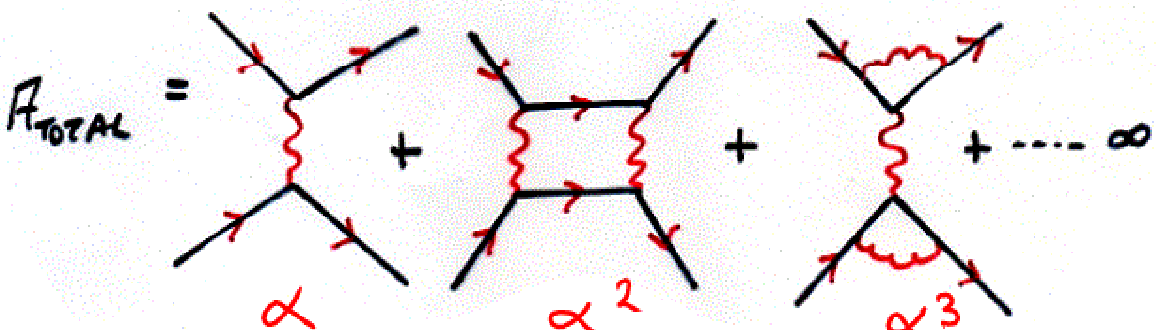
$$\text{PROBABILITY} \sim \left|\sqrt{\alpha}\,\sqrt{\alpha}\cdot Z\sqrt{\alpha}\right|^2 \sim \alpha^3 Z^2$$

$$\sim e^6 Z^2$$

THIS DEPENDENCE ON Z^2 IS SEEN FOR "PAIR PRODUCTION"

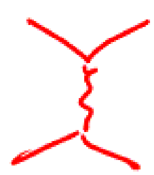
PERTURBATION EXPANSION

SAW → IN Q.M. HAVE TO TAKE ACCOUNT OF
ALL POSSIBLE WAYS A PROCESS CAN OCCURR



α	α^2	α^3
$1/137$	$(1/137)^2$	$(1/137)^3$
7×10^{-3}	5×10^{-5}	4×10^{-7}

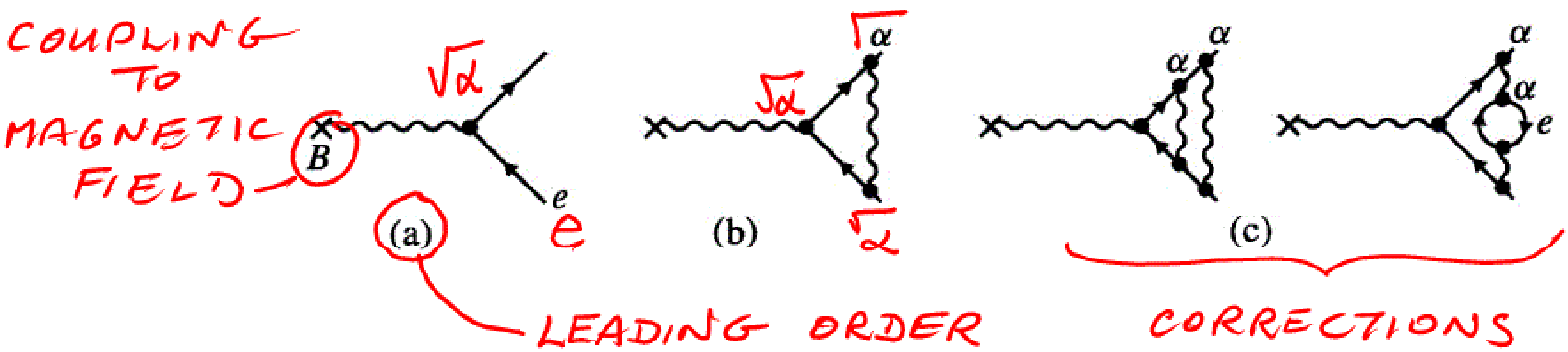
- HAVE TO JOIN INCOMING & OUT GOING LINES BY ALL POSSIBLE VIRTUAL LINES
- IN EM A GOOD APPROXIMATION IS JUST FIRST TERM



BORN APPROXIMATION

- ONLY WORKS BECAUSE $\alpha \ll 1$
- FOR STRONG FORCE $\alpha_s \gg 1$ " CORRECTIONS" DOMINATE $\sum$

ELECTRON / MUON MAGNETIC MOMENT



- HIGHER ORDER CORRECTIONS – SENSITIVE TEST OF THEORY

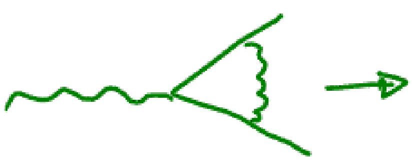
- MAGNETIC MOMENT OF CLASSICAL SPINNING PARTICLE

$$\bar{\mu} = \frac{e\hbar}{2mc} \cdot \bar{\sigma}$$

- FOR A DIRAC FERMION $\bar{\mu} = g \cdot \frac{e\hbar}{2mc} \cdot \bar{\sigma}$ ← SPIN

$g = 2$ ELECTRON

$g = 1$ CLASSICAL

 $\rightarrow$ $\bar{\mu} = g\left(1 + \frac{\alpha}{2\pi}\right)\frac{e\hbar}{2mc}\bar{\sigma}$

$g \Rightarrow 2\left(1 + \frac{\alpha}{2\pi}\right)$; $\frac{g-2}{2}$ "ANOMALOUS" MAGNETIC MOMENT IN ADDITION TO DIRAC

SMALL CLARIFICATION

FOR ELECTRON $\bar{\mu}_e = g \cdot \frac{e}{2m_e c} \cdot \hbar \, \bar{\sigma}$

g - FACTOR
=1 CLASSICALLY
=2 FOR ELECTRON

μ_B

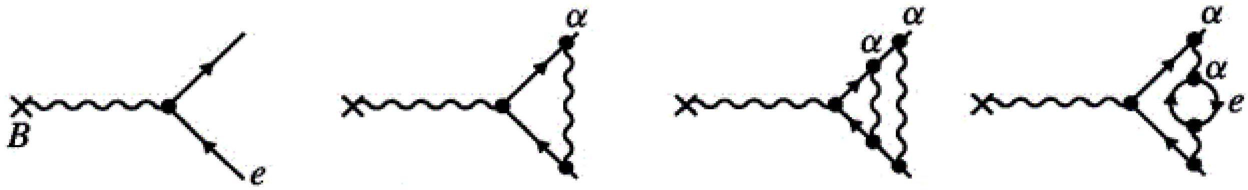
ANGULAR MOMENTUM

$$\bar{\mu}_e = \underbrace{2(1+a_e)}_{g} \mu_B \bar{\sigma}$$

$$g = 2 + 2a_e$$

$$\frac{g-2}{2} = a_e = \frac{\alpha}{2\pi} + \cdots\cdots$$

PRECISION OF M.M. PERTURBATION EXPANSION



(a) (b) (c)

$$\frac{g}{2} = 1 + \alpha/2\pi - 0.328478966\left(\frac{\alpha}{\pi}\right)^2 + 1.1765\left(\frac{\alpha}{\pi}\right)^3 - 0.8\left(\frac{\alpha}{\pi}\right)^4$$

$$= 1.00\,11\,59\,652\,307\,(110)$$ THEORY

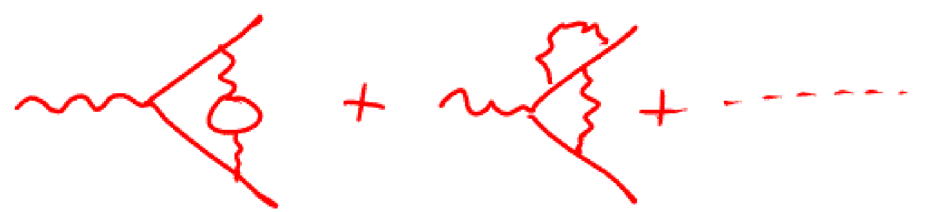
$$1.00\,1159\,652\,193\quad(10)$$ EXPERIMENT

CAN MAKE ARBITRARILY ACCURATE PREDICTION

$$a_e = \frac{g-2}{2} =$$

$$\frac{\alpha}{2\pi}$$

$$-0.328478966\left(\frac{\alpha}{\pi}\right)^2$$



$$+(1.1765)\left(\frac{\alpha}{\pi}\right)^3$$

72 DIAGRAMS

$$-(0.8)\left(\frac{\alpha}{\pi}\right)^4$$

891 DIAGRAMS

$$a_e^{TH} = 11596522044 \times 10^{-13}$$

$$a_e^{EXP} = 11596521884 \times 10^{-13}$$

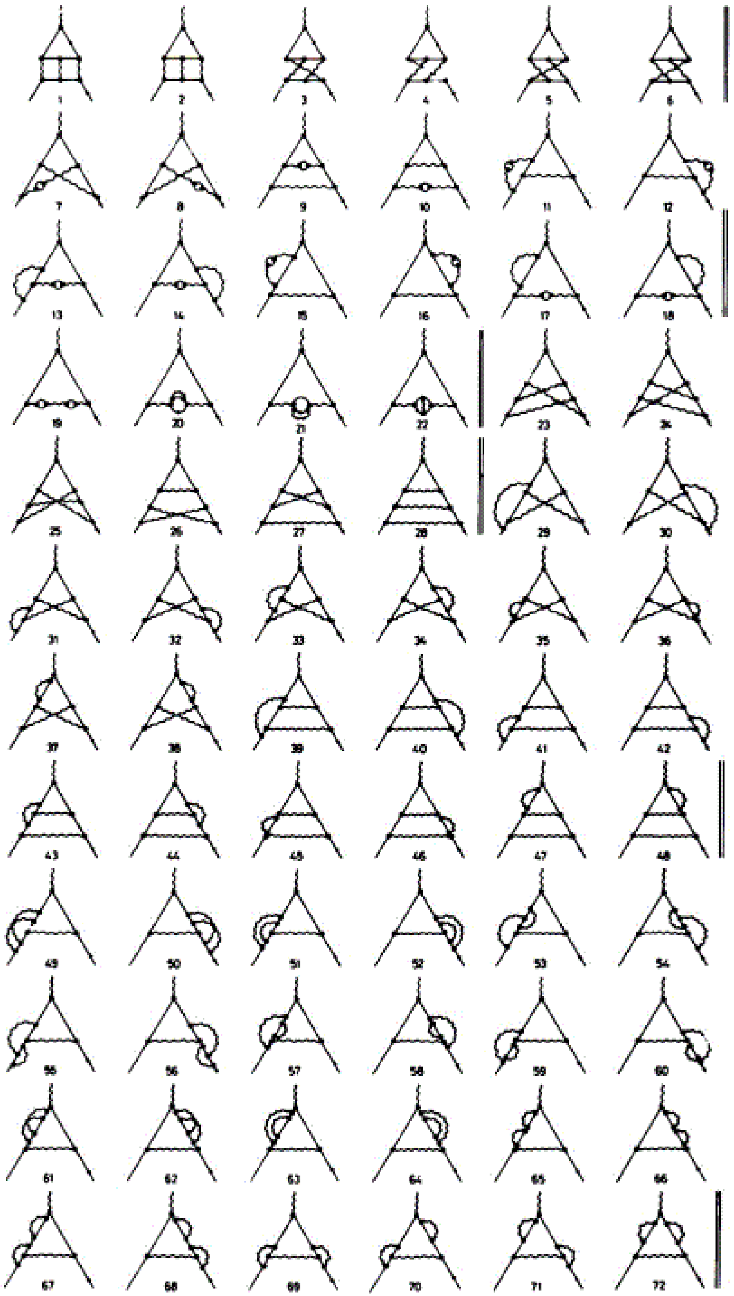


Fig. 8.2 The Feynman graphs which have to be evaluated in computing the α^3 corrections to the lepton magnetic moments (after Lautrup *et al.* 1972).

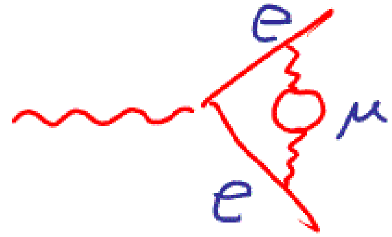
72 $\mathcal{O}(\alpha^3)$

DIAGRAMS

CORRECTIONS BEYOND e & γ

• CONTRIBUTION TO e MAGNETIC MOMENT FROM μ, τ, HADRONS, WEAK BOSONS.

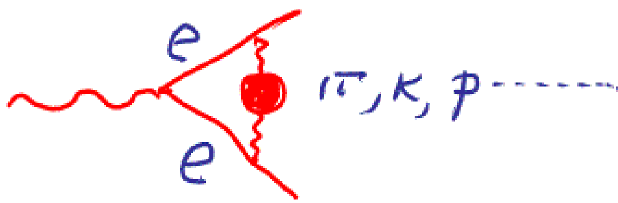
$$\delta(a_e)_\mu = \left(\frac{\alpha}{\pi}\right)^2 \frac{1}{45}\left(\frac{m_e}{m_\mu}\right)^2$$



2.8×10^{-13}

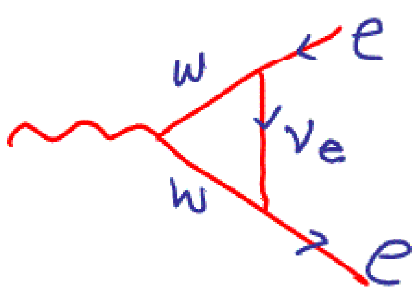
$\delta(a_e)_\tau$ 0.1×10^{-13}

$\delta(a_e)_{HAD}$



16×10^{-13}

$\delta(a_e)_{WEAK}$



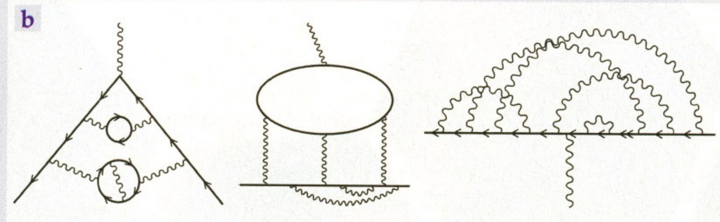
0.5×10^{-13}

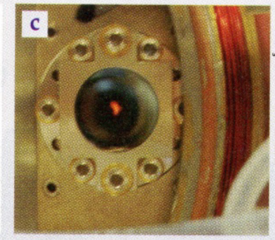
PLUS ANYTHING ELSE WHICH EXISTS

quick study

The standard model's greatest triumph

Gerald Gabrielse





FRANÇOIS BIRABEN

d

Predicted: $\mu/\mu_B = -1.001\,159\,652\,181\,78\,(77)$
Measured: $\mu/\mu_B = -1.001\,159\,652\,180\,73\,(28)$

Unprecedented confrontation of theory and experiment. **(a)** Our Penning trap shown here suspended a single electron for the months it took to measure its magnetic moment **μ**—the most precisely measured property of an elementary particle. **(b)** The magnetic moment is also the quantity most precisely predicted by the standard model of particle physics. The prediction requires the calculation of nearly 14 000 integrals. These Feynman diagrams represent three of those. **(c)** Fluorescing rubidium atoms are used to measure the fine-structure constant α, which gives the strength of the electromagnetic interaction. The measured α and the standard-model calculation are the essential inputs for the precise prediction. **(d)** The predicted and measured values of μ agree to an astounding part per trillion. Both values shown here are divided by the Bohr magneton μ_B defined in the text. Parentheses denote uncertainties in the rightmost two digits.

The spin frequency is proportional to B, which must then be measured to extract μ. Fortunately, the cyclotron frequency, $\omega_c = eB/m$ for an electron with charge $-e$ and mass m, is also proportional to B, so it can be used as an internal magnetometer. The electron is kept cold—with a temperature less than 0.1 degree above absolute zero—to keep the cyclotron motion in its quantum ground state. As with spin flips, a measurable one-quantum excitation of the cyclotron motion, which increases the energy by $\hbar\omega_c$, requires an appropriate driving force. Excitations take place more frequently as the drive frequency approaches ω_c.

Eliminating B from $\hbar\omega_s = -2\mu B$ and $\omega_c = eB/m$ gives the magnetic moment as a ratio of the two measurable frequencies, $\mu/\mu_B = -\omega_s/\omega_c$. The Bohr magneton $\mu_B = e\hbar/(2m)$ is the magnetic moment for circular electron motion with angular momentum $\hbar$. The magnetic moment μ is negative—

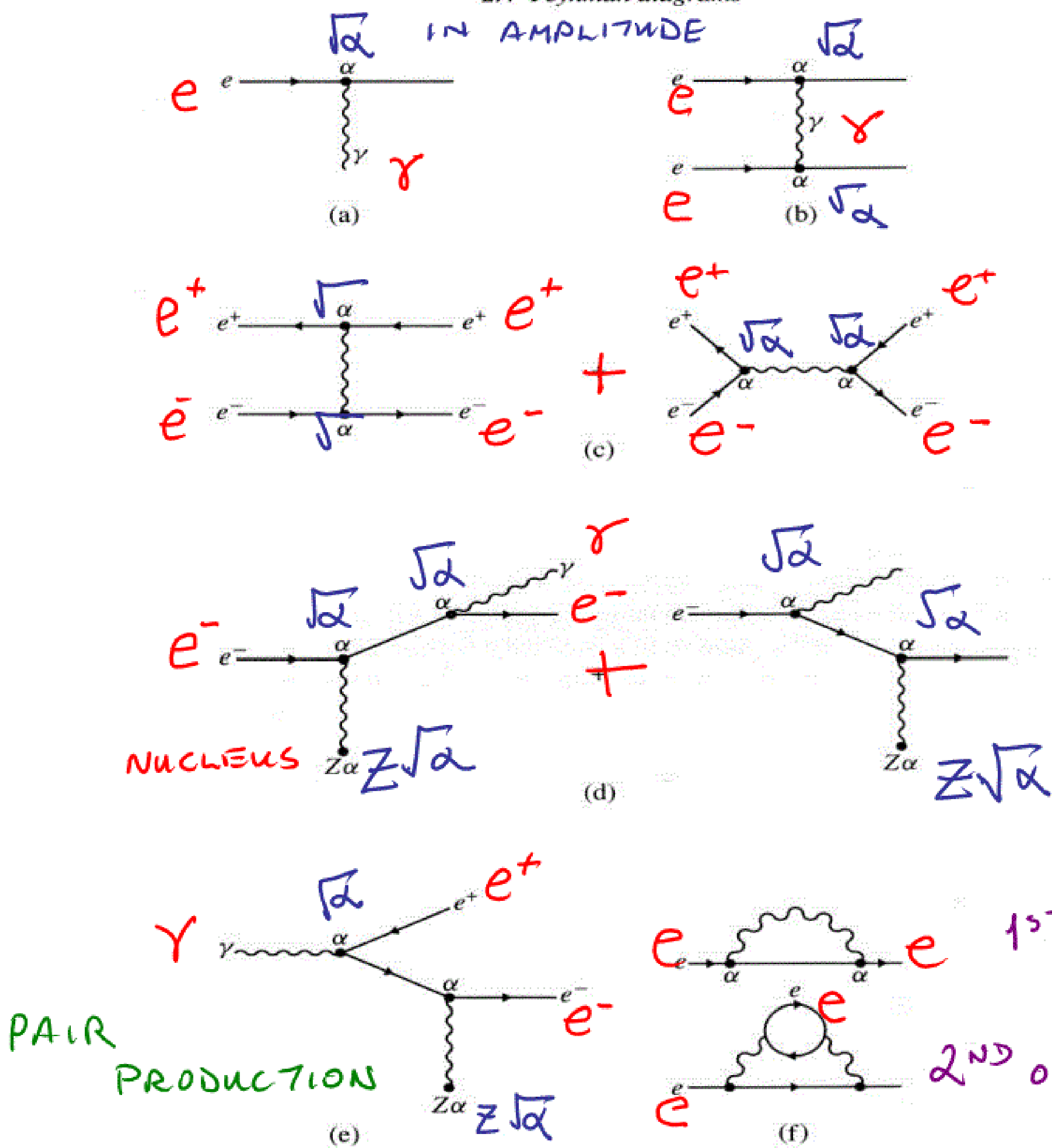
(Hadrons are heavy particles whose internal structure would need to be known here if the correction weren't so small.) The standard-model weak-interaction contribution to the electron moment, a_{weak}, is smaller than the measurement precision.

Quantum electrodynamics gives a_{QED} as a power series in the fine-structure constant, $\alpha \equiv e^2/(4\pi\varepsilon_0\hbar c) \approx 1/137$, which is a measure of the strength of the electromagnetic interaction in the low-energy limit. Specifically,

$$a_{\text{QED}}(\alpha) = C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + C_{10}\left(\frac{\alpha}{\pi}\right)^5 + \ldots.$$

Each of the five displayed terms is much smaller than the previous one, but all are needed to achieve the measurement precision of the magnetic moment.

Q.E.D.



(a) (b) (c) (d) (e) (f)

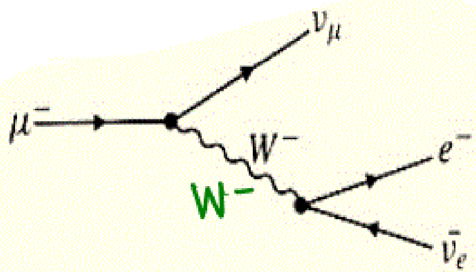
TWO WAYS IT CAN HAPPEN

$A = (A_1 + A_2)$

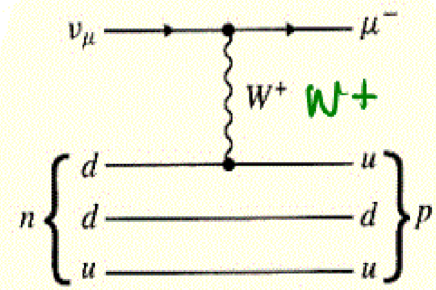
BREMSTRAHLUNG

CORRECTIONS TO BARE ELECTRON MASS

WEAK INTERACTION

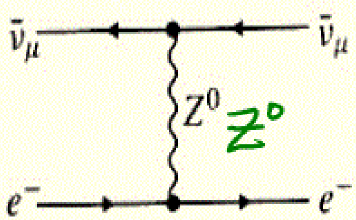


(b) $\mu^- \to \nu_\mu \bar{\nu}_e e^-$

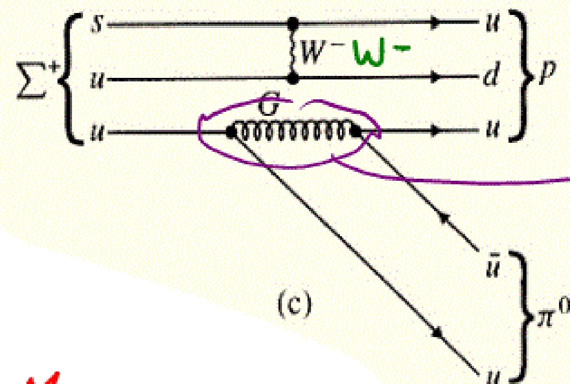


(a)

$\nu_\mu n \to \mu^- p$



$\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$



(c)

$\Sigma^+ \to p \pi^0$

STRONG INTERACTION

CLOSELY RELATED TO E.M.

EM	WEAK
γ^0	W^+
$m = 0$	Z^0
	W^-
	$M \approx 90\,\text{GeV}$

$$\frac{e^2}{q^2} \to \frac{g^2}{q^2 - M^2_{W,Z}}$$

MASS OF EXCHANGE

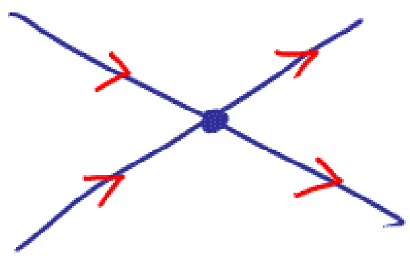
$g \approx e$

IF $g = e$ WHY IS THE WEAK INTERACTION WEAK?

EM $\dfrac{e^2}{q^2} \longrightarrow \dfrac{g^2}{q^2 - M_{W,Z}^2}$ WEAK

FOR $q^2 \ll M_W^2 \longrightarrow \dfrac{g^2}{M_W^2}$

THIS LOOKS LIKE



POINT LIKE
NO q^2 DEP

4-FERMION INTERACTION

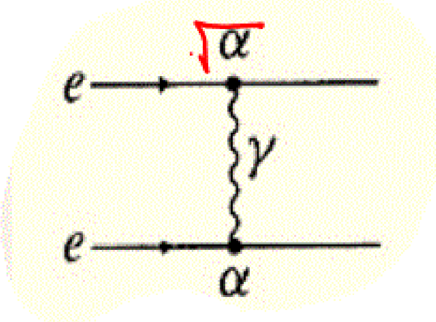
$G \equiv \dfrac{g^2}{M^2} \approx 10^{-15}\ \mathrm{GeV}^{-2}$

REASON FOR WEAKNESS

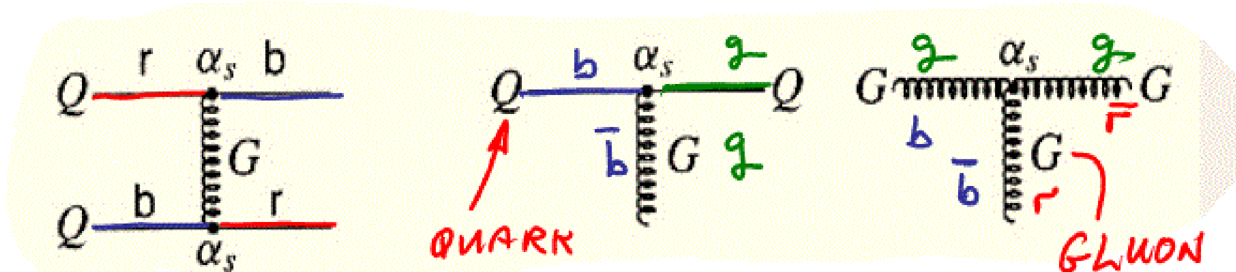
• NOT WEAK WHEN $q^2 \sim M_{W,Z}^3$

STRONG FORCE ≡ COLOR FORCE = QUANTUM CHROMO-DYNAMICS

QED



QCD

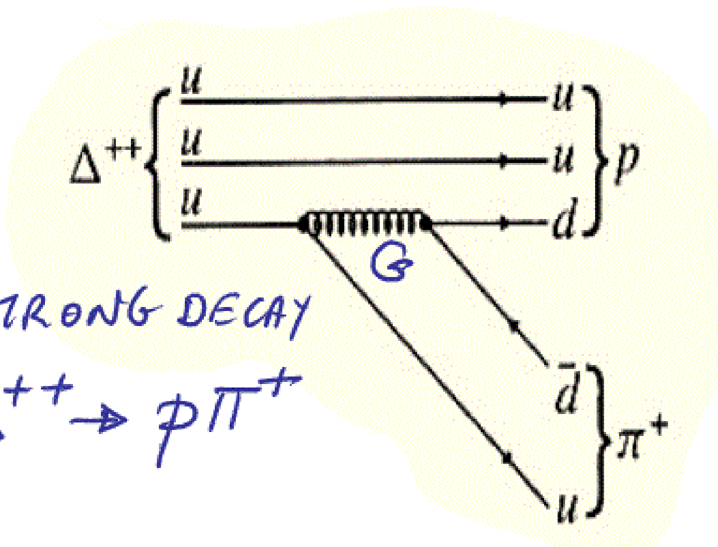


$\sqrt{\alpha_s} \leftarrow$!

COLOURED QUARKS

BICOLOURED GLUONS

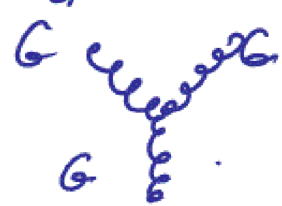
• CHARGE e ⟶ CHARGE RED, BLUE, GREEN



STRONG DECAY

$\Delta^{++} \rightarrow p\pi^+$

• GLUONS CARRY COLOR



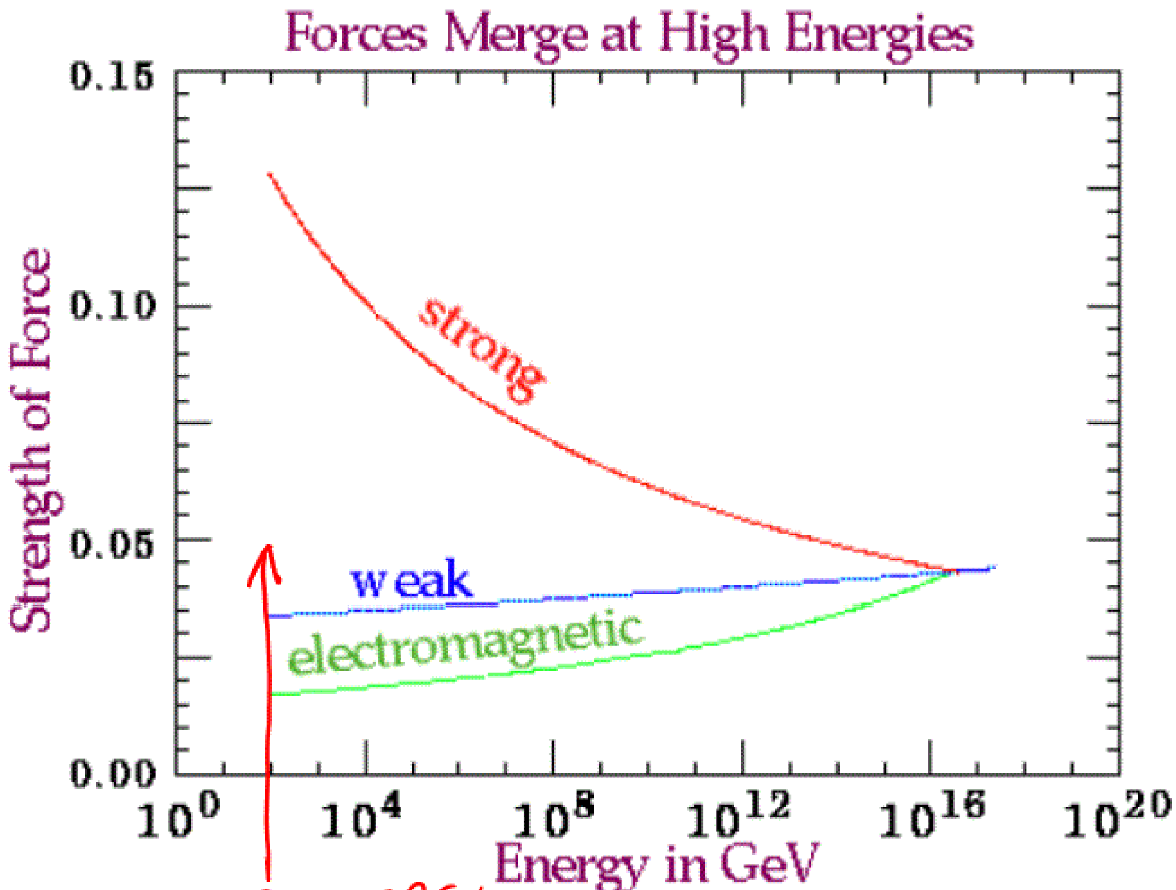
• GLUONS CHANGE COLOR OF QUARKS

• γ DOES NOT CARRY ELECTRIC CHARGE

DOES NOT EXIST

WHY IS THE STRONG FORCE STRONG?

QCD IS "ASYMPTOTICALLY FREE"



OUR ENERGY SCALE

$$\alpha_s(q^2) = \frac{1}{B \ln q^2/\Lambda}$$

SCALE OF STRONG FORCE

NOTE THAT STRENGTH OF ALL FORCES VARY WITH ENERGY SCALE

	1	3	8
GAUGE BOSONS →	γ	$W^{\pm} Z^0$	GLUONS

GAUGE SYMMETRY GROUP → $U(1) \times SU(2) \times SU(3)$

Table 2.2. *Fundamental interactions ($Mc^2 = 1$ GeV)*

	Gravitational	Electromagnetic	Weak	Strong
field boson	graviton	photon	$W^{\pm}, Z$	gluon
spin-parity	2^+	1^-	$1^-, 1^+$	1^-
mass, GeV	0	0	$M_W = 80.2$ $M_Z = 91.2$	0
range, m	∞	∞	10^{-18}	$\leq 10^{-15}$
source	mass	electric charge	'weak charge'	'colour charge'
coupling constant	$\frac{G_N M^2}{4\pi\hbar c} = 5 \times 10^{-40}$	$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137}$	$\frac{G(Mc^2)^2}{(\hbar c)^3} = 1.17 \times 10^{-5}$	$\alpha_s \leq 1$
typical cross-section, m^2		10^{-33}	10^{-39}	10^{-30}
typical lifetime, s		10^{-20}	10^{-10}	10^{-23}

?